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THE EFFECT OF ASYMMETRY ON RESONANT TUNNELING *

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ABSTRACT

Resonant tunneling experiments on multibarrier coupled heterostructures probe the quasistationart nature of the states of the corresponding one dimensional potential. This work considers the effect of asymmetric one dimensional multibarrier potentials on resonant tunneling. It is shown, by using the properties of the propagator of the system, that this effect may lead to novel resonance phenomena and affects the lifetime of the quasistationary states of the system. The above considerations are illustrated by a simple analytical solvable model.

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In recent times there has been an increasing interest in studying electronic transport in double barrier heterostructures [1]. It is well known, since the pionnering work by Tsu and Esaki [2] and Chang, Esaki and Tsu [3], that resonant tunneling plays the relevant role for perpendicular electronic transport in the above systems, which proceeds essentially in one dimension. The problem corresponds to the scattering of electrons by a one dimensional potential and the quantity of interest is the transmission coefficient from which the I-V characteristics may be obtained. Actually provided the electronic mean free path is larger than the length of the system, one may restrict onself to consider only elastic processes. This is the case for typical parameters of the problem [4].

The physics of one dimensional resonant tunneling for a symmetric potential is well understood: It corresponds to a situation where a particle of well defined energy E is incident upon the system, formed by two consecutive barriers of equal height greater than E and a classically allowed region between them. At nearly all incident energies the particle is almost totally reflected. However in a small discrete number of energy intervals, characterized by energies E_r and corresponding widths Γ_r, the particle is actually transmitted to the other side of the system. This is the phenomenon of resonant tunneling which manifests itself as a peak of unity height in a plot of the transmission coefficient vs energy. Actually full transmission is achieved at $E = E_r$ and the corresponding time scale of the event is given by $t \sim \hbar / \Gamma_r$, which reflects the quasistationary nature of the process. During this time a large electronic probability density builds up in the well due to constructive interference between the waves leaking of the first barrier and those being reflected off the second one. The case of multiple barriers and wells is similar except that the sequence of internal reflections leading to the formation of a resonance is more complex. Actually the experimental situation in multibarrier heterostructures corresponds to asymmetric potentials. An interesting point here is that in general asymmetric potentials lead to a reduction of the heights of the transmission peaks at resonance energies. The theoretical explanation of that fact is usually made in terms of the transmission

coefficients through each barrier [5]. However such a procedure gives no insight on the underlying resonant processes Motivated by the above situation, in this work I consider an approach to resonant tunneling aimed to show how resonance levels are affected by asymmetric potentials. In particular I discuss how the lifetime of the levels is affected by the asymmetry of the potential. A crucial feature of the approach is that it takes into account the quasistationary nature of resonant levels. This is usually ignored. The levels of the system are commonly calculated as if they were bound states [1,3]. On the other hand, the usual procedure to obtain the lifetime of a resonant electron is by measuring the FWHM, i.e. the full width at half maximium of the transmission peak obtained by solving numerically the Schrodinger equation of the problem. However as shown below, in the case of asymmetric potentials the lifetime of the resonant electrons is not given by the FWHM of the corresponding reduced transmission peak.

A convenient way to approach the problem is via the outgoing Green function or propagator of the problem because then one may consider the well known relationship between resonances and the complex poles of the propagator to describe processes near resonance energy [6]. Here it is well known that for every resonance the real and imaginary parts of the complex pole correspond respectively to the resonant energy and the energy width described above. Let us therefore consider an arbitrary potential extending through a region of finite length L i.e. $V(x) = 0$ for $x \leq 0$ and $x \geq L$. The solutions of the Schrodinger equation for scattering from the left of a wave of energy $E = k$ with units $\hbar = 2m = 1$, may be written outside the interaction range in the usual form $\Psi(k,x) = \exp(ikL) + r(k) \exp(-ikL)$; $x \leq 0$ and $\Psi(k,x) = t(k) \exp(ikL)$; $x \geq L$, where $r(k)$ and $t(k)$ are respectively the reflection and transmission amplitudes. Here I consider the effective mass approximation and for simplicity I do not include the mass variation in the different regions. Using Green's theorem between the equation for $\Psi(k,x)$ and that one for the full propagator $G^{+}(x,x';k)$ of the problem, which obeys outgoing boundary conditions at the end points of the system, results in the following relationship for the wave function along the internal region [7],

$$\Psi(k,x) = 2ik\, G^{+}(0,x;k)\ ;\ 0 \leq x \leq L \qquad (1)$$

By considering the solution of the wave function at $x = L$ gives the transmission amplitude as

$$t(k) = 2ik\, G^{+}(0,L;k)\ \exp(-ikL) \qquad (2)$$

The above expression is very interesting because it relates the transmission amplitude with the propagator evaluated at the point $x = 0$ and $x' = L$. Similarly the expression for the reflection amplitude reads,

$$r(k) = 2ik\, G^{+}(0,0;K) - 1 \qquad (3)$$

which depends only on the propagator evaluated at $x = x' = 0$. For an electron approaching the system from the right it is easily seen that the transmission and reflection propagators are given respectively by $G^{+}(L,0);k)$ and $G^{+}(L,L;k)$. The relevant point is that near a complex pole $k = a - i\, b$ one may write the propagator as [6,7],

$$G^{+}(x,x';k) \simeq u_n(x)u_n(x')/2k_n(k - k_n) \qquad (4)$$

It is well known that the functions $u_n(x)$ obey the Schrodinger equation with complex eigenvalues $k_n^2 = \mathcal{E}_n - i\,\Gamma_n/2$ with $\mathcal{E}_n = a_n^2 - b_n^2$ and $\Gamma_n = 4\, a_n\, b_n$ and fulfill outgoing boundary conditions [6]. Using Green's theorem between the equations for $u_n(x)$ and $u_n^*(x)$ and using the corresponding boundary conditions gives an interesting relation for the width b in terms of the value of the resonant eigenstate at the end points of the system,

$$b_n = \left(|u_n(0)|^2 + |u_n(L)|^2 \right) /2 \qquad (5)$$

The above relationship defines the partial widths $b_n^o = |u_n(0)|^2$ and $b_n^L = |u_n(L)|^2$. The relevant point here is that the partial widths represent the coupling of the resonant eigenstate with the end points of the system. Using (4) and (5) into (2) gives the transmission coefficient near resonance as,

$$|t(k)|^2 \simeq k^2 \, b_n^o \, b_n^L \, / \, \left(a_n^2 \left[(k - a_n)^2 + (b_n^o + b_n^L)^2/4 \right] \right) \qquad (6)$$

The above equation generalizes the usual expression for the transmission coefficient near resonance. The difference corresponds to the appearence in (6) of the partial widths b and b . Actually for a symmetric potential it is straightforward to see that the system looks the same independently of being approached from the left or from the right. This is because $|G^+(0,0,k)|^2 = |G^+(L,L;k)|^2$ and therefore $b_n^o = b_n^L$ which leads to the usual expression for the transmission coefficient near resonance. Clearly for the asymmetric case one may have $b_n^o \neq b_n^L$ and as a consequence it is easy to convince oneself, by inspection of (6), that the value of the transmission peak will depend on the ratio between b_n^o and b_n^L. The underlying resonant process may be understood by using (4) and (2) into (1) which allows to write the probability density for the electron inside the system, near resonance, as

$$|\Psi(k,x)|^2 \simeq k^2 \, b_n^o \, |u_n(x)|^2 / \left(a_n^2 \left[(k - a_n)^2 + (b_n^o + b_n^L)^2 / 4 \right] \right) \qquad (7)$$

It straightforward to see that the ratio between the partial widths will determine whether there is a small or a large probability for the electron inside the system. The case $b_n^o \gg b_n^L$ is particularly interesting because from (6) it corresponds to very small transmission. However from (7) one finds a large probability for the electron inside the system. This case corresponds to a situation where the incident electron forms a long lived state within the system that decays through the incident direction. This process may be called resonant reflection [7].

In order to illustrate the above points I consider the analytical treatment of a simple double barrier system. This is the double delta shell potential $V(x) = A_o \delta(x) + A_L \delta(x-L)$. Actually it is well known that the in the above model each delta may be seen as the result of the parameters of a square barrier of height V and width d tending to infinity such that the product Vd remains constant corresponding precisely to the intensity A of the delta function. For an electron approaching the system from the left, the expressions for the transmission and reflection propagators follow from the corresponding integral equation in terms of the free particle Green's function $G_0^+(x,x';k) = \exp(ik|x-x'|)/2ik$. They are easily seen to correspond to

$$G^+(0,L;k) = 2ik \exp(-ikL) / J(k) \quad (8)$$

and

$$G^+(0,0;k) = 2ik + A_L (\exp(2ikL) - 1) / J(k) \quad (9)$$

where $J(k) = [(2ik) - (A_o + A_L)2ik - A_o A_L (\exp(2ikL) - 1)]$. The poles of $G^+(x,x';k)$ correspond to the zeroes of $J(k))$. Since sharp resonances require of sufficiently high barriers, it is

easy to see that under the condition where both A_o and A_L are much larger than unity the following approximate analytic solutions are obtained for the poles $k_n = a_n - i b_n$,

$$k_n \simeq n\pi [1 - (A_o + A_L)/ A_o A_L]/ L - i [n\pi (A_o + A_L)/ A_o A_L]^2/ L^3 \quad (10)$$

where n=1,2,3,..... The above expressions are a good approximation for the first levels. Using (5) and (10) allows to write the partial decay widths b and b , respectively as

$$b_n^o \simeq (n\pi)^2 (1/A_o^2 + 1/A_o A_L)/L^3 ;$$

$$b_n^L \simeq (n\pi)^2 (1/A_L^2 + 1/A_o A_L)/L^3 \quad (11)$$

One sees that the partial widths, which represent the coupling of the corresponding resonant eigenstate to the end points of the system, depend on the parameters involving both barriers. Since the Schrodinger equation is analytic in the parameters A_o and A_L, one may consider the limit in which both A_o and A_L tend to an infinite value. That leads to the elementary text book result for the box of infinite walls with eigenvalues $k_n = n\pi / L$. In the symmetric case $A_o = A_L = A$ the poles move near,

$$k_n \simeq n\pi (1 - 2/A) / L^2 - i\, 4 (n\pi / A)^2 / L^3 \quad (12)$$

From (11) one sees that $b_n^0 = b_n^L$ and hence from (6) it follows, as expected, that $|t|^2 = 1$ at the resonance value $k = a_n$. The above situation is independent of well width L and depends only on the equality of the barrier intensities. An interesting situation occurs for the asymmetric case $A_0 \neq A_L$. Actually if $A_L / A_0 \gg 1$ one sees from (10) that the pole moves to a value essentially independent of A_L,

$$k_n \approx n\pi \, (1 - 1 / A_0) / L^2 - i \, (n\pi / A_0)^2 / L^3 \qquad (13)$$

The above condition means, using (11), that $b_n^0 \gg b_n^L$ which leads, from (6), to a reduced value for the transmission peak. The same conclusion is obtained from the exact expressions for the transmission and reflection propagators given by (8) and (9). One sees that the transmission Green function becomes proportional to $1 / A_L$, which reduces the value for the transmission, whereas the reflection Green function becomes essentially independent of A_L. The above example illustrates that an asymmetric potential may sustain resonance poles which do not lead to a significant resonant tunneling effect. As pointed out above this corresponds to the occurrence of reflection resonances. In the present example this process follows from the fact that the

second barrier is much larger than the first one. The resonant electron forms a long lived state of lifetime $t_n \approx (4 a_n b_n^0)^{-1}$ that decays through the incident direction. Alternatively if $A_0 / A_L \gg 1$, a similar analysis shows that both the transmission and reflection propagators become proportional to $1 / A_0$. This corresponds to the usual reflection situation in which the electron remains essentially outside the system. The above considerations may be further substantiated by using (7). For a particle approaching the system from the right the transmission propagator remains the same, as may be seen from (8). However the reflection propagator becomes different. One has to interchange in (9) A_0 with A_L and as a consequence the above discussion becomes reversed. Summarizing: The main result of this work is that for an arbitrary potential the transmission peak at resonance depends on the ratio of the corresponding partial decay widths (7). For symmetric potentials this leads to the familiar resonant tunneling situation. However for asymmetric potentials the partial decay widths are in general different and therefore responsible for the observed reduction of the transmission peak. As a consequence the corresponding FWHM does not provide the lifetime of resonance levels for asymmetric potentials. It is also found for this case that the underlying resonant processes may lead to novel phenomena as resonant reflection.

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