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A NON - STATIC BAG MODEL FOR THE ROPER RESONANCES

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Abstract :

We solve the M.I.T. bag equations for Fermions in the limit of small fluctuations and quantize the solution. We get a non static bag model which provides a satisfactory interpretation of the Roper resonances if the time averaged radius of the cavity is about 1 fm.

The Roper resonances pose a difficult problem for the bag model[1]. They have been identified in several channels (N1440, Δ1600, Λ1600, Σ1660) and are assigned to the (56) representation of the spin-flavour group[2]. Their characteristic feature is that they stand at a much lower energy than the other positive parity states. The difference between their average energy ($\sim$ 1580 MeV) and the average energy of the correspond g ground states (N940, Δ1232, Λ1116, Σ1193) is $\Delta E \sim$ 410 MeV, while the next group of positive parity states stand at an average excitation energy $\Delta E' \sim$ 640 MeV. This large splitting is difficult to understand in the static bag model. In first approximation the states are degenerate and the splitting due to the effect of gluons and pions is too small if one uses standard values of the coupling constants[3]. For this reason it has been suspected for a long time that the Roper resonances are breathing modes which cannot be described in the static approximation. There have been several attempts[4] to build a non static model but most of them have introduced extra ingredients to reach this goal. The only work using the general M.I.T. bag equations[5] is that of *de Grand and Rebbi*[6] *but they have considered only Bosonic bags. In order* to make contact with the real world they have used the result of their Boson calculation of the breathing motion to mix static configurations of the Fermionic bag. This approach is rather frustating. Moreover the eigen-frequencies of the Bosonic and Fermionic bags are already different in the static case (π / R_0 against $2.043 / R_0$ for the lowest mode, with R_0 the radius of the cavity) and therefore one may, a priori, expect differences in the non static case too. In this letter, we propose a non static model which has only the M.I.T. bag equations as input and which treats the quarks as Fermions. It provides a satisfactory interpretation of the Roper resonances if the time averaged value of the cavity radius is about 1 fm.

The M.I.T. bag equations[5] for a quark field confined inside a surface S with normal n are :

$$\gamma \cdot \partial \Psi = 0 \qquad \text{inside S} \tag{1}$$

$$(1 - i\gamma \cdot n)\Psi = 0 \qquad \text{on S} \tag{2}$$

$$2B + n \cdot \partial \bar{\Psi}\Psi = 0 \qquad \text{on S} \tag{3}$$

with B the vacuum pressure[5] (Dirac matrices and metric are those of ref.7). The spinor field $\Psi(\vec{r},t)$ carries color and flavor indices and here only (u, d, s) quarks are considered. We assume that gluon, pion, and strange quark mass effects are not essential for the understanding of the Roper resonances. This means that we adopt the Cloudy Bag Model point of view[7] which considers that the basic structure of the baryons is contained in eqs.(1,2,3) and we postpone the evaluation of the corrections for future work. We choose a spherical surface with a time dependent radius R(t). The normal is ($\hat{r} = \vec{r} / r$) :

:

$$n^0 = \dot{R}/\sqrt{1-\dot{R}^2} \qquad \vec{n} = \hat{r}/\sqrt{1-\dot{R}^2} \qquad (4)$$

so that eqs.(2,3) become :

$$\left[\sqrt{1-\dot{R}^2} + i\vec{\gamma}\cdot\hat{r} - i\gamma^0\dot{R}\right]\Psi = 0 \qquad \text{at } \vec{r} = \hat{r}R(t) \qquad (5)$$

$$2B\sqrt{1-\dot{R}^2} + \dot{R}\frac{\partial}{\partial t}\bar{\Psi}\Psi + \frac{\partial}{\partial r}\bar{\Psi}\Psi = 0 \quad \text{at } \vec{r} = \hat{r}R(t) \qquad (6)$$

Eq.(5) implies :

$$\bar{\Psi}\Psi\left(\hat{r}R(t), t\right) = 0 \qquad (7)$$

On the one hand eqs.(6,7) determine implicitly the surface when the field is known, and on the other hand the field must satisfy the boundary condition (5) which depends upon the surface. Therefore finding exact solutions is far from trivial and, as in ref.[5], we resort to a perturbative method to solve eqs.(1,5,6). The latter have at least one exact solution which is the static one, defined by (Ψ_0, R_0):

$$\Psi_0(\vec{r},t) = \sum_{m,f,c} b^{(0)}_{mfc}\, \mathcal{N}_0^{-1}\; \varphi^m\!\left(\frac{\Omega_0}{R_0}\vec{r}\right) e^{-i\frac{\Omega_0}{R_0}t}\, \chi_f\, \chi_c \qquad (8)$$

with :

$$\varphi^m(\vec{x}) = \begin{bmatrix} j_0(x) \\ i\vec{\sigma}\cdot\hat{r}\; j_1(x) \end{bmatrix} \chi_m^{\frac{1}{2}} \Big/ \sqrt{4\pi} \qquad (9)$$

$$\mathcal{N}_0^{-1} = j_0(\Omega_0)^{-1}\left[\Omega_0 / 2R_0^3\,(\Omega_0 - 1)\right]^{\frac{1}{2}} \qquad (10)$$

:

$$R_0 = \left[\Omega_0 N / 4\pi B\right]^{\frac{1}{4}} \qquad (11)$$

In the preceding equations, Ω_0 is the lowest solution of $j_0(\Omega_0) = j_1(\Omega_0)$ $\Omega_0 = 2.043$, and $\chi^{\frac{1}{2}}_m, \chi_f, \chi_c$ stand for the spin, flavor and color wave functions. After quantization the coefficients $b^{(0)}_{mfc}$ become destruction operators for a quark in the lowest mode and[+] $N = \sum_{m,f,c} b^{(0)*}_{mfc} b^{(0)}_{mfc}$ is the number of quarks. We expand the unknown function (Ψ, R) about the static solution :

$$\Psi = \Psi_0 + \Psi_1 \qquad (12)$$

$$R(t) = R_0 + R_1(t) \qquad (13)$$

and assume that (Ψ_1, R_1) are small fluctuations with respect to (Ψ_0, R_0) Then, to first order in (Ψ_1, R_1) we get the following equations :

$$i\gamma\cdot\partial\Psi_1 = 0 \quad , \quad r < R_0 \qquad (14)$$

$$(1 + i\vec{\gamma}\cdot\hat{r})\left(R_1 \frac{\partial\Psi_0}{\partial r} + \Psi_1\right) - i\gamma^0 \dot{R}_1 \Psi_0 = 0 \;,\; \vec{r} = \hat{r} R_0 \qquad (15)$$

$$R_1 \frac{\partial^2}{\partial r^2} \bar{\Psi}_0 \Psi_0 + \frac{\partial}{\partial r}\left(\bar{\Psi}_0 \Psi_1 + \bar{\Psi}_1 \Psi_0\right) = 0 \quad , \; \vec{r} = \hat{r} R_0 \qquad (16)$$

We look for a solution of the form :

$$\Psi_1(\vec{r},t) = \sum_{m,f,c} \int d\omega \, b^{(1)}_{mfc}(\omega) \, \mathcal{N}_0^{-1} \, \varphi^m\!\left(\frac{\Omega_0+\omega}{R_0}\vec{r}\right) e^{-i\frac{\Omega_0+\omega}{R_0}t} \chi_f \chi_c \qquad (17)$$

$$R_1(t) = R_0 \int d\omega \, e^{-i\frac{\omega}{R_0}t} R_1(\omega) \qquad (18)$$

(+) The * denotes complex conjugation of a number or hermitian conjugation for an operator.

with $R_1^*(\omega) = R_1(-\omega)$ so that $R_1(t)$ is real. In eq.(17) only spin 1/2, positive parity solutions of the Dirac equation have been retained. This is not a real limitation in so far as including other solutions does not generate new non static solutions. This is due to the choice of a spherical shape. If we insert eqs.(17, 18) in eqs.(15, 16) we get the equations which determine $b_\alpha^{(1)}(\omega)$ and $R_1(\omega)$, $(\alpha \sim m, \ell, c)$:

$$d(\omega)\, b_\alpha^{(1)}(\omega) = t(\omega)\, j_0(\Omega_0)\; R_1(\omega)\, b_\alpha^{(0)} \qquad (19)$$

$$R_1(\omega) = \left[v\, j_0(\Omega_0)\, N \right]^{-1} \left[u(\omega) \sum_\alpha b_\alpha^{(0)*} b_\alpha^{(1)}(\omega) + u(-\omega) \sum_\alpha b_\alpha^{(1)*}(-\omega)\, b_\alpha^{(0)} \right] \qquad (20)$$

with :

$$v = 4\,(4\Omega_0 - 5) \quad ,$$

$$s(\omega) = j_0(\Omega_0 + \omega) + j_1(\Omega_0 + \omega) \quad ,$$

$$d(\omega) = j_0(\Omega_0 + \omega) - j_1(\Omega_0 + \omega) \quad ,$$

$$t(\omega) = \omega + 2\,(\Omega_0 - 1) \quad ,$$

$$u(\omega) = t(\omega)\, s(\omega) + 2\, d(\omega) \qquad (21)$$

Equations (19, 20) have static solutions corresponding to $d(\omega) = 0$ with the roots $\Omega_0 + \omega = 5.4,\ 8.6,$ etc... In this case the fluctuation $R_1(\omega)$ is zero and eq.(20) constrains the field coordinates $b_\alpha^{(1)}(\omega)$ to satisfy :

$$\sum_\alpha b_\alpha^{(0)*} b_\alpha^{(1)}(\omega) = \sum_\alpha b_\alpha^{(1)*}(\omega)\, b_\alpha^{(0)} = 0 \qquad (22)$$

The implications of these constraints have been studied in an earlier paper[8]. The result is that the static excited states obtained by promoting one quark from the mode Ω_0 to the mode $\Omega_0+\omega$ are unphysical when the configuration has the spin flavor symmetry (56). This prevents one from interpreting the Roper resonances as radial excited states of the static bag and has motivated the present investigation of non static solutions. The latter must satisfy $d(\omega)\neq 0$ and therefore eq.(19) determines $b_\alpha^{(1)}(\omega)$ as a function of $b_\alpha^{(0)}$ and $R_1(\omega)$. If we multiply eq.(19) by $b_\alpha^{(0)*}$ and sum over α we get :

$$\sum_\alpha b_\alpha^{(0)*}\, b_\alpha^{(1)}(\omega) = d^{-1}(\omega)\, t(\omega)\, j_0(\Omega_0)\, N\, R_1(\omega) \qquad (23)$$

Inserting this expression in eq.(20) then leads to the eigenvalue equation :

$$R_1(\omega) = \sigma^{-1}\left[d^{-1}(\omega)\, u(\omega)\, t(\omega) + d^{-1}(-\omega)\, u(-\omega)\, t(-\omega)\right] R_1(\omega) \qquad (24)$$

Therefore the frequencies of the breathing modes are solution of :

$$\sigma = d^{-1}(\omega)\, u(\omega)\, t(\omega) + d^{-1}(-\omega)\, u(-\omega)\, t(-\omega) \qquad (25)$$

The roots of this equation are symmetrical with respect to zero and we denote them ω_n with the convention $\omega_n > 0$ if $n > 0$. The values of ω_n are determined numerically and the first ones are : $\omega_1 = 1.97$, $\omega_2 = 5.2$, $\omega_3 = 5.7$.

When $n > 3$ the roots are complex. This does not mean that the bag is unstable against the fluctuations of the surface but simply that the zero order solution (8) is not general enough. Above a critical frequency it should be replaced by a solution containing higher static frequencies. The problem then becomes utterly complicated, particularly concerning the quantization of the model. However this involves only high frequency solutions while the lowest one is the most interesting physically. For the latter the zeroth order solution (8) should be sufficient. It is interesting to notice that our numerical result for the lowest breathing mode is nearly the same as the one obtained by J.D.Breit[9] in a model based on the Friedberg-Lee soliton[10]. In this model there are more degrees of freedom than in the bag model and the approach of ref.[9] is very different from the one presented here so that it is difficult to settle whether the numerical coincidence is fortuitous or not.

To determine the non static fluctuations we note that from eq.(24) $R_1(\omega)$ is non zero only when $\omega = \omega_n$ and since $R_1(\omega)^* = R_1(-\omega)$ the solution can be written as follows :

$$R_1(\omega) = \sum_{n>0} \left[\omega_n / 8N\Omega\right]^{\frac{1}{2}} \left[\delta(\omega-\omega_n) C_n + \delta(\omega+\omega_n) C_n^*\right] \qquad (26)$$

where the (arbitrary) factor $\left[\omega_n / 8N\Omega_o\right]^{\frac{1}{2}}$ has been introduced for later convenience. At the classical level, the C_n are just arbitrary complex numbers. From eqs.(22, 26) we deduce the expression for the field coordinate $b_\alpha^{(1)}(\omega)$:

$$b_\alpha^{(1)}(\omega) = j_o(\Omega_o) \sum b_\alpha^{(o)} \left[\omega_n / 8N\Omega_o\right]^{\frac{1}{2}} (\cdots)$$

$$\left[\delta(\omega-\omega_n) d^{-1}(\omega_n) t(\omega_n) C_n + \delta(\omega+\omega_n) d^{-1}(-\omega_n) t(-\omega_n) C_n^*\right] \qquad (27)$$

It is clear from eq.(27) that the independent coordinates are $b_{\alpha}^{(\circ)}$ and C_n and not $b_{\alpha}^{(\ell)}(\omega)$. To find the quantization rules we first compute the energy H of the solution. It is convenient to use the virial theorem[5] :

$$H = 4B < V > \tag{28}$$

where $< V >$ is the time averaged value of the volume. From eqs.(11, 18, 26) we get (we anticipate normal ordering for C_n and C_n^*) :

$$H = \frac{4}{3}\frac{N\Omega_\circ}{R_\circ} + \sum_{n>0}\frac{\omega_n}{R_\circ} C_n^* C_n \tag{29}$$

The first term in eq.(29) is the static solution energy and the second one is the energy due to the fluctuations of the surface. To determine the Poisson Brackets (P.B.) of the coordinates C_n and C_n^* we require that the equation of motion for $R_1(t)$ be satisfied in the Hamilton form :

$$\dot{R}_1(t) = \left\{ R_1(t) , H \right\}_{P.B.} \tag{30}$$

This is realized if we choose as non zero P.B. :

$$\left\{ C_n^* , C_{n'} \right\}_{P.B.} = i\, \delta(n,n') \tag{31}$$

It is clear that the fluctuations of the radius must be bosons. Therefore we quantize the C_n with commutation rules which we determine from the above P.B.

and the correspondance principle. Consequently the non zero commutators are :

$$[C_n , C^*_{n'}] = \delta(n,n') \qquad (32)$$

For the quark operator $b^{(o)}_\alpha$ the appropriate non zero anticommutation rules are as usual :

$$[b^{(o)*}_\alpha , b^{(o)}_{\alpha'}]_+ = \delta(\alpha,\alpha') \qquad (33)$$

To check the consistency of the quantization scheme, we should verify that Heisenberg equation of motion is satisfied by the field. Using eqs.(12, 17, 27, 29, 31, 32), it is straightforward to show that this is the case up to terms which are of higher order in the fluctuation Ψ_1 .

The commutation rules (32) and the Hamiltonian (29) allow us to interpret the operators C_n as creation operators of the mode n . Therefore the spectrum of the model in the N = 3 sector has the following structure : The ground state is the color singlet obtained by acting on the vacuum with the appropriate combination of 3 operators $b^{(o)}_\alpha$. It belongs to the (56) representation of SU(6) and we denote it : $|(G.S)\ (56)\rho>$ where ρ is a set of additional quantum numbers. The simplest excited states have one breathing mode occupied. They have the form :

$$|(n)\ (56)\ \rho> = C^*_n\ |(G.S.)\ (56)\rho> \qquad (34)$$

Since the operator C_n is manifestly SU(6) invariant the excited states also have the (56) symmetry. From eq.(29) their excitation energy is $\Delta E = \omega_n / R_o$

We do not consider states which have more than one breathing mode occupied. That would not be consistent with our initial assumption that the field fluctuation (which is proportional to C_n , see eqs.(17, 27)) can be treated to first order.

In this model the Roper resonances are described by the first non static excited state which stands at $\Delta E_1 = 1.97/R_0 \sim 400$ MeV if $R_0 \sim 1$ fm. Hence we get nice agreement with experiment for a radius which is in the range of the accepted values[7]. For the same (+) value of R_0 the first radial excitation of the static model appears at $\Delta E_1' = (5.4 - 2.043)/R_0 \sim 660$ MeV and therefore is likely to stand among the positive parity states at 610 MeV average excitation.

The calculation presented above is approximate and we do not claim it is the full story. In particular we have not taken into account pionic, gluonic and strange quark mass effects. This is justified in so far as our primary goal in this letter is to interpret the basic structure of the spectrum rather than its details. An estimate of these corrections is in progress and preliminary results indicate that the pionic effects are moderate[11]. Also the correction due to the center of mass motion has not been considered but one can expect that the effects in the ground state and the excited states cancel to a large extent.

(+) If we let the radius depend on the state, then $\Delta E_1' \sim 610$ MeV. However it is preferable to consider that this variation can be neglected because it would lead to severe orthogonality problems. In particular the charge operator is not diagonal between states which have a different radius !

Finally we would like to make a comment about an alternative explanation[12] of the low energy of the Roper resonances. This explanation invokes the very low frequency ($\sim 1.2 / R_0$) of the static P3/2 mode[13] which allows (with a reasonable radius $R_0 \sim 1.15$ fm) the interpretation of the Roper resonances as static states with one quark in the lowest mode and two quarks in the P3/2 mode. There are several arguments against this interpretation :

i) The internal pressure of the P3/2 mode is not isotropic. It does not match with a spherical shape and therefore the low value of the frequency must not be taken seriously.

ii) The configuration with two P3/2 quarks allows many more states than the (56) representation while only the states belonging to the latter are observed at low energy.

iii) If this interpretation was correct then one should observe negative parity states around $1.2 / R_0 \sim 200$ MeV excitation which is ruled out by experiment (the average excitation energy of the first such states is about 430 MeV).

In fact it is well known that in the bag model the P3/2 mode is much too low (this is probably related to point i)) and causes serious problems in the negative parity spectrum[14,15]. It would be misleading to interpret the Roper resonances thanks to this well recognized drawback of the static spherical bag.

In this letter we have shown that the dynamics of the non-static bag may be a natural explanation of the large splitting between the Roper resonances and the other excited states. A pleasant feature of the model we have developed is that the breathing motion is explicit and its experimental implications can be computed thanks to the expression of the field (eqs.(17, 27)) and of the radius (eqs.(18, 26)). In particular a preliminary investigation suggests that the magnetic form factor of the transition (N $\longrightarrow$ Roper) may be an interesting test of the model provided it can be determined with enough accuracy both in the space-like and in the time-like regions. This point is under study[16].

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