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E.H. Saidi

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RENORMALIZABLE $N = 2$ SUPERSYMMETRIC AND GAUGE INVARIANT INTERACTIONS FROM THE $N = 2$ HARMONIC SUPERSPACE WITH CENTRAL CHARGES *

E.H. Saidi **
International Centre for Theoretical Physics, Trieste, Italy.

ABSTRACT

The $N = 2$ harmonic-superspace in the presence of central charges is developed. Renormalizable interactions unusual in $N = 2$ supersymmetric theories, are derived in a consistent way. Symmetries generated by the central charges are discussed. A certain equivalence between $N = 2$ harmonic superspace with and without central charges is established. A non-abelian generalization of the model is given.

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** On leave of absence from Laboratoire de Physique Theorique, Faculte de Sciences, P.O. Box 1014, Av. Ibn Battouta, Rabat, Morocco.

1. INTRODUCTION

Recently, $N = 2$ extended supersymmetric theories have been of interest in particle physics. The interest is both from the theoretical [1],[2],[3],[4] and also from the phenomenological [5],[6] points of view. For the first approach this comes as a natural consequence of the important recent developements in theoretical physics namely, the harmonic superspace (H-S) techniques [1],[3],[9], [10] and the discovery of anomaly cancellations in superstring theories [11],[12] which are believed to play some role for unification of YM theories and supergravity [13]. Concerning the $N = 2$ extended supersymmetric interacting theories, which are actually related to our present interest, it is worth mentioning the work of the authors in Ref. [1] (hereafter referred to as Gikos) for $N = 2$ (H-S) offering a powerful technique to study the $N = 2$ supersymmetric theories [14],[15],[16]. Much progress has been done in these aspects, in particular the unconstrained off-shell formulations of all $N = 2$ supersymmetric theories (hypermultiplets SYM and SG) [1] and their covariant quantizations [10],[15],[17]. However, this formulation leads, in general, apart from usual gauge interactions known in $N = 2$ supersymmetric ordinary theories [18], to a non-renormalizable global self-hypermultiplet interactions [1],[10],[16] and, furthermore, does not give any information on a eventual role of the central charge z nor on the massive version of the theories. A partial study of this problem has been done in Ref. [16].

In this present work, we shall focus our attention on these two points. We will try to generalize the $N = 2$ (H-S) formulation in presence of central charges and examine how one can get global renormalizable interactions and massive $N = 2$ supersymmetric (eventually gauge invariant) theories from this technique. This actually generalizes our previous results [19],[20].

This study is presented as follows. In Sec. 2, we recall the basic tools for $N = 2$ supersymmetry with central charges in detail. Then we generalize and reformulate in a covariant form the $N = 2$ (H-S) with central charges (H-S-C-C). This gives an explicit realization of the $N = 2$ supersymmetry with central charges z. In Sec. 3, we derive the most general global renormalizable $N = 2$ supersymmetric (eventually gauge invariant) superpotential by solving the central charges eigenstate equations. This approach recovers our previous works and answers the questions raised there. Also it creates other open questions especially the connection between the central charges symmetry (z-symmetry) and renormalizability and furthermore the connection between Fayet-Sohnius (F-S) hypermultiplet and Howe-Stelle-Townsend (H-S-T) hypermultiplet.

Some new features are found such as the passage from $N = 2$ (H-S-C-C) to $N = 2$ (H-S) without central charges; also we present a natural proof of the choice $\xi^{(q-3)} = \eta^{(q-3)} = \Delta^{(q-3)} = 0$ introduced before formally and finally the z-symmetry and the Gikos quartic self coupling (F-S) hypermultiplet [1],[10],[16] are discussed.

In Sec. 4, we extend our previous results to a non-abelian gauge theory in some detail and write down the full $N = 2$ supersymmetric, gauge invariant, SU(2) invariant renormalizable action with its global Yukawa couplings which are really the new features in the renormalizable global sector of $N = 2$ supersymmetric theories. Gauge transformations are also given.

Sec. 5 is devoted to conclusion, results and remarks.

2. GENERALITIES ON THE N=2 SUPERSYMMETRIC ALGEBRA WITH CENTRAL CHARGES IN THE N = 2 (H-S)

The $N = 2$ extended supersymmetric algebra [7],[8],[21] is a $\mathbb{Z}_2$ graded Lie algebra [22] consisting of an odd part containing a pair of $d = 4$ Majorana spinors Q^i_α, an even part formed of the usual Poincaré generators $(P_\mu, M_{\mu\nu})$ and SU(2) symmetry generators rotating the two fermionic operators Q^i_α. The graded Lie commutation relations of the odd generators which are actually anticommutators are summarized as follows *):

$$\begin{aligned} \{Q^i_a, \bar{Q}_{\dot{a}j}\} &= -2\sigma^\mu_{a\dot{a}} P_\mu \epsilon^i{}_j \\ \{Q^i_a, Q^j_b\} &= \epsilon_{ab} Z^{ij} \\ \{\bar{Q}_{\dot{a}i}, \bar{Q}_{\dot{b}j}\} &= \epsilon_{\dot{a}\dot{b}} \bar{Z}_{ij} \end{aligned} \tag{2.1}$$

where a, $\dot{a} = 1, 2$ are the usual Weyl dotted and undotted indices and the extra index $i = 1, 2$ is the SU(2) symmetry index. The 2×2 matrices ϵ, are the antisymmetric tensors with $\epsilon_{12} = \epsilon^{21} = -1$. Finally $z^{ij}(\bar{z}_{ij})$ are 2×2 antisymmetric complex matrix operators and are known as central charges. For convenience, we shall write them in the equivalent following form:

$$\begin{aligned} Z^{ij} &= 2\sqrt{2}\, i\, Z^{(-)} \epsilon^{ij} \\ \bar{Z}_{ij} &= -2\sqrt{2}\, i\, \bar{Z}^{(+)} \epsilon_{ij} \end{aligned} \tag{2.2}$$

where $z^{(-)}$ (and its conjugate $\bar{z}^{(+)}$) is a $\mathbb{C}$-operator, SU(2) singlet, and satisfies an abelian algebra [7] exactly as the translations P_μ in the Minkowski space-time, namely:

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*) Our notations are the same as those of Wess and Bagger [21].

$$[Z^{(+)}, Z^{(-)}] = [Z^{(-)}, \bar{Z}^{(+)}] = [\bar{Z}^{(+)}, \bar{Z}^{(+)}] = 0 \tag{2.3}$$

The extra indices *) (+) and (-) mean that $z^{(-)}$ and $\bar{z}^{(+)}$ turn out to be in the differential representation, exactly the light cone derivatives as we shall see, i.e. these operators can be defined as translations (2.2) along (two) extra directions. We shall adopt this technique by following the work of Ref. [16] which we will generalize and reformulate in an esthetic and covariant form.

The algebra (2.1) with its central charges defined in (2.2) can be adapted to the $N = 2$ (H-S). In the differential representation, we introduce two extra "space-time" coordinates: y^α, $\alpha = 0,1$ with translation operators along these two directions as the two central charges (2.2). Thus the new analytic subspace (A-S) of the (H-S) with central charges becomes (A-S-C-C)

$$\hat{Z}_A = (\hat{X}_A, u^\pm, \theta^+, \bar{\theta}^+) \tag{2.4}$$

where

$$\hat{X}_A = (x^\mu_A, y^\alpha_A) \; ; \; \mu = 0,\dots 3 \quad , \; \alpha = 0, 1$$

with

$$\begin{aligned} x^\mu_A &= x^\mu - 2i\, \theta^{(+}\sigma^\mu\bar{\theta}^{-)} && (1) \\ y^\alpha_A &= y^\alpha + \bar{\eta}^+ \gamma^\alpha \eta^- && (2) \end{aligned} \tag{2.5}$$

and where $\sigma^0 = I_{2\times 2}$ and σ^i the usual 2×2 Pauli matrices. The $\eta^\pm$ is defined as a $d = 2$ dimensional spinor

$$\begin{aligned} \eta^\pm &= \begin{pmatrix} \theta^\pm_a \\ \bar{\theta}^\pm_{\dot{a}} \end{pmatrix} = \begin{pmatrix} \theta^i_a \\ \bar{\theta}^i_{\dot{a}} \end{pmatrix} u^\pm_i = \eta^i u^\pm_i \\ \bar{\eta}^\pm &= \eta^{\pm T} \gamma^0 \end{aligned} \tag{2.6}$$

which looks like a $d = 2$ Majorana spinor, but it is not since its components are $d = 4$ Weyl spinors. The $d = 2$ γ. matrices are given by:

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*) These indices should not be confused with the $\mathrm{U}(1)$ charges of the analytic subspace [1].

$$\gamma^0 = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} \; ; \; \gamma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \; ; \; -i\gamma^0\gamma^1 \equiv \gamma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{2.7}$$

satisfying the $d = 2$ Euclidean Clifford algebra

$$\{ \gamma^\alpha , \gamma^\beta \} = 2\,\delta^{\alpha\beta} \tag{2.8}$$

Putting (2.6) and (2.7) in $(2.5)_2$, we recover the notations of Refs.[16]. Indeed:

$$\begin{aligned} y_A^0 &= y^0 + \eta^{+T}\eta^- = y^0 + (\theta^+\theta^- + \bar\theta^+\bar\theta^-) \equiv x_A^6 \text{ for ref [16]} \\ y_A^1 &= y^1 + \bar\eta^+\gamma^1\eta^- = y^0 + i(\theta^+\theta^- - \bar\theta^+\bar\theta^-) \equiv x_A^5 \end{aligned} \tag{2.9}$$

The supersymmetric transformations in the (A-S-C-C) are given by

$$\begin{aligned} \delta x_A^\mu &= -2i\,[\epsilon^-\sigma^\mu\bar\theta^+ + \theta^+\sigma^\mu\bar\epsilon^-] \\ \delta y_A^\alpha &= 2\,\bar\xi^-\gamma^\alpha\eta^+ \\ \delta\eta^\pm &= \xi^\pm = \begin{pmatrix} \epsilon^\pm \\ \bar\epsilon^\pm \end{pmatrix} \end{aligned} \tag{2.10}$$

The corresponding analytic superfields $q-u^c(1)$ charged: $\tilde\phi^{(q)}(\hat z_A)$ are defined as usual [1] except now there is a dependence on the extra coordinates y^α namely:

$$\begin{aligned} \tilde\Phi^{(q)}(\hat z_A) = \; & \tilde\varphi^{(q)} + \sqrt2\,\theta^+\tilde\psi^{(q-1)} + \sqrt2\,\bar\theta^+\tilde\chi^{(q-1)} + \theta^{+2}\tilde F^{(q-2)} + \bar\theta^{+2}\tilde G^{(q-2)} + \theta^+\sigma^\mu\bar\theta^+\tilde A_\mu^{(q-2)} \\ & + \sqrt2\,\theta^{+2}\bar\theta^+\tilde\zeta^{(q-3)} + \sqrt2\,\bar\theta^{+2}\theta^+\tilde\xi^{(q-3)} + \theta^{+2}\bar\theta^{+2}\Delta^{(q-4)} \end{aligned} \tag{2.11}$$

where each component $\tilde{\mathcal{C}}^{(q')}$ of the expansion (2.11) is actually a function of all the bosonic coordinates, in particular the y^α coordinates:

$$\tilde{\mathcal{C}}^{(q')} = \tilde{\mathcal{C}}^{(q')}(x_A, y_A, u^\pm) \tag{2.12}$$

and which can be expanded in a harmonic expansion as follows:

$$\tilde{\mathcal{C}}^{(q')}(x_A, y_A, u^\pm) = \sum_{n=0}^{\infty} C^{(i_1 \dots i_{n+q'}, j_1 \dots j_n)}(x_A, y_A)\, u^+_{i_1} \dots u^+_{i_{n+q'}} u^-_{j_1} \dots u^-_{j_n} . \tag{2.13}$$

where the symmetrization operation; (...), is taken with one unit strength for convenience. In the next section, we shall see how to handle the y^α dependence, but before let us complete this section by giving the supersymmetric generators, Q, and all the variety of covariant derivatives in the (HSCC) (2.4). To achieve that, it is useful to work with the light cone formalism for the y coordinates. It is defined as *):

$$\begin{aligned} y_A^{(\pm)} &= \frac{1}{\sqrt2}\,(y_A^0 \pm i y_A^1) \\ \partial^{(\pm)} &= \frac{1}{\sqrt2}\,(\partial_A^0 \mp \partial_A^1) \\ \gamma^{(\pm)} &= \frac{1}{\sqrt2}\,(\gamma^0 \pm i\gamma^1) \\ \gamma^{(\pm)}\gamma^{(\mp)} &= (1 \pm \gamma^3) \\ \{\gamma^+, \gamma^-\} &= 2\,I \\ \gamma^+ &= i\sqrt2 \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \; ; \; \gamma^- = -i\sqrt2 \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \end{aligned} \tag{2.14}$$

Using these relations (2.14), the supersymmetric generators Q^i and $\bar Q_i$ may be written in the $N = 2$ (H-S-C-C) as follows:

$$\begin{aligned} Q^+ &\equiv Q^i u_i^+ = -i\frac{\partial}{\partial\theta^-} - 2\sigma^\mu\bar\theta^+\partial_\mu^A - 2i(\gamma^0\gamma^{(-)})\eta^+\partial^{(+)} \\ Q^- &\equiv Q^i u_i^- = i\frac{\partial}{\partial\theta^+} \\ \bar Q^+ &\equiv \bar Q^i u_i^+ = i\frac{\partial}{\partial\bar\theta^-} - 2\theta^+\sigma^\mu\partial_\mu^A - 2i(\gamma^0\gamma^{(+)})\eta^+\partial^{(-)} \\ \bar Q^- &\equiv \bar Q^i u_i^- = -i\frac{\partial}{\partial\bar\theta^+} \end{aligned} \tag{2.15}$$

These generators satisfy the $N = 2$ supersymmetric algebra (2.1) adapted to the $N = 2$ (H-S-C-C) namely

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*) These indices should not be confused with $U^c(1)$ charges of the analytic subspace [1].

$$\begin{aligned}
\{Q^+_\alpha, \bar{Q}^-_{\dot\alpha}\} &= 2i\sigma^\mu_{\alpha\dot\alpha}\partial^A_\mu \\
\{Q^-_\alpha, \bar{Q}^+_{\dot\alpha}\} &= -2i\sigma^\mu_{\alpha\dot\alpha}\partial^A_\mu \\
\{Q^+_a, Q^-_b\} &= 2\sqrt{2}\,\epsilon_{ab}\,\partial^{(-)} \\
\{\bar{Q}^+_{\dot a}, \bar{Q}^-_{\dot b}\} &= -2\sqrt{2}\,\epsilon_{\dot a\dot b}\,\partial^{(+)}
\end{aligned} \tag{2.16}$$

Comparing with (A_1), one recognizes

$$\begin{aligned}
iZ^{(-)} &= \partial^{(-)} \equiv \frac{\partial}{\partial y^+} \\
i\bar{Z}^{(+)} &= -\partial^{(+)} \equiv -\frac{\partial}{\partial y^-}
\end{aligned} \tag{2.17}$$

which corresponds, as already mentioned, to translation operators along the extra directions parametrizing a 2-dimensional plane or a disc $\mathcal{B}^1(0,R)$ with a very large radius R.

The spinorial covariant derivatives $D^\pm$ and $\bar{D}^\pm$ are given by:

$$\begin{aligned}
D^+ &\equiv D^i u^+_i = \frac{\partial}{\partial\theta^-} \\
D^- &\equiv D^i u^-_i = -\frac{\partial}{\partial\theta^+} + 2i\sigma^\mu\bar\theta^-\partial^A_\mu - 2(\gamma^0\gamma^{(-)})\eta^-\partial^{(+)} \\
\bar{D}^+ &\equiv \bar{D}^i u^+_i = \frac{\partial}{\partial\bar\theta^-} \\
\bar{D}^- &\equiv \bar{D}^i u^-_i = -\frac{\partial}{\partial\bar\theta^+} - 2i\theta^-\sigma^\mu\partial^A_\mu + 2(\gamma^0\gamma^{(+)})\eta^-\partial^{(-)}
\end{aligned} \tag{2.18}$$

satisfying the algebra (A_2).

Finally the harmonic covariant derivatives are also modified to:

$$\begin{aligned}
\tilde{D}^{\pm\pm} &= D^{++} + \bar\eta^\pm\gamma^\alpha\eta^\pm\partial^A_\alpha && (1) \\
D^{\pm\pm} &= \partial^{\pm\pm} - 2i\theta^+\sigma^\mu\bar\theta^+\partial^A_\mu && (2) \\
D^0 &= \partial^{+-} - \partial^{-+} && (3)
\end{aligned} \tag{2.19}$$

with

$$\partial^{\pm\pm} = u^{\pm i}\frac{\partial}{\partial u^{\mp i}} \quad ; \quad \partial^{\pm\mp} = u^{\pm i}\frac{\partial}{\partial u^{\pm i}} \tag{2.20}$$

Now we want to explore the y^α dependence and study how to get a renormalizable gauge invariant coupling in $N = 2$ supersymmetric theories using the (H-S) techniques.

3. RENORMALIZABLE SELF INTERACTING HYPERMULTIPLETS IN THE (H-S-C-C) OR HYPER ϕ^3 THEORY

Let us first of all recall that the $N = 2$ on shell scalar multiplet consists of four real scalar fields and two Weyl spinors; $(0^4, 1/2^2)$ [8],[21]. According to their hierarchy, one can construct from this $N = 2$ scalar multiplet two different sets. First, the so called Fayet-Sohnius (F-S) hypermultiplet [23] described in the $N = 2$ (H-S) by a one $U^c(1)$ charged analytic superfield (ASF) $\tilde\phi^+$ given by (2.11, q = 1). This hypermultiplet describes a complex $SU(2)$ doublet of scalar fields: φ^i; $i = 1, 2$ and a Dirac spinor $SU(2)$ singlet $\psi_D = (\psi, \bar\chi)$. The other hypermultiplet, called Howe -Stelle-Townsend (H-S-T) hypermultiplet [24],[1] described by a zero $U^c(1)$ charged ASF Ω given by:

$$\begin{aligned}
\Omega(z_A) &= \omega + \sqrt{2}\theta^+\zeta^- + \sqrt{2}\bar\theta^+\bar\zeta^- + \tfrac{1}{2}\theta^{+2}H^{--} + \tfrac{1}{2}\bar\theta^{+2}\bar{H}^{--} + i\theta^+\sigma^\mu\bar\theta^+B^{--}_\mu \\
&\quad + \sqrt{2}\,\theta^{+2}\bar\theta^+\bar\xi^{(-3)} + \sqrt{2}\,\bar\theta^{+2}\theta^+\xi^{(-3)} + \theta^{+2}\bar\theta^{+2}\Delta^{(-4)} \\
&= \overset{*}{\bar\Omega}(z_A)
\end{aligned} \tag{3.1}$$

It consists of an $SU(2)$ real scalar singlet ω_0, an $SU(2)$ real scalar triplet $\omega^{(ij)}$ and an $SU(2)$ complex isodoublet of Weyl spinors ζ^i. All the other fields in (3.1) are auxiliary and play different second roles, namely closing off shell the $N = 2$ supersymmetric algebra (2.16) and keep off shell the harmonic expansions type (2.13).

Since we have given enough details on Ω superfields, let us complete this discussion by giving its free action S_Ω:

$$S_\Omega = \int d^4x\, du\, \mathcal{L}_\Omega \tag{3.2}$$

with

$$\mathcal{L}_\Omega = \int d^4\theta^+\, \Omega\, D^{++2}\, \Omega \tag{3.3}$$

or equivalently by integrating with respect to θ^+'s

$$\begin{aligned}\mathcal{L}_\Omega = & -6(\partial_\mu\omega)^2 + 2i\,\partial^{++}\varsigma^-\sigma^\mu\partial_\mu\bar\varsigma^- + \tfrac{1}{4}(\partial^{++2}H^{--})H^{--} \\ & +\sqrt{2}\,\partial^{++}\varsigma^-\partial^{++}\xi^{(-3)} + \sqrt{2}\,\partial^{++}\bar\varsigma^-\partial^{++}\bar\xi^{(-3)} \\ & -2\,\partial^{++}\omega\,\partial^{++}\Delta^{(-4)}\end{aligned} \tag{3.4}$$

From this Lagrangian one sees that the auxiliary fields H, ξ and Δ constitute two categories. First, the H^{--} auxiliary fields which appear quadratically in (3.4) and having two mass dimensions are necessary either to close off shell the supersymmetric algebra and contribute in general to the scalar potential of the theory. Therefore, they play a crucial role in the interacting supersymmetric theories exactly as in the N = 1 case. The second class consists of auxiliary fields (ξ' and Δ' in (3.1)) having mass-dimension higher than two (in general different from two as it is the case in the W-Z gauge) and appear linearly in (3.4). They give actually constraints on the degrees of freedom constituting Ω (3.1) and (2.13). Indeed, by eliminating this class of fields, one obtains [19]

$$\partial^{++2}\omega = \partial^{++2}\varsigma^- = 0 \tag{3.5}$$

and by solving these constraints, we find:

$$\begin{aligned}\omega(x_A,u^\pm) &= \frac{1}{2\sqrt{3}}\,\omega^0 + \frac{1}{2\sqrt{3}}\,\omega^{+-} \\ \varsigma^+(x_A,u^\pm) &= \varsigma^i\, u^+_i\end{aligned} \tag{3.6}$$

As we see, the constraints (3.5) put the harmonic expansions (2.13) on shell. Therefore the choice in (2.11) and (3.1)

$$\xi^{(q-3)} = \eta^{(q-3)} = \Delta^{(q-4)} = 0 \tag{3.7}$$

is a natural choice.

Putting (3.6) and (3.7) in (3.4), we get:

$$\begin{aligned}\mathcal{L}_\Omega = & -\tfrac{1}{2}(\partial_\mu\omega^0)^2 - 2(\partial_\mu\omega^{+-})^2 + 2i\,\varsigma^+\sigma^\mu\partial_\mu\bar\varsigma^- \\ & +\tfrac{1}{2}\bar H^{++}H^{--}\end{aligned} \tag{3.8}$$

or equivalently by integrating over the $u^\pm$'s:

$$\begin{aligned}\mathcal{L}_\Omega = & -\tfrac{1}{2}(\partial_\mu\omega^0)^2 - \tfrac{1}{2}\partial_\mu\omega^{(ij)}\partial_\mu\omega_{(ij)} + i\,\varsigma^i\sigma^\mu\partial_\mu\bar\varsigma_i \\ & +\tfrac{1}{8}\bar H^{(ij)}H_{(ij)}\end{aligned} \tag{3.9}$$

which describes the free (H-S-T) hypermultiplet. This is large enough for the (H-S-T) ASF Ω for our present study.

Before going ahead, let us also remark that extended massive supersymmetric representations are closely related to the central charges z [7],[8],[21],[25] so that the z operators are generally introduced whenever dealing with massive N-extended supersymmetric theories. In our case, we shall focus our attention on this fact to derive the most general renormalizable superpotential in N = 2 (H-S-C-C) which turns out to be a generalization of our previous model [19] obtained from general arguments and requirements namely: renormalizability, $U^c(1)$ neutrality and the gauge (3.7). This construction will allow us: first to understand the origin of the extra symmetries observed in Refs. [19],[20]. Second, it permits us also to well understand the source and the compatibility of the gauge choice (3.7) in renormalizable interactions and finally, it explains the consistency of our previous model and the existence of the invariant supersymmetric subspace ($\int d^2\theta^+$ and $\int d^2\bar\theta^+$) of the invariant analytic subspace (Gikos-superspace)[1] itself. Now, we go to the details.

To start, consider the set of analytic superfields type (2.11); $\tilde\Phi^{(q)}(\hat z_A)$ which define a $z^{(-)}$ and $\bar z^{(+)}$ eigenstate basis namely:

$$Z^-\,\tilde\Phi^{(q)}(y^\pm_A, z_A) = z^-(z_A)\,\tilde\Phi^{(q)}(y^\pm_A, z_A) \tag{3.10}$$

$$\bar Z^+\,\tilde\Phi^{(q)}(y^\pm_A, z_A) = -\bar z^+(z_A)\,\tilde\Phi^{(q)}(y^\pm_A, z_A) \tag{3.11}$$

We note by the way that (3.11) is not the conjugate of (3.10); $\tilde{\phi}^{(q)}$ is the same in both of them, the minus sign in (3.11) is introduced for convenience and finally ζ^- and $(\bar{\zeta}^+)$ are, respectively, eigenvalues (analytic superfields) of z^- and $\bar{z}^+$. The integration of (3.10) gives

$$\tilde{\Phi}^{(q)}(y^{\pm}_A, z_A) = e^{i\zeta^- y^+_A}\, \phi_1(y^-_A, z_A) \qquad (3.12)$$

and similarly for (3.11), we obtain

$$\tilde{\Phi}^{(q)}(y^{\pm}_A, z_A) = e^{i\zeta^+ y^-_A}\, \phi_2(y^+_A, z_A) \qquad (3.13)$$

so that the general solution of (3.10) and (3.11) reads as:

$$\tilde{\Phi}^{(q)}(y^{\pm}, z_A) = e^{i(\zeta^- y^+ + \zeta^+ y^-)}\, \phi^{(q)}(z_A) \qquad (3.14)$$

where $\phi^{(q)}(z_A)$ is actually an ordinary analytic superfield [1]. In terms of the normal coordinates y^α; (3.14) may be rewritten as

$$\tilde{\Phi}^{(q)}(y^{\pm}, z_A) = e^{i(\zeta^\alpha(z_A)\, y_\alpha)}\, \phi^{(q)}(z_A) \qquad (3.15)$$

where $\zeta^\alpha = (\zeta^0, \zeta^1)$ are $d = 2$ vector, zero $U^c(1)$ charged, ASF having one mass-dimension since y^α has a dimension of length. This is, in fact obvious since the z operators have one mass dimension according to (2.1). We note, also, by the way, that the ζ^α's are rotated by an SO(2) (or SO(1,1)) symmetry which is the underlying symmetry of the y^α plane. Indeed:

$$\begin{aligned} y'_\alpha &= O_\alpha{}^\beta y_\beta \\ \zeta'^\alpha &= O^\alpha{}_\beta \zeta^\beta \qquad O \in SO(2) \\ \zeta'^\alpha y'_\alpha &= \zeta^\alpha y_\alpha \end{aligned} \qquad (3.16)$$

The relations (3.14) and (3.15) admit an extra symmetry, connected with the central charge transformations, which have been observed in Ref. [20]. It is easily recognizable in the light cone notations (2.14). It is given by

$$\begin{aligned} \zeta^{0\prime} &= \zeta^0 + W \qquad (1^{\text{rst}}\ \text{translation}) \\ \zeta^{1\prime} &= \zeta^1 - iW \qquad (2^{\text{nd}}\ \text{translation}) \end{aligned} \qquad (3.17)$$

so that

$$\zeta^{-\prime} = \zeta^{0\prime} - i\zeta^{1\prime} = (\zeta^0 - i\zeta^1) + (W - W) = \zeta^- \qquad (3.18)$$

and $$\zeta^{+\prime} = \bar{\zeta}^{-\prime} = \zeta^+$$

Thus the symmetry signaled in Ref. [20] and present also in Ref. [19] is in fact the central charges transformations, which correspond to the translations (3.17).

Now, since the ζ^α's are one mass dimension, real analytic superfields and since they have zero $U^c(1)$ charges imply automatically that these ζ^α ASF's describe in fact: either two (H-S-T) hypermultiplets Ω^α (3.1) or one (H-S-T) hypermultiplet Ω and a constant which is nothing but the mass of the $\phi^{(q)}(z_A)$ ASF, as we shall see. Consequently, (3.15) becomes:

$$\tilde{\Phi}^{(q)}(y, z_A) = e^{ig\Omega^\alpha \cdot y_\alpha}\, \phi^{(q)}(z_A) \qquad (3.19)$$

or

$$\tilde{\Phi}^{(q)}(y, z_A) = e^{i[m y_1 + g\Omega y_0]}\, \phi^{(q)}(z_A) \qquad (3.20)$$

The relations (3.19) and (3.20) actually mean that an analytic superfield $\tilde{\phi}^{(q)}$, in particular the (F-S) hypermultiplet (q=1), in presence of the central charges $z^{(-)}(\bar{z}^{(+)})$ is equivalent to one of the following:

i) A massless ASF $\phi^{(q)}$ (respectively ϕ^+) in the absence of central charges interacting with two massless (H-S-T) ASF's Ω_α,

ii) A massive ASF $\phi^{(q)}(\phi^+)$ in the absence of the z's interacting with only one massless (H-S-T) hypermultiplet Ω_α.

Indeed, following Refs. [1], [10] and [16], the free action describing the (F-S) hypermultiplet ϕ^+ Eq.(2.11); q=1) may be generalized as follows:

$$S_{\tilde{\Phi}^+} = \frac{1}{\pi R^2} \int d\mu^{(-4)}\, \overset{*}{\tilde{\Phi}}{}^+ \tilde{D}^{++} \tilde{\Phi}^+ \qquad (3.21)$$

where the integration measure is

$$d\mu^{(-4)} = d^4x_A\, d^2y_A\, du\, d^4\theta^+ \qquad (3.22)$$

and where the integration over the even variables is done on the $M_4 \times S^2(0,1) \times \mathcal{D}^1(0,R)$ manifold namely for the y^α coordinates, for instance:

$$\int_{\mathcal{D}^1(0,R)} d^2y_A = \pi R^2 \qquad (3.23)$$

with

$$\mathcal{D}^1(0,R) = \{ y \mid 0 \leq y^\alpha y_\alpha \leq R^2 \} \qquad (3.24)$$

Now, putting $(2.19)_1$ and 3.19, for instance, in (3.21), we find

$$S_{\phi^+} = S_{\phi^+} + S_{int} \qquad (3.25)$$

where

$$S_{\phi^+} = \int d^4x_A\, du\, \mathcal{L}_{\phi^+} \qquad (3.26)$$

with

$$\mathcal{L}_{\phi^+} = \int d^4\theta^+\, \overset{*}{\tilde{\phi}}{}^+ D^{++}\phi^+ \qquad (3.27)$$

which is the usual Gikos term [1],[10] and which can be written explicitly exactly as we have done for (3.3) in the sense that remarks and particularly the gauge choice (3.7) still hold. The interacting term in (3.25) can be divided into two parts as follows:

$$\mathcal{L}_{int} = ig \int d^4\theta^+\, \bar{\eta}^+ \gamma^\alpha \eta^+\, \overset{*}{\tilde{\phi}}{}^+ \Omega_\alpha \phi^+ \qquad (3.28)_1$$

$$\mathcal{L}'_{int} = \frac{ig}{\pi R^2} \int d^4\theta^+ \left\{ \int_{\mathcal{D}^1} d^2y_A\, \overset{*}{\tilde{\phi}}{}^+ \phi^+ (D^{++}\Omega^\alpha)\, y_{A\alpha} \right\} \qquad (3.29)$$

Before continuing, we would like to remark that since we decided to give to the Ω_α's a physical meaning, namely a propagating degrees of freedom, we have to add to the action (3.13) a kinetic term for Ω_α exactly as one did in gauge theories. Therefore, the full action S, is given by

$$S = S_\Omega + S_{\phi^+} + S_{int} \qquad (3.30)$$

where S_Ω is given in (3.2+3).

The action (3.30) presents a global U(1) symmetry

$$\begin{aligned} \phi^{+\prime} &= e^{-i\alpha\Lambda}\, \phi^+ \\ \Omega'_\alpha &= \Omega_\alpha \\ \overset{*}{\tilde{\Lambda}} &= \Lambda\,; \quad D^{++}\Lambda = 0 \end{aligned} \qquad (3.31)$$

Also, it is interesting to note that is renormalizable by construction (3.21). However, the term (3.29) presents many remarkable features, in particular, it looks non-renormalizable since it presents a dimensional coupling constant. We shall return to it in more detail later. For this moment, let us examine first the coupling terms in (3.17) namely:

$$ig \int d^4\theta^+\, \bar{\eta}^+ \gamma^\alpha \eta^+\, \overset{*}{\tilde{\phi}}{}^+ \Omega_\alpha \phi^+ \qquad (3.28)_2$$

Using (2.6) and (2.9), one can establish:

$$\begin{aligned} \bar{\eta}^+ \gamma^\alpha \eta^+ \Omega_\alpha &= \eta^{+T} \gamma^0 \gamma^\alpha \eta^+ \Omega_\alpha \\ &= (\theta^{+2} + \bar{\theta}^{+2})\Omega_0 + i(\theta^{+2} - \bar{\theta}^{+2})\Omega_1 \\ &= (\Omega_0 + i\Omega_1)\delta^2(\theta^+) + (\Omega_0 - i\Omega_1)\delta^2(\bar{\theta}^+) \end{aligned} \qquad (3.32)$$

Thus (3.28) becomes *)

$$\mathcal{L}_{int} = -g \int d^2\theta^+\, \overset{*}{\tilde{\phi}}{}^+ (\Omega_1 - i\Omega_0)\phi^+ + (h - U(1)).c \qquad (3.33)$$

which turns out to have more than one meaning. First, we get a renormalizable trilinear coupling between two (F-S) ASF's ϕ^+ and one (H-S-T) hypermultiplet Ω_α twice. These interactions will generate global Yukawa couplings in general absent in $N = 2$ extended supersymmetric ordinary theories. Second, there exists an invariant subspace in the analytic subspace (Gikos superspace [1]) which closes under $N=2$ supersymmetric transformations. These transformations were given explicitly in Ref. [20]. Finally, it implies in some sense the gauge choice (3.7). Indeed, the interaction (3.33) is given explicitly by:

*) According to Ref. [17], the delta-function type $\delta^2(\theta^+)$ is not well defined in the (H-S)! For our discussion $\delta^{(2)}(\theta^+)$ means simply $\theta^{+2} = \theta^{a+}\theta^+_a$.

$$
\begin{aligned}
\mathcal{L}_{int} \;=\; & -m\,(\psi\chi+\bar\psi\bar\chi) \;+\; m\left[\varphi^{+}(\bar F^{-}-\bar G^{-})-\bar\varphi^{+}(F^{-}-G^{-})\right] \\
& + ig\omega\left[(\psi\chi-\bar\psi\bar\chi)+\varphi^{+}(\bar F^{-}+\bar G^{-})+\bar\varphi^{+}(F^{-}+G^{-})\right] \\
& - ig\,(\psi\varsigma^{-}+\bar\chi\bar\varsigma^{-})\bar\varphi^{+} \;+\; ig\,(\chi\varsigma^{-}-\bar\psi\bar\varsigma^{-})\varphi^{+} \\
& + \tfrac{1}{2}g\,\bar\varphi^{+}\varphi^{+}\,(H^{--}+\bar H^{--})
\end{aligned} \tag{3.34}
$$

For simplicity we have chosen $\Omega_1 = m/g$ and $\Omega_0 = \Omega$.
Thus the expression (3.34) does not depend at all on the auxiliary fields $\xi^{(q-3)}$, $\eta^{(q-3)}$ and $\Delta^{(q-4)}$, i.e. one can get the same result as (3.34) if these last fields are set to zero and consequently the gauge choice (3.7). This fact would be affected if the Grassmann measure is given by $d^4\theta^+$ which is the case for non-renormalizable interactions.

Furthermore, the interactions (3.34) turn out to be exactly those found in our previous model (up to the coeff i, m = 0) of Ref. [19] and also the mass term of Ref. [20] where Ω_0 is set to zero and $\Omega_1 = m$. Finally, we come back to the term (3.29) namely

$$
\mathcal{L}'_{int} \;=\; \frac{ig}{\pi R^2}\int d^4\theta^+\left\{\int_{D^1} d^2y_A\, \breve{\Phi}^{+}\Phi^{+}(D^{++}\Omega_\alpha)\cdot y^\alpha_A\right\} \tag{3.29$_2$}
$$

The first striking observation is that, if in $(3.29)_2$ we replace

$$
i\,D^{++}\Omega_1 \;=\; \breve{\Phi}^{+}\Phi^{+}
$$

and integrate only on an interval of length equal to $\Delta y_1 = R$; $y_2 = 0$ for instance, (the central charges symmetry (z-symmetry) is actually broken) we obtain:

$$
\mathcal{L}'_{int} \;=\; \tfrac{1}{2}gR\int d^4\theta^+\,(\breve{\Phi}^{+}\Phi^{+})^2 \tag{3.35}
$$

Thus $\mathcal{L}'_{int}$ becomes exactly the quartic non-renormalizable (F-S) self-coupling introduced in Refs. [1],[10] and generalized in Ref. [16] namely

$$
\mathcal{L}_{Gikos} \;=\; \lambda\int d^4\theta^+\,(\breve{\Phi}^{+}\Phi^{+})^2 \tag{3.36}
$$

Comparing (3.35) with (3.36), one discovers that λ is related to g linearly:

$$
\lambda = \tfrac{1}{2}gR \quad ; \quad [\lambda] = [R] = -1 \tag{3.37}
$$

We think that this is not a matter of coincidence, although we do not understand the real connection between the (F-S) and (H-S-T) hypermultiplets, and the relation between the z symmetry and renormalizability. Now we arrive at the y integration in (3.29) keeping the z symmetry, we have:

$$
\int_{D^1} d^2y\;\; \Omega^\alpha y_\alpha \;=\; \int_0^R r^2 dr \int_0^{2\pi} d\beta\, |\vec\Omega|\cdot\cos\beta \tag{3.38}
$$

with

$$
\begin{aligned}
0 \leq |\vec y| &= r \leq R \\
|\vec\Omega|^2 &= \Omega_0^2 + \Omega_1^2
\end{aligned}
\quad , \quad \beta = (\widehat{\vec\Omega,\vec y}) \tag{3.39}
$$

$$
(3.38) \;=\; |\vec\Omega|\cdot\frac{R^4}{4}\int_0^{2\pi} d\beta\,\cos\beta \;=\; 0
$$

Therefore on the disc $\mathcal{D}^1(0,R)$ (and also the circle $C^1(0,R)$), the term (3.29) vanishes due to the z symmetry and therefore does not contribute. Miracle! This is in fact not surprising since we have started from a renormalizable theory (3.21), it should be preserved as long as the theory is not affected. Accordingly, we can conclude at least two things: First, the z-symmetry is the generator of renormalizable trilinear Yukawa couplings as well as mass terms. Second, non-renormalizable terms (3.35) seems to correspond to a broken z-symmetry. The last point needs a careful and separate study. Progress in this aspect will be reported elsewhere.

Finally, and before closing this section, one can evaluate explicitly [26] the scalar potential by solving the equations of motion of the auxiliary fields F^+, G^+ and H^{++} from (3.26) and (3.33+34) and putting them back in:

$$
U(F,G,H) \;=\; \bar F^{+}F^{-} + \bar G^{+}G^{-} + \tfrac{1}{2}\bar H^{++}H^{--} \tag{3.40}
$$

and check that the minimum of this potential is exactly zero and therefore $N = 2$ supersymmetry is preserved.

In the next section, we will try to extend these results to the gauged theory. However, since we have already examined the abelian case [10],[20], we shall give the results only in non-abelian massive theories.

4. NON ABELIAN MASSIVE HYPER ϕ^3 THEORY

In this section, we shall set $\Omega_1 = m$ and Ω_0 equal to a (H-S-T) hypermultiplet describing the physical degrees of freedom and study in the first part the generalization of the hyper ϕ^3 theory to a global non-abelian theory. Later we will examine its gauge version.

4.1 Non abelian global hyper ϕ^3 theory

The global U(1) transformations (3.31) can be generalized straightforwardly to the non-abelian case. Indeed, consider n (F-S) hypermultiplets in the

$$\tilde{\Phi}^+_I(\vec{z}_A) \quad ; \quad I = 1 \ldots n \qquad (4.1)$$

fundamental representation of a simple (or semi-simple) Lie group G, say, for instance, SU(n). The analysis (3.15-21) can be translated here without difficulty. As a result, we find that (3.19) and (3.20) may be generalized as:

$$\tilde{\Phi}^{(q)}_I(\hat{z}_A) = \exp\left\{ ig\left[y^a\Omega_a(z_A) + \eta y^a \Sigma_a(z_A)\right]\right\}_{IJ} \phi^{(q)}_J(z_A) \qquad (4.2)$$

$$\tilde{\Phi}^{(q)}_I(\hat{z}_A) = \exp\left\{ ig y^0\Omega_0(z_A) + i y^0 m + i g' y^a \Sigma_a(z_A)\right\}_{IJ} \phi^{(q)}_J(z_A) \qquad (4.3)$$

where

$$\Omega_{IJ}(z_A) = \Omega^r T^r_{IJ} \; ; \; \mathrm{Tr}\,\Omega = 0$$
$$\Sigma_{IJ}(z_A) = \Sigma(z_A)\,\delta_{IJ} \; ; \; \overset{*}{\tilde{\Sigma}} = \Sigma \qquad (4.4)$$

describe, respectively, a set of (H-S-T) hypermultiplets in the adjoint representation of G and a singlet (H-S-T) hypermultiplet Σ transforming in the trivial representation of G which does not rotate the SU(n) vector γ_I. The operators T^r; r = 1, ...dim G are hermitian group generators in the fundamental representation satisfying the Lie brackets of the Lie algebra G namely

$$[T^r, T^s] = f^{rs}{}_t\, T^t$$

with

$$\mathrm{Tr}(T^r T^s) = \delta^{rs} \qquad (4.5)$$

We note by the way that more higher real representations of G can be added, but we will limit ourselves by studying the adjoint one only. The extension to the singlet is automatically realized. The action (3.25) may be generalized as follows:

$$S = \frac{1}{\pi R^2}\int d\mu^{(-4)}\left\{ \mathrm{tr}\left[\overset{*}{\tilde{\Phi}}{}^+ \tilde{D}^{++}\Phi^+\right] + \mathrm{Tr}\left[\Omega_a D^{++}\Omega_a\right]\right\} \qquad (4.6)$$

and using (4.3) where we put $\Sigma = 0$, we obtain as before:

$$\begin{aligned} S &= \int d^4x\, du\, \mathcal{L} && (1) \\ \mathcal{L} &= \mathcal{L}_0 + \mathcal{L}_{int} && (2) \end{aligned} \qquad (4.7)$$

with

$$\mathcal{L}_0 = \int d^4\theta^+ \left\{ \mathrm{Tr}\left[\Omega_a D^{++}\Omega_a\right] + \mathrm{tr}\left(\overset{*}{\phi}{}^+ D^{++}\phi^+\right)\right\} \qquad (4.3)$$

and

$$\mathcal{L}_{int} = -\int d^2\theta^+\, \mathrm{tr}\left[\overset{*}{\phi}{}^+ m\,\phi^+ - ig\,\overset{*}{\phi}{}^+\Omega\,\phi^+\right] + (h\text{-}U(1))\text{-}c \qquad (4.9)$$

The Lagrangian $(4.7)_2$ is invariant under the following non-abelian global transformations:

$$\begin{aligned} \Phi'^+_I &= \left[\exp(2i\alpha\Lambda)\right]_{IJ} \Phi^+_J \\ \overset{*}{\Phi}{}'^+_I &= \overset{*}{\Phi}{}^+_J\left[\exp(-2i\alpha\Lambda)\right]_{JI} \\ \Omega'_{IJ} &= \left[\exp(2i\alpha\Lambda)\,\Omega\,\exp(-2i\alpha\Lambda)\right]_{IJ} \\ \overset{*}{\tilde{\Lambda}} &= \Lambda = \Lambda^r T^r, \quad \text{global parameters} \\ D^{++}\Lambda &= 0 \end{aligned} \qquad (4.10)$$

The corresponding term to (3.29) vanishes here also as long as the y-integration is taken on all the disc $\mathcal{D}^1(0,R)$, i.e. the z-symmetry is still preserved. Furthermore, the superpotential (4.9), once again, does not depend at all on the $\xi^{(q-3)}$, $\eta^{(q-3)}$ and $\Lambda^{(q-4)}$ and leads to the same gauge (3.7). In this gauge the action $(4.7)_2$ becomes:

$$
\begin{aligned}
\mathcal{L} = \; & \mathrm{Tr}\left[-\tfrac{1}{2}(\partial_\mu \omega^0)^2 - 2(\partial_\mu \omega^{+-})^2 + 2i\, S^+ \sigma^\mu \partial_\mu \bar{S} + \tfrac{1}{2} \bar{H}^{++} H^{--} \right] \\
& + \mathrm{tr}\left[-2\, \partial_\mu \bar{\varphi}^+ \partial^\mu \varphi^- - i\, \psi \sigma^\mu \partial_\mu \bar{\psi} + i\, \chi \sigma^\mu \partial_\mu \bar{\chi} + \bar{F}^+ F^- + \bar{G}^+ G^- \right] \\
& - \mathrm{tr}\left[m(\psi \chi + \bar{\psi}\bar{\chi}) + m\left(\varphi^+ (\bar{F}^- - \bar{G}^-) - \bar{\varphi}^+ (F^- - G^-) \right) \right] \\
& ig\, \mathrm{tr}\left[(\psi \omega \chi - \bar{\psi} \omega \bar{\chi}) + \left(\varphi^+ \omega (\bar{F}^- + \bar{G}^-) \right) + \left(\bar{\varphi}^+ \omega (F^- + G^-) \right) \right] \\
& - ig\, \mathrm{tr}\left[(\psi S^- + \bar{\chi} \bar{S}^-) \bar{\varphi}^+ - (\chi S^- - \bar{\psi} \bar{S}^-) \varphi^+ \right] \qquad (4.11) \\
& + \tfrac{i}{2} g\, \mathrm{tr}\left[\bar{\varphi}^+ (H^{--} + \bar{H}^{--}) \varphi^+ \right]
\end{aligned}
$$

where we have assumed $m_{IJ} = m_I \delta_{IJ}$ and where for instance

$$
\omega_{IJ} = \omega_n T^n_{IJ} = \frac{1}{2\sqrt{3}} \left[\omega^0_n + 2\, \omega^{+-}_n \right] T^n_{IJ} . \qquad (4.12)
$$

Similarly as before, one can replace the auxiliary fields F^+, G^+, H^{++} in (4.12) by their equations of motion and obtain a renormalizable Lagrangian depending only on interacting physical degrees of freedom and check also as a consequence of $N = 2$ supersymmetry that Dirac spinor $(\psi, \bar{\lambda})$ and the SU(2) complex scalar doublet have the same mass. After that one can go ahead and compute the U's integration which is cumbersome but easy to perform in the gauge (3.7). One finds as a consequence of the global SU(2) symmetry [19] that the SU(2) triplet of the (H-S-T) hypermultiplet does not give any Yukawa couplings. Now we come to introduce gauge interactions.

4.2 Non-abelian gauged massive hyper ϕ^3 theory

We give hereafter the extension of the preceeding expressions to the gauged group version. As it is well known [1],[10], the gauge interactions can be directly obtained by covariantizing the harmonic derivatives $(2.19)_2$ namely

$$
D^{++} \longrightarrow \mathcal{D}^{++} = D^{++} - 2i\alpha V^{++} \qquad (4.13)
$$

where α is the gauge coupling constant and $V^{++} = V^{++}_r T^r$ are the gauge potentials. Therefore the action (4.6-9) becomes:

$$
\begin{aligned}
\mathcal{L} = \; & \int d^4\theta^+ \left\{ \mathrm{Tr}\left[\Omega (D^{++} + 2i\alpha V^{++})^2 \Omega \right] + \mathrm{tr}\left[\bar{\Phi}^+ (D^{++} + 2i\alpha V^{++}) \phi^+ \right] \right\} \\
& + m_I \left(\int d^2\theta^+ - \int d^2\bar{\theta}^+ \right) \bar{\Phi}^+_I \Phi^+_I - ig \left(\int d^2\theta^+ + \int d^2\bar{\theta}^+ \right) \mathrm{tr}\left(\bar{\Phi}^+ \Omega \phi^+ \right) \\
& + \mathcal{L}_G . \qquad (4.14)
\end{aligned}
$$

where $\mathcal{L}_G$ is the pure gauge part Lagrangian. It is given by

$$
\mathcal{L}_G = \int d^2\bar{\theta}^+ d^2\bar{\theta}^- \, \mathrm{Tr}\left[W W \right] \qquad (4.15)
$$

with $W = W^r T_r$ being the field strength. For more details on this gauge part, see for instance Refs. [1],[10],[15]. We shall quote here only the final result in component fields:

$$
\begin{aligned}
\mathcal{L}_G = \; & \mathrm{Tr}\Big[-\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} + 2i\, \bar{\lambda}^+ \bar{\sigma}^\mu \nabla_\mu \lambda^- + C \nabla_\mu \nabla^\mu \bar{C} + \alpha\sqrt{2}\, \bar{C} \{ \bar{\lambda}^+, \bar{\lambda}^- \} \\
& + \alpha\sqrt{2}\, C \{ \lambda^+, \lambda^- \} + \alpha^2 \bar{C} [\bar{C}, C] C + \tfrac{4}{9} D^{++} D^{--} \Big] \qquad (4.16)
\end{aligned}
$$

where

$$
\nabla_\mu \cdot = \partial_\mu \cdot + i\alpha [V_\mu, \cdot] \qquad (4.17)
$$

and $F_{\mu\nu}$ is the field strength for non-abelian gauge theories. The action (4.14) is invariant under the following set of gauge transformations;

$$
\begin{aligned}
\Phi^{+\prime}_I &= \left[\exp(-2i\alpha\Lambda) \right]_{IJ} \Phi^+_J \\
\bar{\Phi}^{+\prime}_I &= \bar{\Phi}^+_J \left[\exp(2i\alpha\Lambda) \right]_{JI} \\
\Omega' &= \exp(-2i\alpha\Lambda)\, \Omega\, \exp(2i\alpha\Lambda) \\
V^{++\prime} &= \exp(-2i\alpha\Lambda) \left[V^{++} - D^{++} \right] \exp(2i\alpha\Lambda) \qquad (4.18) \\
m' &= m
\end{aligned}
$$

The action can be written more explicitly in the gauge (3.7). In the harmonic-space notation, it has the covariant form [A3]

$$\mathcal{L}_T = \mathcal{L}_G + \mathcal{L} \qquad (4.19)$$

$$\begin{aligned}
\mathcal{L} = \; & \mathrm{Tr}\Big\{-\tfrac{1}{2}(\nabla_\mu \omega_0)^2 - 2(\nabla_\mu \omega^{+-})^2 + 2i\, \varsigma^+ \sigma^\mu \nabla_\mu \bar{\varsigma}^- + \tfrac{1}{2}\bar{H}^{++}H^{--}\Big\} \\
& + \mathrm{tr}\Big\{-2\,\overline{\mathcal{D}_\mu \varphi^+}\mathcal{D}^\mu \varphi^- - i\mathcal{D}_\mu \psi \sigma^\mu \bar{\psi} + \chi \sigma^\mu \overline{\mathcal{D}_\mu \chi} + \bar{F}^+ F^- + \bar{G}^+ G^-\Big\} \\
& - \alpha\sqrt{2}\,\mathrm{tr}\Big\{\varphi^+ C \bar{F}^- + \bar{\varphi}^+ C G^- - \bar{\varphi}^+ \bar{C} F^- - \varphi^+ \bar{C} G^-\Big\} \\
& + \alpha\sqrt{2}\,\mathrm{tr}\Big\{\psi^a \bar{C} \chi_a + \bar{\psi}_{\dot a} C \bar{\chi}^{\dot a}\Big\} \\
& - 2i\alpha\sqrt{2}\,\mathrm{tr}\Big\{\bar{\varphi}^+ \lambda^- \psi + \varphi^+ \bar{\lambda}^- \bar{\chi} + \chi \lambda^- \varphi^- - \bar{\psi} \bar{\lambda}^- \varphi^+\Big\} \\
& + m_{IJ}\Big\{(\bar{F}^- - \bar{G}^-)_I \varphi^+_J - (F^- - G^-)_I \bar{\varphi}^+_J - \psi_I \chi_J - \bar{\psi}_I \bar{\chi}_J \\
& + \frac{ig}{2\sqrt{3}}\,\mathrm{tr}\Big\{\big[\bar{\varphi}^+ \omega^0 (F^- - G^-) - (\bar{F}^- - \bar{G}^-)\omega^0 \varphi^+\big] \\
& \qquad + 2\big[\bar{\varphi}^+ \omega^{+-}(F^- - G^-) - (\bar{F}^- - \bar{G}^-)\omega^{+-}\varphi^+\big] \\
& \qquad + \big[\bar{\psi}_{\dot a}\omega^0 \bar{\chi}^{\dot a} - \psi^a \omega^0 \chi_a\big] + 2\sqrt{3}\big(\bar{\varphi}^+ \bar{\varsigma}^- \bar{\chi} - \bar{\varphi}^+ \varsigma^- \psi\big) \\
& \qquad + 2\sqrt{3}\big[\varphi^+ \bar{\varsigma}^- \psi - \varphi^+ \varsigma^- \chi\big] \\
& \qquad + \sqrt{3}\;\bar{\varphi}^+\big[H^{--} - \bar{H}^{--}\big]\varphi^+ \Big\} \\
& + \text{(H-S-T) - gauge field couplings.}
\end{aligned} \qquad (4.20)$$

where

$$\mathcal{D}_\mu = \partial_\mu - i\alpha\, v_\mu^r T_r \qquad (4.21)$$

is the gauge covariant derivative and where for instance

$$\begin{aligned}
\mathrm{Tr}\{(\partial_\mu \omega_0)^2\} &= \partial_\mu \omega^{0,r} \partial^\mu \omega^{0,s}\, \mathrm{Tr}(T_r T_s) \\
&= \partial_\mu \omega^{0,r} \partial^\mu \omega^0_r
\end{aligned} \qquad (4.22)$$

and also

$$\mathrm{tr}\{\bar{\varphi}^+ \omega^{+-}(F^- - G^-)\} = \bar{\varphi}^{+I} \omega_I^{+-J}(F^- - G^-)_J \qquad (4.23)$$

with

$$\omega^{+-}{}_I{}^J = \omega^{+-}_r\, (T^r)_I{}^J$$

The next step is to eliminate the auxiliary fields F^+, G^+, H^{++} and D^{++} through their equations of motion, and finally integrate over the harmonic variable. This will lead to a manifestly SU(2) invariant, gauge invariant and renormalizable Lagrangian, describing a massive SU(n) vector (F-S) hypermultiplet interacting with a set of ($n^2 - 1$ = dim SU(n)) massless (H-S-T) hypermultiplets. We shall leave the calculations, which are tedious but straightforward *) to the interested reader. However, we would like to note that this action contains all the necessary interactions for doing phenomenology especially global Yukawa couplings (4.14) N = 2 supersymmetric and gauge invariant usually absent in the N = 2 ordinary formalism. However, the price to pay for this is the introduction of a large number of particles required by N = 2 supersymmetry and as a counterpart a very restricted number of free parameters. The scalar potential U is obtained as usual [19]. It is actually given by

$$U(F,G,H,D) = \bar{F}^{+I}F^-_I + \bar{G}^{+I}G^-_I + \tfrac{1}{2}\bar{H}^{++r}H^{--}_r + \tfrac{4}{9}D^{++r}D^{--}_r \qquad (4.24)$$

where I = 1,...n and r = 1... $n^2 - 1$ and where F,G,H,D are given by their equations of motion from (4.19). As a consequence of N = 2 supersymmetry, one can check that (4.24) is positively defined and admits a vanishing minimum.

*) In the abelian case see Refs. [19],[20]

6. CONCLUSION

In this paper we have studied essentially three points: First we develop the formalism of $N = 2$ (H-S) with central charges. Second, we derive the most general renormalizable couplings covering our previous model from the $N = 2$ (H-S) with central charges (H-S-C-C). This allowed us to well understand the origin of extra symmetries remarked before. Third, we work out the generalization of the previous results to non-abelian gauge theories with explicit details.

To achieve all this, we have first introduced a convenient covariant formalism dealing with central charges. In this language, the two central charges turn out to be exactly the light cone derivatives in the y plane, a fact which allows us to understand both the SO(2) symmetry and the translation symmetry. In this context, we found as an interesting result that: The dynamic of an analytic superfield in the (H-S-C-C) is equivalent either to: 1) a massless ASF in the usual $N = 2$ (H-S) (without central changes) interacting with two massless (H-S-T) hypermultiplets through a renormalizable coupling as long as the z-symmetry is preserved. 2) A massive ASF in $N = 2$ (H-S) interacting with only one (H-S-T) hypermultiplet. Furthermore, this construction has allowed us to:

i) obtain the most general renormalizable interactions (Yukawa coupling) usually absent in the $N = 2$ ordinary formalism and necessary for doing phenomenology;

ii) Recover all our previous construction;

iii) Give a natural proof to the gauge choice (3.7) formally introduced before and in which all harmonic expansions are on shell;

iv) Show that there exists an invariant subspace of the (A-C) found by Gikos as long as we deal only with renormalizable interactions (gauge interactions and hyper ϕ^3 theory).

v) Contains previous approach to the $N = 2$ supersymmetry using $N = 1$ superfield [12] technics and finally contains a non-renormalizable term which looks like the Gikos one and which vanishes as a consequence of the z-symmetry. We still do not understand whether there exists a connection between the Gikos term and the z-symmetry which seems have a deep relation with renormalizability. This point and others such as, for instance, the extension of these results to $N = 3$ extended supersymmetry, quantization and calculation of some loop corrections to see whether there exist other cancellations in the renormalizable sector are actually under study.

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APPENDIX

A.1 N = 2 supersymmetric algebra in the N = 2 (H-S-C-C)

Using the harmonic notations namely:

$$A^{\pm} = A^{i} u_{i}^{\pm}$$

the z_2 graded Lie algebra (2.1) using the definitions (2.2), may written as follows:

$$\{ Q_a^{\pm} , \bar{Q}_{\dot{b}}^{\mp} \} = \pm 2 \sigma^{\mu}_{a\dot{b}} P_{\mu}$$

$$\{ Q_a^{+} , Q_b^{-} \} = 2\sqrt{2}\, i\, Z^{(-)} \epsilon_{ab}$$

$$\{ \bar{Q}_{\dot{a}}^{+} , \bar{Q}_{\dot{b}}^{-} \} = 2\sqrt{2}\, i\, \bar{Z}^{(+)} \epsilon_{\dot{a}\dot{b}}$$

all others are identically zero.

A.2 N = 2 supersymmetric algebra for the spinorial covariant derivatives in the N = 2 (H-S-C-C)

$$\{ D_a^{+} , \bar{D}_{\dot{a}}^{-} \} = -2i\, \sigma^{\mu}_{a\dot{a}} \partial_{\mu}^{A}$$

$$\{ D_a^{-} , \bar{D}_{\dot{a}}^{+} \} = 2i\, \sigma^{\mu}_{a\dot{a}} \partial_{\mu}^{A}$$

$$\{ D_a^{+} , D_b^{-} \} = -2\sqrt{2}\, \epsilon_{ab}\, \partial^{(-)}$$

$$\{ \bar{D}_{\dot{a}}^{+} , \bar{D}_{\dot{b}}^{-} \} = 2\sqrt{2}\, \epsilon_{\dot{a}\dot{b}}\, \partial^{(+)}$$

A.3 θ^{+} expansions of the ASF's ϕ^{+} and Ω in the gauge (3.7)

$$\phi^{+}(x_A, u^{\pm}, \theta^{+}, \bar{\theta}^{+}) = \varphi^{+} + \sqrt{2}\,\theta^{+}\psi + \sqrt{2}\,\bar{\theta}^{+}\bar{\chi} + \theta^{+2}F^{-} + \bar{\theta}^{+2}G^{-} + \theta^{+}\sigma^{\mu}\bar{\theta}^{+}A_{\mu}^{-}$$

$$\Omega(x_A, u^{\pm}, \theta^{+}, \bar{\theta}^{+}) = \omega + \sqrt{2}\,\theta^{+}\zeta^{-} + \sqrt{2}\,\bar{\theta}^{+}\bar{\zeta}^{-} + \theta^{+2}\frac{H^{--}}{2} + \bar{\theta}^{+2}\frac{\bar{H}^{--}}{2} + i\,\theta^{+}\sigma^{\mu}\bar{\theta}^{+}B_{\mu}^{--}.$$

In non-abelian gauge theories each component de ϕ^{+} is a vector, whereas each one of Ω is in fact an $n \times n$ matrix, for instance:

$$(\zeta_a^{+})_{IJ} = (\zeta_a^{i})_{IJ}\, u_i^{+} = \zeta_a^{i,r}(T_r)_{IJ}\, u_i^{+}$$

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