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PERTURBATIVE AND GLOBAL ANOMALIES IN SUPERGRAVITY THEORIES *

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ABSTRACT

Perturbative and global anomalies in supergravity theories are reviewed. The existence of a matter and gauge coupled supergravity theory in six dimensions with $E_6 x E_7 x U(1)$ symmety and highly nontrivial anomaly cancellations is emphasised. The possible string origin of this theory is posed as an open problem, study of which may lead to discovery of new ways to construct/compactify heterotic superstrings.

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1. Introduction

1. Anomalies in supergravity theories [1] have been studied since supergravity was discovered ten years ago [2]. However, the study of anomalies in Kaluza-Klein supergravities is a relatively recent phenomenon [3].

Although Grim and Marculescu [4] in1974, and Hosoda, Kozakai and Lee [5] in 1980 considered gauge anomalies in higher than four dimensions, anomalies in the context of Kaluza-Klein theories were considered first by Frampton[6], Frampton and Kephart [7] and Sierra and Townsend [8]in 1983. The purely gravitational anomalies were first computed in an influential paper by Alvarez-Gaume and Witten [3]. These authors gave a complete formula for gravitational, gauge and mixed anomalies in arbitrary dimensions. In the same paper they showed that the chiral N=2, d=10 supergravity is anomaly free.

In 1984, Green and Schwarz [9] found the dramatic result that of all Yang-Mills coupled N=1, d=10 supergravities, only those with SO(32), E(8)XE(8), $E(8)XU(1)^{248}$ and $U(1)^{496}$ are anomaly free. This was due to a new anomaly cancellation mechanism which required the modification of the theory by certain higher derivative terms. In the case of SO(32), it was shown that these anomaly cancellations could be incorporated in type I superstring [9]. This, in fact, started the revival in string theories. Soon after, the heterotic string with SO(32) or E(8)XE(8) symmetry was constructed [10].

2. The anomalies considered above are <u>perturbative</u> anomalies, i.e. anomalies in infinitesimal gauge and/or coordinate transformations . Anomalies in large gauge and/or coordinate transformations that cannot be reached continuously from the identity can also arise. Following Witten, we shall refer to these as <u>global</u> anomalies [11][12][13][14].

Witten has derived a general formula for global anomalies [12]. Using this formula, he has shown that chiral N=2, d=10 supergravity [15], and N=1, d=10 supergravity coupled to SO(32) or E(8)XE(8) Yang-Mills [16], are free from global anomalies [12][14].

3. Are there more anomaly free supergravity theories? It is reasonable to ask this question, because given a supergravity theory, if it happens to be the low energy limit of some superstring theory, for the anomaly freedom of the latter, it necessary that the perturbative and global anomalies of this supergravity theory be absent.

Supergravity theories we are interested in must be chiral. This is so because, presumably, only chiral theories can give rise to chirality in d=4 upon spontenaous compactification.

So far it has been shown that:

(a) N=1, d=8 Einstein/Yang-Mills supergravity [17] with chiral couplings is anomalous[18] (The maximal N=2, d=8 supergravity is vectorlike [19]).

(b) N=6, d=6 chiral supergravity with Sp(2)xSp(1) automorphism group is anomalous [20]. (The maximal N=8, d=6 supergravity with the automorphism group Sp(2)xSp(2) constructed by Tanii [22] is vectorlike with respect to the fundamental local symmetries).

(c) N=4, d=4 chiral supergravity coupled to 21 tensor multiplets is anomaly free [22]. (This theory is obtained from K_3 compactification of the anomaly free chiral N=2, d=10 supergravity theory).

(d) A large class of N=2, d=6 supergravities obtained from K_3 compactification of the anomaly free N=1, d=10 supergravities with SO(32) or E(8)XE(8) symmetry, are anomaly free [23]. (Note that all N=2, d=6 supergravities are chiral).

(e) A class of N=2, d=6 supergravities which do not seem to be related to any

compactification of anomaly free d=10 supergravities, are anomaly free [24][18][20]. Among these, by far the most nontrivially anomaly free theory has $E_6 \times E_7 \times U(1)$ symmetry [24]. (Here U(1) is the automorphism group).

4. Anomalies in d=4 supergravities should be considered in the context of a given compactification of a higher dimensional supergravity theory. Witten [25] has shown that a certain topological condition, which is needed for the SO(32) symmetric anomaly free N=1, d=10 supergravity theory to make sense, also ensures (perturbative)anomaly freedom in d=4. As far as the global anomalies are concerned, it is hoped that they give new restrictions on possible compactifications [12].

5. Anomalies in d=2 conformal supergravities play a special role in superstring theories. Although we shall not consider these anomalies here, it is a remakable fact that the necessity for SO(32) or E(8)XE(8) invariance in the heterotic string, can be derived by requiring the absence of the global gravitational anomaly in the underlying chiral N=1 conformal supergravity theory [10][13].

In the following, we first review some general formulae for perturbative and global anomalies, and the Green-Schwarz mechanism for anomaly cancellations. We then apply these formulae to:

(a) Chiral N=2, d=10 supergravity.

(b) N=1, d=10 supergravity coupled to E(8)XE(8) Yang-Mills.

(c) Chiral N=4, d=6 supergravity coupled to 21 tensor multiplets.

(d) N=2, d=6 supergravity coupled to $E_6 \times E_7 \times U(1)$ Yang-Mills and hypermatter.

These examples are chosen to illustrate the parallel between the cases (a), (b) and the cases (c), (d). It should be emphasized, however, that although case (c) is related to case (a) through K_3 compactification, no relation is known between the case (d) and (b).

2. Perturbative Anomalies and Green-Schwarz Mechanism

1. It is well known that, gauge, gravitational and mixed anomalies in 2n-dimensions are characterized by an 2n-form, $\Omega_{2n}{}^1$, derivable from an invariant (2n+2)-form, Ω_{2n+2} (R,F), which is a polynomial polynomial in Riemann and Yang-Mills curvature 2-forms, R and F, respectively [26]. The derivation goes through the descent equations [27]:

$$\Omega_{2n+2} = d\,\Omega_{2n+1}{}^0 \qquad (1\text{ a})$$

$$\delta\,\Omega_{2n+1}{}^0 = d\,\Omega_{2n}{}^1 \qquad (1\text{ b})$$

Eq. (1a) holds because Ω_{2n+2} is closed, and (1b) since Ω_{2n+2} is invariant. The anomaly is given by the variation of the one loop effective action, Γ, under gauge and/or Lorentz transformations (gravitational anomalies are equivalent to Lorentz anomalies [28]):

$$\delta\Gamma = \int \Omega_{2n}{}^1 \qquad (2)$$

The fact that $\delta\Gamma$ obeys the Wess-Zumino consistency condition can be easily shown. (This is equivalent to showing that $\delta_{BRS}{}^2\,\Gamma = 0$). The topological interpretation of (2), however, is more subtle; a detailed exposition has been given by Alvarez-Gaume and Ginsparg [29].

2. For anomaly freedom, there are two possibilities. The obvious possibility is that,

when the contribution of all chiral fields is taken into account $\Omega_{2n}{}^1$ sums up to zero. The second possibility is the Green-Schwarz mechanism [9], and works as follows. Suppose that the theory contains a p-form tensor field B, and that the "anomaly polynomial" Ω_{2n+2} factorizes as follows

$$\Omega_{2n+2} = \Omega_{p+2}\,\Omega_{2n-p} \qquad (3)$$

Then the anomaly is given by

$$\delta\Gamma = \int \Omega_p{}^1\,\Omega_{2n-p} \qquad (4\text{ a})$$

$$= \int \Omega_{p+2}\,\Omega_{2n-p-2}{}^1 \qquad (4\text{ b})$$

Here "= " means equality upto an irrelevant local counterterm $\delta\,(\,\Omega_{p+1}{}^0\,\Omega_{2n-p-1}{}^0\,)$, which can always be absorbed into $\delta\,\Gamma$.

Note that $(\,\Omega_{p+1}{}^0\,,\ \Omega_{2n-p-1}{}^0)$ and $(\,\Omega_p\,,\Omega_{2n-p-2}{}^1\,)$ are defined by descent equations (as in eq. (1)) from $(\,\Omega_{p+2}\,,\ \Omega_{2n-p}\,)$, respectively.

To cancel the anomaly, one adds the following counterterm to the action

$$\Delta L = B\ \Omega_{2n-p} \qquad (5)$$

and, furthermore, let B transform under gauge and/or Lorentz transformations as follows

$$\delta B = -\ \Omega_p{}^1 \qquad (6)$$

It is clear that the anomalies now cancel, i.e. $\delta(\Gamma + \Delta L) = 0$. Note also that the gauge invariant field strength for B is given by

$$H = dB + \Omega_{p+1}{}^{0} \qquad (7)$$

In 2n dimensions, the p-form tensor field B is <u>on-shell</u> equivalent to a dual (2n-p)-form tensor field, say C. In that case, the Green-Schwarz anomaly cancellation mechanism is evidently again operative [18][30] ; (5),(6) and (7) must simply be replaced by

$$\Delta L = C\, \Omega_p \quad ; \quad \delta C = -\Omega_{2n-p}{}^{1} \quad ; \quad H = dC + \Omega_{2n-p+1}{}^{0} \qquad (8)$$

To summarize, the Green-Schwarz mechanism for anomaly cancellations works provided that the theory contains a p-form tensor field, and that the anomaly polynomial factorizes as in (4).

3. In concluding this section, we list the contributions of various chiral fields (most often encountered in supergravity theories in 2n (=2,6,10) dimensions) to the anomaly polynomial Ω_{2n+2} [3]. (We use the notation and conventions of [29]):

$$\Omega_{2n+2}(\text{Spinor}) = A(R)\,\mathrm{ch}(F) \qquad (9\,a)$$

$$= \left[1 + 1/48\,\mathrm{tr}\,R^2 + 1/5760\,(\mathrm{tr}\,R^4 + 5/4\,(\mathrm{tr}\,R^2)^2) + \right.$$

$$\left. + 1/362880\,(\mathrm{tr}\,R^6 + 21/16\,\mathrm{tr}\,R^4\,\mathrm{tr}\,R^2 + 35/64\,(\mathrm{tr}\,R^2)^3) + \cdots \right] \mathrm{tr}\,e^F.$$

$$\Omega_{2n+2}(\text{Gravitino}) = A(R)\,(2n-1 + \mathrm{tr}\,(\cos R - 1))\,\mathrm{ch}\,F \qquad (9\,b)$$

$$= \left[(2n-1) + (2n-25)/48\,\mathrm{tr}\,R^2 + 1/5760\,((2n+239)\mathrm{tr}\,R^4 + \right.$$

$$+ 5(2n-47)/4\,(\mathrm{tr}\,R^2)^2) + 1/362880\,((2n-505)\mathrm{tr}\,R^6 +$$

$$\left. + 21(2n+215)/16\,\mathrm{tr}\,R^4\,\mathrm{tr}\,R^2 + 35(2n-73)/64\,(\mathrm{tr}\,R^2)^3) + \ldots \right] \mathrm{tr}\,e^F.$$

$$\Omega_{2n+2}(\text{Self-dual tensor}) = -L(R)/8 \qquad (9\,c)$$

$$= -1/8 + 1/48\,\mathrm{tr}\,R^2 + 1/5760\,(28\,\mathrm{tr}\,R^4 - 10\,(\mathrm{tr}\,R^2)^2) +$$

$$+ 1/362880\,(496\,\mathrm{tr}\,R^6 - 294\,\mathrm{tr}\,R^4\,\mathrm{tr}\,R^2 + 35\,(\mathrm{tr}\,R^2)^3] + \cdots$$

Here, A(R) is the Dirac genus, ch (F) is the Chern character and L(R) is the Hirzebruch polynomial, and "..." refers to terms which are irrelevant to ten and lower dimensional spacetimes. (The interested reader can find them, for example, in [29]). The traces involving R are over the fundamental representation of the Lorentz group in 2n-dimensions, while the traces involving F are over a given representation of the Yang-Mills gauge group which the fermions are in.

It is important to note that the spinors and the gravitini are complex Weyl with the same handedness, while the self-dual tensor is real.

It is clear that in 2n-dimensions one must consider only the 2n-forms on the right hand side of (9). For example, one can read off the contribution of a left handed Dirac spinor in a given representation of a Yang-Mills gauge group, to the purely gravitational, purely gauge and all possible gauge-Lorentz mixed anomalies in 2,4,6,8 and 10 dimensions, simply from (9 a)!

3. Witten's Formula for Global Anomalies

The global anomaly is defined by the change in the effective action under the diffeomorphism of spacetime M, which is not continously connected to the identity [12]

$$\Delta I = I(g_{\mu\nu}{}^{\pi}) - I(g_{\mu\nu}) \quad (10)$$

Absence of global anomalies requires that $\Delta I = 0 \bmod 2\pi i$.

It makes sense to look at the global anomalies after establising the absence of perturbative anomalies. Assuming that perturbative anomalies cancel a la Green-Schwarz (see (3), (5), (6) and (7)), Witten [12] has shown that the global anomaly is given by the following formula:

$$\Delta I = 2\pi i\,[\,N_D \text{ index (D)} + N_R \text{ index (RS)} - N_S\,\sigma/8 - \int_B \Omega_{p+2}\,\Omega_{2n-p} + \int_{\partial B} H\,\Omega_{2n-p}\,] \quad (11)$$

Here, N_D is the number of complex Weyl spinors, N_R is the number of complex Weyl Rarita-Schwinger fields (i.e. gravitini), N_S is the number of real self-dual tensors; index (D) and index (RS) are the indices of Dirac and Rarita-Schwinger operators defined on an (2n+2)-dimensional spin manifold B <u>with boundary</u>.

The boundary of B is the mapping cylinder $(M \times S^1)_\pi$, defined by multiplying M by the unit interval I=[0,1] and gluing together the top and bottom of $M \times I$ by identifying (x,0) with (π(x),1) for any $x \in M$. (If $N_S \neq 0$, then to avoid subtleties associated with the zero modes of the self-dual tensor fields, we consider 2n-dimensional spacetimes M with vanishing n-th Betti number [12]). That such B exists is assumed. Given an M, this assumption must

be justified.

If anomalies cancel without Green-Schwarz mechanism, i.e. if the anomaly polynomial Ω_{2n+2} vanishes identically, then the last two terms in (11) are absent.

The indices and the signature on B are not simply given by the usual integrals of polynomials in the curvatures because B has boundary. However, for our purposes it is often sufficient to know whether the indices are even or odd, and whether the signature is divisible by 8.

The all important relation $dH = \Omega_{p+2}$, which follows from (7), is extended to B. Howevever, in general, this relation holds <u>locally</u> on B. If it holds globally as well, then the last two terms in (11) sum up to zero by Stoke's theorem.

4. Perturbative and Global Anomalies in 6 and 10 Dimensions

4.1. Chiral N=2, d=10 Supergravity

The chiral fields contained in this theory are: A complex left-handed gravitino (N_R=1), a complex right handed spinor (N_D=-1), and a real fourth rank tensor field with self-dual field strength (N_S=1). From (9a), (9 b) and (9 c), one finds that the anomaly polynomial $\Omega_{12}=0$. (Thus the perturbative anomalies are absent without the need to use the Green-Schwarz mechanism). This remarkable anomaly cancellation was first discovered by Alvarez-Gaume and Witten [3].

As for the global anomaly : First of all, a twelve dimensional spin manifold B which bounds the mapping cylinder $(M \times S^1)_\pi$ does exist. (This is so because the spin cobordism group is trivial in eleven dimensions [14]). Second, from (11), one finds that the global anomaly now reads $\Delta\,\Gamma = (\sigma/8) \bmod 2\pi i$. Computing the signature on an arbitrary B is a difficult task. Taking M to be S^{10}, Witten [12] showed that the mapping cylinder $(S^{10} \times S^1)_\pi$ bounds a spin manifold B of signature divisible by 8. In showing this it is convenient to utilize the fact that [12] $\partial B = (S^{10} \times S^1)_\pi = S^{10} \times S^1 + (S^{11})_\pi$, where $(S^{11})_\pi$ is one of the 991 exotic 11-spheres. Here, "+" refers to connected sum, i.e. cut out and discard an 11-disk from both $S^{10} \times S^1$ and $(S^{11})_\pi$, then paste the manifolds together along these boundaries.

In summary, the N=2, d=10 supergravity is perturbatively anomaly free without the Green-Schwarz mechanism, and moreover globally anomaly free when formulated on S^{10}.

4. 2 . N=1, D=10 Supergravity Coupled to E(8)XE(8) Yang-Mills

The chiral fields in this theory are [16]: A left-handed Majorana-Weyl gravitino, a right-handed Majorana-Weyl spinor and left-handed Majorana-Weyl gauginos in the adjoint representation of E(8)XE(8) .

From (9 a) and (9 b), one finds the result [9][31] $\Omega_{12} = \Omega_4\, \Omega_8$, where

$$\Omega_4 = \mathrm{tr}\, R^2 - 1/30\, \mathrm{Tr}\, F^2$$

$$\Omega_8 = 945\, [\, \mathrm{tr}\, R^4 + 1/4\, (\mathrm{tr}\, R^2)^2 - 1/30\ \mathrm{tr}\, R^2\, \mathrm{Tr}\, F^2 - 1/900 (\mathrm{Tr}\, F^2)^2 + 1/3\, \mathrm{Tr}\, F^4\,] \qquad (12)$$

Here, Tr is over the adjoint representation of E(8)XE(8) . Now, since the theory also contains a two form field, in accordance with our discussion in Section 2, due to the highly nontrivial

factorization $\Omega_{12} = \Omega_4\, \Omega_8$, perturbative anomaly freedom in this theory is guarantied by the Green-Schwarz mechanism.

We now turn to global anomalies. Since $N_R = 1/2$, $N_D = (-1+496)/2$ and $N_S = 0$, recalling also the fact that in 12-dimensions index(D) and index(RS) are even [12], from (11) it follows that $\Delta\,\Gamma = 2\pi i\, (-\int_B \Omega_4\, \Omega_8 + \int_{\partial B} H\, \Omega_8) \bmod 2\pi i$. Here, B is a twelve dimensional manifold with boundary $\partial B = (M \times S^1)_\pi$. Now, Witten [14] has shown that H can be defined to obey the relation $dH = \Omega_4$ <u>on B</u> . (The proof is rather subtle, and is explained in detail by Witten in [14]). Consequently, by Stoke's theorem, $\Delta\,\Gamma = 0 \bmod 2\pi i$, and therefore there are no global anomalies.

3. *Chiral N=4, d=6 Supergravity Coupled to 21 Tensor Multiplets*

This theory can be obtained by compactification of the chiral N=2, d=10 supergravity on K_3 [22]. The automorphism group is Sp(2). The chiral fields are: Two complex left handed gravitini, and five real self-dual tensors coming from supergravity multiplet; and 2x21 right-handed spinors and 21 real anti-self-dual tensors coming from the tensor multiplets. Consequently, from (9 a), (9 b) and (9 c) one finds that the total anomaly polynomial sums up to zero: $\Omega_8 = 0$. Therefore, this theory is perturbatively anomaly free [22] without Green-Schwarz mechanism, just as in the case of chiral N=2, d=10 supergravity.

Now, let us consider the global anomaly. First, we note that an eight dimensional spin manifold which bounds the mapping cylinder $(M \times S^1)_\pi$ does exists because the spin cobordism group is trivial in seven dimensions. Moreover, since $N_R = 2$, $N_R = -42$ and $N_S = (-5+21)$, from (11) it immediately follows that $\Delta \Gamma = 0 \mod 2\pi i$. Therefore, this theory is free from global anomalies as well [20]. (Note that, here, the knowledge of the signature is not needed. Therefore, the simplifying assumption that spacetime is S^6 is not necessary).

4. *N=2, d=6 Supergravity Coupled to $E_6 \times E_7 \times U(1)$ Yang-Mills and Hypermatter*

General couplings of N=2, d=6 supergravity to Yang-Mills and hypermultiplets have been described in [32]. The fact that an $E6 \times E7 \times U(1)$ symmetric theory with a particular hypermatter content is anomaly free by Green-Schwarz mechanism was found in [24]. The chiral field content of this theory is the following: A complex left-handed gravitino, a complex right-handed spinor, complex left-handed gauginos in the adjoint representation of $E_6 \times E_7 \times U(1)$, and pseudo-real right-handed hyperinos (i.e. the fermions of the hypermultiplet) in 912 dimensional representation of E_7. The U(1) group is the gauged automorphism group.

After some algebra (and group theory), from (9 a) and (9 b) one finds that, highly nontrivially, the anomaly polynomial factorizes as $\Omega_8 = \Omega_4{}^{(1)} \Omega_4{}^{(2)}$, where

$$\Omega_4{}^{(1)} = -1/12 \,\mathrm{tr}\, R^2 + 1/12 \,\mathrm{tr}\, F_6{}^2 - 1/8 \,\mathrm{tr}\, F_7{}^2 + 3 F_1{}^2$$
$$\Omega_4{}^{(2)} = 3/4 \,\mathrm{tr}\, R^2 + 1/4 \,\mathrm{tr}\, F_6{}^2 + 3/8 \,\mathrm{tr}\, F_7{}^2 + 3 F_1{}^2 \qquad (13)$$

Here, F_6, F_7 and F_1 denote the E_6, E_7 and U(1) field strength two-forms, respectively, and the traces are over the fundamental representations.

Now, since the anomaly polynomial Ω_8 factorizes as above, and moreover the theory

contains a two form field, the Green-Schwarz mechanism is operative, and the theory is free from perturbative anomalies.

Now, let us consider the global anomalies. Since $N_R = 1$, $N_D = (78+133+1)-(912/2)$ and $N_S = 0$, from (11) one finds $\Delta \Gamma = 2\pi i \left(-\int_B \Omega_4{}^{(1)} \Omega_4{}^{(2)} + \int_{\partial B} H\, \Omega_4{}^{(2)} \right) \mod 2\pi i$. Recall that $dH = \Omega_4{}^{(1)}$. In general we do not know whether this relation can be extended to B. However, taking spacetime to be S^6, following Witten's argument for a similar situation in ten dimensions [12], one can proceed as follows. First, because $\pi_6(E_6 \times E_7 \times U(1)) = 0$, global gauge anomalies cannot arise. Thus, we need only consider the global gravitational anomaly

$$\Delta \Gamma = 2\pi i \left(1/16 \int_B (\mathrm{tr}\, R^2)^2 + 3/4 \int_{\partial B} H \,\mathrm{tr}\, R^2 \right) \mod 2\pi i \qquad (14)$$

Next, one uses the fact that $\partial B = (S^6 \times S^1)_\pi = S^6 \times S^1 + (S^7)_\pi$, where $(S^7)_\pi$ is one of the 27 exotic 7-spheres. Thus, B can be chosen to be a connected sum of $S^6 \times$ 2-disk and and eight dimensional parallalizable manifold whose boundary is $(S^7)_\pi$. (The existence of the latter is guarantied by a theorem due to Milnor [33]). On $S^6 \times$ 2-disk it is easy to show that $\mathrm{tr}R^2 = 0$. Moreover, on a parallalizable manifold one can always choose a connection such that $R=0$. As (14) does not depend on the choice of connection, it then follows that $\Delta \Gamma = 0 \mod 2\pi i$, i.e. there is no global anomaly.

5. Comments

The anomaly cancellations in the $E_6 \times E_7 \times U(1)$ theory are highly nontrivial. It is unlikely that this is a pure accident. Can there exist an anomaly free heterotic superstring theory in six dimensions which can give rise to the anomaly free $E_6 \times E_7 \times U(1)$ theory in low energy limit? This is an open question, study of which may very well lead to discovery of new ways to construct/compactify heterotic superstring theories.

In fact, Narain [34] has shown that the nonchiral N=4, d=6 supergravities coupled to certain rank 20 Yang-Mills groups do correspond to the massless sector of a heterotic string theory in d=6. Moreover, Narain, Sarmadi and Witten [35] have shown that these string theories can be obtained from the toroidal compactification of the usual d=10 heterotic string. The problem is to find an anomaly free and "stringy" supersymmetry breaking mechanism which, in the present case, would break N=4 down to N=2. It would be interesting to see whether such a mechanism could accommodate the $E_6 \times E_7 \times U(1)$ theory.

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