Fermi National Accelerator Laboratory

FERMILAB-Pub-86/63-A
April 28, 1986

PARKER LIMIT FOR MONOPOLES WITH LARGE MAGNETIC CHARGE

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Abstract

The survival of galactic magnetic fields places a limit on the flux of magnetic monopoles, the so-called "Parker limit." Previous discussions of the Parker limit have assumed that the charge of the monopole is the Dirac value, $g_{Dirac} = 2\pi/e$. However, if the grand unified group is broken by Wilson lines, as is assumed in some superstring models, the minimum value of the magnetic charge is not the Dirac quantum, but an integer multiple of it. In this brief report we investigate the dependence of the Parker limit on the charge of the magnetic monopole.

Operated by Universities Research Association Inc. under contract with the United States Department of Energy

One of the most interesting predictions of grand unified theories (GUTS) is the existence of field configurations corresponding to magnetic monopoles. In standard GUTS the breaking of the grand unified symmetry is via the Higgs mechanism. With such symmetry breaking, it is possible to have non-trivial topologies for the gauge orientation of the vacuum expectation value of the Higgs field. One such example of a non-trivial topology corresponds to a magnetic monopole, i.e. a solution with Coulombic magnetic field $\mathbf{B} = g\mathbf{x}/r^3$ for $|r| \to \infty$.[1] In these theories, the magnetic monopole has a mass of $m_M \simeq M_{GUT}/e$, where M_{GUT} is the GUT symmetry breaking scale. For GUTS such as $SU(5)$ or $SO(10)$, this mass is about 10^{16}GeV. The minimum magnetic charge of GUT monopoles is the Dirac quantum, $g = g_{Dirac} = 2\pi/e$. Although monopoles with charge greater than the Dirac quantum exist in these theories, they are expected to be unstable and decay to the minimum-charge monopoles.

The discovery of a magnetic monopole would be of tremendous significance. Not only would such a discovery be important for particle physics, but it would also have profound implications for cosmology, as the very early Universe ($t \leq 10^{-34}$sec) is the only possible source of such massive particles. At present there are many ongoing or planned experiments to look for superheavy cosmic-ray monopoles.[2] A benchmark value of the flux of magnetic monopoles is the upper limit obtained by requiring that the magnetic monopoles moving through the galaxy do not drain the galactic magnetic field faster than astrophysical processes can regenerate it.[3] For magnetic monopoles of unit Dirac charge and moving initially with $v \simeq 10^{-3}c$ (the galactic virial velocity), the Parker limit is[4]

$$\begin{aligned} F_M &\leq 10^{-15} cm^{-2} s^{-1} sr^{-1} && (m_M \leq 10^{17} GeV) \\ F_M &\leq 10^{-15} (m_M/10^{17} GeV) cm^{-2} s^{-1} sr^{-1} && (m_M \geq 10^{17} GeV). \end{aligned} \tag{1}$$

[If galactic monopoles have large-scale coherent motions, they can possibly help to maintain the galactic magnetic field, and the Parker bound may be evaded. For a discussion of the possibility, see Wasserman, et al.[5] In this report we will also mention the effect of varying the magnetic charge on the coherent motions.]

Recent work on unified theories with extra dimensions has led to the discovery of an additional mechanism for symmetry breaking.[6] In theories with extra dimensions, the vacuum geometry is of the form $M^4 \times B$, where M^4 is four-dimensional Minkowski space, and B is a compact internal manifold. The most promising model at present is the superstring theory with gauge group $E_8 \times E_8$.[7] If B has $SU(3)$, $U(3)$, or $O(6)$ holonomy, and the spin connection of B is embedded in one of the E_8 factors, the E_8 will be broken to a subgroup G, which is E_6, $O(10) \times U(1)$, or $O(10)$.[6] The symmetry can be broken further by means of Wilson lines if $\pi_1(B)$ is non-trivial. The Wilson lines may be thought of as Higgs bosons in the adjoint representation of G, but with some fundamental differences. The difference of interest here, is the fact that the minimum magnetic charge when symmetry breaking is done by Wilson lines is not the Dirac value $(2\pi/e)$ as with Higgs symmetry breaking, but rather $k(2\pi/e)$, where k is an integer.[8]

The production of monopoles in the early Universe has been considered for GUT monopoles [10] and Kaluza-Klein monopoles.[9] In the case of GUT monopoles, the standard cosmology predicts an abundance of monopoles far in excess of that allowed by the present mass density of the Universe. The expected abundance of Kaluza-Klein monopoles is far more difficult to estimate, but could be as large as that for GUT monopoles. Of course, the monopole glut can be turned into a famine by inflation. In either case, it might seem unlikely that monopoles are present today in an abundance accessible to observation. Nevertheless, it is possible that some magnetic monopoles were produced in the very early stages of the big bang, survived annihilation and inflation (by being produced after inflation), and would be present in the Universe today. The surviving monopoles would today be quite cold, and would have velocities determined by the galactic virial velocity, $v \simeq 10^{-3}c$. Even if monopoles are not bound to the galaxy, $v \simeq 10^{-3}c$ would still be the relevant velocity, as this is about the peculiar velocity of our galaxy with respect to the $3K$ microwave background. In the absence of theoretical guidance, we will take the relic flux to be a free parameter, and the local monopole velocity to be $O(10^{-3})c$.

The application of the Parker limit to monopoles of arbitrary mass and unit Dirac charge was discussed in detail by Turner, Parker, and Bogdan[4] (hereafter TPB). Here, we will review some of their assumptions and results relevant for the extension of their calculations to include monopoles of magnetic charge not equal to the Dirac quantum.

Observations of the galactic magnetic field suggest an average B-field strength of 3×10^{-6}G, with a typical coherence length of $l_c = 300pc (\simeq 10^{21}cm)$, a spatial extent of $R_B = 30kpc (\simeq 10^{23}cm)$ from the center of the galaxy, and a regeneration time (via dynamo action) for the galactic magnetic field of $t_{reg} = 30Myr$. We refer the reader interested in more details and references to TPB.

The magnetic force on a monopole of charge $g = kg_{Dirac}$ is

$$F_{mag} = gB \simeq 0.06eV cm^{-1} kB_3, \tag{2}$$

where $B = B_3(3 \times 10^{-6}G)$. The energy gain by a monopole (initially at rest) traversing a field B of coherence length l_c is

$$\Delta E = gBl_c \simeq 0.6 \times 10^{20} eV kl_{21} B_3, \tag{3}$$

where $l_c = l_{21} 10^{21} cm$. Note that ΔE is proportional to k, and independent of m_M. The final velocity of the monopole is

$$v_{mag} = 10^{-3} c (kl_{21} B_3 / m_{17})^{1/2}, \tag{4}$$

where m_{17} is $m_M / 10^{17} GeV$.

Now consider monopoles initially not at rest. Monopoles with initial velocity $v \geq v_{mag}$ will suffer only a small change in velocity, while monopoles with initial velocity $v \leq v_{mag}$ will suffer a large change in velocity and will emerge with $v \simeq v_{mag}$ after traversing the magnetic field region. The Parker bound will depend upon whether v is larger or smaller than v_{mag}. We consider the two cases in turn.

(1) $v_{mag} > v$. Upon encountering the first B-field region, the monopole will be accelerated to $v \simeq v_{mag}$. In subsequent encounters with B-field regions, the monopole will gain or lose an energy given by Eq.(3). On average, the monopole traverses a distance comparable to the diameter of the galactic magnetic field region ($2R_B$) before leaving the galaxy. In its journey it traverses $2R_B/l_c$ coherent regions, gaining an energy of

$$\Delta E_{TOTAL} = (2R_B/l_c)^{1/2} gBl_c \simeq 6 \times 10^{21} eV\ k\ B_3 (l_{21} R_{23})^{1/2} \tag{5}$$

($R_{23} = R_B / 10^{23}$ cm). The survival of the galactic magnetic field requires

$$F_M \times (\pi sr) \times (4\pi R_B^2) \times \Delta E_{TOTAL} \leq (B^2/8\pi)(4\pi R_B^3/3) t_{reg}^{-1}. \tag{6}$$

This results in the flux limit

$$F_M \leq 10^{-15}\ k^{-1}\ cm^{-2} sr^{-1} sec^{-1}, \tag{7}$$

for the fiducial values of B, R_B, l_c, and t_{reg}. Note that the limit is proportional to k^{-1}, i.e., as k increases the limit becomes more stringent.

(2) $v_{mag} < v$. In order to have $v_{mag} < v \simeq 10^{-3}c$, the monopole mass must satisfy $m_M > 10^{17}\ k\ GeV$. Note that the critical mass increases in proportion to k. As TPB point out, the energy gained by a monopole in traversing a coherent field region is a second order effect; to lowest order, on average an isotropic flux of monopoles undergoes no net gain of energy. To second order, the average energy gain per monopole is proportional to $m_M(\Delta v)^2$, where

$$\Delta v \simeq (gB/m_M) l_c/v. \tag{8}$$

This leads to a change in the magnetic field energy of

$$\Delta E \simeq 2 \times 10^{19} eV\ k^2\ B_3^2 l_{21}^2 / m_{17}. \tag{9}$$

Note that ΔE is proportional to k^2, and inversely proportional to m_M. The number of monopoles which pass through a coherent field region per time is $F_M \times (4\pi l_c^2) \times (\pi sr)$. If we require the total field energy in the coherent field region, $(B^2/8\pi)(4\pi l_c^3/3)$, to have a dissipation time less than t_{reg}, the monopole flux bound becomes

$$F_M \leq 10^{-15}\ k^{-2}\ m_{17} cm^{-2} sr^{-1} s^{-1}. \tag{10}$$

Note that the flux bound is proportional to k^{-2}.

The results of the limits in the two regions are given in the two figures. To summarize the results, in the region $v_{mag} > v$ (which applies for $m_M < 10^{17}k$ GeV) the flux limit is proportional to k^{-1}, in the region $v_{mag} < v$ the flux limit is proportional to k^{-2}:

$$\begin{aligned} F_M &\leq 10^{-15} k^{-1} cm^{-2} s^{-1} sr^{-1} & (m_M \leq 10^{17}\ k\ GeV) \\ F_M &\leq 10^{-15} (m_M/10^{17} GeV) k^{-2} cm^{-2} s^{-1} sr^{-1} & (m_M \geq 10^{17}\ k\ GeV). \end{aligned} \tag{11}$$

As expected the flux bounds are more stringent for $k > 1$.

As mentioned above, it may be possible to evade the Parker limit if the monopoles undergo coherent motions. A necessary, but not sufficient, condition for coherent monopole motions is that the phase velocity, v_{ph}, of the monopoles associated with their coherent motions be greater than their internal velocity dispersion, $\langle v^2\rangle^{1/2}$. The phase velocity is

$$v_{ph} \simeq (\omega_p/2\pi)l, \tag{12}$$

where $\omega_p^2 = 4\pi g^2 n_M/m_M$ is the monopole plasma frequency and l is the spatial scale associated with the coherent monopole motions. The condition $v_{ph} > \langle v^2\rangle^{1/2}$ leads to the *lower* bound to the monopole flux

$$\begin{aligned} F_M &\geq \frac{1}{4} m_M \langle v^2\rangle^{3/2}(gl)^{-2} \\ &\geq 10^{-12} m_{17}(kpc/l)^2\ k^{-2}\ cm^{-2} sr^{-1} sec^{-1}, \end{aligned} \tag{13}$$

where we have used the velocity dispersion appropiate for a galactic halo of monopoles, $\langle v^2\rangle^{1/2} \simeq 10^{-3}c$. Note that increasing k decreases the flux needed to sustain coherent monopole motions.

Finally, from Eq.(9), the formula for the energy gain by a monopole with $v > v_{mag}$, we can estimate the time it takes for monopoles in the galactic halo to gain enough energy to evaporate from the halo. The evaporation time is about $t_{evap} \simeq 10^{14} m_{17}^2 k^{-2} sec$. In order that the galactic halo population of monopoles have an evaporation time longer than the age of the Universe requires $m_M \geq 3 \times 10^{18} k$GeV. The mass increases with k.

This work was supported in part by the Department of Energy and NASA.

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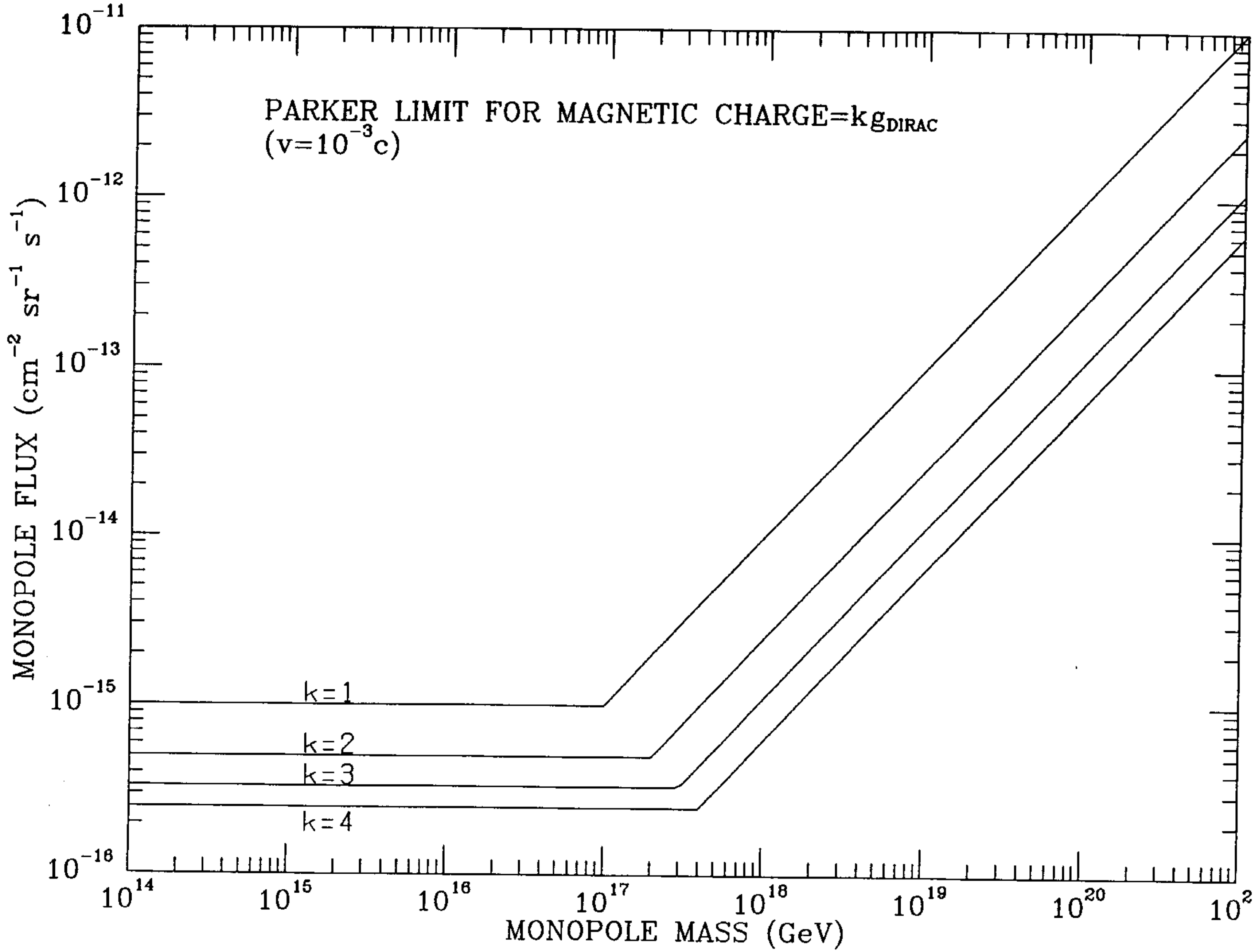
Figure Captions

Figure 1. The Parker limit for magnetic monopoles of magnetic charge $g = kg_{Dirac} = k(2\pi/e)$, with $k = 1, 2, 3, 4$.

Figure 2. The Parker Limit for magnetic monopoles of magnetic charge $g = kg_{Dirac} = k(2\pi/e)$, with $k = 10^0, 10^1, 10^2, 10^3$.



PARKER LIMIT FOR MAGNETIC CHARGE=kg_{DIRAC}
($v=10^{-3}c$)
k=1
k=2
k=3
k=4
MONOPOLE FLUX (cm^{-2} sr^{-1} s^{-1})
10^{-11}
10^{-12}
10^{-13}
10^{-14}
10^{-15}
10^{-16}
10^{14}
10^{15}
10^{16}
10^{17}
10^{18}
10^{19}
10^{20}
10^{21}
MONOPOLE MASS (GeV)

FIGURE 1

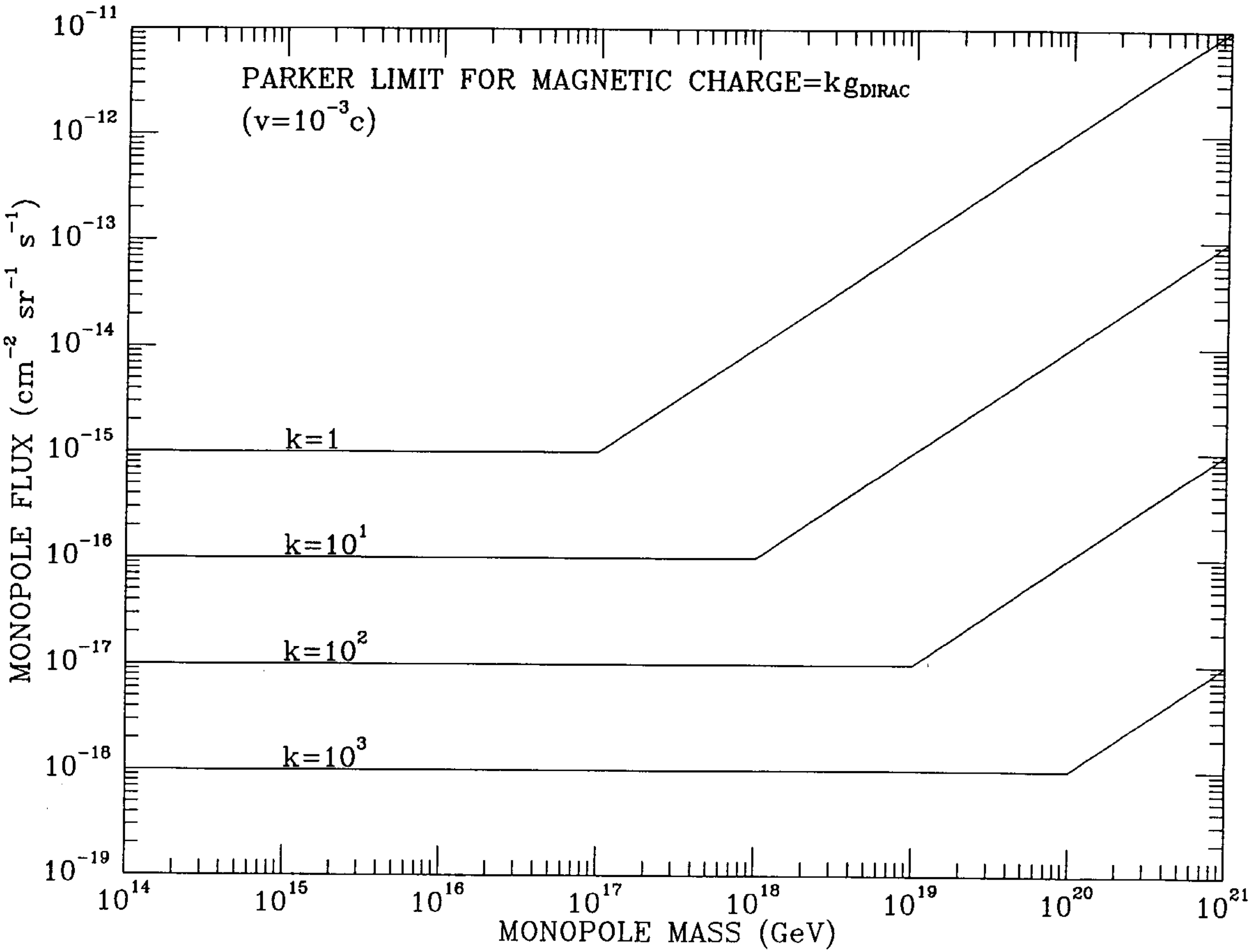

PARKER LIMIT FOR MAGNETIC CHARGE=kg$_{DIRAC}$
(v=10^{-3}c)
k=1
k=10^1
k=10^2
k=10^3
MONOPOLE FLUX (cm^{-2} sr^{-1} s^{-1})
MONOPOLE MASS (GeV)
10^{-11}
10^{-12}
10^{-13}
10^{-14}
10^{-15}
10^{-16}
10^{-17}
10^{-18}
10^{-19}
10^{14}
10^{15}
10^{16}
10^{17}
10^{18}
10^{19}
10^{20}
10^{21}

FIGURE 2