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HELICITY CONTENT AND TOKAMAK APPLICATIONS OF HELICITY

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ABSTRACT. Magnetic helicity is approximately conserved by the turbulence associated with resistive instabilities of plasmas. To generalize the application of the concept of helicity, the helicity content of an arbitrary bounded region of space will be defined. The definition has the virtues that both the helicity content and its time derivative have simple expressions in terms of the poloidal and toroidal magnetic fluxes, the average toroidal loop voltage and the electric potential on the bounding surface, and the volume integral of **E·B**. The application of the helicity concept to tokamak plasmas is illustrated by a discussion of so-called MHD current drive, an example of a stable tokamak q profile with q less than one in the center, and a discussion of the possibility of a natural steady-state tokamak due to the bootstrap current coupling to tearing instabilities.

I. INTRODUCTION

The helicity of a magnetic field is defined as $K \equiv \int \mathbf{A}\cdot\mathbf{B}d^3x$, with $\mathbf{A}$ the vector potential of the magnetic field, $\mathbf{B}=\nabla\times\mathbf{A}$. The concept of helicity was introduced into plasma physics during the 1950's by Elsasser[1] and Woltjer.[2] The importance of helicity derives from its conservation properties in fluid type plasma turbulence. In a turbulent resistive fluid, the dissipation of magnetic energy is enhanced by a much greater factor by the turbulence than is the dissipation of the helicity.[3,4] This property of the helicity was exploited by Taylor in his well-known theory of the reversed field pinch.[5] Taylor found the minimum energy state of the magnetic field consistent with helicity conservation, a so-called Taylor minimization. This state must be stable both to ideal instabilities and to resistive instabilities that grow on a rapid time scale compared to the characteristic dissipative, or skin, time of the magnetic field. Although Taylor's theory explains the major features of experiments on the reversed field pinch and spheromak devices, it does not reproduce the major features of tokamak experiments. Apparently the reversed field pinch and spheromak are sufficiently susceptible to resistive instabilities to validate the assumption that all the magnetic energy that is accessible within the helicity constraint is dissipated by the turbulence associated with these instabilities. The tokamak is apparently too stable to resistive instabilities for that to be a good assumption, at least for the plasma as a whole. Nevertheless, we will give some examples of the application of a Taylor type analysis to tokamak problems.

The concept of helicity appears ill-defined due to the gauge freedom of the vector potential. Indeed, the helicity density $\mathbf{A}\cdot\mathbf{B}$ has essentially no meaning due to the freedom of gauge. Suppose the original vector potential is $\mathbf{A}_0$ and $\mathbf{A}=\mathbf{A}_0+\nabla G$. Then one can make $\mathbf{A}\cdot\mathbf{B}$ zero throughout a neighborhood of any point in space by an appropriate choice of G. The problem of gauge freedom is significantly alleviated if only single-valued vector potentials are used. By this we mean that $\mathbf{A}(\mathbf{x})$ is a well-defined function throughout space. The so-called Poincaré lemma[6] guarantees that such vector potentials always exist for globally divergence-free fields. With single-valued vector potentials, there is no gauge problem, provided the volume of integration in the definition of helicity is bounded by a magnetic surface, as was assumed in Taylor's original work. (The normal $\hat{\mathbf{n}}$ to a magnetic surface is orthogonal to the magnetic field, $\hat{\mathbf{n}}\cdot\mathbf{B}=0$, at every point on the surface.) The result that a change in gauge does not change $\int \mathbf{A}\cdot\mathbf{B}d^3x$ follows obviously from $\mathbf{B}\cdot\nabla G=\nabla\cdot(G\mathbf{B})$.

The concept of helicity is important not only when the integration volume is bounded by a magnetic surface, but also when the bounding surface has an arbitrary shape and time dependence. By an arbitrary bounding surface, we actually mean the surface is either simply connected or topologically toroidal. Unfortunately, the problem of gauge does not disappear with an arbitrary bounding surface. In order to eliminate the dependence on the gauge, Bevir and Gray[7] introduced a modified form of the helicity invariant,

$$K_0 = \int \mathbf{A}\cdot\mathbf{B}d^3x - (\oint \mathbf{A}\cdot d\mathbf{x}_\theta)(\oint \mathbf{A}\cdot d\mathbf{x}_\varphi), \qquad (1)$$

which is the original helicity invariant $\int \mathbf{A}\cdot\mathbf{B}d^3x$ minus the product of a poloidal and a toroidal loop integral of the vector potential in the bounding surface. Bevir and Gray assumed that the bounding surface was a magnetic surface, so the use of a single-valued vector potential would have eliminated the need for the modification; their form of the invariant is independent of gauge only if the bounding surface is a magnetic surface. Nevertheless, their modified invariant K_0, which we will call the helicity content, is the proper definition of the helicity contained in an arbitrary, time-dependent, bounded region of space, provided the gauge is chosen correctly. The helicity content is essentially an integration of poloidal magnetic flux in the volume with respect to the toroidal flux. One can frequently gain insight into the helicity equations by letting $K_0=K_v+2l_hI\Psi$, with K_v the helicity content of the vacuum field, I the toroidal current in the volume, Ψ the toroidal flux in the volume, and l_h the helicity inductance. The helicity inductance has the same units as, and is closely related to, the inductances for magnetic flux and energy.[8] Each of these inductances is determined by the profile of the plasma current.

The important feature of the helicity content is its time derivative.

The time derivative of the helicity content consists of three terms. Each of these terms has arisen earlier in the theory of helicity under more specialized circumstances. The importance of the present work demonstrates that these three terms retain their simple form for a general bounding surface. One term, $2\int \mathbf{E}\cdot\mathbf{B}d^3x$, is a volume integral. This term gives the dissipation of helicity by resistivity. With the standard Ohm's law, $\mathbf{E}\cdot\mathbf{B}$ is $\eta\mathbf{j}\cdot\mathbf{B}$. The fact that helicity dissipation is linear in the current $\mathbf{j}$, while magnetic energy dissipation is quadratic, accounts for the smaller enhancement of helicity dissipation by turbulence than energy dissipation.[3,4] In a plasma with turbulent magnetic fields, the parallel current density tends to be quite spiky. Helicity conservation in a turbulent plasma is essentially a statement that $\int\eta\mathbf{j}\cdot\mathbf{B}d^3x$ can be accurately evaluated using a spatially averaged, or mean, current density. The resistance of a plasma can be defined as $R_\Omega \equiv (\int\eta\mathbf{j}\cdot\mathbf{B}d^3x)/I\Psi$. The other two terms in the time derivative of the helicity content are surface terms. One is $2V\Psi$, with V the spatially averaged toroidal loop voltage on the surface and Ψ the toroidal magnetic flux inside the surface. This term accounts for the so-called F-Θ pumping or MHD current drive, which was first proposed by Bevir and Gray.[7] The standard method of maintaining a plasma current by transformer action can be thought of as balancing the dissipation of helicity in the plasma volume by the loop voltage V. The last term in the time derivative of the helicity content has been discussed by Berger and Field[9] and by Jensen and Chu.[10] This term, $2\int\Phi\mathbf{B}\cdot d\mathbf{a}$, is an integration of the electric potential Φ in the bounding surface with respect to the magnetic flux $\mathbf{B}\cdot d\mathbf{a}$ penetrating the surface. The creation and maintenance of helicity by an electrostatic potential has been studied experimentally at Los Alamos as part of the spheromak program.[11]

The concept of helicity in fluid mechanics is essentially identical to that in plasma physics. In fluid mechanics, the helicity is $\int\mathbf{v}\cdot\boldsymbol{\omega}d^3x$, with $\boldsymbol{\omega}\equiv\nabla\times\mathbf{v}$ and $\mathbf{v}$ the fluid velocity. If there is a well-defined enthalpy, $dw=(dp)/\rho$, then the Navier-Stokes equation,

$$\partial\mathbf{v}/\partial t + \mathbf{v}\cdot\nabla\mathbf{v} = -\nabla w - \upsilon\,\nabla\times(\nabla\times\mathbf{v}), \qquad (2)$$

is the fluid "Ohm's law." To see this, let

$$\mathbf{E} \equiv -\,\partial\mathbf{v}/\partial t - \nabla(w + v^2/2), \qquad (3)$$

which is equivalent to Faraday's law, then

$$\mathbf{E} + \mathbf{v}\times\boldsymbol{\omega} = \upsilon\,\nabla\times\boldsymbol{\omega}, \qquad (4)$$

which has the form of the standard Ohm's law of plasma physics with the resistivity η replaced by the viscosity υ. The equivalent of the magnetic field energy in fluid mechanics is the enstrophy $\int(\omega^2/2)d^3x$; so the equivalent of a Taylor minimization is a minimization of the enstrophy holding the helicity constant. The energy of an incompressible fluid $\int(\rho v^2/2)d^3x$ is dissipated, in an unbounded region with uniform viscosity, at the rate $\int\upsilon\omega^2d^3x$. The equivalent of the fluid energy for a magnetic field $\int(A^2/2)d^3x$ has no name and no apparent applications, due to the complexity of its time derivative and the absence of definiteness which arises from the freedom of gauge. Constraints on the effect of turbulence on the Ohm's law of plasma physics[4] have an analogue in the effect of fluid turbulence on the Navier-Stokes equation.

To avoid confusion, it should be noted that the connection between Sec. II on the helicity content and Sec. III on the application of the concept of helicity to tokamaks is somewhat looser than one might at first suspect. Although the concept of helicity content and the mathematics of Sec. II help clarify the generality of the formalism, the actual examples could have been derived using the more restricted formalisms which previously existed in the literature. Sec. II contains much more mathematical detail than the other parts of the paper, but it need not be mastered to understand the other sections.

II. HELICITY CONTENT

Consider the helicity content of an arbitrary bounded region of space. The goal is to express the helicity content and its time derivative in terms of physically meaningful quantities: the poloidal and toroidal magnetic fluxes, the loop voltage, and the electric potential. The region boundary is assumed to have no special orientation with respect to the magnetic field lines, and the boundary is assumed to evolve in time. There are three tasks that must be performed to reach the goal. First, a suitable definition of the poloidal and toroidal fluxes must be found. This definition involves the magnetic vector potential. Second, the time derivative of the vector potential in an arbitrary time-dependent coordinate system is required. Third, the helicity content must be defined and its time derivative calculated.

Consider the task of finding a suitable definition of the poloidal and toroidal magnetic fluxes. Although the definition of the fluxes should relate to the boundary, this relation is non-trivial, since the boundary need not be a magnetic surface. The starting point for the definition is the expression for the vector potential in a general set of toroidal coordinates. Let ρ,θ,φ be any well-behaved set of toroidal coordinates, with θ a poloidal angle, φ a toroidal angle, and ρ a radial coordinate. The vector potential of an arbitrary magnetic field can then be written as (Appendix (I))

$$\mathbf{A} = \psi_* \nabla\theta_* - \chi_* \nabla\varphi \ , \qquad (5)$$

with $2\pi\psi_*$ a toroidal flux and $2\pi\chi_*$ a poloidal flux.[12] The flux functions ψ_* and χ_* are well-behaved functions of position. The boundary of the region can be specified by $\psi_*=\overline{\psi}_b(\theta_*,\varphi,t)$. It turns out to be inconvenient and unnecessary for the boundary function $\overline{\psi}_b$ to depend on the poloidal angle θ_*. If a new poloidal angle is introduced, $\theta=\theta_*+\omega(\theta_*,\varphi,t)$, then the poloidal angle dependence of boundary function can be eliminated. Choose ω and a new radial coordinate ξ so that

$$\overline{\psi}_b(\theta_*,\varphi,t) = \psi_b(\varphi,t)\,(1 + \partial\omega/\partial\theta_*) \quad \text{and} \quad \xi \equiv \psi_*/\overline{\psi}_b. \qquad (6)$$

Equation (5) for the vector potential can then be written as

$$\mathbf{A} = \xi\, \psi_b(\varphi,t)\, \nabla\theta - \chi\, \nabla\varphi \ , \qquad (7)$$

with

$$\chi \equiv \chi_* + \xi\, \psi_b(\varphi,t)\, \partial\omega/\partial\varphi. \qquad (8)$$

The toroidal flux function ψ is defined as $\xi\psi_b(\varphi,t)$ and the boundary poloidal flux function $\chi_b(\theta,\varphi,t)$ is defined as $\chi(\xi,\theta,\varphi,t)$ at $\xi=1$. Equations (6) and (7) are not changed if an arbitrary function of ψ and t is added to ω. By choosing this arbitrary function appropriately, an additional condition, the φ independence of $\int\chi_b d\theta$, will be imposed.

The second task is to calculate the time derivative of the vector potential in an arbitrary, time dependent, coordinate system. The purpose of this calculation is to relate the time derivatives of the fluxes to **E**·**B** and to the electric field in the bounding surface. Let $\mathbf{x}(\xi^1,\xi^2,\xi^3,t)$ give the spatial location defined by arbitrary, but well-behaved, coordinates ξ^1,ξ^2,ξ^3. The vector potential can be written as

$$\mathbf{A} = A_1\, \nabla\xi^1 + A_2\, \nabla\xi^2 + A_3\, \nabla\xi^3. \qquad (9)$$

In Appendix (II), we show that the time derivative of **A** is

$$\partial\mathbf{A}(\mathbf{x},t)/\partial t = [\Sigma\, \partial A_i(\xi,t)/\partial t\, \nabla\xi^i] + (\partial\mathbf{x}/\partial t)\times\mathbf{B} - \nabla[(\partial\mathbf{x}/\partial t)\cdot\mathbf{A}]. \qquad (10)$$

Faraday's law says that the electric field is related to the vector potential by

$$\mathbf{E} = -\,\partial\mathbf{A}(\mathbf{x},t)/\partial t - \nabla\Phi. \qquad (11)$$

The electric field in the ξ^i coordinates $\mathbf{E}_\xi$ is

$$\mathbf{E}_\xi = \mathbf{E} + (\partial\mathbf{x}/\partial t)\times\mathbf{B}; \qquad (12)$$

using equations (10) and (11),

$$\mathbf{E}_\xi = -\,[\partial A_i(\xi,t)/\partial t]\, \nabla\xi^i - \nabla\, \Phi_\xi\ , \qquad (13)$$

with $\Phi_\xi = \Phi - (\partial \mathbf{x}/\partial t)\cdot\mathbf{A}$ the electric potential in the ξ^i coordinate system. In other words, the covariant components of the electric field look the same in any coordinate system. It should be noted that $\partial\mathbf{x}/\partial t$ is the velocity through ordinary space of a point in the ξ^i coordinates and that $\mathbf{E}\cdot\mathbf{B} = \mathbf{E}_\xi\cdot\mathbf{B}$. The parallel component of the electric field is the same in any coordinate system.

The final task of this section is to define the helicity content and to calculate its time derivative. To motivate the definition, consider $\mathbf{A}\cdot\mathbf{B}$ with $\mathbf{A}$ given by Eq. (7), $\psi = \xi\psi_b$, and $\mathbf{B} = \nabla\times\mathbf{A}$,

$$\mathbf{A}\cdot\mathbf{B} = [\psi(\partial\chi/\partial\psi) - \chi]\,[(\nabla\psi \times \nabla\theta)\cdot\nabla\varphi]. \qquad (14)$$

Using $d^3x = d\psi d\theta d\varphi/[(\nabla\psi\times\nabla\theta)\cdot\nabla\varphi]$, the helicity $K = \int \mathbf{A}\cdot\mathbf{B}\,d^3x$ satisfies

$$K = -\,2\int\chi\; d\psi d\theta d\varphi + (\text{surface terms}). \qquad (15)$$

The helicity content K_0 is defined as

$$K_0 \equiv 2\int(\chi_b - \chi)\; d\psi d\theta d\varphi. \qquad (16)$$

Using the vector potential of Eq.(7), one can show that the helicity content, as defined in Eq.(16), is identical to the modified helicity invariant of Bevir and Gray, Eq.(1).

To calculate the time derivative of K_0, we will first calculate the parallel component of the electric field in the ξ,θ,φ coordinates using Eqs. (12), (13), and (7). One obtains

$$\mathbf{E}\cdot\mathbf{B} = -\,\xi\,(\partial\psi_b/\partial t)\,\mathbf{B}\cdot\nabla\theta + (\partial\chi/\partial t)\,\mathbf{B}\cdot\nabla\varphi - \nabla\cdot(\Phi_\xi\mathbf{B})\,, \qquad (17)$$

with ψ_b a function of φ and t, and with χ a function of ξ, θ, φ, and t. By taking the curl of Eq.(7), one can show that

$$\mathbf{B}\cdot\nabla\varphi = \psi_b\,(\nabla\xi\times\nabla\theta)\cdot\nabla\varphi \quad\text{and}\quad \mathbf{B}\cdot\nabla\theta/\mathbf{B}\cdot\nabla\varphi = (\partial\chi/\partial\xi)/\psi_p. \qquad (18)$$

This implies that $d^3x=(\psi_b/\mathbf{B}\cdot\nabla\varphi)d\xi d\theta d\varphi$, and that the integral of $\mathbf{E}\cdot\mathbf{B}$ over the bounded region can be written as

$$\int \mathbf{E}\cdot\mathbf{B}d^3x+\int\Phi_\xi\mathbf{B}\cdot d\mathbf{a} =-\int\xi(\partial\psi_b/\partial t)(\partial\chi/\partial\xi)d\xi d\theta d\varphi+\int(\partial\chi/\partial t)\psi_b d\xi d\theta d\varphi. \quad (19)$$

The first integral on the right hand side can be integrated by parts,

$$\int\xi(\partial\psi_b/\partial t)(\partial\chi/\partial\xi)d\xi d\theta d\varphi = \int(\chi_b-\chi)(\partial\psi_b/\partial t)d\xi d\theta d\varphi. \quad (20)$$

The time derivative of K_0 can now be calculated. Using ξ,θ,φ coordinates,

$$K_0 = 2\int(\chi_b - \chi)\,\psi_b\, d\xi d\theta d\varphi. \quad (21)$$

Therefore,

$$dK_0/dt=2\int(\partial\chi_b/\partial t)\psi_b d\xi d\theta d\varphi-2\int[(\partial\chi/\partial t)\psi_b-(\chi_b-\chi)(\partial\psi_b/\partial t)]d\xi d\theta d\varphi. \quad (22)$$

On the right hand side of Eq. (22), the last integral is, except for a factor of two, equal to the right hand side of Eq.(19), but the first integral requires further reduction. Define the loop voltage $\tilde{V}(\theta,t)$ as

$$\tilde{V} \equiv \oint (\mathbf{E}_\xi+\nabla\Phi_\xi)\cdot(\partial\mathbf{x}/\partial\varphi)\, d\varphi \;, \quad (23)$$

with the loop integral taken in the boundary surface. Equations (13) and (7) imply that

$$\tilde{V}(\theta,t) = \oint (\partial\chi_b/\partial t)\, d\varphi. \quad (24)$$

The average loop voltage V is defined as

$$V(t) \equiv \oint \tilde{V}\, d\theta/2\pi. \quad (25)$$

Finally the toroidal flux content Ψ of the bounded region is defined by

$$\Psi(t) \equiv \oint \psi_p(\varphi,t)\, d\varphi. \qquad (26)$$

If we use the condition that the θ integral of χ_b is independent of φ, then we find that the first integral on the right hand side of (22) has the form

$$\int (\partial\chi_b/\partial t)\psi_b d\xi d\theta d\varphi = V\,\Psi, \qquad (27)$$

and the time derivative of the helicity content is

$$dK_0/dt = 2\,V\,\Psi - 2\int \mathbf{E}\cdot\mathbf{B}\, d^3x - 2\int \Phi_\xi \mathbf{B}\cdot d\mathbf{a}, \qquad (28)$$

which is the desired expression. It should be noted that the equation for the helicity evolution was derived using only Maxwell's equations and is, therefore, valid for any model of the plasma.

III. APPLICATION OF HELICITY TO TOKAMAK PROBLEMS

The concept of helicity has found its primary application in the theory of the reversed field pinch and spheromak devices. There are, however, a number of applications of the helicity concept to the tokamak device. Three of these will be given here.

Taylor showed that inside a perfectly conducting toroidal surface, the magnetic field with minimum energy at a given helicity satisfies the equation

$$\nabla \times \mathbf{B} = k\,\mathbf{B}\ , \qquad (29)$$

with k independent of position.[5] The field which satisfies Eq. (29) may not be unique, since more that one k may exist for which the field obeys the boundary conditions and has the given helicity. Letting Θ be the poloidal field at the bounding surface divided by the spatially averaged toroidal field, one can show that non-uniqueness requires $\Theta \gtrsim 1$. Indeed, Taylor found[5] that non-uniqueness occurs in a cylinder for $\Theta > 1.6$. In a tokamak, $\Theta \ll 1$, so there is a unique k for which the helicity constraint is satisfied. In other words, a zero pressure plasma in a tokamak must be stable to all ideal and resistive instabilities if $j_{\parallel}/B$ is spatially uniform. Consider for simplicity a tokamak with a very large aspect ratio so that the field can be approximated by that in a cylinder,

$$\mathbf{B} = B_{\varphi}\,\hat{z} + (\hat{z}/R) \times \nabla\chi(r,\theta) \quad , \qquad (30)$$

with $2\pi R$ the periodicity length of the cylinder and B_{φ} a constant. The symmetry of the tokamak becomes symmetry in the $\hat{z}$ direction, and the magnetic surfaces are given by the constant χ surfaces $\mathbf{B}\cdot\nabla\chi=0$. Define the local radius of a surface r_0 so that r_0^2 is proportional to χ,

$$\chi=(kRB_{\varphi}/4)r_0^2. \qquad (31)$$

Equation (29) becomes

$$\nabla^2 r_0^2(r,\theta) = 4, \quad \text{so} \quad r_0^2 = r^2 + \alpha_m r^m \cos(m\theta). \qquad (32)$$

The bounding magnetic surface can be defined as $r_0(r=a,\theta)$, or $r_0=a[1+\delta_m\cos(m\theta)]$. The bounding magnetic surface is distorted from a circle by the δ_m's . Assuming $\delta_m<<1$,

$$\alpha_m = 2\,\delta_m / a^{(m-2)}. \qquad (33)$$

The toroidal flux $2\pi\psi$ can be written

$$\psi = (B_{\varphi}/4\pi) \int(\partial r^2/\partial\chi)d\chi d\theta. \qquad (34)$$

The safety factor $q\equiv d\psi/d\chi$ is then equal to

$$q = [1 + m(m-1)\,\delta_m^2\,(r_0/a)^{2(m-2)}]\,q_c \quad , \qquad (35)$$

with $q_c\equiv 2/kR$, which is the value the safety factor would have in a circular plasma with the same current density. For $m\geq 3$, the central q value can be less than unity and increase through unity even with $j_{\parallel}/B$

constant. It is relatively easy to find stable current profiles in tokamaks with $q>1$. Therefore, one can hope to find current profiles that vanish at the plasma edge, but have a central q value well below unity due to shaping. With triangularity, one should be able to stabilize q values as low as $1-6\delta_3^2$. The usual analysis of the $m=1$ instability in tokamaks is clearly invalid unless the parameter $\delta^2/(1-q_0)$ is much less than one, with δ the distortion of the $q=1$ surface and q_0 the central q value.

The second application of helicity is to the possible existence of an intrinsically steady-state tokamak equilibrium due to the interaction of the bootstrap current and tearing modes. Bickerton, Connor, and Taylor[13] noted in 1971 that the neoclassical bootstrap current could greatly reduce the difficulty of maintaining the tokamak current by either a current-drive scheme or by a loop voltage. But they also found that "a completely bootstrapped tokamak is not possible." The interaction of helicity conserving tearing modes with the bootstrap effect may make a completely bootstrapped tokamak possible. The fundamental limitation on the use of the bootstrap effect for maintenance of the current in a tokamak is that the bootstrap effect creates no poloidal magnetic flux. Instead, the poloidal magnetic flux is pushed out of regions of high plasma pressure. This poloidal diamagnetism is large when the poloidal beta is of the order of the square root of the aspect ratio. In tokamaks of the usual aspect ratio, the poloidal diamagnetism can be very large due to the bootstrap effect, even when the overall, or toroidal, diamagnetism is small. That is, the bootstrap effect can greatly modify the q profile of a tokamak with a high poloidal beta so that q is large in the central region of the tokamak and is non-monotonic. Such non-monotonic, or "double-q", profiles tend to be unstable to tearing modes. These tearing modes may be beneficial, since they create additional poloidal magnetic flux. Indeed, the bootstrap effect and the associated tearing modes could, in principle, maintain the tokamak current without any externally applied loop voltage or current drive power.

The relevant neoclassical equations[14] can be rewritten in the form

$$\frac{1}{r}\frac{d(rB_\theta)}{dr} = \mu_0 j_\varphi \quad , \qquad (36)$$

$$E_\varphi + v_r B_\theta = \eta j_\varphi \quad \text{, and} \tag{37}$$

$$\sqrt{\varepsilon}\, \frac{dp}{dr} = -\left(j_\varphi - \frac{1-\sqrt{\varepsilon}}{\eta} E_\varphi\right)B_\theta \quad . \tag{38}$$

The local inverse aspect ratio is proportional to $\sqrt{\varepsilon}$, the effective pressure gradient is

$$dp/dr \approx (T_e + T_i)\, dn/dr + n\, dT_e/dr + n\, dT_i/dr, \tag{39}$$

the Spitzer parallel resistivity is η, and the effective radial velocity is

$$v_r \approx (\Gamma/n) + (Q_e/nT_e) + \sqrt{m_e/m_i}\, (Q_i/nT_i). \tag{40}$$

In an actual neoclassical calculation, each term in dp/dr and v_r is multiplied by a constant factor. The particle flux is Γ, and Q_e and Q_i are the electron and ion heat fluxes. In the presence of anomalous transport phenomena, only that fraction of each flux that is associated with neoclassical phenomena would enter the expression for the effective radial velocity. The most important parameter associated with the neoclassical equations is essentially the ratio of the magnetic diffusion, or skin, time to an effective confinement time,

$$\kappa \equiv \int_0^r (\mu_0/\eta)\, v_r\, dr. \tag{41}$$

A strong bootstrap effect corresponds to $\kappa >> 1$. Close to the magnetic axis, the sources of particles and heat can be assumed spatially uniform so that $v_r \propto r$, and $\kappa \propto r^2$. Letting $\mu_0 I \equiv 2\pi r B_\theta$, one can solve Eqs. (36) and (37) under the assumption of uniform sources. The solution is

$$I = (2\pi r/\mu_0 v_r)\, (e^\kappa - 1)\, E_\varphi, \tag{42}$$

which demonstrates that the bootstrap effect does not modify the electric field required to drive the central current; at r=0, the current density is $j_\varphi = E_\varphi/\eta$. A strong bootstrap effect, $\kappa >> 1$, distorts the current distribution, $j_\varphi=(dI/dr)/(2\pi r)$, into a hollow profile and makes the safety factor, $q\propto r^2/I$, non-monotonic.

Equation (38) gives the pressure gradient that is associated with a strong bootstrap effect, $E_\varphi \ll \eta j_\varphi$. In the limit of a strong bootstrap effect, $dp/dr = [(1/r^2)\, d(r^2 B_\theta{}^2/2\mu_0)/dr]/\sqrt{\varepsilon}$, so that the poloidal beta satisfies $\beta_\theta \approx 1/\sqrt{\varepsilon}$. This means that if the bootstrap effect is strong in a central region of a tokamak, then the drop in the plasma pressure across that region must be of the order of $B_\theta{}^2/(2\mu_0\sqrt{\varepsilon}\,)$.

The tendency of the bootstrap effect to produce a non-monotonic q profile may make the tokamak self-maintaining through the action of tearing modes. Assume that there is strong tearing mode activity in a tokamak unless $dq/dr>0$, and that $r=b$ is the outer edge of a tearing mode unstable central region. In the region $r<b$, both j_φ and q are made spatially uniform by the tearing modes. The helicity dissipation rate, $\int \mathbf{E}\cdot\mathbf{B}d^3x$, is not enhanced by the tearing modes. The standard neoclassical equations are derived under the assumption that $B_\varphi \gg B_\theta$; so $\mathbf{E}\cdot\mathbf{B}\approx E_\varphi B_\varphi$. The presence of tearing modes for $r<b$ implies that Eq. (37) becomes

$$\int_0^b v_r\, B_\theta\, r dr = \int_0^b \eta j_\varphi\, r dr \qquad (43)$$

under the assumption that the externally applied E_φ is zero. If we assume a uniform source of heat and particles, then v_r can be set from the condition that $(dq/dr)_b=0$, which implies $I'/I=2/b$ and $\kappa'=2/b$ at $r=b$ with $\kappa'=(\mu_0/\eta)v_r$. Equation (43) gives a condition for poloidal flux maintenance through the bootstrap effect such that $2\langle\eta\rangle=\eta_b$, with $\langle\eta\rangle$ the volume average of the resistivity inside $r=b$. In other words, one must assume the tearing activity in the region $r<b$ is sufficiently strong to flatten the toroidal current profile but sufficiently weak to allow a drop inside that region of both the pressure, of the order of $B_\theta{}^2/(2\mu_0\sqrt{\varepsilon}$), and the electron temperature, so that $2\langle\eta\rangle=\eta_b$. If these conditions

are met, the tokamak should maintain its current without any external current drive.

The final application of the helicity concept to tokamaks is the maintenance of the tokamak current with an oscillatory loop voltage. This is just a variant of the F-Θ current drive scheme for reversed field pinches, which was originally proposed by Bevir and Gray.[7] Tokamak current drive using helicity arguments has been discussed by Bellan[14] and by Stix and Ono.[16] The basic equation for F-Θ current drive,

$$dK_0/dt = 2\,(V - R_\Omega I)\,\Psi, \qquad (44)$$

follows from Eq.(28) with

$$R_\Omega \equiv (\int \mathbf{E}\cdot\mathbf{B}d^3x)/I\Psi \qquad (45)$$

the resistivity of the plasma (in Ohms); I the toroidal plasma current (in amperes), $I \equiv \oint d(\varphi/2\pi)\int \mathbf{j}\cdot d\mathbf{a}_\varphi$; and V the toroidal loop voltage. The standard Ohm's law implies

$$R_\Omega = (\int \eta \mathbf{j}\cdot\mathbf{B}d^3x)/I\Psi. \qquad (46)$$

In a steady-state plasma, $\nabla \times \mathbf{E}=0$, which implies that $\mathbf{E}=V(\nabla\varphi/2\pi)+\nabla\Phi$, with the loop voltage V a constant, and $V=R_\Omega I$; so the power dissipation $\int \mathbf{E}\cdot\mathbf{j}d^3x$ is given by $VI=R_\Omega I^2$. It is useful to express the helicity content K_0 in terms of the helicity inductance l_h by

$$K_0 = K_v + 2l_h I\Psi \ , \qquad (47)$$

with K_v the helicity content of the vacuum magnetic field (K_v is zero for a tokamak but not for a stellarator). The helicity inductance is closely related to the internal energy inductance and the internal flux inductance of the plasma.[8] If one assumes the tokamak current profile is determined by the edge value of q, the safety factor, then in a circular plasma, l_h/R is a function of RI/Ψ alone, with R the major radius. The decay time of the plasma current is defined by

$$\tau \equiv l_h/R_{\Omega}. \qquad (48)$$

In a plasma with constant toroidal flux but zero loop voltage, the plasma current decays as $I=I_0\exp(-t/\tau)$ if l_h is a constant.

To illustrate the concept of MHD current drive, assume that τ is a constant and write Eq. (44) as

$$dK_0/dt = 2V\Psi - K_0/\tau. \qquad (49)$$

Oscillate the loop voltage and the toroidal flux so that $V=V_1\cos(\omega t)$ and $\Psi=\Psi_0+\Psi_1\cos(\omega t)$, then (50)

$$\frac{K_0}{\tau} = \frac{\cos(\omega t) + \omega\tau \sin(\omega t)}{1 + (\omega\tau)^2} 2\Psi_0 V_1 + [1 + \frac{\cos(2\omega t) + 2\omega\tau \sin(2\omega t)}{1 + (2\omega\tau)^2}] \Psi_1 V_1$$

using Eq.(49). Although the time average loop voltage is zero, the plasma maintains a positive definite helicity content provided

$$\frac{\Psi_1}{\Psi_0} \geq \frac{2}{[1+(\omega\tau)^2]^{1/2}} \frac{[1 + (2\omega\tau)^2]^{1/2}}{[1+(2\omega\tau)^2]^{1/2} - 1} . \qquad (51)$$

For plasma maintenance, one needs not only $K_0 \geq 0$, but also $\Psi_1 \leq \Psi_0$. To satisfy both conditions simultaneously, one requires $\omega\tau \geq 2.326$. Assuming these conditions are satisfied the oscillatory loop voltage and the oscillation on the toroidal flux produce an effective rectified loop voltage $\overline{V}$

$$\overline{V} = \langle V\Psi\rangle/\langle\Psi\rangle \ , \qquad (52)$$

with $\langle\cdots\rangle$ denoting a time average. For the example of cosine oscillations which was studied above, $\overline{V}=V_1\Psi_1/2$. In the limit of $\omega\tau \gg 1$, so that the oscillations in Ψ and K_0 are small, the rectified voltage can be thought of as an ordinary loop voltage for either maintaining or ramping-up the plasma current. Oscillations in the toroidal flux content of the plasma Ψ can be easily produced in a tokamak by varying the vertical, or horizontal, magnetic field, which will push the plasma off and onto a magnetic or material limiter and thereby change the cross-sectional area of the plasma.

The critical assumption in the analysis of the last paragraph is the rigidity of the current profile. In particular, the magnetic axis must respond as the flux and voltage are oscillated. Let $\bar{\chi}=2\pi\chi$ be the poloidal flux and $\bar{\chi}_0(t)$ be the poloidal flux enclosed by the magnetic axis. Then Faraday's law implies that $d\bar{\chi}_0/dt=\oint \mathbf{E}\cdot d\mathbf{x}$, with the line integral taken along the magnetic axis. Using the standard Ohm's law, $d\bar{\chi}_0/dt=2\pi R\eta j_0$, with j_0 the current density on the magnetic axis. Unless j_0 reverses sign, the flux enclosed by the axis increases without limit. This result is essentially Cowling's theorem.[17] Resistive MHD plasma turbulence, such as tearing modes, can produce an effective Ohm's law

$$\mathbf{E}\cdot\mathbf{B} = \eta\mathbf{j}\cdot\mathbf{B} - \nabla\cdot[\lambda\nabla(j_{\parallel}/B)] \quad , \tag{53}$$

with λ, the viscosity of the parallel current, a positive definite coefficient determined by the turbulence.[4] The implication of this Ohm's law is that a steady-state requires $d(j_{\parallel}/B)/d\psi=\alpha j_{\parallel}/B$ at the axis, with α positive and proportional to η/λ. For MHD turbulence to maintain the poloidal flux $\bar{\chi}_0$, the current density must increase away from the axis, but if the turbulence effects are strong, $\eta/\lambda<<1$, the increase in the current density can be arbitrarily small. Rigid current profiles, such as those which give $j_{\parallel}/R$ as a function of RI/Ψ alone, imply three-dimensional turbulence, including turbulence in the vicinity of the magnetic axis.

The energetics of time dependent fields implies another serious limitation on MHD current drive. Assume the bounding surface is outside the plasma but that it is a magnetic surface. The electric field on the boundary can be written as

$$\mathbf{E} = d\chi_b/dt\ \nabla\varphi - d\psi_b/dt\ \nabla\theta - \nabla\Phi \quad , \tag{54}$$

with χ_b and ψ_b functions of t alone. The magnetic field can be written as[18]

$$\mathbf{B} = (\mu_0/2\pi)\,[\,G(t)\,\nabla\varphi + I(t)\,\nabla\theta\,]\ , \qquad (55)$$

with G the number of amperes of poloidal current outside the boundary and I the number of amperes of toroidal current inside the boundary. The energy flowing across the boundary is given by

$$dU/dt = -\int \mathbf{E}\times\mathbf{B}\cdot\nabla\psi\ J\,d\theta\,d\varphi\ , \qquad (56)$$

with J the Jacobian for ψ,θ,φ coordinates and with the sign chosen so that an inward flow of energy is positive. One then finds

$$dU/dt = IV + G\,d\Psi/dt\ , \qquad (57)$$

with $V=2\pi(d\chi_b/dt)$ the loop voltage and Ψ the toroidal flux. Assume the plasma has constant inductances and vary the current I holding the toroidal flux Ψ fixed. Then

$$d(l_h I^2/2)/dt = IV - R_\Omega I^2. \qquad (58)$$

The magnetic energy is

$$W=W_v + l_w I^2/2\ , \qquad (59)$$

with W_v the vacuum field energy; so

$$d[(l_h/l_w)W]/dt = dU/dt - R_\Omega I^2. \qquad (60)$$

Suppose l_h/l_w is less than one, as it is for a centrally peaked current profile. Then there is a contradiction, since the magnetic energy W increases faster than the energy input, unless

$$R_\Omega I^2 \geq [(l_w/l_h) - 1]\,dU/dt. \qquad (61)$$

If one assumes l_h/l_ψ is greater than one, as it is for an outwardly peaked current profile, then one obtains a similar contradiction if energy is taken out of the system, unless

$$R_\Omega I^2 \geq [(l_h/l_\psi) - 1] \, |dU/dt|. \qquad (62)$$

The inequalities suggest that a current profile can only be rigid on a resistive time scale, roughly $\tau = l_h/R_\Omega$, unless $l_h = l_\psi$. Although the inequalities do not preclude MHD current drive in a device with $l_h \neq l_\psi$, they do make one believe MHD current drive is difficult (since $\omega\tau$ must be greater than 2.326 in the simple model) and that large amplitude oscillations are required. A similar conclusion can be drawn from the Ohm's law of Eq.(53). The viscosity λ serves only to flatten the current profile; so a rigid, but peaked, current profile must be maintained in part by the resistivity η.

IV. DISCUSSION

The fundamental equation of the paper is the expression for the time derivative of the helicity content of an arbitrary volume of space. The time derivative can be written as

$$dK_0/dt = 2\,(V - R_\Omega I + \Phi_l)\,\Psi, \qquad (63)$$

with $V(t)$ the surface averaged toroidal loop voltage, $I(t)$ the total toroidal current, $\Psi(t)$ the toroidal flux, and $\Phi_l(t)$ electric potential which is linked with the magnetic flux

$$\Phi_l \equiv -(\int \Phi \, \mathbf{B} \cdot d\mathbf{a})/\Psi. \qquad (64)$$

The resistivity of the plasma is $R_\Omega \equiv (\int \mathbf{E} \cdot \mathbf{B} d^3x)/I\Psi$. In steady-state, but not in time dependent, plasmas, $R_\Omega I^2$ is the rate of energy dissipation.

The helicity content K_0 is defined in Eq. (16), in terms of an integral over the poloidal flux in the plasma with respect to the toroidal flux. Eq. (63) can be written in a form which resembles familiar electrical equations by the use of the helicity inductance

$$l_h \equiv (K_0-K_v)/I\Psi \quad , \qquad (65)$$

with K_v the helicity of the field for I equal to zero. The inductance l_h is closely related to the internal inductances for poloidal flux l_χ and energy l_w , and is determined by the plasma current profile. The helicity inductance is smaller than the energy inductance for centrally peaked current profiles, and is larger than the energy inductance for outwardly peaked current profiles.

The important feature of the helicity is that its dissipation rate is negligibly enhanced by MHD type turbulence. This means that a spatially averaged, or smoothed, current density can be used to evaluate the plasma resistance R_Ω as well as the helicity inductance l_h. The poloidal field energy can, nonetheless, be dissipated at an arbitrarily rapid rate by the turbulence. Fast relaxation processes reduce the magnetic field energy while conserving the helicity.

Earlier work has emphasized the importance of the helicity concept to the reversed field pinch and spheromak devices. Three applications of the helicity concept to the tokamak device have been given. The applications are the stability, even the resistive stability, of the m=1 mode in plasmas with a non-circular q=1 surface; the potential for a tokamak to be intrinsically steady-state due to the interaction of the bootstrap current and tearing modes; and current drive by oscillating the plasma size and the loop voltage, the so-called MHD current drive.

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Appendix A: REPRESENTATION OF THE VECTOR POTENTIAL

The proof that any magnetic field has a vector potential of the form of Eq.(4) has been given previously.[12] The proof requires the so-called Poincaré lemma[6] that any globally divergence-free field has a well-behaved, single-valued vector potential. In addition, one uses the theorem from vector representation theory that an arbitrary vector can be represented in a well-behaved coordinate system, such as the ρ,θ,φ coordinates, as

$$\mathbf{A} = A_\rho\, \nabla\rho + A_\theta\, \nabla\theta + A_\varphi\, \nabla\varphi \;, \qquad \text{(AI-1)}$$

with A_ρ, A_θ, and A_φ well-behaved functions of position. By defining a function G so that $\partial G/\partial\rho = A_\rho$, one finds that $\mathbf{A}$ can be written in the form, the so-called canonical form

$$\mathbf{A} = \psi\, \nabla\theta - \chi\, \nabla\varphi + \nabla G \;, \qquad \text{(AI-2)}$$

with $\psi \equiv A_\theta - \partial G/\partial\theta$ and $\chi \equiv A_\varphi - \partial G/\partial\varphi$. Clearly ψ, χ, and G are well-behaved functions of position. Since a gradient, such as ∇G, does not alter the magnetic field, $\mathbf{B} = \nabla\times\mathbf{A}$, the term ∇G can be dropped, giving Eq.(5).

The magnetic field associated with the canonical form for the vector potential is

$$\mathbf{B} = \nabla\psi \times \nabla\theta + \nabla\varphi \times \nabla\chi. \qquad \text{(AI-3)}$$

To calculate the toroidal flux, we use the element of toroidal area in ψ,θ,φ coordinates

$$da_\varphi = \frac{\nabla\varphi}{[(\nabla\psi\times\nabla\theta)\cdot\nabla\varphi]}\, d\psi d\theta. \qquad \text{(AI-4)}$$

The toroidal flux is

$$\int \mathbf{B}\cdot d\mathbf{a}_\varphi = \int d\psi d\theta; \qquad \text{(AI-5)}$$

so $2\pi\psi$ is the toroidal flux inside a constant ψ surface. The element of poloidal area in χ,θ,φ coordinates is

$$da_\theta = \frac{\nabla\rho}{|\nabla\chi\times\nabla\theta)\cdot\nabla\varphi|}\, d\chi d\varphi. \qquad \text{(AI-6)}$$

The poloidal flux outside a constant χ surface is $2\pi\chi$.

Appendix B: TIME DERIVATIVE OF A

Let $\mathbf{x}(\xi^1,\xi^2,\xi^3,t)$ define the spatial position corresponding to a point in the ξ^1,ξ^2,ξ^3 coordinates. Any vector can be written as

$$\mathbf{A} = \Sigma\, A_j\, \nabla\xi^j. \qquad \text{(AII.1)}$$

The time derivative of $\mathbf{A}(\mathbf{x},t)$ is

$$\partial\mathbf{A}(\mathbf{x},t)/\partial t = \Sigma(\partial A_j(\mathbf{x},t)/\partial t)\nabla\xi^j - \Sigma(\partial\xi^j(\mathbf{x},t)/\partial t)\nabla A_j + \nabla[\Sigma A_j(\partial\xi^j(\mathbf{x},t)/\partial t)]. \qquad \text{(AII.2)}$$

The chain rule implies

$$(\partial A_k/\partial t)_\mathbf{x} = (\partial A_k/\partial t)_{\xi^i} + \Sigma(\partial A_k/\partial\xi^j)(\partial\xi^j/\partial t)_\mathbf{x}\ , \qquad \text{(AII.3)}$$

as well as $\nabla A_k = \Sigma(\partial A_k/\partial\xi^j)\nabla\xi^j$. This means that

$$\partial\mathbf{A}(\mathbf{x},t)/\partial t = \Sigma(\partial A_j/\partial t)_{\xi^i}\nabla\xi^j + \Sigma(\partial\xi^j/\partial t)\ [(\partial A_k/\partial\xi^j) - (\partial A_j/\partial\xi^k)]\ \nabla\xi^k + \nabla[\Sigma A_j(\partial\xi^j/\partial t)]. \qquad \text{(AII.4)}$$

The relation between $\partial\mathbf{x}/\partial t$ and $\partial\xi^j/\partial t$ can be derived from the chain rule

$$(\partial\mathbf{x}/\partial t)_{\mathbf{x}} = (\partial\mathbf{x}/\partial t)_{\xi^i} + \Sigma(\partial\mathbf{x}/\partial\xi^j)\partial\xi^j/\partial t, \qquad \text{(AII.5)}$$

and the obvious relation $(\partial\mathbf{x}/\partial t)_{\mathbf{x}}=0$. The orthogonality theorem of partial differentiation theory gives a universal and important relation between $\partial\mathbf{x}/\partial\xi^i$ and $\nabla\xi^i$,

$$(\partial\mathbf{x}/\partial\xi^i)\cdot\nabla\xi^j = \delta_i{}^j \;, \qquad \text{(AII.6)}$$

with $\delta_i{}^j$ the Kronecker delta. These relations, as well as $\mathbf{B}=\nabla\times\mathbf{A}$, can be used to prove that

$$\partial\mathbf{x}/\partial t \times \mathbf{B} = \Sigma_{jk}(\partial\xi^j/\partial t)[\partial A_k/\partial\xi^j - \partial A_j/\partial\xi^k]\,\nabla\xi^k. \qquad \text{(AII.7)}$$

It is then easy to obtain the desired relation,

$$\partial\mathbf{A}(\mathbf{x},t)/\partial t = [\Sigma\,(\partial A_j/\partial t)_{\xi^i}\,\nabla\xi^j] + (\partial\mathbf{x}/\partial t)\times\mathbf{B} - \nabla[(\partial\mathbf{x}/\partial t)\cdot\mathbf{A}]. \qquad \text{(AII.8)}$$

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