RELATIVISTIC-IN Variant statistical theory and its application to multiple processes
Ю.М. Синюков

Релятивистско-инвариантная статистическая теория и ее приложение к множественным процессам

Предложено релятивистско-инвариантное обобщение статистической теории идеальных газов. Развит ковариантный метод статистической суммы, позволяющий находить статистические и термодинамические свойства газов на любой гиперповерхности в произвольной инерциальной системе отсчета. Обсуждаются следствия развитой теории для статистических моделей множественного рождения адронов.

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Relativistic-Invariant Statistical Theory and Its Application to Multiple Processes

The relativistic-invariant generalization of the ideal gas statistical mechanics is suggested. The covariant partition function's method is developed. The statistical and thermodynamical properties of gases are found on any hypersurface in arbitrary inertial reference frame. The consequences of the developed theory for statistical models of hadron multiparticle production are discussed.

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RELATIVISTIC-IN Variant STATISTICAL THEORY
AND ITS APPLICATION TO MULTIPLE PROCESSES

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When studying high energy density processes the dominant methods of statistical physics. For a given class of problems statistical theory relativization distinguishes two aspects. The first one suggests the account for the relativistic character of motion of thermolized matter being as a whole at rest. The corresponding generalization for an ideal gas of relativistic particles was made in 1911 by Juttner [1]. The second aspect according to the inertial observers equivalence principle requires the consideration of relativistic motion of an equilibrium statistical system (gas) as a whole. An attempt to develop the corresponding generalization for an ideal gas of relativistic particles was made by Touschek in 1968 [2]. A practical necessity in a completely relativized equilibrium statistical mechanics has appeared only recently in view of the study of transitions between hadronic and chromodynamic phases of matter [3] and description of multiparticle production processes in high-energy collisions of hadrons and heavy ions [4, 5].

According to Ref. [2] the relativistic representation for a partition function of an ideal N-particle Boltzmann gas of particles with a mass $m$ has the following form

$$Z_N = \frac{1}{N!} \int_{i=1}^N d^3 p_i V \frac{u \cdot p_i}{(2\pi)^3} \exp \left(-\frac{u \cdot p_i}{T}\right) \delta(p_i^2 - m^2), \quad (1)$$

where $p_i$ are the particles momenta, $u^\mu$ is the four-dimensional velocity of motion of the gas as a whole with velocity $\vec{v}$, $V$ is the volume of the gas in the rest frame. Here and further on the asterisk marks the quantities measured in a rest frame of the gas on a constant time hypersurface.

As was shown in Ref. [6] the use of the covariant formulation of statistical mechanics [2] for describing the thermodynamical quantities on surfaces of the constant time $\Sigma_t: t = const$ in moving inertial frames leads to the results contradicting the statements of relativistic mechanics of continuous media. In Refs. [6] and the following cycle of papers [7-9] the method enabling one to overcome the difficulty of the covariant approach [2], has been proposed. Its essence consists in a consequent consideration of the non-closedness of a statistical system cau-
separated by its interaction with external bodies (for example, container walls). In this approach the representation for the partition function of ideal Bose-, Fermi- and Boltzmann-gases on surfaces of constant time $\Sigma_t$ in an arbitrary inertial frame of reference has been found. For a $N$-particle of Boltzmann-gas this representation can be written as

$$Z_N(\Sigma_t) = \frac{1}{N!} \int \prod_{i=1}^{N} d^3p_i \frac{V_n}{u^o(\lambda f)^3} \exp \left( -\frac{u \cdot \hat{p}_i}{T} \right).$$

The expression (2) leads to the thermodynamical and statistical characteristics of gas which are different from those obtained by means of (1). The present paper shows that this difference is to be accounted for by physical nonequivalence of the systems investigated in Refs. [2] and [6-9].

To begin with, we note that the transformation properties of physical values can not be determined unless the space-time character of the measuring procedure is prescribed. For example, the moving body length experiences the Lorentz contraction when its space-time coordinates are measured simultaneously in the frame of reference where this body moves. In the other case when an observer defines the position of a body in the Minkowski space in the space-time points corresponding to the equal-time coordinates of the body at rest its length will now be transformed as a zero component of a four-vector. This is due to the fact that the positions of the moving body surface ends are considered at different times. In the first case the body is referred to the hypersurface $\Sigma_t^t : t = \text{const}$ of the constant time in the moving reference frame, in the second one - to the hypersurface $\Sigma_{t^*} : t^* = \text{const}$. In coordinates of the moving frame the latter hypersurface has the form

$$\Sigma_{t^*}(t, \lambda) : t = u \lambda^4 + \text{const}.$$  

The above examples show that from the relativistic point of view a single hypersurface on which a physical system is considered is impossible to be fixed. Any such a fixation (for example, $\Sigma_{t^*} : t^* = \text{const}$) contradicts the relativity principle. On the other hand, since the equilibrium statistical system is non-closed, thermodynamical values referred to different hypersurfaces do not coincide in any chosen frame of reference. Due to this fact the transformational properties of statistical
values associated with different hypersurfaces, for example, determined in synchronous measurements in different reference frames, are nontrivial \[6].

Thus, the relativistic invariant description of a statistical system should involve two physical parameters: the velocity of a statistical system \(\mathbf{V} (\mathbf{U}^\mu)\) and the position of hyperplane on which the system is considered. This position will be given by the normal vector to hyperplane \(\mathbf{N}^\mu\).

In the following consideration we obtain a relativistic description of Bose and Boltzmann ideal gases on an arbitrary hypersurface in an arbitrary reference frame. We apply the developed covariant method of a partition function to studying the spectra of secondary particles in the hydrodynamical theory of multiple production of hadrons.

2. Let us consider an equilibrium gas of free uncharged scalar particles with mass \(m\) in a rectangular box of volume \(V = a_1 a_2 a_3\).

Since the equation of motion is covariant
\[
(\Box - m^2) \psi = 0
\]
the boundary conditions should also be of covariant form.

Following [8,9] we give a covariant generalisation of periodic boundary conditions:
\[
\psi(x + a_1 f_{(i)}^1) = \psi(x + a_2 f_{(i)}^2) = \psi(x + a_3 f_{(i)}^3) = \psi(x)
\]
where three space-like vectors \(f_{(i)}^1\) define the directions of periodicity in Minkowski space, \(a_i\) are the corresponding periods. 4-vectors are given such that in a rest frame of rectangular box containing the field, the boundary conditions be reduced to periodic ones in space coordinates directed along the edges of a box
\[
f_{(i)}^1 = (0,1,0,0); \quad f_{(i)}^2 = (0,0,1,0); \quad f_{(i)}^3 = (0,0,0,1).
\]
We set a covariant scalar product
\[
\langle \psi_1, \psi_2 \rangle = i \int (\bar{\psi}_1 \frac{\partial}{\partial \mu} \psi_2) d\sigma^\mu
\]
where the surface \(t^\mu = \text{const}\) is fixed by three carrier vectors \(f_{(i)}^1, f_{(i)}^2, f_{(i)}^3\). The integral (6) is taken in
The field operator \( \Phi(x) \) satisfying (3), (4) is expressed by means of the creation and annihilation operators with standard commutation relations and a complete orthonormalized set of eigenfunctions of the momentum operator

\[
\Phi(x) = \sum f \left[ a^+_f \Phi_f(x) + a^+_f \Phi_f(x) \right] = \Phi^{(+)}(x) + \Phi^{(-)}(x) \tag{7}
\]

where

\[
\rho_f^2 = \frac{2\pi \ell^2}{a^2}, \quad \rho^2 = \sqrt{\rho^2 + m^2}; \quad \ell = (\ell, \ell, \ell), \quad \ell' = \ell' \ell \ldots \tag{9}
\]

The basis wave functions (8) are defined invariantly in any reference frame. The invariant Fock space of the states

\[
|0\rangle, \quad a^+_f |0\rangle, \ldots, \prod_\kappa \frac{1}{\sqrt{N_\kappa}} (a^+_\kappa)^N |0\rangle \tag{10}
\]

is constructed by means of the corresponding operators \( a^+_f \). The locally conserved dynamical variables \( T^\mu_\nu(x) \) are expressed via the field operator \( \Phi(x) \) in a usual manner. The current of uncharged scalar particles has the form [10]:

\[
j^\mu(x) = j^\mu(x', x') \bigg|_{x' = x'} = \Phi^{(-)}(x) \frac{d}{d\nu} \Phi^{(+)}(x) \bigg|_{x' = x'}; \quad \nabla^\nu j^\mu = 0 \tag{11}
\]

The integral quantities, namely, the operator of a number of particles crossing the hypersurface \( \Sigma \) and the energy-momentum operator \( P^\mu \) on this surface are defined in a standard way.

\[
N(\Sigma) = \int d\sigma_x j^\mu(x) \quad P^\mu(\Sigma) = \int_{\Sigma} d\sigma_x T^\mu_\nu(x) \tag{12}
\]

It is easy to see that only when the surface \( \Sigma \) in coordinates of the rest frame coincides with that of a constant time \( \Sigma = \Sigma_t \), the number of particles and energy-momentum operators are represented as simple-mode sums. At \( \Sigma \neq \Sigma_t \), such a simple representation doesn't take place. The relation of this fact to closedness of a statistical system was considered in detail in Ref. [9]. Following these works in order to introduce the operators \( N(k, \Sigma) \) of a number of particles with the momentum \( k \), we consider the Wigner representation of bilinear operators.

The covariant modification of the Wigner representation for
periodic operator functions can be written as [9]:

\[ j^\mu(x,k) = \int \frac{d\theta_{\alpha \beta}}{2\pi^2} \int \int \int \frac{d^3\theta}{8V} e^{iky} j^\mu(x, y) \]  

(13)

where \( \vec{f}^{\mu} = \vec{f}_{ij} \) are 4-vectors of periodicity directions (5), \( \vec{f}_{\alpha \beta} \) is the time-like vector orthogonal to them. The integration in Eq. (16) is performed with respect to scalar variables \( \theta_{\alpha \beta} \) which are the projections of splitting variables \( y \) on orthonormalized vectors \( \vec{f}_{ij} \). The form of the covariant Wigner representation (13) is connected with the periodicity of the function \( j^\mu(x, y') \) of variables \( \theta_{ij} = y \cdot \vec{f}_{ij} \) with periods \( 2a_i \). The inverse transformation to (13) looks as follows

\[ \int dq_z \sum_{k_z} j^\mu(x, k_z) = j^\mu(x) \]  

(14)

where \( q_z^{ij} = k_z^{ij} = \frac{2\pi}{a_i} \). In the thermodynamic limit of \( V \rightarrow \infty \) the Wigner transformations (13), (14) take a standard form [11].

In accordance with the basis principles of quantum theory only the distribution functions either in the coordinate or in momentum space are meaningful. Integrating \( j(x,k) \) over the hyperplane bounded by the world lines of a box (periodicity cells) one finds, in a certain reference frame, the operator of number of particles with momentum \( k \) crossing the hypersurface \( \Sigma \) :

\[ N(k, \Sigma) = \int d\sigma_{kj} j^\mu(x, k). \]  

(15)

In order to calculate statistical averages we introduce a density matrix. The density matrix given in the invariant basis (10) has an invariant form. Therefore the statistical operator in any inertial system is reduced by its definition in the rest frame

\[ \rho = e^{-\frac{1}{T} (H^*-\mu \tilde{Q})} \]  

(16)

where

\[ H^* = \int d^3x \tilde{T}^{\mu\nu}(x^*). \]  

(17)
me. \( \mu \) is the chemical potential. In our case \( \mu = 0 \).

Using the density matrix (16) we find for the average occupation numbers of one-particle levels \( \rho \) (9) on the space-like hyperplane \( \Sigma \) with the normal vector \( n^\mu \) in the reference frame where the gas moves as a whole with 4-velocity \( u^\mu \).

\[
\overline{N}(\rho, \Sigma) = \frac{\rho \cdot n}{u \cdot \rho \cdot u \cdot n} \left( z^4 \exp \frac{\rho \cdot u}{T} - 1 \right)^{-1},
\]

where the fugacity \( z^4 \rightarrow 1 \).

The most simple way to describe the Boltzmann gas using the formalism developed for Bose-particles is to consider a priori a single-particle system. The statistical average of (15) in single-particle states reads as

\[
\overline{N}_1(\rho, \Sigma) = \frac{1}{z^4} \frac{\rho \cdot n}{u \cdot \rho \cdot u \cdot n} \exp \left( -\frac{\rho \cdot u}{T} \right).
\]

Let us determine the correlation function for Bose-particles in the occupation numbers space of momenta

\[
R(\rho_{\xi}, \rho_{\xi}, \Sigma) = \sum \rho N(\rho_{\xi}, \Sigma) N(\rho_{\xi}, \Sigma) - \overline{N}(\rho_{\xi}, \Sigma) \overline{N}(\rho_{\xi}, \Sigma)
\]

In the approximation of large volume \( V_\xi \) we find

\[
R(\rho_{\xi}, \rho_{\xi}, \Sigma) = \frac{1}{2} \left[ \frac{\rho_{\xi} \cdot n}{u \cdot \rho_{\xi} \cdot n \cdot u \cdot \rho_{\xi}^o} \right] \left[ \text{ch} \frac{\rho_{\xi} \cdot u}{T} - 1 \right] \delta_{\xi, \xi^o}
\]

When calculating the correlator the contribution of nondiagonal elements into the current (15) is taken into account. The appearance of these elements is due to nonclosedness of a statistical system. According to the formula that follows from (9) and the transition at \( V_\xi \rightarrow \infty \) the intermediate summation over \( \rho \) on the level is replaced by the integration over \( \rho \) :

\[
\sum \rho \rightarrow \int d^3 \rho \frac{V_\xi}{(2\pi)^3} \frac{u \cdot \rho}{\rho^o}.
\]

The relations (18), (19), (21) show that the distribution and correlation functions are invariants only at a fixed surface. These functions are isotropic in \( \xi \) (at \( \xi_i \rightarrow -\xi_i \)) just on the surface \( \Sigma = \Sigma_{i \xi} : t = \text{const} \). On other hypersurfaces \( \Sigma \neq \Sigma_{i \xi} \) the distribution and correlation functions are anisotropic. In the rest frame of a gas this anisotropy manifests itself simply as anisotropy with respect to the momenta of particles.

It should be noted that a total number of particles in the gas is an invariant value independent of the choice of hyper-
plane \( \Sigma \)
\[
\overline{N}(\Sigma) = \sum \overline{N}(p, \Sigma) = \sum \left( z^{-1} \exp \frac{p \cdot u}{T} - 1 \right)^{-1} = \overline{N}.
\]  

(23)

This statement follows from easily verified relation
\[
\lambda_e + \lambda_{-e} = 2, \quad \lambda_e = \frac{p \cdot n}{u \cdot u}, \quad -\ell = (-e_1, -e_2, -e_3).
\]  

(24)

The expressions for the gas energy and momentum on the space-like hyperplane \( \Sigma \) result from (21) or directly from (18):
\[
\rho^\mu(\Sigma) = \sum p_e^\mu \overline{N}(p_e, \Sigma) = \rho^\mu + \overline{P} V(u^\mu - \frac{n^\mu}{u \cdot n}).
\]  

(25)

where
\[
\rho^\mu = \sum p_e^\mu \overline{N}(p_e, \Sigma_{t_e}), \quad \overline{P} = \frac{1}{3} \sum \frac{\left( \overline{p}_{e^\mu} \right)^2}{\rho_e^\mu(\Sigma_{t_e})} \overline{N}(p_e, \Sigma_{t_e})
\]  

(26)

the thermodynamic energy and momentum are 4-vector depending on the choice of hyperplane \( \Sigma \) to which a statistical system is referred to. The relations (26) give the energy determined in a standard ray by synchronous measurements in the rest frame of a thermodynamical system, the value \( \overline{P} \) determines the invariant pressure [6].

3. This section deals with the construction of the covariant partition function method for calculating the thermodynamic quantities of ideal gases on an arbitrary hypersurface in an arbitrary inertial reference frame.

The partition function \( Z \) determined according to (16) is invariant and can be given for a Bose-gas as follows:
\[
Z_1 = \prod (1 - z_e \exp (- \frac{p_e \cdot u}{T}))^{-1} = \prod (1 - z_e \exp (- \frac{p_e \cdot u}{T})),
\]  

(27)

where \( z_e = z = 1 \), \( \Lambda_e(u, n) \) are such numbers that \( \Lambda_e + \Lambda_{-e} = 2 \). We represent the statistical function on the surface \( \Sigma \) in such a form that the calculation of \( \overline{N}(p_e, \Sigma) \) should be made according to the classical partition function method
\[
\overline{N}(\Sigma, p_e) = \frac{\partial}{\partial z_e} \ln Z = \Lambda_e \left( z^{-1} \exp \frac{p \cdot u}{T} - 1 \right)^{-1}
\]  

(28)
Comparing (28) and (18) we obtain for the space-like surface
or the light cone surface

\[ \Lambda_e(u, n) = \frac{\rho \cdot n}{u \cdot n u \cdot \rho} = \lambda_e. \]  

(29)

In this case the desired condition (24) is satisfied which implies the invariance of a total number of particles and its independence of the choice of a surface. As can be easily seen the condition \( (24) \) is also satisfied on the arbitrary time-like hyperplane \( \Sigma : |n^0| < |\vec{n}| \). In this case, however, in the region of momenta for which \( \lambda_e < 0 \) or equivalently

\[ n \cdot u_e < 0, \quad \mu_e = \frac{\rho \cdot n}{m} \]

the numbers of particles become negative \( \bar{N}(\Sigma, \rho_e, \lambda_e < 0) < 0 \). The condition (30) means that the trajectory of a classical particle intersects the surface \( \Sigma \) outside the hypersurface, see Fig.1. Such particles are not involved in the initial system when the latter is analyzed in one periodicity cell. These are given a repeated account in the presence of walls retaining a statistical system. As seen from (24) and Fig.1 negative numbers of particles with the momentum \( \rho_e \) contributing to a total number of particles are compensated by the same "non-physical" addition of numbers of particles with the momentum \( \rho_e \) which are not involved in the initial system (see Fig.1). For these particles the conditions resulting from (30) are satisfied

\[ \lambda_e > 2 ; \quad -\bar{N}(\Sigma, \rho_e, \lambda_e < 0) = \bar{N}(\Sigma, \rho_e, \lambda_e > 2) - \bar{N}(\Sigma, \rho_e, \lambda_e = 2) \]

(31)

For a box with walls an additional contribution to a number of particles with momentum \( \rho_e \) also corresponds to a multiple accounting one and the same classical particle intersecting the surface \( \Sigma \) due to the reflection from the walls (see Fig.1). In accordance with Eq. (31) the exclusion of nonphysical contributions to numbers of particles when choosing the time-like surface is guaranteed by the addition of the condition \( O < \Lambda_e(u, n) < 2 \) to that in (31). In this case on an arbitrary hypersurface \( \Sigma \) \( \Lambda_e \) is of the form

\[ \Lambda_e(u, n) = \lambda_e \Theta(\lambda_e) \Theta(2 - \lambda_e) + 2 \Theta(\lambda_e - 2), \]

(32)
Thus, the relativistic partition function for an ideal
Roose gas moving with velocity $U^\nu$ on the arbitrary hyperplane $\Sigma$ (with the normal $n^\nu$) reads as
\begin{equation}
Z(\Sigma, z, \beta) = \prod_{\epsilon} \left[ 1 - \frac{Z}{z} \exp(-\beta \cdot \rho) \right] = \Lambda(\rho, \beta)
\end{equation}
where $\Lambda(\rho, \beta)$ is given according to (32). This partition function is invariant and independent of the choice of hypersurface.
$N(\Sigma, \rho)$ is calculated by (28). A total number of particles has the following representation
\begin{equation}
\overline{N}(\Sigma) = \frac{1}{2} \partial_{\rho^\nu} \ln Z(\Sigma, \rho^\nu, \beta) \Big|_{\rho^\nu = \frac{\mu^\nu}{T}} = \text{inv}
\end{equation}
The correlator is calculated according to
\begin{equation}
\overline{N}(\Sigma, \rho) \overline{N}(\Sigma, \rho) = \left\{ \left[ \frac{\partial}{\partial z} \left[ \frac{\partial}{\partial z} \right] \right] Z(\Sigma, \rho, \beta) \right\} \Big|_{\rho^\nu = \frac{\mu^\nu}{T}}
\end{equation}
and on the space-like surface coincides with (20), (21).

Thermodynamic energy and momentum in a moving system on the hyperplane $\Sigma$ are determined by the relations
\begin{equation}
P^\nu(\Sigma, U) = -\frac{\partial}{\partial \beta^\nu} \ln Z(\Sigma, \beta, z) \Big|_{\beta^\nu = \frac{\mu^\nu}{T}}
\end{equation}
On any fixed hyperplane the values $P^\nu$ constitute a 4-vector but its quantity due to unclosedness of the system changes under transition from one hypersurface to another. The thermodynamic energy-momentum systems (36) coincide with (25) only on the space-like surface (just on which the relations (25) were obtained). On the time-like surface the results (25) do not take place since they contain multiple contributions of the same particles. In this case the energy-momentum is defined only by the relation (36). It represents the energy-momentum of particles which can be measured when the system breaks (i.e. the external wall pressure is removed) on the time-like surface.

The entropy is also expressed by the partition function (33) being, as easily seen, the invariant
\begin{equation}
S = -\rho \ln \rho = \left\{ \ln Z(\Sigma, \beta, z) - \beta^\nu \frac{\partial}{\partial \beta^\nu} \ln Z(\Sigma, \beta, z) \right\} \Big|_{\beta^\nu = \frac{\mu^\nu}{T}}
\end{equation}
The invariant pressure is also determined by the partition function of a grand canonical ensemble (33)
The partition function of a particle in a box is described using the same arguments. For the distribution function we find by means of (19)

\[ Z_1(\Sigma, \beta) = \prod \Lambda_e(\nu, n) \exp(-\rho_e \beta) |_{\beta = \frac{\mu}{T}}. \tag{39} \]

The partition function for a \( N \)-particle Boltzmann gas is calculated as a product of one-particle values (39) with a factor \( (N!)^{-1} \).

Going over to the thermodynamic limit \( V \to \infty \) by means of the transition rules (22) we find for the Boltzmann- and Bose-gases

\[ Z_N(\Sigma, \beta) = \frac{1}{N!} \left( \prod_i d^4 \rho_i \frac{2V}{(2\pi)^3} u \cdot \rho \Lambda_{\rho}(\nu, n) \delta(\rho_i^2 - m^2) \right) e^{-\rho \cdot \rho} |_{\beta = \frac{\mu}{T}}. \tag{40} \]

\[ \ln Z(\Sigma, \beta, \rho) = -\int d^4 \rho \frac{2V}{(2\pi)^3} u \cdot \rho \Lambda_{\rho} \ln(1 - \rho \cdot \rho \delta(\rho^2 - m^2)) |_{\beta = \frac{\mu}{T}, \rho \cdot \rho = \tilde{\rho}}. \tag{41} \]

The statistical and thermodynamical characteristics of the gases are found on the basis of equations of this section with the replacement \( \delta e \to \delta \rho \) and passage from an ordinary derivative in \( \delta e \) to a functional one.

4. Here we consider some problems concerning the application of developed theory to physics of multiparticle production processes.

One of the most wide-spread approaches in studying the processes of multiple hadron production in hadron-hadron and nuclear-nuclear collisions at high energies is the use of hydrodynamic approximation. This approach proposed by L.D. Landau suggests that the time evolution of multiparticle production process consists of three stages:

1) formation of initial conditions;
2) expansion of relativistic fluid;
3) the decay of the fluid elements into final particles which takes place on a hypersurface defined from the condition for the temperature of the fluid element to be equal to critical one \( T_c \), \( T_c \approx \frac{m \rho}{5} \), \( m \rho \) is the \( \rho \)-meson mass.
this surface the system is an ideal gas of $\Pi$-mesons freely expanding into vacuum.

Here we examine directly the third stage of the process setting aside the first two stages. Let the hyperplane of decay $\Sigma_c$ be $t_c = f(\mathcal{X})$. According to the assumption $/3/$ each element of this hypersurface is occupied by a Bose-gas in the equilibrium (local) state. We find the partition function of this system using the results obtained in the previous section.

The partition function defined by (41) is given on the hyperplane with a normal vector $\mathcal{N}^\mu$. The four-vector of the site of this hypersurface cut by the world lines of fluid element is as follows

$$
\int \frac{d\sigma^\mu}{\Delta V^\mu} = \Delta \Sigma^\mu = \Delta \Sigma n^\mu = \frac{\Delta V^\mu \cdot n^\mu}{u \cdot n}, \quad (42)
$$

where $\Delta V^\mu$ is the volume of fluid element measured in its system.

Therefore assuming the function $t_c = f(\mathcal{X})$ to be sufficiently smooth we can write the partition function of the Bose gas being in the local thermodynamic equilibrium on the hypersurface $\Sigma$ with the 4-vector $N^\mu(\mathcal{X}) = \frac{u^\mu(\mathcal{X})}{T_c}$ changing smoothly along the hypersurface as

$$
\ln Z(\Sigma, \beta^\mu(\mathcal{X}), \mathcal{Z}) = \int d^4 p \int d\sigma^\mu [\rho^\mu \theta(\lambda_p)\theta(2-\lambda_p) + u^\mu(\mathcal{X}) \rho \cdot u(\mathcal{X})].
$$

$\theta(\lambda_p-2) \frac{2\pi}{(2\pi)^3} \delta(p^2-m^2) \ln [1 - \frac{\rho}{\rho^\mu(\mathcal{X})} \exp(-\beta^\mu(\mathcal{X}) \rho^\mu)] \left| \beta^\mu(\mathcal{X}) = \frac{u(\mathcal{X})}{T_c}, \mathcal{Z}_x = 1 \right. \quad (43)

where $g$ is the number of internal degrees of freedom of particles, $\lambda_p$ is defined according to (24)

$$
\lambda_p = \frac{p \cdot n(\mathcal{X})}{u(\mathcal{X}) \cdot n(\mathcal{X}) \cdot u(\mathcal{X}) \cdot p}. \quad (44)
$$

In order to find the distribution function of particles in the momenta for a locally-equilibrium system defined by the partition function (43) we use the relation (28). Then in virtue of the additivity property for the distribution function in $\Sigma^\mu$ we obtain

$$
\frac{\rho \cdot dN^\mu(\mathcal{X})}{d^4 p} = \rho \frac{\partial}{\partial \rho} \delta \ln Z(\Sigma_c) \bigg|_{\lambda_p=1} = \int d\sigma^\mu [\rho^\mu \theta(\lambda_p)\theta(2-\lambda_p) + u^\mu(\mathcal{X}) \rho \cdot u(\mathcal{X})]. \quad (45)
$$

$$
\rho^\mu \cdot u(\mathcal{X}) \theta(\lambda_p-2) \left( \frac{\partial}{\partial \rho} \left( \exp \beta^\mu(\mathcal{X}) \rho - 1 \right) \right)^{-1}
$$
The correlations of particle densities in the space of momenta on the decay surface in the continuous momentum limit are also additive in $\Sigma$ and can be obtained from the partition function (43) using (35), (45): 

$$\rho \frac{dN}{d^3p} = \rho \frac{dN}{d^3p} = \int d\Omega \left[ \rho \Theta(\xi) \Theta(\xi - 1) \right] \frac{1}{(2\pi)^3} \left( \exp \frac{\xi}{T} - 1 \right)^{-1}$$

According to the statement 3) in the hydrodynamic theory of multiparticle production particles preserve their momenta which they had on the hypersurface $\Sigma$. Then $\rho = \frac{\rho}{T}$. If the decay hypersurface has no time-like pieces which is inherent for exactly solvable models, then the distribution function of secondary particles is of the form

$$\rho \frac{dN}{d^3p} = \int d\Omega \rho \frac{dN}{d^3p} = \rho \frac{dN}{d^3p}$$

As to the correlation of identical particles, it should be noted that in detecting the particles far off from the decay hypersurface $\Sigma$, it is necessary to take into account the particle interference arising from different sites of a locally equilibrium system. The relativized Hanbury-Brown and Twiss method will be considered in detail elsewhere.

The expression for $dN/d^3p$ was obtained earlier in Refs. [4, 5] and in [5] where the analysis was made from the viewpoint of the theory developed here, the local anisotropic effects in the multiparticle production processes were revealed. Indeed, as follows from the formulas (47), (42) in the rest frame of a certain fluid element in the vicinity of the point $X$, the distribution function of the decay product of this element turns to be anisotropic

$$\rho \frac{dN}{d^3p} |_{X} = \frac{\rho}{(2\pi)^3} \left( \exp \frac{\rho}{T} - 1 \right)^{-1}$$

Such anisotropy is due to the fact that the decay surface in the rest frame of a fluid element does not coincide with the surface of a constant time in this system $n_x \neq (1, \vec{0})$.

It exemplifies the necessity to construct statistical systems
Of special interest are the processes in which the transition of quark-gluonic matter into a hadron gas occurs partially or completely on time-like surfaces. Recently such processes have been associated with the presence of shock deflagration or detonation waves in quark-gluon plasma formed in high-energy hadron-nuclear collisions. In this case the distribution functions are described by relations (45) for the statistical theory on the time-like surfaces. A detailed analysis of such situations is impossible in the framework of this paper; at the same time from (45) there follows a significant qualitative conclusion concerning such processes. The presence of $\theta$-functions in the expressions for the distribution functions (45) and correlations (46) results in characteristic bends in the momentum dependence of corresponding values. Experimental detection of such bends can turn out to be an important criterion for choosing a specific mechanism of multiparticle production processes.

5. Our formulation of statistical theory of ideal gases proves to be covariant and relativistically-invariant since it enables one to describe a statistical system in an arbitrary inertial reference frame on an arbitrary (including time-like) hypersurface. The developed relativistic method of the partition function having for the Boltzmann and Bose gases the form of (33), (40), (41) and (43) allows one to find all thermodynamical and statistical characteristics of a system on an arbitrary hypersurface in the Minkowskii space in a simple way such as in the classical theory. In virtue of the representations developed here the distinction in the expressions for relativistic partition function (1) and (2) becomes transparent. According to a general expression (40) for the partition function of the Boltzmann gas, the partition function (1) describes the gas in an arbitrary inertial system on the hypersurface $\Sigma_t: t = \text{const}$ where the gas moves with 4-velocity $U^\mu$. The partition function (2) describes the gas in the same system on the hypersurface $\Sigma_t: t = \text{const}$ where $\eta^\mu = (1, \vec{0})$. In this sense the partition functions (1) and (2) describe various objects with different distribution functions, energies etc., but of the same temperature, entropy and pressure. The
necessity to consider statistical systems on a set of different hypersurfaces is connected, first of all, with the relativistic theory in which simultaneity is a conventional notion.

At the same time we can now turn to a nonrelativistic limit and consider nonrelativistic statistical systems on the surfaces \( \Sigma \) in a moving Galilean frame of reference. Using the relation (40) it is not difficult to find the particles distribution in a gas moving with velocity \( \mathbf{U}_0 \) on the surface \( t = \mathbf{A} \cdot \mathbf{x} \). The corresponding analogue of the Maxwell-Boltzmann distribution is of the form

\[
\frac{dN}{d^3\mathbf{v}} = D N \exp \left[ - \frac{m(\mathbf{v}^2 - \mathbf{V}_0^2)}{2kT} \left( \lambda_v \Theta(\lambda_v) \Theta(\lambda_v - 2\lambda_v) + 2\Theta(\lambda_v - 2) \right) \right], \tag{49}
\]

where \( \lambda_v = \frac{1 - \mathbf{A} \cdot \mathbf{v}}{1 - \mathbf{V}_0 \cdot \mathbf{A}} \), \( N \) is the total number of particles in a gas. \( D \) is the normalized multiplier. At \( \mathbf{A} = 0 \) the function (49) transforms into the known Maxwell-Boltzmann distribution function in a moving frame of reference.

The corresponding distribution function (49) is directly observable in the processes accompanied by the system decay ("removal" of external walls, pressure etc.) which occur not simultaneously but on a certain surface \( t = \mathbf{A} \cdot \mathbf{x} \) in spacetime. Quite naturally, it is not the purpose of our paper to describe such specific physical processes. Here we would like only to emphasize that nonclosedness of an equilibrium system leading to nontrivial physical effects in a relativistic situation manifests itself also in nonrelativity.

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Fig. 1. Kinematics of the intersection of flows of classical particles with velocities $\frac{1}{2} < \nu_\alpha < 1$ and $\nu_\alpha = -\nu_\alpha$ on the time-like surface $\Sigma$ with the 4-vector of normal $n^{\mu} = (1, \alpha)$ in the two-dimensional Minkowskii space. The vertical straight lines $x = 0$ and $x = \alpha$ are the trajectories of the rest "box" walls. The dotted lines indicate geometric regions of formation of particle flows intersecting repeatedly the hypersurface $\Sigma$ due to "reflection" from the walls: $\nu_\alpha \rightarrow \nu_\alpha \rightarrow \nu_\alpha$. 
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