EVOLUTION OF ENVELOPE SOLITONS OF IONIZATION WAVES

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Introduction

The pulse-like nonlinear waves are well known as the solitons, one of which is the envelope soliton that a pulse-like wave envelope behaves like a particle. There are many theoretical investigations for the envelope solitons which can be described by the nonlinear Schrödinger equation, while there are not so many experimental investigations. Generally many plasma waves are stable for the perturbed amplitude-modulation.

The evolutions of envelope solitons were observed in the ionization waves[1,2] which were excited in the positive column of He glow discharge. In the present paper, we report the propagation of envelope soliton of the ionization wave by numerically solving from the basic equations under a boundary condition.

Theoretical Procedures

Assuming the quasi-charge neutrality, we write the hydrodynamic equations in the one-dimenticnal system by the forms[3],

$$\frac{\partial n}{\partial t} - b_i \frac{\partial}{\partial x}\left(\frac{T_e}{e}\frac{\partial n}{\partial x}\right) = n\left(Z - \frac{1}{\tau_0}\right) \quad (1)$$

$$\frac{T_e}{n}\frac{\partial n}{\partial x} + \frac{\partial}{\partial x}\left(\frac{n b_e T_e}{j}\frac{\partial T_e}{\partial x}\right) + \frac{3}{2}\frac{\partial T_e}{\partial x} = \frac{en}{j}H - \frac{j}{n b_e}, \quad (2)$$

where b_i and b_e are the mobilities of the ions and electrons, j the electron current density, n the charge particle density, T_0 the electron temperature, H the electron energy loss, and τ_0 the particle lifetime. Assuming a constant τ_0, the ionization rate Z is expressed by

$$Z = \tau_0^{-1} \exp\left[V_1\left(T_{eo}^{-1} - T_e^{-1}\right)\right]. \quad (3)$$

We can put $V_1/T_{eo} \gg 1$ in the He glow discharge, where V_1 is the ionization potential and the suffix 0 corresponds to the steady-state value. Linearizing eq.(3), the following equations are deduced from eqs.(1) and (2) as

$$\frac{\partial}{\partial \eta}\left(N\frac{\partial T}{\partial \eta}\right) - \frac{\partial \ln N}{\partial \eta} = \frac{T_{eo}}{V_1}\left(N - N^{-1}\frac{3}{2}\frac{\partial T}{\partial \eta}\right) \quad (4)$$

$$\frac{\partial N}{\partial \theta} - N\left\{\exp(T) - 1\right\} = C\frac{\partial^2 N}{\partial \eta^2}, \quad (5)$$

where $T = V_1(T_e - T_{eo})/T_{eo}^2$, $N = n/n_0$, $\eta = eE_0V_1x/T_{eo}^2$, $\theta = t/\tau_0$, $c = b_i \tau_0 eE_0^2 V_1^2/T_{eo}^3$. Since $c \ll 1$ and $V_1/T_{eo} \gg 1$, we can put the right-hand sides of eqs.(4) and (5) as nearly zero,

$$\frac{\partial}{\partial x}\left(N\frac{\partial T}{\partial x}\right) - \frac{\partial \ln N}{\partial x} = 0 \quad (6)$$

$$\frac{\partial N}{\partial t} - N\left\{\exp(T) - 1\right\} = 0, \quad (7)$$

where $X \equiv \eta$, $t \equiv \theta$.

The nonlinear dispersion relation including the effect of the wave amplitude ϕ is generally given by

$$\omega = \omega(k, |\phi|^2), \quad (8)$$

where ω is the frequency and k the wavenumber. N and T in eqs.(6) and (7) are expanded by a form as[4]

$$U = U_0^{(0)} + \sum_{\alpha=1}^{\infty}\sum_{\ell=-\infty}^{\infty} U_\ell^{(\alpha)} \exp[i\ell(kx - \omega t)], \quad (9)$$

where ℓ and α are the integers, $U_\ell^{(\alpha)}$

is evaluated as the order of ε^{α} and ε is a small parameter. Substituting eq.(9) into eqs.(6) and (7), and including the terms up to $U_1^{(3)}$, a nonlinear dispersion relation is deduced as

$$\omega = \frac{1}{k} - \left(\frac{11}{6k} + \frac{1}{3k^2} + \frac{1}{2k^2}i\right)|\phi|^2. \quad (10)$$

If $|\phi|^2 = 0$, a linear dispersion relation is obtained for the ionization wave with the backward wave property.

The nonlinear Schrödinger equation is derived from eq.(10) by neglecting the imaginary term.

$$i\frac{\partial \phi}{\partial \tau} + P\frac{\partial^2 \phi}{\partial \xi^2} + Q|\phi|^2\phi = 0, \quad (11)$$

where $\tau = t$, $\xi = X - v_g t$, v_g the group velocity, $P = 1/2(\partial^2\omega/\partial k^2)$, $Q = -(\partial\omega/\partial|\phi|^2)$. The soliton solution of eq.(11) is given by

$$\Phi = A_m\,\mathrm{sech}\left\{\sqrt{\frac{2Q}{P}}A_m\xi\right\}\cos\{(\omega_0 + Q/2\,A_m)t - k_0(\xi + v_g t) + \varphi_0\}, \quad (12)$$

where A_m is the initial wave amplitude and φ_0 the phase constant.

Numerical Calculation and Its Results

To solve eqs.(6) and (7) numerically under some initail and boundary conditions, we assume

$$T(0 - v_g t) = T(L - v_g t) = 0$$
$$\left.\frac{\partial T}{\partial X}\right|_{X=0-v_g t} = \left.\frac{\partial T}{\partial X}\right|_{X=L-v_g t} = 0, \quad (13)$$

where L is the total length of positive column. It the initial value of N(0) is given with a required form, the value of T(0) can be computed from eqs.(6) and (7). The next value of N(Δt) can be computed by using the above T(0), and then the value of T(Δt) can be evaluated by using N(Δt). We can calculate the time variation of T and N by repeating the above procedures, where we take Δt and Δx as 0.01 and 0.02 respectively. Typical patterns of a propagating wave packet and an envelope soliton are shown in Fig.1 (a) and (b) respectively, where $k = 2/3\pi$ is used. The nonlinear dispersion relation is shown in Fig.2, where $|\phi|^2$ is the para-

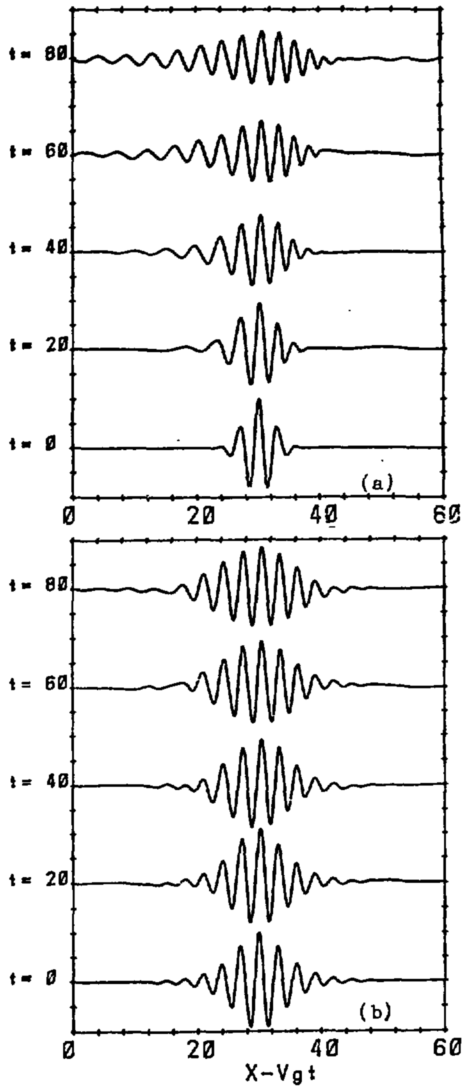

Fig.1 Propagation of a wave packet (a) and an envelope soliton (b), where $k = 2/3\pi$, $|\Phi|^2 = 0.01$

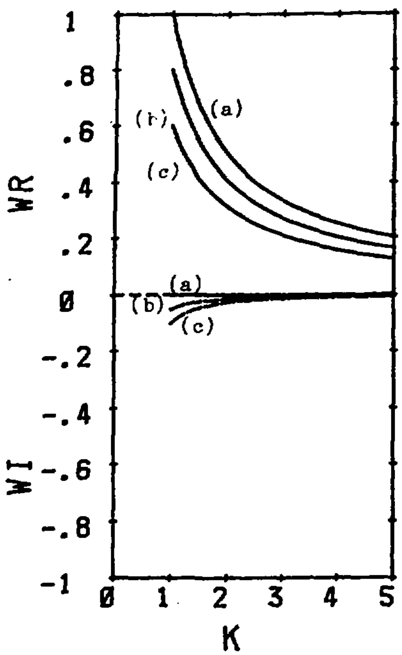

Fig.2 Nonlinear dispersion relation expressed in eq.(10), where (a) corresponds to $|\Phi|^2 = 0$, (b) to $|\Phi|^2 = 0.1$, and (c) to $|\Phi|^2 = 0.2$ respectively. WR expresses the real part of ω and WI the imaginary.

meter.

Discussion

The coefficients of the nonlinear Schrödinger equation for $k = 2/3\pi$ and $\Phi = 0.1$ are evaluated from eq. (10) as $P = 0.109$ and $Q = 0.912 + 0.051\,i$. Then we can expect the formation of envelope soliton, since $PQ > 0$.

When a suitable wave packet is given as the initial condition, the wave packet expands its width according to the linear theory, as is shown in Fig.1 (1). On the other hand, using the envelope soliton solution of eq.(12) as the initial condition, the envelope remains its initial wave form with a slightly damped. The damping of the envelope soliton corresponds to including the imaginary term in eq.(10). This can be expected by adding a imaginary term, which is assumed to be linearly proportional to Φ, to the right-hand side of eq.(11).

Concluding Remarks

The propagation of envelope solitons is demonstrated by numerically solving the basic equations describing the ionization wave. The envelope soliton pattern remains its wave form during propagation, while the wave packet spreads.

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