SATURATION OF ION-ACOUSTIC TURBULENCE

V.YU.Bychenkov, O.M.Gradov
P.N.Lebedev Physical Institute, Moscow, USSR

In spite of the qualitative conformity of the distributed quasi-stationary (in $\vec{K}$ - space) spectra [1,2] of the ion-acoustic turbulence (IAT) to the experimental data the absence of the consistent nonstationary IAT theory, generally speaking, kept open the question of establishing such distributed spectra realization and, consequently, whether such spectra actually conforms to the experiments. We have investigated the evolution of IAT with considerably exceeded threshold of the ion-acoustic instability and the growth rate as a function of the wave number beeing sufficiently broad so that the exitation of the long-wave ion sound become available. It has been shown that IAT relaxation leads to the quasistationary noise distributions characterized by the distributed according to wave numbers as and angles spectra. This spectra conforms to the stationary theory [1,2] .

As in [1,2] we took into consideration both the scattering of the electrons on the IAT and the induced scattering of the waves by the ions. Then for $\mathcal{N}(\theta, K, t)$- the number of ion-acoustic quanta is the following:

$$\frac{\partial \mathcal{N}(\theta,K,t)}{\partial t} = 2\Gamma_{NL}(\theta,K,t)\mathcal{N}(\theta,K,t) + 2\gamma_s(K)\mathcal{N}_{sp}(K)[1+\delta(K)+\delta_{st}(K)] + 2\mu\gamma_0 r_{De}^{-2}\frac{\partial^2\mathcal{N}(\theta,K,t)}{\partial K^2} \qquad (1)$$

Here Γ_{NL} - nonlinear growth rate [1]

$$\Gamma_{NL}(\theta,K,t) = \gamma_s(K)\left[\cos\theta\sqrt{1+K^2r_{De}^2}\,u(\theta,t)-1-\delta_{st}(K)-\delta(K)\right] + K^2V_{Ti}^2(1+K^2r_{De}^2)^{3/2}\frac{\partial}{\partial K}K^4(1+K^2r_{De}^2)^{3/2}\cdot\int_{-1}^{1}d\cos\theta'\,Q(\sin\theta,\cos\theta')\mathcal{N}(\theta',K,t)/4\pi n_e \varkappa T_e ,$$

$$\gamma_s(K) = \gamma_0 K r_{De}(1+K^2r_{De}^2)^{-2}, \quad \gamma_0 = \sqrt{\pi/8}\,\omega_{Li}^2/\omega_{Le},$$

$$\delta_{st}(K) = \delta_0 (K r_{De})^{-1}(1+K^2r_{De}^2)^3, \quad \delta(K) = \delta\exp\left[\frac{K^2 r_{De}^4}{2r_{Di}^2(1+K^2r_{De}^2)}\right],$$

$$u(\theta,t) = \frac{2}{\pi}\int_0^{\sin\theta}\frac{d\cos\theta'}{\sqrt{\sin^2\theta-\cos^2\theta'}}\;\frac{K_N+\sin^{-1}\theta'\cdot\chi_1(\theta',t)}{\chi_2(\theta',t)+K_N/K_{st}} ,$$

$$\chi_n(\theta,t) = \frac{V_{Ti}^2}{\gamma_0}\int_0^{\sin\theta}d\cos\theta'\int_0^{\infty}\frac{K^4dK}{4\pi n_e\varkappa T_e}\,\mathcal{N}(\theta',K,t)\cdot\left(\frac{\cos\theta'}{\sin\theta}\right)^n(1+K^2r_{De}^2)^{\frac{n-5}{2}}(\sin^2\theta-\cos^2\theta')^{-\frac{1}{2}}, \quad n=1,2$$

the last term in (1) models small ($\mu \ll 1$) terms which are accounted by the four-waves proceses, N_{sp} is the thermal noise, θ is the angle between $\vec{K}$ and the force vector $\vec{F} = e\vec{E} - n_e^{-1}\nabla n_e \varkappa T_e$ which generates the turbulence. The values δ and δ_0 characterize the ion Cherenkov and collision damping, K_N is the Knudsen turbulent number which is proportional to F ; K_{st} is determined by the usual Knudsen number, Q(x,y) is the kernel of the nonlinear interaction (see [1]).

Eq.(1) with $\tau = 2\gamma_0 t$, $x = \cos\theta$, $y = K r_{De}$ was numerically solved for the function

$$w(x,y,\tau) = (V_{Ti}^2/\gamma_0 r_{De}^5)(y/\sqrt{1+y^2})(\mathcal{N}(\theta,K,t)/4\pi n_e\varkappa T_e)$$

which is proportional to the spectral energy density of the ion sound. It was used $w(x,y,0) = \alpha = 10^{-3}+10^{-5}$, $\delta < 1, \delta_0 < 1$ and the condition $K_{st} \gg 1$, corresponding to the considerable exceeding of the long-wave ion sound threshold. After a short period of time $\tau \sim (y\cdot K_{st})^{-1}\cdot 10$ the distribution w became independent on the primary thermal noise and the primary growth rate i.e. on the α and K_{st} parameters. In the large region of $\mu = (0{,}1 \div 5\ 10^{-4})$ the results didn't depend on this value.

In the relaxation turbulence process the IAT saturation was observed. If $K_N < 1$ the saturation time is of the order of magnitude of $y(\delta + \varepsilon)^{-1}$, where $\varepsilon = (8K_N/3\pi)\cdot \ln(1/K_N)$ was introduced in [1] as

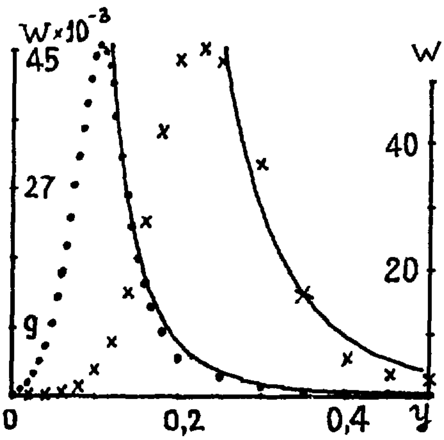

Fig. 1.
The quasistationary spectra for $K_N = 70$, $\delta = 0.18$, $\delta_0 = 0.5$ (points, $y_m = 0.11$); $K_N = 0.4$, $\delta = 10^{-4}$, $\delta_0 = 0.1$ (crosses, $y_m = 0.23$) and the KP spectrum (solid curves). $X = 1$.

a characteristic of the induced scattering efficiency. When $K_N \gg 1$ this time is of the order of $(y\cdot\sqrt{K_N})^{-1}$ magnitude. In the saturation regime the energy density of the noise $\mathcal{E}^2/4\pi$ corresponds to the interpolative dependencies $\mathcal{E}^2/4\pi \propto K_N(\delta+\varepsilon)^{-1}$ if $K_N < 1$ and $\mathcal{E}^2/4\pi \propto \sqrt{K_N}$ if $K_N \gg 1$. Under saturation regime the spectral IAT distribution has the maximum for a long waves $y_m \ll 1$. The energy in

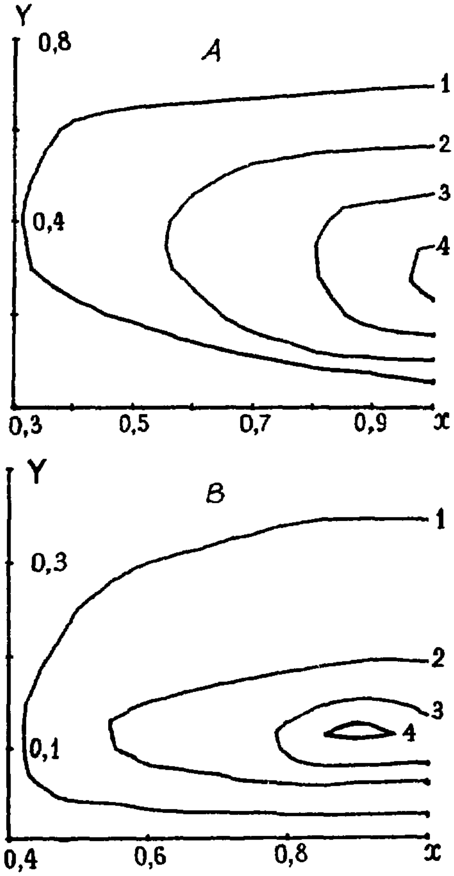

Fig. 2.
The curves w=const for $K_N = 70$, $\delta = 0.18$, $\delta_0 = 0.5$
A-τ=2.5: 1- w=10, 2- w=400, 3- w=1200, 4- w=2800
B-τ=58 (the quasistationary distribution): 1- w=300, 2- w=10^4, 3- w=$2.8\cdot10^4$, 4- w=$4\cdot10^4$.

the short-wave region $y \gtrsim 1$ is small due to the induced scattering as well as to the ion Cherenkov damping. The energy decreasing under $y < y_m$ is due to the collision damping of the ion sound. In the region $1 > y > y_m$ the spectrum is close to the Kadomtsev-Petviashvili (KP) spectrum (look Fig. 1).

The curves w =const is given on Fig. 2 for the case $K_N \gg 1$. Fig. 2 demonstrates decreasing of wavelength IAT and also the fact that on the initial stage of relaxation the turbulent pulsation concentrates along the force $\vec{F}$ which generates instability. In the saturation regime the angle distributions are essentially different for $K_N < 1$ and $K_N \gg 1$. So the quasistationary noise distibution for $K_N \gg 1$ has the maximum in the direction which differs from the direction $X=1$ corresponding to the noise maximum for $K_N < 1$ and coincidencing with the direction of the vector $\vec{F}$. This is in accordance with the quasistationary theory [1, 2]. We have discovered that the displacement of the angle maximum of the quasistationary spectrum increases with K_N increasing and reaches (if K_N is very large) the value [2] $\theta \simeq 35^{\circ}$.

Finally it shoued be mentioned that as the result of that numerical calculations similarity of K-dependencies of the noise for different angles in the reqion $y > y_m$ has been obtained. So there is a possibility of the aproximative represetation of the numerical solution for the quasistationary spectra as a product of two functions one of which depends only on the wave number and another depends only on the angles. It confirms the possibility of using of the variable separation method in the analitic quasistationary spectra theory [1,2].

1. V.Yu.Bychenkov, V.P.Silin, Sov. Phys. JETP 55 (1982) 1086.
2. V.Yu.Bychenkov, O.M.Gradov, V.P.Silin, Fiz. Plasmy 10 (1984) 33.
3. E.D.Volkov, N.F.Perepelkin, V.A.Suprunenko, E.A.Sukhomlin, Collective phenomena in a current-carrying plasma, p.21, Kiev, "Naukova Dumka".