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LINEAR STABILITY OF TEARING MODES

By

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ABSTRACT

This paper examines the stability of tearing modes in a sheared slab when the width of the tearing layer is much smaller than the ion Larmor radius. The ion response is nonlocal, and the quasineutrality retains its full integral form. An expansion procedure is introduced to solve the quasineutrality equation in powers of the width of the tearing layer over the ion Larmor radius. The expansion procedure is applied to the collisionless and semi-collisional tearing modes. The first order terms in the expansion we find to be strongly stabilizing. The physics of the mode and of the stabilization is discussed. Tearing modes are observed in experiments even though the slab theory predicts stability. It is proposed that these modes grow from an equilibrium with islands at the rational surfaces. If the equilibrium islands are wider than the ion Larmor radius, the mode is unstable when Δ' is positive.

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I. INTRODUCTION

There have been numerous analyses of tearing mode stability, starting with the classic work of H. P. Furth, J. Killeen, and M. N. Rosenbluth[1] (the F.K.R. theory). Present-day, magnetically confined plasmas, however, are not adequately described by the resistive Magnetohydrodynamic (MHD) equations. Drake and Lee[2] identified the semi-collisional regime, where parallel diffusion of the electrons limits the width of the electron current layer to one which is smaller than the ion Larmor radius. In almost all present-day tokamak experiments the large-scale (e.g., $m = 2$, $n = 1$) tearing modes are in the semi-collisional regime.

The main thrust of this paper is to show that in the analysis of the tearing mode stability the use of the 'unmagnetized' ion response is incorrect. When the full nonlocal ion response is included, the mode is strongly stabilized. References 3 and 4 use the quasineutrality, integral equation for the collisionless case. We introduce a new iterative scheme to solve this integral equation analytically so as to obtain explicit dispersion relations for both the collisionless and semi-collisional tearing modes.

The stabilizing term arises from the ion polarization current outside the electron current layer. A simple physical estimate of this term gives the order of magnitude of the actual stabilization. Drake et al.[5] treated the case where $T_i = 0$. This case produces some but not all of the effects we have observed. When $T_i = 0$, the ion Larmor radius is much less than the current layer width so that the dominant stabilization, which comes in our analysis (where $T_i \approx T_e$) from the partially magnetized region, is absent.

Tearing modes are observed in present-day tokamaks even though slab theory predicts these modes to be stable. We believe this apparent contradiction can be explained if the rational surfaces are broken into magnetic islands in the equilibrium. The breakup of the rational surfaces may be caused by some nonaxisymmetry of the equilibrium. If the islands caused by the breakup are larger than the ion Larmor radius, and therefore the current layer, we expect the islands to be unstable for realistic values of Δ' defined in Reference 1. This conjecture is discussed in Sec. VI. We do not include toroidal effects and these may have a profound effect on stability.

In Sec. II we discuss the geometry for both modes. In Sec. III we examine the collisionless tearing mode and develop our mathematical technique. In Sec. IV we treat the semi-collisional mode for its solution (mathematical details are relegated to appendices). In Sec. V we give simple physical derivations of the physical processes. The nonlinear aspects are discussed in Sec. VI. Comparison with other work is given in the conclusion.

II. SLAB GEOMETRY

In this section we describe the basic geometry and electrodynamics of the tearing mode. The plasma response for each mode is given in Secs. III and IV. We consider a simple sheared slab model without curvature. (The effects of curvature will be presented elsewhere.) The slab is used here to obtain the inner solution near the rational surface. These solutions are to be matched to outer solutions in which realistic geometry is included.

Consider a sheared slab with an equilibrium magnetic field given by

$$\underset{\sim}{B}_0 = B_0 \left(\hat{\underset{\sim}{z}} + \frac{x}{\ell_s} \hat{\underset{\sim}{y}}\right) . \tag{1}$$

The equilibrium density and temperature vary in the x direction. We look for a mode with a rational surface at x = 0, and therefore take the z derivative of the mode to be zero where B is in the z direction at the rational surface.

For the perturbation we use the vector and scalar potentials defined by

$$\nabla \times \underset{\sim}{A} = \underset{\sim}{B} \quad , \tag{2a}$$

$$\frac{i\omega \underset{\sim}{A}}{c} - \nabla\phi = \underset{\sim}{E} \quad , \tag{2b}$$

$$\nabla \cdot \underset{\sim}{A} = 0 \quad , \tag{3}$$

where the perturbed quantities vary as exp $(-i\omega t + iky)$. The outer solution is represented by the jump in the logarithmic derivative of B_x, Δ'. The matching between the inner and outer solution is achieved through Δ' by

$$\Delta' = \left(\frac{d}{dx} \ln A_{z_{outer}}\right)_{x=-0}^{x=+0} = \left(\frac{d}{dx} \ln A_{z_{inner}}\right)_{-\infty}^{+\infty} \quad . \tag{4}$$

It is also important that $E_\parallel \to 0$, as $x \to \infty$ so that the ideal MHD solution in the outer region is approached as $x \to \infty$ in the inner solution. This is obtained automatically in our solutions.

Because the width of the mode greatly exceeds the Debye length, we can use the quasineutrality condition

$$n_i = n_e \quad . \tag{5}$$

In the low β plasma, only $A_\parallel$, the parallel component of $\underset{\sim}{A}$, survives. From Ampere's law,

$$(\nabla^2 \underset{\sim}{A})_\parallel = -\frac{4\pi}{c} J_\parallel \quad . \tag{6}$$

In both modes we consider $\partial/\partial x >> k_y$, so that Ampere's Law becomes

$$\left(\frac{\partial^2 A_\parallel}{\partial x^2}\right) = -\frac{4\pi}{c} J_\parallel \quad . \tag{7}$$

We shall expand in powers of β_p in which to lowest order, A_z is a constant (constant 'ψ' of FKR). Schematically, Eq. (5) becomes an equation for ϕ in terms of a known $A_\parallel$, and Eq. (7) along with the boundary condition Eq. (7), determines $A_\parallel$ to next order in β_p. Equation (7) can be integrated once to yield

$$\Delta' \; A_z(\infty) = -\frac{4\pi}{c} \int_{-\infty}^{\infty} dx \; J_\parallel \quad . \tag{8}$$

Equations (5) and (8) are solved for the collisionless and semi-collisional plasma response in Secs. III and IV, respectively.

In Sec. V we present a simple physical picture of the dominant physical processes. In the collisionless theory, the electron response to $E_\parallel$ is determined by electron inertia at $x = 0$. For $x \neq 0$, $k_\parallel \neq 0$, the electron response becomes dominated by electron pressure forces. This determines the extent of the electron current layer whose width is given by the position at which

$$\omega = k_\parallel \, v_{the} \; , \tag{9}$$

where $k_\parallel = kx/\ell s = \underset{\sim}{k} \cdot \underset{\sim}{b}_o$ and $v_{the} \approx (2T_e/m)^{1/2}$ is the electron thermal velocity. The width δ of the mode is thus

$$\delta = \frac{\omega \; \ell_s}{V_{the} k} \; . \tag{10}$$

For $x < \delta$ an electron with parallel velocity v_{the} 'sees' a DC electric field for a period of the wave, while for $x > \delta$ an electron 'sees' a rapidly oscillating AC electric field for a period of the wave. Clearly, the current response is much larger in the DC region. When $x \sim O(\rho_i)$, $E_{\parallel} \to 0$. This is because the ions moving perpendicular to $\underset{\sim}{B}$ cannot neutralize the charge from electron current flow parallel to the field. Therefore, the electrons build up a potential which shorts out $E_{\parallel}$.

For the semi-collisional mode, parallel electron diffusion limits the width of the mode. Essentially, the electron dynamics are as follows: $E_{\parallel}$ drives the electron current, building up density and temperature gradients along the field lines, while parallel diffusion relaxes these gradients. The inner width of the mode Δ is determined by the x for which a typical electron diffuses one parallel wave length in a period of the wave, i.e., where

$$\omega = \frac{k_{\parallel}^2 v_{the}^2}{\nu_e} \quad . \tag{11}$$

The width Δ is thus

$$|\Delta| = \left(\frac{m_e \nu_e \omega \ell_s^2}{k^2 T_e}\right)^{1/2} = \left(\frac{\nu_e}{\omega}\right)^{1/2} \delta \quad . \tag{12}$$

As in the collisionless mode, when $x \sim O(\rho_i)$, $E_{\parallel} \to 0$ due to 'shorting' out of $E_{\parallel}$ by electrons.

In the modes considered we must have $\omega \sim \omega^*$; otherwise, as we shall show, the parallel current in the electron layer is larger than the left-hand side of Eq. (8). As pointed out in Ref. 2., in most experiments

$$\rho_i \gg |\Delta|, \qquad \rho_i \gg \delta \,. \tag{13}$$

For this reason the ion dynamics in the inner layer behave predominantly as though they were unmagnetized. However, the most important damping mechanism for this mode involves the effect of finite (not infinite) ion Larmor radius. If the density of the plasma is raised while keeping everything else fixed, the semi-collisional mode passes to the collisional mode of F.K R.[1] in which $\Delta \gg \rho_i$. In this collisional limit the width of the mode is determined not by Δ, which is large, but by the distance at which $E_\parallel$ goes to zero as x increases.

III. DYNAMICS OF COLLISIONLESS TEARING MODE

A. The Linearized Equations

The procedure for finding the electron and ion current and density responses is standard[2] and only outlined here. The normalized equilibrium distributions are

$$f_{o\alpha}\,(H,\, P_y,\, P_z,) \simeq f_{m\alpha}\Big[1 + \frac{P_z U_{z\alpha}}{T_\alpha} + \frac{P_y U^u_{y\alpha}}{T_\alpha} + \frac{P_y U_{y\alpha}}{T_\alpha^2}\,\big(H - \frac{3T}{2}\big)\Big] \quad , \tag{14}$$

where α is a species label, q_α the signed charge, and

$$H_\alpha = \frac{m_\alpha v_\alpha^2}{2} \,, \tag{15a}$$

$$\underset{\sim}{P}_a = m_\alpha \underset{\sim}{v}_\alpha + q_\alpha \frac{\underset{\sim}{A}_o(x)}{c} \,, \tag{15b}$$

$$f_{m\alpha} = \Big(\frac{m_\alpha}{2\pi T_\alpha}\Big)^{3/2} \times \exp\Big(-\frac{H_\alpha}{T_\alpha}\Big) \quad , \tag{15c}$$

$$U^n_{y\alpha} = \frac{cT_\alpha}{q_\alpha B_o} \frac{\partial \ln n_o}{\partial x} = \frac{cT_\alpha}{q_\alpha B_o} \frac{1}{a} \quad , \tag{15d}$$

$$U^T_{y\alpha} = \eta_\alpha U^n_{y\alpha} \quad , \tag{15e}$$

$$\eta_\alpha = \frac{\partial \ln T_\alpha}{\partial \ln n_o} \quad , \tag{15f}$$

$$U_{ze} = \frac{-cB_o}{4\pi n_o e \ell_s} \quad , \tag{15g}$$

$$U_{zi} = 0 \quad . \tag{15h}$$

The perturbed electron distribution function can be obtained from the linearized Vlasov equation averaged over the equilibrium field gyrophase angle[2]

$$\tilde{f}_e = \frac{ie\tilde{E}_\parallel v_\parallel}{\omega - k_\parallel v_\parallel} \frac{\partial f_o}{\partial H} + ie \frac{(\tilde{\phi} - v_\parallel \tilde{A}_\parallel)}{(\omega - k_\parallel v_\parallel)} k_y \frac{\partial f_o}{\partial P_y} \quad , \tag{16}$$

The term involving $\partial f_o / \partial P_z$ is small and has been dropped. Taking moments of (3.3), we obtain the current and density perturbations

$$J_{ze} \approx J_\parallel = \frac{i\omega^2_{pe}}{4\pi\omega} E_\parallel \left[\frac{\omega - \omega^*}{\omega} z'(|s|) + \eta_e \frac{\omega^*}{\omega} \frac{|s|}{2} z''(|s|)\right] \frac{\omega^2}{k^2_\parallel v^2_{the}} \quad , \tag{17}$$

$$\hat{n}_e = \frac{-k_\parallel J_{ze}}{\omega_e e} + \frac{k^2_{De} \omega^*}{4\pi\omega e} \hat{\phi} \quad , \tag{18}$$

where $\parallel$ refers to the component parallel to the equilibrium magnetic field, and

$$s = \frac{\omega}{k_\parallel v_{the}} = \frac{\delta}{x} \quad , \tag{19a}$$

$$\omega^* = k\, U^n_{ye} \quad , \tag{19b}$$

$$z(s) = \frac{-1}{\pi^{1/2}} \int_{-\infty}^{\infty} du\, \frac{\exp(-u^2)}{s-u} \quad . \tag{19c}$$

The ion response is obtained from either the gyrokinetic equation or by simple orbit integration. We assume that $\omega >> k_{\parallel} v_{thi}$ ($\omega = k_{\parallel} v_{thi}$ at a much greater distance from the rational surface than ρ_i), and that J_{zi} is negligible. The ion density response is

$$\frac{\tilde{n}_i}{n_o} = \frac{e}{T_i} \left\{ -\phi(x) + \int_{-\infty}^{\infty} dp\, \exp(ipx)\phi(p) \left[\left(\frac{\omega^*}{\omega} + 1 \right) \Gamma_o - \frac{\omega^*}{\omega} \frac{\eta_i}{2} \frac{p^2 \rho_i^2}{2} \left(\Gamma_o - \Gamma_i \right) \right] \right\} , \tag{20}$$

where

$$\Gamma_n = \exp\left(\frac{-p^2 \rho_i^2}{2} \right) I_n \left(\frac{p^2 \rho_i^2}{2} \right) \quad , \tag{21}$$

and $I_n(x)$ is the modified Bessel function of order n. Quasineutrality is obtained by combining Eqs. (20), (17), and (18) to give

$$\int_{-\infty}^{\infty} dp \Psi(p) F(p) \exp(ipx) = \frac{\delta}{x} \frac{1}{2} \left(A_z - \frac{x\Psi}{\delta} \right) \left[z'(|s|) \left(1 - \frac{\omega_i^*}{\omega} \right) + \frac{\omega^*}{\omega} \frac{\eta_e}{2} |s| z''(|s|) \right], \tag{22}$$

where

$$\Psi = \frac{ck\phi\delta}{\ell_s \omega} = \frac{c\phi}{v_{the}} \quad , \tag{23}$$

and

$$F(p) = \left(\frac{\omega^*}{\omega} + \frac{T_e}{T_i}\right)\left(\Gamma_o - 1\right) - \frac{\omega^*}{\omega}\frac{\eta_i}{2}\, p^2\rho_i^2(\Gamma_o - \Gamma_i) \ . \tag{24}$$

Equation (22) is grouped so that the parallel response of the electrons is on the right-hand side and the perpendicular response of both species is on the left-hand side (the ion response is due entirely to perpendicular motion). Ampere's law, Eq. (7), can be written as

$$\left(\frac{\omega^*}{\omega}\right)^2 \frac{1}{\beta_p}\frac{d^2A_\parallel}{dx^2} = \frac{1}{\delta^2}\frac{\delta}{x}\int dp\ \Psi(p)\ F(p)\exp(ipx) \quad , \tag{25}$$

where

$$\beta_p = 4\pi\frac{n_oT_e}{B^2}\frac{\ell_s^2}{a_s^2} \quad .$$

We rewrite Eq. (22) as

$$-\frac{x}{\delta}\left(A_\parallel - \frac{x}{\delta}\Psi\right)\sigma\left(\frac{x}{\delta}\right) = \int dp\ \Psi(p)\ F(p)\exp(ipx) \quad , \tag{26}$$

where the scaled conductivity is

$$\sigma\left(\frac{x}{\delta}\right) = -\frac{1}{2}\left[\left(1 - \frac{\omega^*}{\omega}\right)\left(\frac{\delta}{|x|}\right)^2 z'\left(\frac{\delta}{|x|}\right) + \frac{\omega^*}{\omega}\frac{\eta_e}{2}\left(\frac{\delta}{|x|}\right)^3 z''\left(\frac{\delta}{|x|}\right)\right] \quad . \tag{27}$$

Equation (26) is an integral equation relating Ψ to $A_\parallel$. It has two limits. When $\rho_i\ d\Psi/dx \gg \Psi$, we are led to the ion adiabatic or unmagnetized response

$$-\left(\frac{\omega^*}{\omega} + \frac{T_e}{T_i}\right)\Psi(x) = \frac{x}{\delta}\left[A_\parallel - \frac{x}{\delta}\Psi(x)\right]\sigma\left(\frac{x}{\delta}\right) \quad . \tag{28}$$

This limit is somewhat subtle and is examined in detail below. The other limit of Eq. (26) is $\rho_i\ d\Psi/dx \ll 1$. In this limit the ions are magnetized, and the right-hand side of Eq. (26) becomes the polarization drift and finite Larmor radius $\underset{\sim}{E} \times \underset{\sim}{B}$ drift of the ions. Thus

$$\left[\frac{T_e}{T_i} + \frac{\omega^*}{\omega}\left(1 + \frac{\eta_i}{2}\right)\right] \rho_i^2\ \frac{d^2\Psi}{dx^2} = \frac{x}{\delta}\left(A_\parallel - \frac{x}{\delta}\ \Psi\right)\ \sigma\left(\frac{x}{\delta}\right) \quad . \tag{29}$$

Since $\delta \ll \rho_i$ in present experiments, Eq. (29) is valid for $x \gg \rho_i$ where $\Psi \sim A_\parallel\ \delta/x$. Returning to Eq. (28), we note the unmagnetized approximation is valid where

$$\Psi \gg \int dp\ \exp(ipx)\Psi(p)\ \Gamma_o\left(\frac{p^2\rho_i^2}{2}\right) . \tag{30}$$

Expressing the integral term on the right-hand side of Eq. (30) as a nonlocal response function we find,

$$\int dp\ e^{ipx}\ \Psi(p)\ \Gamma_o\left(\frac{p^2\rho_i^2}{2}\right) = \frac{1}{2\rho_i}\ \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{\infty} dx'\Psi(x')\exp\left(\frac{x-x'}{\rho_i}\right)^2 K_o\left(\frac{x-x'}{\rho_i}\right)^2 . \tag{31}$$

where K_o is the Bessel function of the second kind.

The response function $\exp[(x-x')^2/\rho_i^{\,2}]K_o[(x-x')^2/\rho_i^{\,2}]$ is approximately constant for $x - x' \sim \rho_i$ and falls off like $\rho_i/(x - x')$ outside this region [see Fig. 1]. We expect $\Psi(x')$ to have structure for $x \sim \delta$ and to fall off like δ/x for $x > \rho_i$. This is because we expect that as $x \to \infty$, $E_\parallel \to 0$. Note also that $\Psi(x)$ is antisymmetric. For $x \ll \delta$, we can estimate Eq. (31) to be of order $x/\rho_i\ \Psi_o$ where $\Psi_o = \Psi(\delta)$. Clearly, for $x \cdot \delta$ it is of order $\Psi_o\delta/\rho_i$ (see Fig. 4). For $x \gtrsim \rho_i$, the inequality in Eq. (30) breaks down: the term on the right-hand side is of order $\Psi_o\ \delta/\rho_i$, and because in this region $\Psi \sim \Psi_o\ \delta/x$, the term on the left-hand side is the same size. We can see the physical origin of this effect from Eq. (31). The ion response function is approximately constant over the whole orbit (distance ρ_i) so that the largest contribution to the density comes from the inner region ($x \sim \delta$) where Ψ is large. The important point in this discussion is that Eq. (28) represents Ψ in terms of $A_\parallel$ to order δ/ρ_i for $x \sim \delta$.

B. The Iterative Method For The Solution of The Integral Equation

By a change of scale Eqs. (25) and (26) can be written as,

$$\left(\frac{\omega^*}{\omega}\right) \frac{1}{\beta_p} \frac{D^2 A_\parallel}{dt^2} = \frac{1}{t} \int dq\ \psi(q)\ H\left(\frac{q}{\varepsilon}\right) \exp(iqt) \tag{32}$$

and

$$-t\left[A_\parallel - t\psi(t)\right]\sigma(t) = \int dq\ \psi(q)\ H\left(\frac{q}{\varepsilon}\right) \exp(iqt) \tag{33}$$

where $t = x/\delta$, $q = p\delta$, $\varepsilon = \delta/\rho_i$, $H(q/\varepsilon) \equiv F(q/\delta)$ and $\psi(q)$ is the Fourier transform of $\psi(t)$. The "eigenvalue" ω is determined by requiring that the solution of Eqs. (32) and (33) for $A_\parallel$ and ψ satisfy the boundary conditions $E_\parallel \rightarrow 0$ and $dA_\parallel/dt = \delta\Delta' A_\parallel$ for $t >> \varepsilon^{-1}$.

Equation (32) is solved by treating β_p as small and expanding. Clearly the lowest order $A_\parallel$ is a constant (we shall compute $A_\parallel$ to order β_p in our final solution). ψ is obtained from Eq. (33) as an asymptotic series in ε. In this expansion $A_\parallel$ is treated as a known function which is approximately constant. The solution of Eq. (33), $\psi(q)$, [which is known in terms of $A_\parallel(t)$] is substituted into Eq. (32) yielding an equation for $A_\parallel$ alone. [The β_p

correction is obtained from Eq. (32) by setting $A_\parallel$ equal to a constant on the right-hand side and integrating with respect to t twice. The dispersion relation is then obtained by writing $A_\parallel$ to order β_p on the right-hand side of Eq. (32) then integrating over all t and matching to Δ'.] In general, Eq. (33) cannot be written as a differential equation in either t or q space. We must employ an iteration method to find an aysmptotic series in ε for ψ, that we now describe.

The internal region (i.e., the region where nonideal MHD effects are important) divides into an inner region where $t \sim O(1)$ and an outer region where $t \sim O(\varepsilon^{-1})$ making the whole problem a three region problem. Henceforth, we shall refer to the "inner region" when $t \sim O(1)$ the "outer region" when $t \sim O(\varepsilon^{-1})$ and the "external region" where $t \gg \varepsilon^{-1}$ (the ideal MHD region). In some approximate sense the outer region corresponds to the rapid variations of $\psi(q)$ for $q \sim O(\varepsilon)$ and the inner region to the smoother variations of $\psi(q)$ on the $q \sim O(1)$ scale. The two regions are connected by the nonlocal ion response so that it is impossible to treat either region independently. The nonlocal response arises as follows: A change in the potential in one region causes a perturbation of the ion orbit which, since the ion orbit can pass through both regions, can affect the ion density in the other region. The potential in this second region must then change to preserve charge neutrality.

There are two approximations that facilitate the solution of Eq. (33). First we have shown [Eqs. (28) and (30)] that in the inner region the dominant ion response is unmagnetized and therefore $\psi(t) \approx \psi_u(t)$ where

$$H(\infty)\psi_u(t) = -\left(\frac{\omega^*}{\omega} + \frac{T_e}{T_i}\right)\psi_u(t) = -t\left[A_\parallel(t) - t\psi_u(t)\right]\sigma(t). \qquad (34)$$

Secondly in the outer region the electron response is Boltzmann since the electrons travel many parallel wavelengths in one period of the mode. Mathematically

$$\lim_{t\to\infty} [t^2\sigma(t)] = \left(1 - \frac{\omega^*}{\omega}\right) = \sigma(\infty) \quad . \tag{35}$$

Subtracting the unmagnetized ion perpendicular response $H(\infty)\psi(t)$ from both sides of Eqs. (33) and rearranging terms we obtain,

$$\left[H(\infty) - t^2\sigma(t)\right]\psi(t) = -t\sigma(t)A_{\parallel} + \int dq\ \psi(q)\left[H(\infty) - H\left(\frac{q}{\varepsilon}\right)\right] \exp(iqt) \quad . \tag{36}$$

The left-hand side of Eq. (36) is the unmagnetized response and parallel electron response to ψ, the first term on the right-hand side is the electron parallel response to $A_{\parallel}$ and the second term is the correction due to the assumption of unmagnetized ions. This term in Eq. (36) contains the convection from the ions propagating from the outside region $[q \sim O(\varepsilon)]$ to the inner region. Note that for $t \sim O(\varepsilon^{-1})$ all three terms in Eq. (36) are, of order ε, but that for $t \sim O(1)$ the magnetization correction is smaller than the other two terms by a factor ε (this has been demonstrated in the previous section). To find the order ε corrections due to magnetization we must know $\psi(q)$ to lowest order for $q \sim O(\varepsilon)$. One may easily verify that even though $\psi(t)$ is $O(\varepsilon)$ for $t \sim O(\varepsilon^{-1})$ the outer region has an order one effect on $\psi(q)$ when $q \sim O(\varepsilon)$. Since $\psi(t) \sim O(\varepsilon)$ and $\psi(t) - \psi_u(t) \sim O(\varepsilon)$ for $t \sim O(\varepsilon^{-1})$, $\psi(q) - \psi_u(q) \sim O(1)$ for $q \sim O(\varepsilon)$. It is therefore not sufficient to substitute $\psi(q) = \psi_u(q)$ in Eq. (36) to find the order ε corrections to $\psi(t)$ for $t \sim O(1)$.

We again rewrite Eq. (33) in order to find $\psi(t)$ in the outer region. Subtracting the electron response in the outer region, $\sigma(\infty)\psi(t)$, from both sides of Eq. (33) we obtain,

$$-t\sigma(t)A_\parallel + \left[t^2\sigma(t) - \sigma(\infty)\right]\psi(t) = \int dq\left[H\left(\frac{q}{\varepsilon}\right) - \sigma(\infty)\right]\psi(q)\,\exp(iqt) \tag{37a}$$

or writing $\psi(t) = \psi_u(t) + [\psi(t) - \psi_u(t)]$ and using Eq. (34),

$$[H(\infty) - \sigma(\infty)]\psi_u(t) + \left[t^2\sigma(t) - \sigma(\infty)\right]\left[\psi(t) - \psi_u(t)\right]$$

$$= \int dq \left[H\left(\frac{q}{\varepsilon}\right) - \sigma(\infty)\right]\psi(q)\,\exp(iqt) \quad . \tag{37b}$$

In Eq. (37a) the first term on the left-hand side is the $A_\parallel$ driven electron parallel response. The electron parallel response to ψ in the inner region is given by $t^2\sigma(t)\psi$ and the $\sigma(\infty)\psi$ terms cancel from both sides. In the outer region the electron parallel response is given by the $\sigma(\infty)\psi$ term on the right-hand side. We write Eq. (37b) in a form that anticipates the fact that $\psi(t) \approx \psi_u(t)$ in the inner region. The left-hand side of Eq. (37b) acts like an inner source which is propagated by the ion response to the outer region.

The second term on the left-hand side of Eq. (37b) is smaller than the first term by a factor ε for all t. We therefore drop it in lowest order. We may then obtain $\psi_0(q)$ (where the subscript stands for order in ε) for all q by inverting the Fourier transform in Eq. (37b). $\psi_0(t)$ the inverse Fourier transform of $\psi_0(q)$ is correct in the outside region to order ε [i.e., $\psi(t) - \psi_0(t) \sim O(\varepsilon^2)$ for $t \sim O(\varepsilon^{-1})$]. This outer potential changes the ion orbit and therefore the ion density by order ε for $t \sim O(1)$. Then an order ε change of the inner potential in the parallel electron response and unmagnetized ion response reestablishes charge neutrality. The order ε change in the inner potential is obtained by substituting $\psi_0(q)$ into the integral and $\psi_1(t)$ in the left-hand side of Eq. (36). $\psi_1(q)$ is obtained by substituting for $\psi(t)$ -

$\psi_u(t)$ to order ε in the left-hand side of Eq. (37b). This step is made to obtain the corrections to the outer region due to the correction to the inner "source." We may combine these operations by substituting for $\psi(t) - \psi_u(st)$ from Eq. (36) into Eq. (37b) and inverting the transform. We obtain

$$\psi(q) - \psi_o(q) = \frac{1}{2\pi[H(q/\varepsilon)-\sigma(\infty)]} \int dt \left[\frac{t^2 b(t)-\sigma(\infty)}{H(\infty)-t^2\sigma(t)}\right] \int dq'\left[H(\infty) - H\left(\frac{q'}{\varepsilon}\right)\right]$$

$$\times \psi(q') \exp(iq't - iqt) \qquad (38)$$

where

$$\psi_o(q) = \left[\frac{H(\infty)-\sigma(\infty)}{H(q/\varepsilon)-\sigma(\infty)}\right] \psi_u(q) \qquad (39)$$

and $\psi_u(q)$ is the Fourier transform of $\psi_u(t)$ defined by Eq. (34).

Physically, Eq. (38) is somewhat obscure. However, mathematically, the iteration scheme is easier to understand in this form. As $H(\infty) - H(q'/\varepsilon)$ is of order one for $q' \sim O(\varepsilon)$ and of order ε for $q' \sim O(1)$, the q' integration in Eq. (38) reduces the order by a factor ε. The t integration does not change the order since it occurs over an interval $[t^2\sigma(t) - \sigma(\infty)] \sim O(1)$. $\psi_n(q)$ is the order ε^n correction to $\psi(q)$. $\psi_{n+1}(q)$ may be obtained by substituting $\psi_n(q)$ into the right-hand side of Eq. (38). The expansion is not strictly in powers of ε. For instance, $\varepsilon\ln\varepsilon$ will enter Eq. (38), since in the q' integration, when $q' >> \varepsilon$, $H(\infty) - H(q/\varepsilon) \sim \varepsilon/q$. The occurrence of the logarithm is a manifestation of the contribution of the potential in the partially magnetized region $\delta < x < \rho_i$. We have loosely referred to order ε when we actually mean order ε and order $\varepsilon\ln\varepsilon$. This does not affect the asymptotic nature of the expansion that the iteration generates. The lowest order approximation to $\psi(q)$ is $\psi_0(q)$. Subsequent iterations are obtained from,

$$\psi_{n+1}(q) = \frac{1}{2\pi[H(q/\varepsilon)-\sigma(\infty)]} \int dt \left[\frac{t^2\sigma(t)-\sigma(\infty)}{H(\infty)-t^2\sigma(t)}\right] \int dq' \left[H(\infty) - H(\frac{q'}{\varepsilon})\right]$$

$$\times \psi_n(q')\exp(iq't - iqt) \quad . \tag{40}$$

Let us briefly review the physical basis of this iteration method. In the inner region the potential scale length is short compared to the ion Larmor radius and the ions behave approximately as though they are unmagnetized with small magnetization corrections. If the inner solution is known to some order, the outer solution can be generated by calculating the effect on the ion orbits from the inner solution and demanding that charge neutrality hold in the outer region. In this part of the iteration the inside acts like a source and the ions propagate the effects of this source to the outer region where the nonlocal part of the ion density response is neutralized by local ion and electron responses. The outer potential (that has been determined) makes an order ε change in the inner ion density via the nonlocal ion response. In order to preserve charge neutrality the inner potential is corrected by an amount of order ε in the parallel electron and unmagnetized ion responses. The potential is then correct to next order in the inner region and the iteration may begin again.

We find that the order ε corrections determine stability since the lowest order frequency is entirely real. Calculating $\Psi(q)$ from Eq. (40) and substituting $\psi(q) = \psi_0(q) + \psi_1(q)$ into Eq. (32) we obtain to order ε,

$$\frac{1}{\beta_p}\left(\frac{\omega^*}{\omega}\right)^2 \frac{d^2A_\parallel}{dt^2} = \frac{H(\infty)}{t}\,\psi_u(t) - \left[H(\infty) - \sigma(\infty)\right] \frac{\psi_u(t)}{A_\parallel} \int dq$$

$$\times \left[\frac{H(\infty)-H(q/\varepsilon)}{H(q/\varepsilon)-\sigma(\infty)}\right] \psi_u(q)\, \exp(iqt) \quad , \tag{41}$$

where we have used

$$\frac{H(q/\varepsilon)}{H(q/\varepsilon)-\sigma(\infty)} = \frac{H(\infty)}{H(\infty)-\sigma(\infty)} + \frac{\sigma(\infty)[H(\infty)-H(q/\varepsilon)]}{[H(q/\varepsilon)-\sigma(\infty)][H(\infty)-\sigma(\infty)]} \quad . \tag{42}$$

The second term on the right-hand side of Eq. (41) is of order ε. Note that for $t \sim \varepsilon^{-1}$ both terms on the right-hand side of Eq. (41) are of order ε^2. The total current is obtained from integrating Eq. (41) over t. The region $t > \varepsilon^{-1}$ contributes an amount of order ε to the total current.

We assume that

$$\beta_p^2 << \varepsilon \tag{43}$$

and ignore $\varepsilon\beta_p$ corrections. Iterating Eq. (41) in β_p once, substituting $A_\parallel(t)$ into the right-hand side, and integrating over t from $t = -\infty$ to $t = +\infty$, we obtain the general dispersion relation

$$\left(\frac{\omega^*}{\omega}\right)^2 \frac{1}{\beta_p} (\Delta' a)\left(\frac{\delta}{a}\right) = H(\infty) \int_{-\infty}^{\infty} dt \left\{\frac{\theta(t)}{t} + \left(\frac{\omega}{\omega^*}\right)^2 \beta_p \left[\int_0^t dt'\, \frac{\theta(t)'}{t'}\right]^2\right\}$$

$$+ [H(\infty) - \sigma(\infty)]2\pi \int_{-\infty}^{\infty} dq\, [\theta(q)]^2 \left[\frac{H(\infty)-H(q/\varepsilon)}{H(q/\varepsilon)-\sigma(\infty)}\right] \quad , \tag{44}$$

where

$$\theta(t) = \frac{\psi_u(t)}{A_\parallel} = \frac{t\sigma(t)}{\left[(\omega^*/\omega)+(T_e/T_i)\right]+t^2\sigma(t)} \tag{45}$$

and θ(q) is the Fourier transform of θ(t). The first term on the right-hand side of Eq. (44) is the current from the inner region. This term dominates the equation. The magnetization correction to the current is given by the third term on the right-hand side of Eq. (44).

C. The Collisionless Tearing Mode

We now apply our iteration scheme to the collisionless tearing mode. Equations (27), (45), and (44) give us the dispersion relation to the lowest significant order needed to determine stability. The dominant current comes from the first term on the right-hand side of Eq. (44), so ω is determined to dominant order by

$$I(\omega_o) = H(\infty) \int_{-\infty/\delta}^{+\infty/\delta} dt \, \frac{\theta(t)}{t} = 0 \; . \tag{46}$$

Using Eq. (27) to express σ(t) and therefore θ(t), we can find I(ω) from Eq. (46). In Appendix A we show that I(ω) is imaginary for ω real and that ω_o [determined by Eq. (46)] is real. In Appendix A we also show that the β_p correction does not affect stability, so it is ignored here.

Equation (46) represents a very important part of the physics of these modes. The dominant current integrated across the inner layer is out of phase with $A_\parallel$, and there is no term in Ampere's law, Eq. (44), to balance this term. Consequently, the dominant integrated current must vanish. Note, however, that the current density is not zero at every point (unless ∇T_e should vanish and then $\omega = \omega^*$ gives $J_\parallel = 0$ everywhere). The inclusion of the δ/ρ_i correction introduces some $J_\parallel$ in phase with $A_\parallel$. The frequency of the mode must then become complex to satisfy Eq. (44). The inclusion of the δ/σ_i and Δ' terms in Eq. (44) is necessary because the mode is marginally stable to zeroth order. Using Eq. (27) and (45), we have

$$\theta(t) = \frac{-\frac{1}{2}\left[\left(1-\frac{\omega^*}{\omega}\right)z'\left(\frac{1}{|t|}\right)+\frac{\omega^*}{\omega}\frac{\eta_e}{2}\frac{1}{|t|}z''\left(\frac{1}{|t|}\right)\right]}{\left(\frac{\omega^*}{\omega}+\frac{T_e}{T_i}\right)-\frac{1}{2}\left[\left(1-\frac{\omega^*}{\omega}\right)z'\left(\frac{1}{|t|}\right)+\frac{\omega^*}{\omega}\frac{\eta_e}{2}\frac{1}{|t|}z''\left(\frac{1}{|t|}\right)\right]} \quad . \tag{47}$$

To calculate the corrections to ω in Eq. (44), we approximate $\theta(t)$ by

$$\theta(t) \approx \frac{1}{2[H(\infty)-\sigma(\infty)]t}\left[\left(1-\frac{\omega^*}{\omega}\right)z'\left(\frac{1}{|t|}\right)+\frac{\omega^*}{\omega}\frac{\eta_e}{2}\frac{1}{|t|}z''\left(\frac{1}{|t|}\right)\right] \quad . \tag{48}$$

This is only valid for $t \gg 1$. But, since the denominator in Eq. (47) does not vanish for ω real, we can obtain qualitatively correct results by using Eq. (48) for all t. Note that we do not use this approximation in Eq. (48) to demonstrate that ω_0 is real, but only to calculate approximate corrections to ω due to ion magnetization. The expression (48) for $\theta(t)$ can be Fourier transformed. The details are given in Appendix B. For $q > 0$

$$\theta(q) = \frac{-i}{[H(\infty)-\delta(\sigma)]\,\pi^{1/2}}\int_0^\infty du\left[\left(1-\frac{\omega^*}{\omega}\right)-\frac{\eta_e}{2}\frac{\omega^*}{\omega}(2u^2-1)\right]\exp-\left(u^2-\frac{iq}{u}\right). \tag{49}$$

We obtain the approximate value for $I(\omega)$ by integrating $\theta(t)/t$ over t. Because of symmetry, we integrate over positive t and multiply by two.

$$I(\omega) \approx \frac{1}{2}\left[\int_0^\infty ds\left(2-\frac{\omega^*}{\omega}\right)z'(s)+\frac{\omega^*}{\omega}\frac{\eta_e}{2}\,s\,z''(s)\right]\frac{\left[(\omega^*/\omega)+(T_e/T_i)\right]}{\left[1+(T_e/T_i)\right]} \quad . \tag{50}$$

Therefore, carrying out this integral explicitly

$$\omega_o \approx \omega^*[1 + (\eta_e/2)] \tag{51}$$

and

$$i\gamma \left|\frac{dI}{d\omega}\right|_{\omega_o} \approx + \pi^{1/2}\left(\frac{\gamma}{\omega^*}\right) \frac{[(\omega^*/\omega_o)+(T_e/T_i)]}{[1 + (T_e/T_i)]} \left(\frac{\omega^*}{\omega_o}\right) \quad , \tag{52}$$

where γ is the growth rate. The root at $\omega = \omega^* T_i/T_e$ has been shown to be stable by Antonsen et al.,[3] and we ignore it here. We can write the dispersion relation from Eq. (3.33) as

$$\frac{[(\omega^*/\omega_o)+(T_e/T_i)]}{[1+(T_e/T_i)]} \left(\frac{\gamma}{\omega^*}\right) \pi^{1/2}\left(\frac{\omega^*}{\omega_o}\right) = \left(\frac{\omega^*}{\omega_o}\right) \frac{1}{\beta_p} (\Delta' a) \frac{\delta^*}{a} + (1 + T_e/T_i)\, R(\omega_o) \tag{53}$$

where $\delta^* = \omega^* \ell s/kv_{the}$ and,

$$R(\omega) = 2\pi \int dq\, [\theta(q)]^2 \left[\frac{H(\infty) - H(q/\varepsilon)}{H(q/\varepsilon) - \sigma(\infty)}\right] \quad . \tag{54}$$

Since $\theta(q)$ is approximately constant for $q \sim O(\varepsilon)$, an approximate form for $R(\omega)$ is

$$R(\omega) = \frac{-2\theta(q=0)}{[1+(T_e/T_i)]} 2\pi \int_0^{+\infty} dq\, \theta(q) \left[H(\infty) - H(q/\varepsilon)\right] \quad , \tag{55}$$

where we have set $H(q/\varepsilon) \approx H(\infty)$ in the denominator because the biggest contribution to the integral comes from the region $\varepsilon < q \ll 1$ where $H(q/\varepsilon) -$

$H(\infty) \sim \epsilon/q$. Using Eqs. (49), (55), (24), and (33) we obtain for the real part of $R(\omega_o)$, $R'(\omega_o)$

$$\left(1 + \frac{T_e}{T_i}\right) R'(\omega_o) = - \frac{[1-(\omega^*/\omega)]}{[1+(T_e/T_i)]^2} \left\{\left(1 - \frac{\omega^*}{\omega}\right)\left[\left(\frac{\omega^*}{\omega} + \frac{T_e}{T_i}\right) W_o - \frac{\omega^*}{\omega}\frac{\eta_i}{2} W_2\right] - \frac{\omega^*}{\omega}\frac{\eta_e}{2}\left[\left(\frac{\omega^*}{\omega} + \frac{T_e}{T_i}\right) W_1 - \frac{\omega^*}{\omega}\frac{\eta_i}{2} W_3\right]\right\} \quad . \tag{56}$$

The quantities W_o, W_1, W_2, and W_3 are evaluated in Appendix C:

$$W_o \equiv \mathrm{Re}\ 2\pi^{1/2} \int_o^\infty dq\left\{\exp - \left[\left(u^2 - \frac{iq}{u} + \frac{q^2}{2\epsilon^2}\right)\right] I_o(q^2/2\epsilon^2)\right\} = \pi^{1/2}\epsilon K_o(\epsilon), \tag{57a}$$

$$W_1 \equiv \mathrm{Re} 2\pi^{1/2} \int_o^\infty dq \int_o^\infty du(2u^2-1)\left\{\exp -\left[u^2 - \frac{iq}{u} + \frac{q^2}{2\epsilon^2}\right]\right\} I_o(q^2/2\epsilon^2) \simeq O(\epsilon), \tag{57b}$$

$$W_2 \equiv \mathrm{Re}\ 2\pi^{1/2} \int_o^\infty dq \int_o^\infty du\ \left[\exp -\left(u^2 - \frac{iq}{u} + \frac{q^2}{2\epsilon^2}\right)\right] \frac{q^2}{\epsilon^2}\left[I_o\left(\frac{q^2}{\epsilon^2}\right) - I_1\left(\frac{q^2}{\epsilon^2}\right)\right] \simeq W_o, \tag{57c}$$

$$W_3 \equiv \mathrm{Re} 2\pi^{1/2}\int_o^\infty dq\int_o^\infty du(2u^2-1)\left[\exp-\left(u^2 - \frac{iq}{u} + \frac{q^2}{2\epsilon^2}\right)\right]\frac{q^2}{\epsilon^2}\left[I_o\left(\frac{q^2}{\epsilon^2}\right)-I_1\left(\frac{q^2}{\epsilon^2}\right)\right] = O(\epsilon) \quad , \tag{57d}$$

where $\epsilon = \delta/\rho_i$, and Re is the real part. Note that the Bessel function

$$K_o(\epsilon) \simeq - \ln\left(\frac{\epsilon}{2}\right). \tag{58}$$

so, by ignoring W_1, and W_3, we have ignored order ε in favor of $\varepsilon \ln \varepsilon$. One may verify that the order ε corrections to Eq. (56) are also stabilizing. For the semi-collisional mode, we include all the order ε corrections. Here we include only the dominant stabilization. With ω_0 given by Eq. (51) and with $\eta_i = \eta_e = 2$, $T_e = T_i$, we find the criterion for stability from Eqs. (53), (56), and (57) to be

$$(\Delta' a) < \frac{\pi^{1/2}}{4} \frac{a}{\rho_i} \left(\frac{\eta_e}{2}\right)^2 \beta_p \ln \frac{2}{\varepsilon} . \tag{59}$$

The right-hand side is typically large because a/ρ_i is very large and $\Delta' a$ is of o der unity. We do not compare this mode to fusion experiments as typically ω^* is less than ν_{ei}, the electron collision frequency, and the mode is in the semi-collisional regime.

If the inequality in Eq. (59) holds, the damping rate is given from Eqs. (53), (56), (57), and (58)

$$\gamma_{damping} \simeq \frac{\omega^* \, \varepsilon \ln(2/\varepsilon) \, (\eta_e/2)^2 [(1-\eta_i/2)/(1+\eta_e/2)+T_e/T_i]}{[1+(T_e/T_i)]\{1/[1+(\eta_e/2)]\}+T_e/T_i]}$$

$$- \frac{\varepsilon}{[1+(\eta_e/2)]} \frac{i}{\beta_p} \frac{(\Delta' a)}{\pi^{1/2}} \frac{\rho_i}{a} \frac{(T_e/T_i+1)}{\{1/[1+(\eta_e/2)]+T_e/T_i\}} . \tag{60}$$

The $1/t$ behavior of Ψ in the outer regions $(t > 1)$ leads to logarithmic behavior. This justifies using the approximation Eq. (48). Thus, the residual ion magnetization in region $1 < t < \rho_i/\delta$ plays a dominant role in stabilizing the mode.

IV. DYNAMICS OF THE SEMI-COLLISIONAL TEARING MODE

In Sec. III we considered tearing modes for plasmas in which collisions are negligible. This is valid for $\nu_e \ll \gamma$ where ν_e is the electron collision rate. We now consider the semi-collisional regime,[2] $\nu_e \gg \omega^*$, $k_\parallel V_{the}$, so that Braginskii's equations[6] for the electrons are valid. ρ_i is still large compared to the inner electrons layer, Δ, and $\nu_i \ll \omega^*$. The ion response is given by Eq. (20). The case $\omega \sim \nu_e$ was treated by Hazeltine et al.,[7] without considering the full ion dynamics. Over the width Δ [see Eq. (12)] of the mode we expect

$$k_\parallel v_{the} \ll \nu_{ei} \quad , \tag{61}$$

$$\omega \sim \frac{k_\parallel^2 v_{the}^2}{\nu_e} \; . \tag{62}$$

So, the ordering becomes

$$\omega \ll k_\parallel v_{the} \ll \nu_{ei} \ll \omega_{ce} \; . \tag{63}$$

A. Evaluating the Conductivity.

To describe the electron response, we use Braginskii's equations.[6] Note that we must drop the terms of order ω/ν_e to be consistent since Braginskii does not include order $(kV_{the}/\nu_e)^2$. The next order corrections to Braginskii's equations are examined by Hassam.[8,9]

Dropping terms of order $a^2 \ell_s^{-2}$ and higher in Braginskii's equations, we obtain the following set of equations to describe the linearized mode: The temperature evolution equation,

$$\frac{3}{2}\, n \left(\frac{\partial T_e}{\partial t} + \underset{\sim}{v} \cdot \nabla T_e\right) + nT_e\, \nabla \cdot \underset{\sim}{v} = -\nabla \cdot q \quad , \tag{64}$$

where

$$\underset{\sim}{q} = -3.2\, \frac{nT_e}{m_e \nu_e}\, \nabla_\parallel\, T_e - \frac{5}{2}\, \frac{nT_e}{m_e \omega_{ce}}\, \underset{\sim}{b} \times \nabla T_e + 0.71\, nT_e\, \underset{\sim}{v}_\parallel \quad ; \tag{65}$$

the continuity equation,

$$\frac{\partial n}{\partial t} + \nabla \cdot \left(n\underset{\sim}{v}\right) = 0 \tag{66}$$

the force balance equation,

$$\frac{\nabla p_e}{m_e n} = -\frac{e}{m_e}\left(\underset{\sim}{E} + \frac{\underset{\sim}{v} \times \underset{\sim}{B}}{c}\right) - \nu_e\left(\frac{\underset{\sim}{v}_\perp}{2} + \underset{\sim}{v}_\parallel\right) - \frac{0.71}{m_e}\, \nabla_\parallel T_i \quad . \tag{67}$$

From Eq. (67) the perpendicular velocity of the electrons is given by,

$$\underset{\sim}{v}_\perp \simeq c\, \frac{\underset{\sim}{E} \times \underset{\sim}{b}}{B} - c\, \frac{\underset{\sim}{b} \times \nabla p_e}{neB} \quad . \tag{68}$$

The corrections to $\underset{\sim}{v}_\perp$ from perpendicular transport [the ν_e term in Eq. (67)] produce changes of order $a^2\, \ell_s^{-2}$ in the final result. The parallel gradients must be evaluated with respect to the perturbed magnetic field. For example,

$$\nabla_\parallel T_e = ik_\parallel\, \tilde{T}_e + b_x\, \frac{\partial T_{eo}}{\partial x} = ik_\parallel \tilde{T}_e + \frac{ikA_\parallel}{B_o}\, \frac{\partial T_{eo}}{\partial x} \quad . \tag{69}$$

Linearizing Eq. (66) and noting that v_o can be ignored, we obtain

$$i\omega \frac{\hat{n}_e}{n_o} + i\omega_n^* \frac{e\phi}{T_e} + ik_\parallel v_\parallel = 0, \tag{70}$$

where as before,

$$\omega_n^* = \frac{-cT_o k}{eB} \frac{\nabla n_o}{n_o} . \tag{71}$$

Eliminating T and $\tilde{n}$ using Eqs. (64) and (70), we obtain

$$\tilde{v}_\parallel = \frac{-e}{\nu_e m_e} \left(\frac{i\omega A_\parallel}{c} - ik_\parallel \phi\right) \frac{\left[A + \left(Bx^2/\Delta^2\right)\right]}{\left[1+5.08\left(x^2/\Delta^2\right)+2.13\ \left(x^4/\Delta^4\right)\right]} , \tag{72}$$

where

$$\frac{1}{\Delta^2} = \frac{ik^2 T_e}{m_e \nu_e \omega \ell_s^2} , \tag{73}$$

$$A = 1 - \frac{\omega^*}{\omega} \left(1 + 1.71\ \eta_e\right) ,$$

$$B = 2.13\left(1 - \frac{\omega^*}{\omega}\right) . \tag{74}$$

Note that Eqs. (70) and (18) are identical. Making the transformation,

$$\frac{\omega}{c} \frac{\Psi}{\Delta} = \frac{k}{\ell_s} \phi \tag{75}$$

and combining Eqs. (70), (72), (5), and (20), we obtain the quasineutrality equation

$$- \frac{x}{\Delta} \left(A_\parallel - \frac{x}{\Delta} \Psi\right) \tilde{\sigma}\left(\frac{x}{\Delta}\right) = \int dp\ \Psi(p)\ F(p)\ e(ipx) , \tag{76}$$

where the rescaled parallel conductivity is given by

$$\tilde{\sigma}\left(\frac{x}{\Delta}\right) = \frac{A + B\left(x^2/\Delta^2\right)}{1 + 5.08\left(x^2/\Delta^2\right) + 2.13\left(x^4/\Delta^4\right)} \quad . \tag{77}$$

Ampere's law, Eq. (7), becomes from Eqs. (72) and (76)

$$\left(\frac{\omega^*}{\omega}\right)^2 \frac{1}{\beta_p} \frac{d^2 A_\parallel}{dx^2} = \frac{1}{\Delta^2}\left(\frac{\Delta}{x}\right) \int dp\ \Psi(p)\ F(p)\ \exp(ipx) \quad . \tag{78}$$

There is a formal similarity between Eqs. (15) and (78) and between Eqs. (16) and (76). Setting $x = t\Delta$ and $q = p\Delta$, we obtain equations formally equivalent to Eqs. (22) and (23), except that the q contour is along (argument q) = $\pi/4$. Note that as before

$$\Psi(q) = \int_{-\infty\sqrt{i}}^{\infty\sqrt{i}} \frac{dt}{2\pi} \exp(iqt)\Psi(t) \quad , \tag{79a}$$

$$\Psi(t) \doteq \int_{¿\infty\sqrt{i}}^{\infty\sqrt{i}} \frac{dq}{2\pi} \exp(iqt)\Psi(q) \quad . \tag{79b}$$

Let $H(q/\varepsilon) \equiv F(q/\Delta)$, then Eq. (78) becomes

$$t\left[A_\parallel - t\Psi(t)\right] \tilde{\sigma}(t) = \int_c dq\ \Psi(q)\ \tilde{H}(q/\varepsilon)\ \exp(iqt) \quad . \tag{80}$$

Equation (78) becomes

$$\left(\frac{\omega^*}{\omega}\right) \frac{1}{\beta_p} \frac{d^2 A_\parallel}{dt^2} = \frac{1}{t} \int_c dq\ \Psi(q)\ \tilde{H}(q/\varepsilon)\ e(iqt) \quad . \tag{81}$$

We can use the scheme developed in Sec. II to solve Eqs. (80) and (81).

The dispersion relation can be written in a form analogous to Eq. (44),

$$(\frac{\omega^*}{\omega})^2 \frac{1}{\beta_p} \Delta' a \left(\frac{\Delta}{a}\right) \approx \tilde{H}(\infty) \int_{-\infty i^{1/2}}^{\infty i^{1/2}} dt \left[\frac{\tilde{\theta}(t)}{t} + \left(\frac{\omega}{\omega^*}\right)^2 \beta_p \left(\int_o^t dt' \frac{\tilde{\theta}(t')}{t'}\right)^2\right]$$

$$+ \left[\tilde{H}(\infty) - \tilde{\sigma}(\infty)\right] 2\pi \int_c dq \left[\tilde{\theta}(q)\right]^2 \left[\frac{\tilde{H}(\infty) - \tilde{H}(q/\varepsilon)}{\tilde{H}(q/\varepsilon) - \tilde{\sigma}(\infty)}\right]$$

$$+ \frac{\tilde{H}(\infty)}{\tilde{H}(q/\varepsilon) - \sigma(\infty)} 2\pi \int_c dq\, \tilde{T}(q)\, \tilde{\theta}(q) \left[\frac{\tilde{H}(\infty) - \tilde{H}(q/\varepsilon)}{\tilde{H}(q/\varepsilon) - \tilde{\sigma}(\infty)}\right] , \qquad (82)$$

where

$$\tilde{\sigma}(\infty) = \left(1 - \frac{\omega^*}{\omega}\right) \text{ and } \tilde{H}(\infty) = -\left(\frac{\omega^*}{\omega} + \frac{T_e}{T_i}\right) \qquad (83)$$

and,

$$\tilde{\theta}(t) \equiv \frac{t\tilde{\sigma}(t)}{\{\omega^*/\omega + [t^2\tilde{\sigma}(t)]\}} = \frac{t}{2.13[1+(T_e/T_i+1)]} \frac{A+Bt^2}{(t^2-\beta_+)(t^2-\beta_-)} \qquad (84)$$

and $\beta_\pm$ are

$$\beta_\pm = \frac{1}{4.26[(T_e/T_i)+1]} \left[-5 \left(\frac{T_e}{T_i} + \frac{\omega^*}{\omega}\right) - 1 + \frac{\omega^*}{\omega}(1 + 1.71\, \eta_e)\right.$$

$$\left. \pm \left\{\left[5\left(\frac{T_e}{T_i} + \frac{\omega^*}{\omega}\right) + 1 - \frac{\omega^*}{\omega}(1 + 1.71\, \eta_e)\right]^2 - 8.52\left(\frac{T_e}{T_i} + 1\right)\left(\frac{T_e}{T_i} + \frac{\omega^*}{\omega}\right)\right\}^{1/2}\right\}.$$

(85)

The dominant term in Eq. (84) is given by

$$G(\omega) = \tilde{H}(\infty) \int_{-\infty\sqrt{i}}^{-\infty\sqrt{i}} dt \frac{\hat{\theta}(t)}{t} = \frac{-[(T_e/T_i)+(\omega^*/\omega)]}{2.13(T_e/T_i+1)} \int_{-\infty\sqrt{i}}^{\infty\sqrt{i}} \frac{A + Bt^2}{(t^2-\beta_+)(t^2-\beta_-)} .$$

(86)

In Appendix F we show that $G(\omega)= 0$ has no unstable roots and one real root, where

$$\omega_o \approx \omega^*(1 + 0.4\ \eta_e). \tag{87}$$

We also show in Appendix E that the β_p term on the right-hand side of Eq. (82) corrects the real frequency but does not affect the stability. We compute the Δ/ρ_i corrections in Eq. (82) to find the damping rate. The evaluation of these terms is straightforward, but messy. Here we present only the results. $\theta(q)$ evaluated from $\bar{\theta}(t)$ in Eq. (84) for $q > 0$ is,

$$\underset{q>0}{\theta(q)} = \frac{-i}{4.26[(T_e/T_i)+1]} \frac{1}{(|\beta_-|-|\beta_+|)} [(A-B|\beta_+|)\exp(-a|\beta_+|)$$

$$- (A - B|\beta_-|)\exp(-q|\beta_-|)] \quad . \tag{88}$$

In evaluating the integrals in Eq. (82) we set $H(q/\varepsilon)$ in the denominator equal to $H(\infty)$. One may verify that this approximation slightly decreases the damping rate. The dispersion relation evaluated from Eq. (82), (88), and (24) is

$$(\Delta' a)\ (\frac{\omega^*}{\omega_o})^2 \frac{1}{\beta_p}(\frac{\Delta}{a}) - \frac{\Delta}{\rho_i} \frac{\pi}{(2.13)^2[(T_e/T_i)+1]^2}\ \{D^2[C_+^2\ Q(2\xi_+)$$

$$+ C_-^2\ Q(2\xi_-) - 2C_+C_-Q(\xi_+ + \xi_-)]$$

$$= \frac{\gamma}{\omega^*} \frac{\omega^*}{\omega_o} \left[\frac{[(T_e/T_i)+(\omega^*/\omega_o)]}{2.13[1+(T_e/T_i)]}\right]^{1/2} \frac{3\pi}{(\xi_+ + \xi_-)} \tag{89}$$

where $C_+ = A-B|\beta_+|$, $C_- = A-B|\beta_-|$, $\xi_+ = |\beta_+|^{1/2}\ 2^{-1/2}$, $\xi_- = |\beta_-|^{1/2}2^{-1/2}$, and $D = (|\beta_- - |\beta_+|)^{-1}$. β_+, β_-, A, and B are evaluated for $\omega = \omega_o$ where ω_o is given approximately by Eq. (87). $Q(\alpha)$ is the function defined by

$$Q(\alpha) = \int_0^\infty dr\ \exp\left[-\frac{\alpha r}{(1+i)}\ 2^{1/2} - \frac{r^2\rho_i^2}{2|\Delta|^2}\right]\left(\frac{\omega^*}{\omega_o}\right) + \left(\frac{T_e}{T_i}\right)I_o \tag{90}$$

$$- (\omega^*\ \eta_i\ r^2\rho_i^2)/(\omega_o 2|\Delta|^2)(I_o - I_1)] \quad ,$$

where the argument of I_o and I_1 is $r^2\rho_i{}^2/2|\Delta\ |^2$. Define

$$M(\alpha) = 2v^2|\Delta|/\rho_i \int_0^\infty dr\ \exp(-\alpha|\Delta|\rho_i^{-1}i^{-1/2}v - v^2)I_o(v^2) \quad . \tag{91}$$

Integrating by parts on the I_1 noting that $I(x) = dI_o(x)/dx$, we obtain

$$Q(\alpha) = \left[\frac{T_e}{T_i} + \frac{\omega^*}{\omega}\left(1 - \frac{\eta_i}{2}\right)\right] M(\alpha) - \frac{\omega^*}{\omega}\frac{\eta_i}{2}\ \alpha\ \frac{dM(\alpha)}{d\alpha} \quad . \tag{92}$$

$M(\alpha)$, evaluated in Appendix D, is

$$M(\alpha) = \frac{e^{-(\alpha^2|\Delta|^2/2\rho_i^2)}}{4\pi^{1/2}} K_o\left(\frac{\alpha^2|\Delta|^2}{4\rho_i^2}\right) \approx -\frac{1}{2\pi^{1/2}}\left(\ln\frac{\alpha|\Delta|}{2\rho_i} + \gamma_E\right) \quad , \tag{93}$$

where γ_E is Euler's constant. Evaluating the growth rate for $\eta_e = \eta_i = 2$ and $T_e = T_i$, we have

$$\frac{|\Delta|}{(2)^{1/2}\rho_i}\left[(\Delta' a)\frac{1}{3.4\ \beta_p}\left(\frac{\rho_i}{a}\right) - \left(\frac{\eta_e}{2}\right)^2 0.24\ \left(0.3\ \ln\frac{\rho_i}{|\Delta|} + 0.8\right)\right] = \frac{\gamma}{\omega^*} \ . \tag{94}$$

Since ρ_i/a is a very small number in most tokamaks (1/300 or less), the first term on the left-hand side of Eq. (94) is small. The semi-collisional tearing mode is thus stable with the damping rate given by the second term on the left-hand side of Eq. (94). We note that this calculation is valid for $|\Delta| < \rho_i$, $\beta_p < 1$, and $\nu_i < \omega^* < \nu_e$. In a discharge with 1 keV < T_e < 8 keV, $\beta_p \approx 1/4$, and $n_e \approx 2 \times 10^{13}$ cm^{-3}, a typical experimental regime for present-day large tokamaks, the large scale $2 < m < 20$ tearing modes lie in the semi-collisional regime described here. These modes are almost always stable since $(\Delta'a)$ must be greater than 100 for the growth term in Eq. (94) to be significant. It is, of course, impossible to say that the modes are always stable for every experiment. However, there are experiments on the Princeton Large Torus and the Tokamak Fusion Test Reactor for which Eq. (94) predicts stability for the tearing mode, yet the tearing modes are observed.

This apparent contradiction can be resolved by changing the original assumptions about the equilibrium upon which the perturbation is superimposed. In particular, we contend that most of the low m and n rational surfaces in a tokamak are broken up in the equilibrium due to resonant interaction with nonaxisymmetry of the coils or other instabilities in the plasma. When the island size or width of the stochastic region at the rational surface is as wide as the current layer Δ, it is inappropriate to consider equilibria with smooth surfaces. We expect the mode to be in a new regime similar to the Rutherford nonlinear regime of the classical tearing mode. We examine this regime briefly in Sec. VI.

V. HEURISTIC DISCUSSION

In this section we give simple physical descriptions of the physical processes important to this mode. The mathematics of Sec. IV is somewhat opaque, and here we wish to clarify the physical picture. We outline the role of ω^* and the origin of the stability.

The parallel temperature and density gradients give rise to the important ω^* effects in the parallel current. The mechanisms for building up parallel gradients are twofold: parallel flows along the unperturbed field lines and tilting the field lines along the equilibrium gradients. The second mechanism we illustrate in some detail. The equilibrium gradients are in the x direction. The perturbed magnetic field in the x direction is

$$\delta B_x = ikA_\parallel \quad . \tag{95}$$

As the field line moves, the electrons also move. The electrons move in the x direction with $\underset{\sim}{E} \times \underset{\sim}{B}$ velocity. The x displacement of the electrons is, therefore,

$$\delta\xi_{xe} = \int dt\, \frac{cik\phi}{B_o} = \frac{k}{\omega}\frac{c\phi}{B_o} \quad . \tag{96}$$

Integrating Eq. (95) along the perturbed field line to find the displacement of the field line, we obtain the displacement of the line relative to the fluid $\delta\xi_x$,

$$\delta\xi_x = \frac{\delta B_x}{ik_\parallel B_o} - \delta\xi_{xe} = \frac{-ick}{k_\parallel \omega B_o}\left(\frac{i\omega A_\parallel}{c} - ik_\parallel\phi\right) = -\frac{-ick}{k_\parallel \omega B_o} E_\parallel \quad . \tag{97}$$

Note how this explicitly demonstrates how the 'slippage' of the field lines is proportional to $E_\parallel$.

Now consider the parallel forces on an electron fluid where for simplicity we set $\nabla T_e = 0$ in the equilibrium.

$$F_{\parallel e} \equiv \nabla_{\parallel} p_e - neE_{\parallel} + R_{\parallel} \quad , \tag{98}$$

where $R_{\parallel}$ is the collisional momentum gained by the electrons. For simplicity ignore $R_{\parallel}$. Also, ignore the contribution to $\nabla_{\parallel} p_e$ from the parallel flows, keeping only the part from perpendicular slippage. We find the driving force is

$$F'_{\parallel e} = ne\left(\frac{\omega^*}{\omega} - 1\right) E_{\parallel} \quad . \tag{99}$$

In the collisionless case this force is balanced by inertia, and in the semi-collisional case, by friction. In the region outside the electron current layer, where $k_{\parallel}$ is larger, the parallel gradients are relaxed by free-streaming in the collisionless case and by parallel diffusion in the semi-collisional case.

The frictional force $R_{\parallel}$ is $m_e n \nu_e \tilde{v}_{\parallel}$ and the parallel pressure gradient due to bunching of the electrons along the field lines is $nT_e k_{\parallel}^2 v \hat{h}_{\parallel}/\omega$. When ∇T_e = 0, the parallel equation of motion for the electron is approximately,

$$m_e(\omega + i\nu_e)\tilde{v}_{\parallel} = \frac{eE_{\parallel}}{i}\left(1 - \frac{\omega^*}{\omega}\right) + \frac{k_{\parallel}^2 T_e}{\omega}\tilde{v}_{\parallel} \quad . \tag{100}$$

The left-hand side of Eq. (100) is the inertial and drag forces, and the second term on the right-hand side is the bunching term. The width of the mode is set by where the bunching is comparable to the left-hand side of Eq. (100). For the collisionless mode where $\nu_e \ll \omega^* \sim \omega$ the width of the mode is

$$\delta \approx \frac{\omega \ell_s}{v_{the} k} = \left(\frac{\omega}{\omega^*}\right) \frac{\ell_s}{a} \rho e \tag{101}$$

and for the semi-collisionless case where $\nu_e >> \omega^* \sim \omega$ the width s is,

$$\Delta \simeq \left(\frac{\nu_e}{\omega}\right)^{1/2} \delta \quad . \tag{102}$$

The jump in the slope of $A_\parallel$, $\Delta' A_\parallel$, over the inner layer is the total current contained in the layer. The order of magnitude of the total current I_{total} is

$$I_{total} = e\tilde{v}_\parallel \Delta \simeq \beta_p \left(\frac{\omega}{\omega^*}\right)^2 \left(1 - \frac{\omega^*}{\omega}\right) \frac{A_\parallel}{\Delta} \quad . \tag{103}$$

Since the $I_{total} >> \Delta' A_\parallel$ in most cases, it is clear the ω must be chosen so that, to lowest order, I_{total} equals zero.

Physically, we now see Eq. (100) as the balancing of the total current generated by the pressure gradients against the current generated directly by the parallel electric field. It is of some interest to set $\nabla T_e = 0$, and ask what happens. We see from Eq. (99) that $\omega = \omega^*$ sets the driving force for the parallel current to zero and therefore $J_\parallel = 0$ everywhere. Furthermore,

$$\frac{n_e}{n_o} = \frac{e\phi}{T} \quad . \tag{104}$$

The electron response is Boltzmann. In this case, the corrections due to ion magnetization are irrelevant since they change $E_\parallel$ while $J_\parallel$ is zero, because $J_\parallel \propto (1 - \omega^*/\omega) E_\parallel$. When $\nabla T_e \neq 0$, the situation is different because of the different dependence on x of the temperature and d nsity gradient driven terms. It is not possible to choose ω so that $J_\parallel = 0$ at every x, but it is possible to choose it so that $\int dx J_\parallel = 0$. The current in some regions of x space is driven by parallel density and temperature gradients, and in some regions directly by $E_\parallel$.

The cancellation depends in this case on the functional dependence of $E_\parallel(x)$ on x. Changing $E_\parallel$ by including ion magnetization will destroy the cancellation and necessitate a new choice of ω to restore the cancellation. We have shown in Secs. III and IV that when $\nabla \ell n\ T_e$ is of order $\nabla \ell n n$, then the mode is stabilized by these magnetization corrections. The stabilization arises from the difference in the current predicted assuming 'unmagnetized' ions and the current predicted assuming 'magnetized' ions (Fig. 2). As we have seen, this difference is large in the outer region where $x \sim \rho_i >> \delta, \Delta$. In the outer region, the quasineutrality condition can be written [see Eq. (29)]

$$\frac{1}{4\pi} \frac{c^2}{V_A^2} \frac{\partial^2 \phi}{\partial x^2} = \frac{-k_\parallel J_\parallel}{\omega} \quad , \tag{105}$$

where we have ignored ω^* terms. The right-hand side is the polarization currents of the ions. Note that the $\underset{\sim}{E} \times \underset{\sim}{B}$ currents cancel. In this region, $E_\parallel \to 0$ so we expect $\phi \sim (\omega A_z \ell_s/ck) 1/x$. Hence, $J_\parallel$ can be estimated as

$$J_{\parallel \mathrm{magnetized}} = \frac{-2c\omega^2 \ell_s^2}{4\pi V_A^2 k^2} \frac{A_z}{x^4} \ . \tag{106}$$

The current in the outer region given by the 'unmagnetized' ions is, in the collisionless case, obtained from Eq. (17),

$$J_{\parallel \mathrm{unmagnetized}} \simeq \frac{2c\omega^2 \ell_s^2}{4\pi V_A^2 k^2} \frac{A_z}{\rho_i^2 x^2} \left(\frac{\eta_e}{2} \frac{\omega^*}{\omega}\right) \ . \tag{107}$$

Hence, the correction to the current in the magnetized case is

$$\frac{4\pi}{c} \int \Delta J_\parallel dx \simeq \frac{-4\ell_x^2 \omega^2 A_z}{V_z^2 k^2 \rho_i^2} \int_{\alpha\rho_i}^{\infty} dx \left(\frac{\rho_i^2}{x^4} + Y \frac{1}{x^2}\right) \tag{108}$$

$Y = (n_e\omega^*/2\omega)$. The cutoff introduced is $\alpha\rho_i$ where α is of order one. It ensures that we include the contribution in the outer region only. Using Eq. (8), Ampere's law, we find the dispersion relation becomes

$$\frac{1}{a}\left(\Delta' a - \frac{4}{3}\frac{\ell_x^2\omega^2 a}{V_A^2 k^2 \rho_i^3}\right)A_z = -\frac{4\pi}{c}\int_{-\infty}^{\infty} dx\, J_{\parallel\ \mathrm{unmagnetized}} . \tag{109}$$

Note that the stabilizing term is of order $\beta_p\ a/\rho_i$, which is generally large. In the semi-collisional case, the magnetized response in the outer region is still given by Eq. (105) as $E_\parallel \to 0$, and Eq. (106) holds in this region. The unmagnetized response is very similar to Eq. (107). We obtain the response from Eq. (74)

$$J_{\parallel\ \mathrm{unmagnetized}} = \frac{c}{4\pi}\frac{\omega^2\ell_s^2}{V_A^2 k^2}\frac{A_z}{\rho_i^2 x^2} . \tag{110}$$

The dispersion relation is given by

$$\frac{1}{a}\left(\Delta' a - \frac{\ell_s^2\omega^2 a}{V_A^2 k^2 \rho_1^3}\right) = -\frac{4\pi}{c}\int_{-\infty}^{\infty} dx\, J_\parallel(x)_{\ \mathrm{unmagnetized}} , \tag{111}$$

almost exactly the same as Eq. (109). The numerical factors are not accurate, so Eq. (109) and (111) are only estimates. Also, we have not accounted for the partially magnetized region which produces the logarithmic factors in Eq. (60) and Eq. (94). Equations (110) and (111) indicate how the corrections due to magnetization tend to overcome Δ' which otherwise would drive the mode unstable.

A primitive picture of the mode is as follows: $E_\parallel$ is applied to the plasma and the electrons respond with a parallel current. In the inner region the ions move unimpeded (on the scale of this region) across the field lines to provide charge neutralization. In the outer region the ions are constrained to move (on the scale of the region) with their polarization drift across the field lines. To ensure neutrality in the outer region, $E_\parallel$ must go to zero; otherwise the electron parallel current builds up charge that the ions are unable to neutralize by perpendicular motion. The total parallel current in the mode must be zero, which makes $\omega \sim O(\omega^*)$. The ion inertia plays the stabilizing role, and 'magnetized' ions are stuck to field lines and are 'shaken' at a frequency ω. The Δ' term is not large enough to drive the 'shaking' of the ions and the mode is stable.

VI. ON THE PRESENCE OF TEARING MODES IN TOKAMAKS

We pointed out in Sec. IV of this paper that we expect tearing modes to be stable in most discharges of present-day large tokamaks. However, large islands are apparently observed in discharges, particularly by Mirnov coil and soft X-ray diagnostics. Our contention, briefly stated in Sec. IV, is that the linear theory presented there is not applicable to these experiments because the rational surfaces of these modes are liable to be broken up with either a stochastic region or an island bigger than the current layer of the linear mode considered. The transition to the nonlinear regime of the tearing mode takes place when the island size is the size of the current layer. At this point the electrons which carry most of the parallel current are flowing around the island. The effective parallel wavelength inside the island is the length along the field line to orbit the island. This length is approximately ℓ_s/kW, where W is the width of the separatrix. The width of the current layer is defined in the collisional case, Eq. (10), to be where the parallel wavelength is equal to the parallel distance travelled by an electron with velocity v_{the} in one period of the wave (approximately ω^{*-1}). In the semi-collisional case, Eq. (11), the width is defined to be where the parallel wavelength is equal to the parallel distance diffused by an electron with velocity V_{the} in one period of the wave. If $W \gg \Delta$ or δ, then the electrons make many orbits of the island in one period of the wave. We, therefore, expect the electrons to have a Boltzmann type of response. Furthermore, we expect a strong flattening of the electron temperature around the island. We have not investigated the regime $\rho_i > W > \Delta$, δ because we expect to find the rational surfaces broken up to the extent that $W \gg \rho_i$ so that the ions are fully magnetized. We do not wish to go into the details of the nonlinear growth, and we provide only simple demonstration of growth from an equilibrium island using resistive MHD. Following Rutherford[10] we consider the development of the mode to be the evolution of islands though a series of equilibriums. Let

$$\Psi = \Psi_o(\underset{\sim}{r}) = \Psi_p(\underset{\sim}{r}) + \Psi_H(\underset{\sim}{r}, t) \tag{112}$$

in the outer region where the island is treated as a perturbation. $\Psi_p(r)$ is the particular solution for the equilibrium with perturbed boundaries[12] and $\Psi_H(\underset{\sim}{r},t)$ is the homogeneous solution. For equilibrium,

$$\left.\frac{\partial^2 \Psi_p}{\partial x^2}\right)_{x=0} = 0 \quad , \tag{113}$$

where x = 0 is the rational surface. $\partial\Psi_H/\partial\underset{\sim}{x}$ has the usual discontinuity

$$\Delta'\Psi_H = \int j_z dx \ . \tag{114}$$

The nonlinear behavior of the island is

$$\frac{\Delta' \Psi_H}{(\Psi_p + \Psi_H)^{1/2}} = \frac{16\pi A}{\eta(2B_o/\ell_s)^{1/2}} \frac{\partial \Psi_H}{\partial t} \quad , \tag{115}$$

where A = 1.[11] Initially, if $\Psi_H << \Psi_p$,

$$\Psi_H \sim \Psi_{Ho} \exp\left[- \eta\left(\frac{2B_o}{\ell_s \Psi_p}\right)^{1/2} \frac{1}{16\pi A} \int_o^t \Delta'(t)\, dt'\right] \tag{116}$$

when $\Psi_H >> \Psi_p$

$$\Psi_H^{1/2} \simeq \eta\left(\frac{\eta B_o}{\ell_s}\right)^{1/2} \int_o^t \Delta'(t)\, dt' \quad . \tag{117}$$

Under these conditions the mode grows and the linear stability plays no role. We have attempted to show that the linear stability of the tearing mode does not preclude the presence of islands in tokamaks since small departures from axisymmetry can break up the surfaces enough to pass into the nonlinear regime.

VII. CONCLUSION

We have shown that the collisionless and semi-collisional tearing modes are strongly stabilized by the magnetization of the ions, Eqs. (59) and (60) for the collisionless case and Eqs. (89) and (94) for the semi-collisional. In both cases we have obtained a solution by asymptotic expansion in the current layer width over the ion Larmor radius. The validity condition for this expansion for the collisionless case ($\omega^* > \nu_{ei}$) is,

$$\frac{\delta}{\rho_i} = \left(\frac{m_e}{m_i}\right)^{1/2} \frac{1}{\varepsilon} << 1 \quad , \tag{118}$$

where ε is the aspect ratio. In present tokamak experiments, $\omega^* < \nu_{ei}$ for the large scale tearing modes, and the semi-collisional regime is appropriate. For this expansion to be valid,

$$\frac{\Delta}{\rho_i} \simeq m^{-1/2} \frac{n^{1/2}}{T^{5/4}} B^{1/2} \left(\frac{\ell_s}{500}\right) << 1 , \qquad (119)$$

where m is the poloidal mode number, ℓ_s the shear length, n the density in units of 10^{14} cm^{-3}, T the temperature in keV, and B the magnetic field in units of 50 kG. The inequality in Eq. (119) is satisfied by most present-day tokamak experiments. The strong temperature dependence of Eq. (119) indicates that future higher temperature experiments may remain in this regime.

It is instructive to compare pictorially the physics of the 'unmagnetized' ion mode[2] and the magnetized ion, $T_i = 0$, mode[4] with the mode of full ion dynamics treated here in the semi-collisional regime. In Fig. 3 we plot $E_\parallel$ for the three cases, and in Fig. 4 we plot the real part of the conductivity $\sigma(x)$ where $J_\parallel = \sigma(x)\ E_\parallel$. Clearly, there are no qualitative differences in the parallel currents since for $x < \delta$, $E_\parallel$ is approximately the same for all cases. However, quantitatively the modes differ, and it is necessary to do the full analysis to obtain the growth or damping rates.

Our analysis correctly examines the regime of interest in present-day experiments and concludes that tearing modes are linearly stable. In Sec. VI we have pointed out that islands may grow in the plasma from an initial width greater than ρ_i. Breakup of low m and n rational surfaces large enough to make island widths of order ρ_i can be achieved by nonaxisymmetry of the coils, or toroidal coupling to unstable modes such as the internal kink.

The inclusion of curvature and toroidal coupling is outside the scope of this work. However, there is evidence[14] that these effects can increase stability in an average, good curvature device. It is possible that toroidal effects could destabilize the linear mode (this is the case for drfit waves[17]), but studies with $T_i = 0$[15,18] indicate otherwise. Since the major equilibrium rational surfaces are expected to be broken up, one might argue that for the large-scale tearing modes the nonlinear calculations[12,16] are more relevant to experiments. Some more analysis is necessary to evaluate the impact of high β_p on our results.

In Sec. V we have outlined the dominant physical processes. We show how the ω^* effects arise from slippage of the field lines through the electrons. We also have shown how the stabilization of the mode arises from the ion inertia manifested as polarization current in the outer region.

The technique used to solve the coupled integral equations, Eqs. (32) and (33), is convenient since we arrive at a dispersion relation, Eq. (44), without solving any differential equations. The same technique may be applicable to a number of problems where two scales are present but where a nonlocal response links the scales. This applies especially, of course, to plasma modes where the ion Larmor radius is larger than a typical length.

The nonlinear dynamics of these modes have recently received some attention.[12,16] We touched only briefly on the physical aspects in Sec. VI.

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APPENDIX A: ZEROTH ORDER DISPERSION RELATION FOR THE COLLISIONLESS MODE

The dominant current in the collisionless mode, corresponding to using the unmagnetized ion response obtained from Eqs. (27), (45), and (46) is,

$$I(\omega) = 2\left(\frac{\omega^*}{\omega} + \frac{T_e}{T_i}\right) \int_o^\infty \frac{ds\ \chi(s)}{[(\omega^*/\omega)+(T_e/T_i)]-\chi(s)} = \left(\frac{\omega^*}{\omega} + \frac{T_e}{T_i}\right) J(\omega) \quad , \tag{A-1}$$

where $s = 1/t$. We have used half of the s plane to avoid the absolute value signs and,

$$\chi(s) = \frac{1}{2}\left[z'(s)\left(1 - \frac{\omega^*}{\omega}\right) + \frac{\omega^*}{\omega}\frac{\eta_e}{2} sz''(s)\right] . \tag{A-2}$$

By expanding the denominator in the integral $I(\omega)$, we can verify that taking the first term to be dominant gives a good approximation. However, we also want to prove that $J(\omega)$ has no roots in the upper half of the plane. Writing the real part $\chi(s)$ as $p(s)$ and the imaginary part as $q(s)$, we note that for ω real

$$p(s) = p(-s) \quad , \tag{A-3}$$

$$q(s) = q(-s) \quad .$$

We note also that for ω real, the real part of the integral is symmetric in s and the imaginary part is antisymmetric. Hence,

$$\mathrm{Real}(J) = \frac{1}{2}\int_{-\infty}^{\infty} ds \frac{\chi(s)}{[(\omega^*/\omega)+(T_e/T_i)]-\chi(s)} \quad . \tag{A-4}$$

[Note that now we are using the analytic function $\chi(s)$ not $\chi(|s|)$.]

The integral (A-1) is evaluated by analytically continuing from $I(\omega)$ from $\omega \approx \infty$ so that the poles, zeros of $[(\omega^*/\omega + (T_e/T_i)] - \chi(s)$, move from the lower-half s plane to the upper-half plane. Since the poles must be symmetric about the imaginary s axis for ω real and the value of the integrand is the same for each pole, (A-4) can be taken along a contour passing above the poles in the upper-half plane. Since the integrand vanishes at infinity in the upper-half plane, we find

$$\text{Real } J(\omega) = 0 \quad . \tag{A-5}$$

Now consider the behavior of the function $J(\omega)$ for complex ω. We find that $J(\omega)$ has only one pole at $\omega = -\omega^* T_i/T_e$. For $|\omega| \to \infty$ we find

$$iJ(\omega) = i \int_0^\infty ds \frac{z'(s)}{[T_e/T_i]-Z'(s)} = C \ , \tag{A-6}$$

where C is real and greater than zero. To determine whether $J(\omega)$ has zeros in the upper half of the ω plane, we use a Nyquist technique. If $J(\omega)$ has n zeros and one pole on the real line, the number of zeros Q of $J(\omega)$ in the upper-half plane is

$$\frac{n-1}{2} = -Q \ . \tag{A-7}$$

We know $n \geq 1$ because there is at least one root near $\omega = \omega^*[1 + (n_e/2)]$; hence Q = 0 and n = 1. There is, therefore, one marginally stable root of $J(\omega) = 0$, and no unstable roots. We note from Eq. (A-1) that another root $\omega = -\omega^*(T_i/T_e)$ is present in the problem since we look for $[(\omega^*/\omega + T_e/T_i)]$ $J(\omega)$ to vanish. This root has been examined by Crew et al.[3] It is not treated correctly by our expansion. Note that for $\omega = [-\omega^*(T_e/T_i)]$, $E_\parallel$ is zero in the inner region.

We can show simply that the β_p correction to the collisionless mode does not affect the stability by noting that the function,

$$\left(\int_o^s \frac{ds'\chi(s')}{[(\omega^*/\omega)+(T_e/T_i)]-\chi(s')}\right)^2 = K(s,\omega) \tag{A-8}$$

has a real part symmetric in s and an imaginary part antisymmetric in s for ω real. Using the same argument to show that $J(\omega)$ is imaginary for real ω, we may show for real ω that

$$\mathrm{Re} \int_o^\infty \frac{ds}{s^2} K(s,\omega) = 0 \ . \tag{A-9}$$

The correction to the real frequency due to finite β_p, $\delta\omega_{\beta p}$ is given by

$$\left[\left(\frac{\omega_o}{\omega^*}\right) \beta_p^2 \int_o^\infty \frac{ds}{s^2} K(s,\omega)\right] \times \left(\frac{dI}{d\omega}\bigg|_{\omega_o}\right)^{-1} = -\delta\omega_{\beta p} \quad . \tag{A-10}$$

APPENDIX B

FOURIER TRANSFORMS OF z FUNCTIONS

Here we obtain expressions for Q(q) for the collisionless modes. From Eqs. (27), (45), and (48), we have

$$\theta(t) \approx \frac{1}{2[H(\infty)-\sigma(\infty)]}\,\frac{1}{t}\left[\left(1-\frac{\omega^*}{\omega}\right) z'\left(\frac{1}{|t|}\right)+\frac{\omega^*}{\omega}\,\frac{\eta_e}{2}\,z''\left(\frac{1}{|t|}\right)\right] \quad . \tag{B-1}$$

Consider

$$Q(q) = -\int_{-\infty}^{\infty} dt\,\frac{1}{t}\exp(iqt)\,\frac{1}{|t|}\,Z\left(\frac{1}{|t|}\right) \tag{B-2}$$

$$= \frac{1}{\pi^{1/2}}\int_{-\infty}^{\infty}\frac{dt}{t}\exp(-iqt)\int_{-\infty}^{\infty}\frac{du}{u}\,\frac{\exp(-u^2)}{(|t|-1/u)} \quad .$$

Let H(x) be the heavyside step function,[13] so that H(x) = 1 for x > 0 and H(x) = 0 for x < 0. We can write

$$\int_{-\infty}^{\infty}\frac{du}{u}\,\frac{\exp(-u)^2}{[|t|-(1/u)]} = H(t)\int_L\frac{du}{u}\,\frac{\exp(-u^2)}{[t-(1/u)]} + H(-t)\int_{AL}\frac{du}{u}\,\frac{\exp(-u^2)}{[t-(1/u)]} \ . \tag{B-3}$$

L denotes the Landau contour, where u passes under the pole at 1/t and AL denotes the contour where u passes over the pole at 1/t. We replace the two contours by one c, which is just above the real u axis for real u < 0 and just below the real u axis for real u < 0. Q(q) can now be written

$$Q(q) = \frac{1}{\pi^{1/2}}\int_{-\infty}^{\infty}\frac{dt}{t}\exp(iqt)\int_c\frac{du}{u}\,\frac{\exp(-u^2)}{[t-(1/u)]} \tag{B-4}$$

$$= -\int_{-\infty}^{\infty}\frac{dt}{t}\exp(iqt) + \frac{1}{\pi^{1/2}}\int_{-\infty}^{\infty} dt\,\exp(-iqt)\int\frac{du\,\exp(-u^2)}{[t-(1/u)]}$$

$$= + i\pi \text{ sign } q - i2\pi^{1/2}\int_c du \exp[(-u^2)-(\frac{iq}{u})][H(-u)H(q) - H(u)H(-q)].$$

We only need the transforms for $q > 0$. After some algebra we find

$$\theta(q) \simeq \frac{-i}{H(\infty) - \delta(\infty)\pi^{1/2}}\int_{-\infty}^{\infty} du \exp[(-u^2)+(\frac{iq}{u})][(1 - \frac{\omega^*}{\omega}) - \frac{\eta_e}{2}\frac{\omega^*}{\omega}(2u^2 - 1)] \quad . \tag{B-5}$$

APPENDIX C: INTEGRALS FOR THE COLLISIONLESS MODE

Here we give the integrals defined by Eq. (57). Note that by integrating by parts

$$W_1 = -\alpha^2 \frac{d}{dx} \frac{W_o}{\alpha} \qquad \text{(C-1)}$$

and

$$W_3 = -\alpha^2 \frac{d}{d\alpha} \frac{W_2}{\alpha} . \qquad \text{(C-2)}$$

Using [8] $I_1(x) = dI_o/dx$, we find

$$W_2 = W_o - W_1 \quad . \qquad \text{(C-3)}$$

We first evaluate W_o and then use Eqs. (C-3), (C-2), and (C-1) to construct W_1, W_2, and W_3. Making the transformations

$$\frac{u}{q} = \frac{k}{\sqrt{2}\varepsilon} \qquad \text{(C-4)}$$

and

$$\frac{q^2}{2\varepsilon^2} = \alpha \quad , \qquad \text{(C-5)}$$

we find W_o is given by

$$W_o = \text{Re}\, \frac{\varepsilon}{\sqrt{2}}\, 2\pi^{1/2} \int_o^{\pi} d\theta \int_o^{\infty} dk\, \exp\left(\frac{-i\sqrt{2}\varepsilon}{k}\right) \int_o^{\infty} d\alpha\, \exp\left[-(k^2+1+\cos\theta)\alpha\right] , \qquad \text{(C-6)}$$

where we have used the integral form of I_o[10]

$$I_o(x) = \frac{1}{\pi}\int_0^{\pi} d\theta \, \exp(-x\cos\theta) \quad . \tag{C-7}$$

Noting that the k integral can be extended to negative infinity we perform the α integral and substitute z = 1/k

$$W_o = \text{Re}\,\frac{\varepsilon\sqrt{2}}{\pi^{1/2}}\int_0^{\pi} d\theta \int_{-\infty}^{\infty} \frac{dz}{2}\,\frac{\exp(-\sqrt{2}\varepsilon z)}{1+z^2(1+\cos\theta)} \quad . \tag{C-8}$$

The z integral is evaluated by closing the z contour with a semicircular contour at infinity in the lower half z plane and by using Cauchy's theorem. Setting θ = 2y,

$$W_o = \varepsilon\pi^{1/2}\int_0^{\pi/2} dy\,\frac{\exp(-\varepsilon/\cos y)}{\cos y} \quad , \tag{C-9}$$

which makes the transformation $(\cos y)^{-1} = \cosh(j)$,

$$W_o = \varepsilon\pi^{1/2}\int_0^{\infty} dj\,\exp[-\varepsilon\cosh(j)] = \varepsilon\pi^{1/2}\,K_o(\varepsilon) \quad , \tag{C-10}$$

where K_o is the modified Bessel function.[10]

APPENDIX D. INTEGRAL FOR SEMI-COLLISIONAL MODE

Here we evaluate the integral M of Eq. (91)

$$M(\alpha) = \int_0^\infty dq\ I_o(q^2)\ \exp - [q^2 + \varepsilon(1-i)q] \quad . \tag{D-1}$$

Using the integral form of I_o, Eq. (C-6), where we write $\varepsilon = \alpha/|\Delta|/\rho_i$,

$$M = \frac{1}{\pi} \int_0^\infty dq \int_0^\pi d\theta\ \exp[-q^2(1 + \cos\theta) - q\ \varepsilon(1 - i)] \quad . \tag{D-2}$$

Let $s = q \cos \theta/2$ and $x = (\cos \theta/2)^{-1}$

$$M = \frac{1}{2\pi} \int_0^\infty \frac{dx\ \exp -i(\varepsilon^2 x^2/2)}{(x^2-1)} \int_0^\infty ds\ \exp -[s + \frac{\varepsilon x}{2}(1 - i)]^2 \tag{D-3}$$

$$\omega = \frac{s + (\varepsilon x/2)(1 - i)}{x\varepsilon} \quad . \tag{D-4}$$

Then,

$$M = \frac{\varepsilon}{2\pi} \int_{(1-i)/2}^\infty d\omega \int_1^\infty \frac{dx\ x}{(x^2-1)^{1/2}} \exp - (i\varepsilon^2 + \omega^2)x^2 \quad . \tag{D-5}$$

Let $x = \cosh \chi$, so that the the χ integral can be done yielding

$$M = \frac{\pi^{-1/2}}{2} \int_{(1-i)/2}^\infty d\omega \frac{\exp-(i/2+\omega^2)}{(i/2+\omega^2)} \varepsilon^2 \quad . \tag{D-6}$$

Let $\omega = (1-i)/2 \sin h\lambda$. Identifying K_o (Appendix C), we find

$$M = \frac{\exp[-(i\varepsilon^2/2)]}{4\pi^{1/2}} K_o(\frac{\varepsilon^2}{4}) \quad . \tag{D-7}$$

For α small we obtain[10]

$$M \approx \frac{1}{2}\,\pi^{1/2}\,\ln\frac{\alpha}{2} \qquad . \qquad \text{(D-8)}$$

APPENDIX E. EVALUATION OF THE REAL FREQUENCY FOR THE SEMI-COLLISIONAL MODE

We wish to determine the dominant frequency given by $G(\omega_0) = 0$, Eq. (86), where

$$G(\omega) = \frac{-[(T_e/T_i)(\omega^*/\omega)]}{2.13[(T_e/T_i)+1]} \int_{-\infty\sqrt{i}}^{\infty\sqrt{i}} dt \frac{A+Bt^2}{(t^2-\beta_+)(t^2-\beta_-)} . \tag{E-1}$$

The integral over t is defined for $|\omega| \to \infty$ where β_+ and β_- are real and negative. For ω finite we analytically continue

$$G(\omega) = -2\pi i \frac{[(T_e/T_i)+(\omega^*/\omega)]}{2.13[(T_e/T_i)+1](\beta_+\beta_-)^{1/2}[(\beta_+)^{1/2}+(\beta_-)^{1/2}]} [A + B(\beta_-\beta_+)^{1/2}]. \tag{E-2}$$

The branches of the square roots must be found by noting that for $|\omega| \to \infty$, $(\beta_-)^{1/2}$ and $(\beta_+)^{1/2}$ are on the positive imaginary axis. From Eq. (88)

$$\beta_-\beta_+ = \frac{[(T_e/T_i)+(\omega^*/\omega)]}{2.13[(T_e+T_i)+1]} , \tag{E-3}$$

$$G(\omega) = -2\pi \left[\frac{[(T_e/T_i)+(\omega^*/\omega)]}{2.13[(T_e/T_i)+1]}\right]\left[\frac{1}{(\beta_+)^{1/2}+(\beta_-)^{1/2}}\right]\{A + B[\frac{[(T_e/T_i)+(\omega^*/\omega)]}{2.13[(T_e/T_i)+1]}]\} . \tag{E-4}$$

Since at $\omega = -\omega^*(T_i/T_i)$ our expansion breaks down, this does not represent a true root. The true roots are then determined by,

$$0 = \frac{1}{\hat{\omega}}[\hat{\omega} - (1+1.71\eta_e) + (2.13)^{1/2}[(\frac{T_3}{T_i})+1]^{-1/2}(\hat{\omega}-1)(\frac{T_e}{T_i}) + (\frac{1}{\hat{\omega}})^{1/2}] \tag{E-5}$$

where $\hat{\omega} = \omega/\omega^*$. By a Nyquist technique, one may verify that Eq. (E-6) has one root on the real line at $\omega \approx \omega^* (1 + 0.4\eta_e)$ and no roots in the upper-half ω plane. (We ignore $\hat{\omega} = \infty$ root.) So the dominant frequency is real.

To evaluate the effect of the β_p term, we note that $\beta_\pm$ for $\omega = \omega_o$ is real and less than zero. We can rotate the t contour onto the real axis as the poles are on the imaginary axis.

The β_p term is manifestly real, and because $dG/d\omega$ and ω_o are real, the β_p correction is real.

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FIGURE CAPTIONS

FIG. 1. The correction to the unmagnetized response in the region $x \lesssim \delta$. The logarithmic singularity gives corrections of order $\delta/\rho_i \ln(\delta/\rho_i)$. In the outer regions $x \gtrsim \rho_i$. The long tail of the Green's function picks up large contributions from the inside regions to violate the inequality in Eq. (30).

FIG. 2. Current J versus x showing the effect of magnetization.

FIG. 3. $E_\parallel$ versus χ for semi-collisional tearing modes with different ion responses: (A) Full ion dynamics (this paper). (B) Unmagnetized ions,[2] and (C) magnetized $T_i = 0$ ions.[5]

FIG. 4. Conductivity $\sigma(x)$ versus x for the semi-collisional mode.

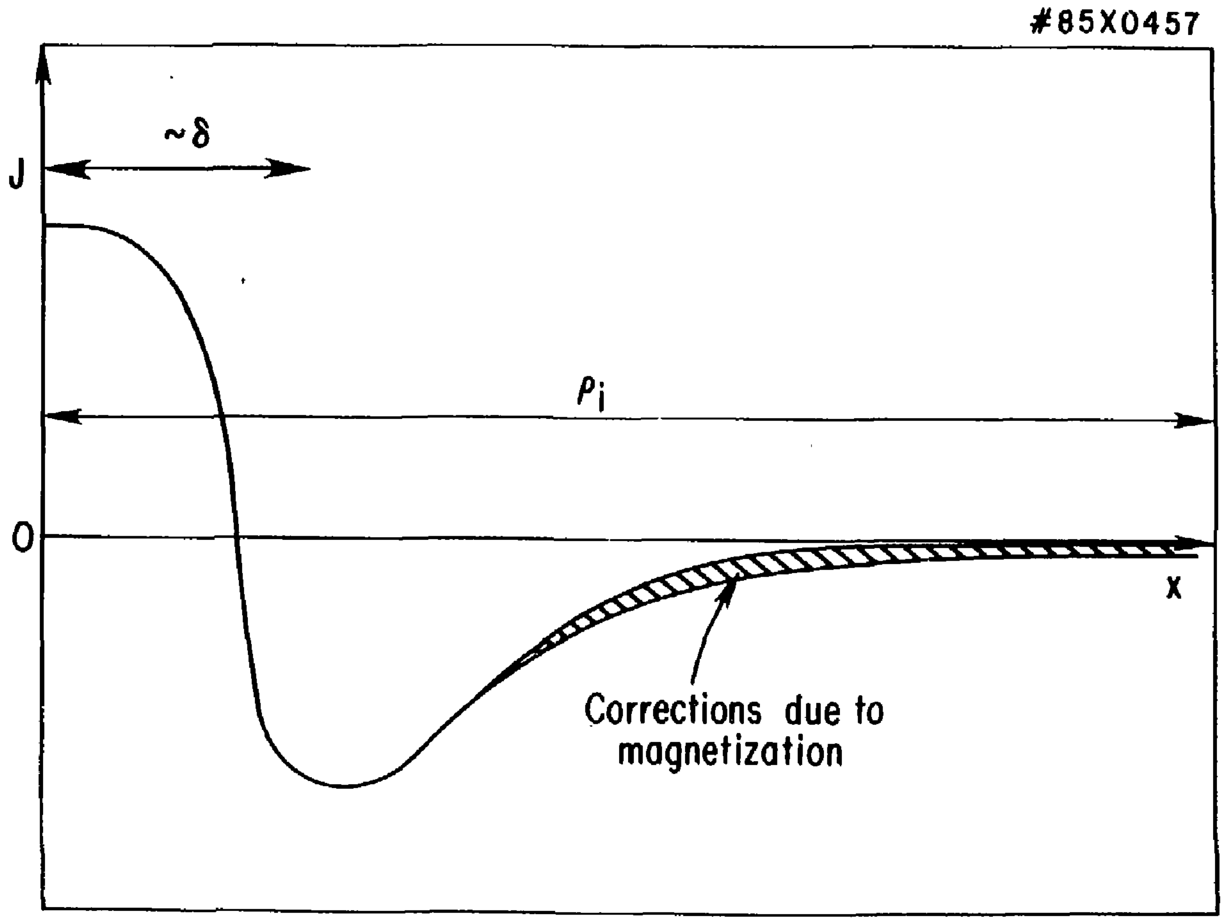

Fig. 1

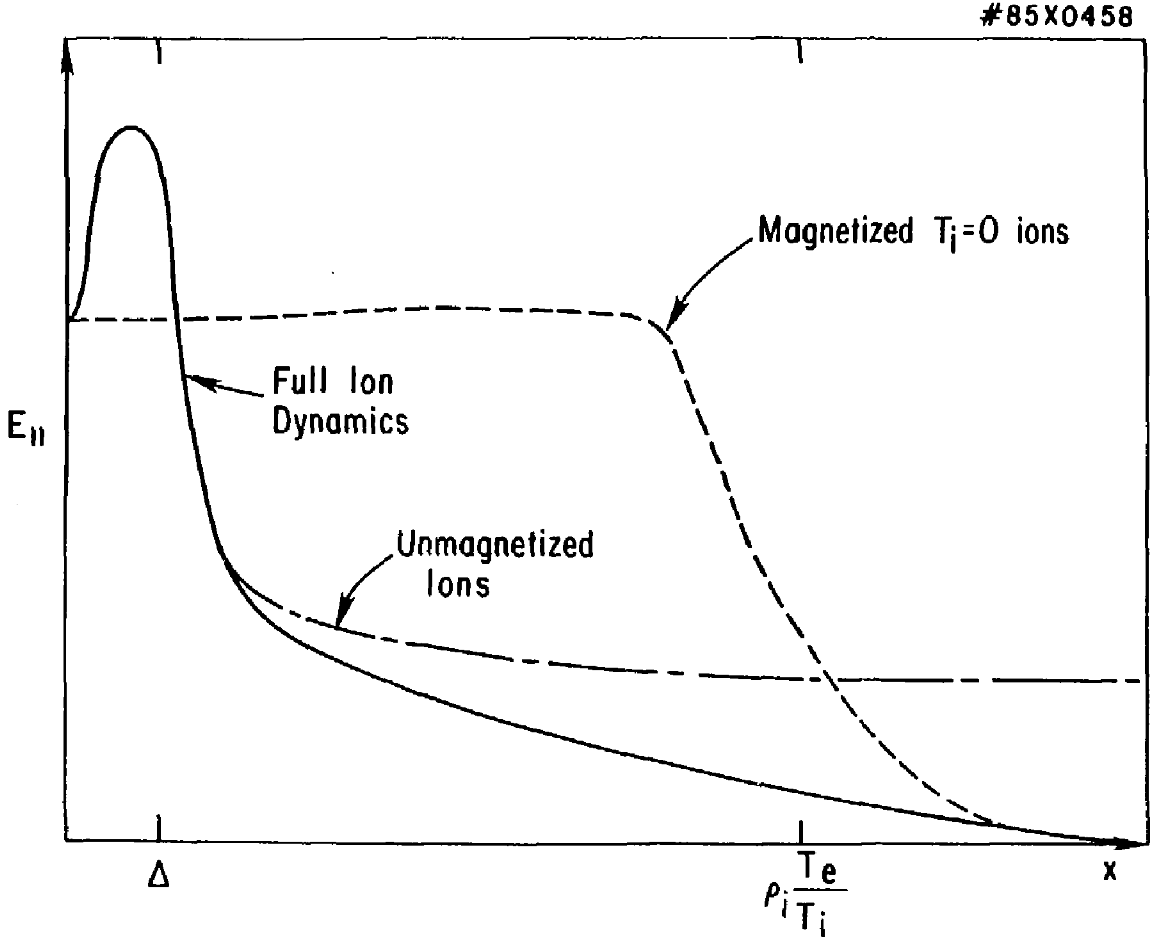
#85X0458
Magnetized $T_i = 0$ ions
Full Ion
Dynamics
$E_{\parallel}$
Unmagnetized
Ions
Δ
$\rho_i \frac{T_e}{T_i}$
x

Fig. 2

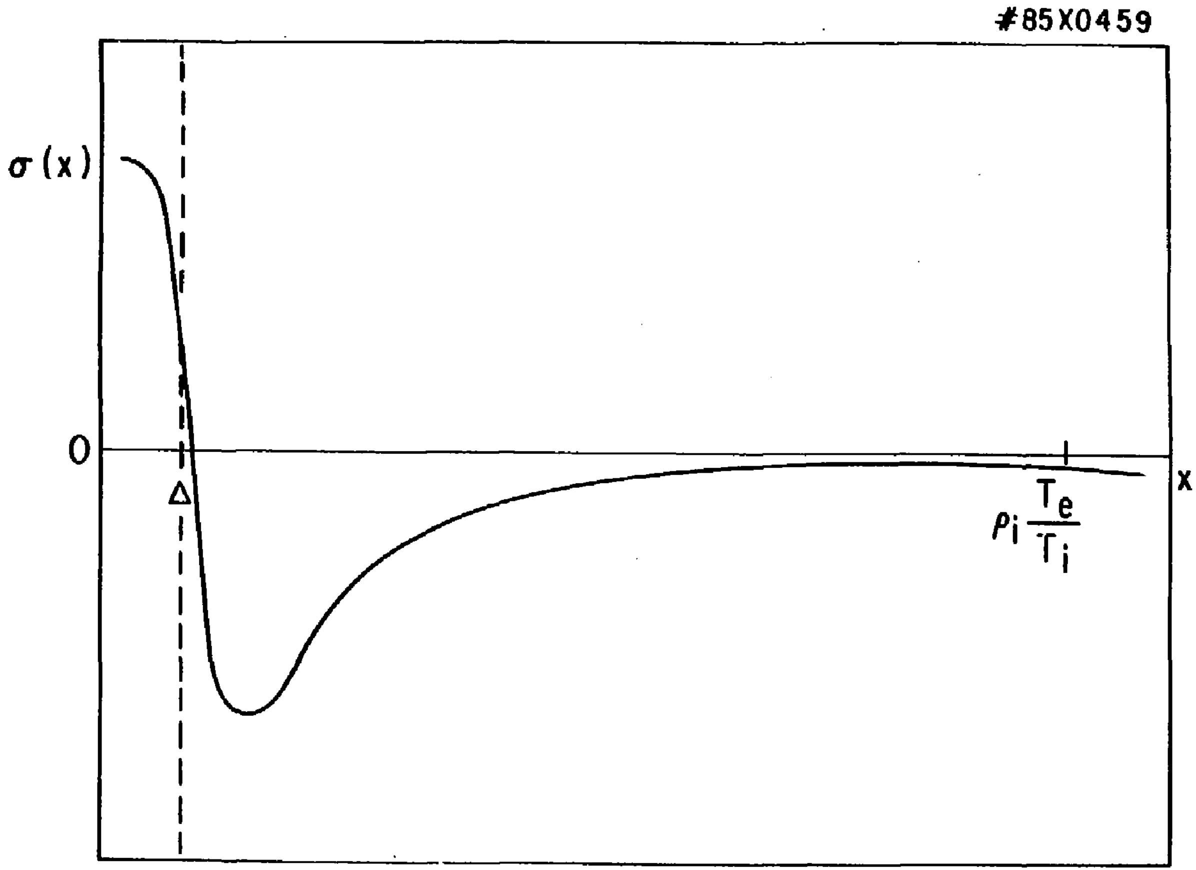

Fig. 3

#85X0460

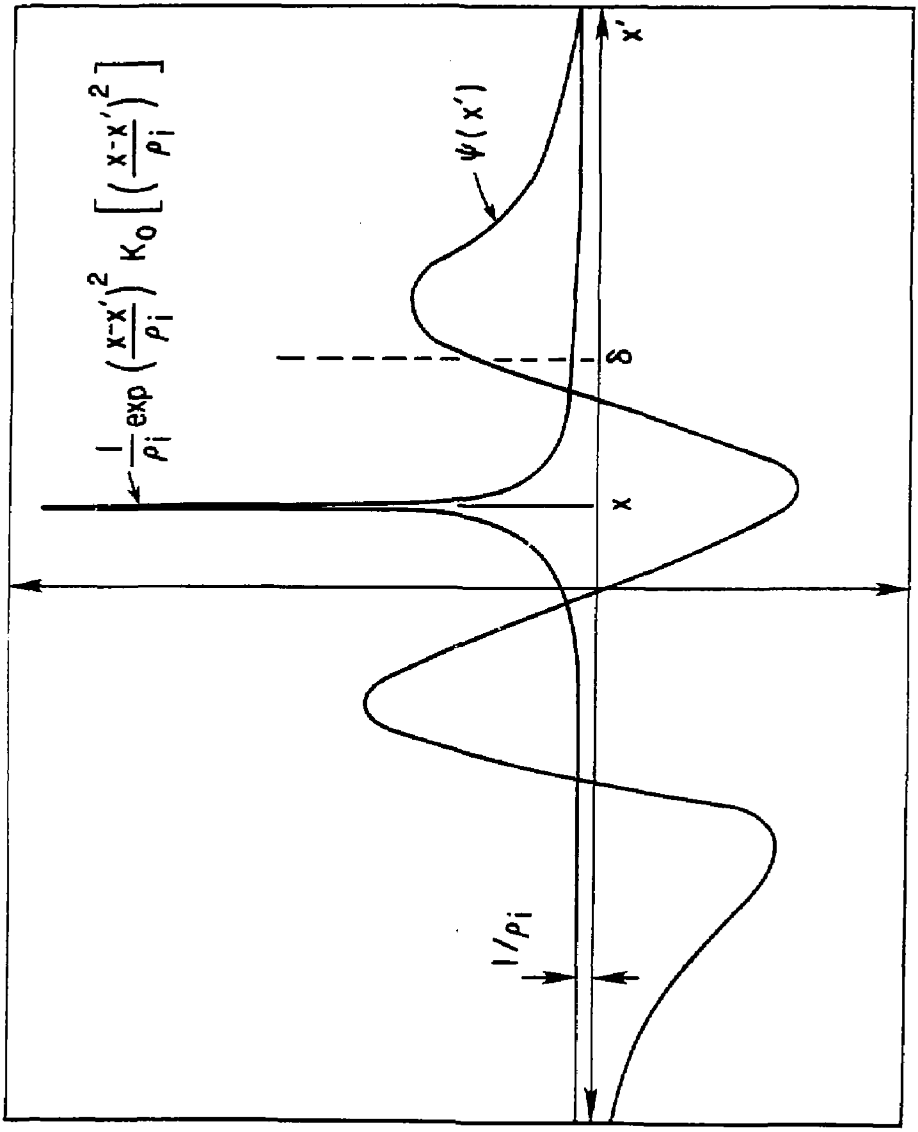

Fig. 4

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