EFFECT OF NEGATIVE IONS ON THE FORMATION OF WEAK ION ACOUSTIC DOUBLE LAYERS

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Using kinetic theory small amplitude double layers, associated with ion acoustic waves in a plasma containing negative species of ions, have been investigated. Analytic solution for the double layer potential has been carried out. The limiting values of the negative ion density for the existence of this type of DL have been calculated and the application of this result to space plasmas is discussed.

A double layer which is a narrow, isolated region of abrupt potential jump of amplitude ψ, due to a localized dipole sheet of space charge surrounded by quasineutral plasmas, has been studied extensively both experimentally [1], and in numerical simulations [2]. Recently, several authors [3-5] have put forward theories of double layers analytically. Most of the authors have shown analytically that the stationary, one dimensional and collision free nature of double layers which are basically a kinetic phenomenon, suggest a description in terms of appropriate solutions of the Vlasov-Poisson system. In the reference [3] slow electron acoustic (SEADL) and slow ion acoustic double layers (SIADL) are investigated and their existences have been explained in the small amplitude limit.

In this paper, we present analytic solution of the small amplitude double layers and solitons propagating near the ion acoustic speed in a model of plasma containing negative ions.

The distribution functions for electrons, positive ions and negative ions, which satisfy the stationary Vlasov equations, are respectively as,

$$f_e(v)=\left(\frac{\delta}{2\pi}\right)^{1/2}\begin{cases}\exp\left[-\frac{\delta}{2}\left(\pm\left(v^2-\frac{2\phi}{\delta}\right)^{1/2}+v_o\right)^2\right]\\ \exp\left[-\frac{\delta u_o^2}{2}\right]\exp\left[-\beta\left(\frac{\delta}{2}v^2-\phi\right)\right]\end{cases} \quad (1)$$

$$f_i(u_i)=A_1\left(\frac{\theta_i}{2\pi}\right)^{1/2}\begin{cases}\exp\left[-\frac{\theta}{2}\left(\pm\left(u_i^2-2(\psi-\phi)\right)^{1/2}+u_{oi}\right)^2\right]\\ \exp\left[-\frac{\theta_i u_{oi}^2}{2}\right]\exp\left[-\alpha_i\theta_i\left(\frac{u_i^2}{2}-(\psi-\phi)\right)\right]\end{cases} \quad (2)$$

$$f_j(u_j)=A_2\left(\frac{q\theta_j}{2\pi}\right)^{1/2}\begin{cases}\exp\left[-\frac{q\theta_j}{2}\left(\pm\left(u_j^2-\frac{2\phi}{q}\right)^{1/2}+u_{oj}\right)^2\right]\\ \exp\left[-\frac{q\theta_j u_{oj}^2}{2}\right]\exp\left[-\alpha_j\theta_j\left(\frac{q}{2}u_j^2-\phi\right)\right]\end{cases} \quad (3)$$

where $\delta = m_e/m_i$, $q = m_j/m_i$, $\theta_i = T_e/T_i$, $\theta_j = T_e/T_j$, $\alpha_i = T_{if}/T_{ir}$, $\alpha_j = T_{jf}/T_{jr}$ and $\beta = T_{ef}/T_{er}$. v_o, u_{oi} and u_{oj} are respectively the drift velocities of the free electrons, positive ions and negative ions respectively and $T_{if}(T_{ir})$, T_{jf} (T_{jr}) and $T_{ef}(T_{er})$ are the temperatures of the free (reflected) positive ions, negative ions and electrons, respectively. α_i, α_j and β are known as the trapping parameters. The electric potential, space and velocities are normalized to T_e/e, $\lambda_D\,(=kT_e/4\pi n_0 e^2)^{1/2}$ and $C_s\,(=kT_e/m_i)^{1/2}$ respectively. Here the subscript j stands for negative ions and other notation is standard.

Following [3] we can write the poisson's equation as

$$\phi'' = A\phi + B_1\phi^{3/2} + B_2[\Psi^{3/2} - (\Psi-\phi)^{3/2}] + C_1\phi^2 + C_2\phi\Psi = -\partial V/\partial\phi \qquad (4)$$

where

$$A = -\frac{1}{2}\left[Z_r'\left(\sqrt{\delta}\, v_0/2\right) + \frac{\alpha}{1-\alpha} Z_r'\left(\sqrt{\frac{\theta_j\delta}{2}}\, u_{0j}\right)\theta_j + \frac{1}{1-\alpha} Z_r'\left(\sqrt{\frac{\theta_i}{2}}\, u_{0i}\right)\theta_i\right]$$

$$B_1 = -\frac{4}{3}\left[b(\beta, \sqrt{\delta}\, v_0) + \frac{\alpha}{1-\alpha} b(\alpha_j, \sqrt{\theta_j\delta}\, u_{0j})\theta_j^{3/2}\right]$$

$$C_1 = \frac{1}{2}\left[G(\sqrt{\delta}\, v_0) + \frac{\alpha}{1-\alpha} G(\sqrt{\theta_j\delta}\, u_{0j})\theta_j^2 - \frac{1}{1-\alpha} G(\sqrt{\theta_i}\, u_{0i})\theta_i^2\right] \qquad (5)$$

$$B_2 = -\frac{4}{3}\frac{1}{1-\alpha} b(\alpha_i, \sqrt{\theta_i}\, u_{0i})\theta_i^{3/2}$$

$$C_2 = \frac{1}{1-\alpha}\theta_i^2\left[G(\sqrt{\theta_i}\, u_{0i}) - \left\{\frac{1}{2} Z_r'\left(\sqrt{\frac{\theta_i}{2}}\, u_{0i}\right)\right\}^2\right]$$

$$\alpha = n_{0j}/n_{0i}$$

where Z_r' is the real part of the Fried-Conte dispersion function. b and G are defined in [3]. Multiplying (4) by ϕ' and then integrating we obtain the "Energy Equation" as

$$\frac{\phi'^2(x)}{2} + V(\phi) = 0 \qquad (6)$$

where $V(\phi)$ is called the "Classical potential" and is given by

$$-V(\phi) = \frac{1}{2}A\phi^2 + \frac{2}{5}B_1\phi^{5/2} + B_2\left\{\phi\Psi^{3/2} - \frac{2}{5}\left[\Psi^{5/2} - (\Psi-\phi)^{5/2}\right]\right\} + \frac{1}{3}C_1\phi^3 + \frac{1}{2}C_2\phi^2\Psi + \cdots \qquad (7)$$

The nonlinear dispersion relation is obtained by imposing the condition $V(\Psi)=0$ in (7) as

$$A + \frac{2}{5}[2B_1 + 3B_2]\Psi^{1/2} + \left[\frac{2}{3}C_1 + C_2\right]\Psi = 0 \qquad (8)$$

Double layer solution is obtained by imposing condition $\partial V/\partial\phi = 0$ at $\phi = \Psi$. Thus we obtain from (4),

$$A + (B_1 + B_2)\Psi^{1/2} + (C_1 + C_2)\Psi = 0 \qquad (9)$$

From (8) and (9), we have,

$$B_1 = B_2 - \frac{5}{3}C_1\Psi^{1/2} \qquad (10)$$

$$A = -2B_2\Psi^{1/2} + \frac{2}{3}C_1\Psi - C_2\Psi \qquad (11)$$

Neglecting the parameters, C_2 and B_2 which comes due to the trapping effects of positive ions and using (10) and (11) we can write (7) as

$$-V(\phi) = \frac{1}{3}C_1\phi^2\left[\sqrt{\Psi} - \sqrt{\phi}\right]^2 \qquad (12)$$

Now, the solutions of (6) is found to be [3-5]

$$\phi(x) = \frac{\Psi}{4}\left[1 + \tanh \kappa x\right]^2 \qquad (13)$$

where $\kappa = \left(\frac{C_1\Psi}{24}\right)^{1/2}$

Following [3] the necessary condition for the existence of DL is that $C_1 > 0$ i.e.

$$\frac{\left(1+\frac{\alpha}{\theta}\right)^2}{1+\alpha} + \frac{3\alpha}{\theta^2} > 3 \qquad (14)$$

From (14) we may conclude that the existence condition of DL in the finite current condition i.e. at $u_{oi} = u_{oj}$ (the electron current is assumed to be negligibly small) satisfied for $\alpha \gtrsim 0.26$ and $\theta \simeq 1$.

Using (10) and (11), the velocity of this type of double layer is found to be

$$u_{oi} = \sqrt{\frac{1}{1-\alpha}\left(1+\frac{\alpha}{\theta}\right)}\left[1+\frac{8}{15}\left\{b\left(\beta, \sqrt{\delta}\, u_o\right) + \frac{\alpha}{1-\alpha}\, b\left(\alpha_j, \sqrt{\theta_j \theta}\, u_{oj}\right)\theta_j^{3/2}\right\}\left(\frac{\psi}{4}\right)^{1/2}\right] \qquad (15)$$

The DL solution in the current zero condition is found to exist for the limiting value of $\alpha \gtrsim \frac{1}{3}$ and $\theta \simeq 1$.

Retaining terms upto second orders in the expresion of $V(\phi)$ in (7) and then using (6) we obtain the solitary wave solution,

$$\phi = \psi\, \mathrm{Sech}^4\left[\left(\left(-\frac{B_1}{20}\right)\sqrt{\psi}\right)^{1/2} x\right] \qquad (16)$$

The soliton solution exist since B_1 is negative.

SUMMARY AND CONCLUSION:

Small amplitude compressive ion acoustic double layers in a plasma containing negative species of ions have been found to exist. It is also found that the DL moves with a velocity faster than the ion acoustic velocity. The limiting values of the densities of negative ions for the existence of analytic DL have been calculated. Small amplitude ion acoustic wave is also shown to exist. Our results may be applicable in the auroral plasma where different species of ions are found to exist. It should be pointed out that in presence of only one (positive) species of ions, no compressive ion acoustic DLs exist [4]. In such a situation only rarefactive DLs are found to exist, when the reflected ions follow a vortex type distribution [5].

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