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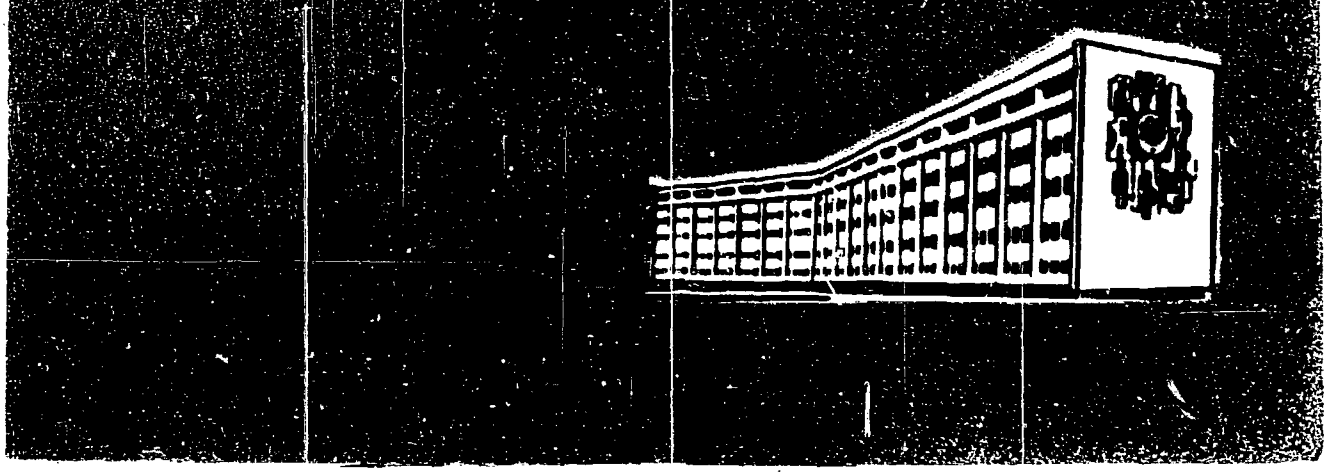
~~THE~~ PION INTERFEROMETRY THEORY FOR THE HYDRODYNAMIC STAGE OF MULTIPLE PROCESSES



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Теория пионной интерферометрии для гидродинамической стадии множественных процессов

Получено описание двухчастичных инклюзивных сечений тождественных частиц в гидродинамической теории множественных процессов. Теория пионной интерферометрии обобщена на случай, когда генерация вторичных частиц происходит на фоне внутреннего релятивистского движения излучающей адронной материи. В рамках релятивистского формализма функций Вигнера установлена связь между корреляционными функциями, соответствующим различным схемам эксперимента.

Makhlin A.N., Sinyukov Yu.M.

The Pion Interferometry Theory for the Hydrodynamic Stage of Multiple Processes

The double pion inclusive cross-section for identical particles is described in hydrodynamical theory of multiparticle production. The pion interferometry theory is developed for the case when secondary particles are generated against the background of internal relativistic motion of radiative matter. The connection between correlation functions in various types of experiment is found within the framework of relativistic Wigner functions formalism.

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Теория пионной интерферрометрии для гидродинамической стадии множественных процессов

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THE PION INTERFEROMETRY THEORY FOR THE HYDRODYNAMIC STAGE OF MULTIPLE PROCESSES

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I. INTRODUCTION

In central hadronic and nuclear collisions at high energies the hadronic matter experiences strong compression in a very short time. The existence of the new quark-gluonic state of the matter is predicted at energy densities more than 2 GeV/fm^3. Secondary particles prove to be the only source of information about the state of hadronic matter during the collision. The most important quantities allowing one to define the state of hadronic matter are its energy, the degree of its thermalization, the volume of the secondary particles generation region, the character of internal collective motion.

The idea of using the identical pion correlations to determine the space-time region of particles generation was proposed in Refs. [1,2,3]. The authors of these papers used the quantum-mechanical version of the intensity interferometry method suggested previously by Hanbury-Brown and Twiss for the purpose of steller angular size determination. In this approach the identity of the particles leads to the interference of their registration amplitudes. The probability of joint registration of two particles with momenta k_1 and k_2 which are emitted at the points x_1 and x_2 grows with respect to the product of their independent registration probabilities:

$$W(k_1, k_2; x_1, x_2) \sim 1 + \cos(k_1 - k_2)(x_1 - x_2). \qquad \text{(A)}$$

While analysing the experimental data one should take into account that the correlations are influenced by the particles interaction in the final states, the possibility of particles emission in the mixture of coherent and incoherent states, the restrictions imposed by conservation laws. These effects are often taken into account by means of phenomenological factor $\Lambda < 1$ before the cosine in the formula (A) [4].

Let's assume that the sources of the particles are independently distributed in the region $\mathfrak{D}$ with the density $\rho(x)$. Then the probability of joint two particles detection with momenta k_1 and k_2 takes the following form:

$$W(k_1, k_2) = \iint_{\mathfrak{D}} d^4x_1 d^4x_2\, \rho(x_1)\, \rho(x_2) \left[1 + \cos(k_1 - k_2)(x_1 - x_2)\right]. \qquad \text{(B)}$$

The option of that or other multiple particles generation process model imposes its own distribution function, and thus defines the functional dependence between pair correlation function and the generation region volume. The authors of the initial paper [1] assumed the $\rho(x)$ to be constant inside the spherical particles generation region. In Ref. [2] the particles were supposed to be emitted off the spherical surface the characteristic time being τ. Later on the pion interferometry theory was improved by changing the quantum-mechanical description in terms of two-particle wave functions for multiparticle S -matrix description in the picture of nuclear collisions based on a cascade theory of intranuclear nucleonic interactions [4].

However both in pp , $p\bar{p}$ and nuclear-nuclear collisions at present-day energies not the pions bremsstrahlung [4] is to be anticipated but initial thermalization of hadronic or quark-gluonic matter with its consequent decay into secondary particles. Locally, equilibrium thermodynamic system formed in this process possesses the collective internal motion as the hot matter clot boardering vacuum is nonstationary. This collective motion is normally described in frames of hydrodynamic theory of multiple particle generation.

In the present paper we develop the correlation method of the secondary particles generation space-time region definition for thermalized systems with internal relativistic hydrodynamic motion. The method is essentially based on the relativistic-invariant statistical description [5] and its formulation in terms of Wigner phase distributions [6].

2. STATEMENT OF THE PROBLEM

Let the stage of hadrons free motion following the hydrodynamic expansion of hadronic matter start from the space-like hypersurface $t = t^c(\vec{x})$ in the laboratory frame $(t, \vec{x})$. Its form can be taken e.g. from the Landau hydrodynamic model [7]. This surface we shall treat as the Caushy surface where the initial data for the free field $\Phi(x)$ are to be given (We'll deal for certainty with the scalar field of one sort of pions: $\Phi(x) = \varphi(x) + \varphi^+(x)$) .

Assume the system to cross the decay surface in the state of local thermodynamic equilibrium. In order to describe this state we demand all the thermodynamic averages of the field operators products like

$$\langle \varphi(\vec{x}_1, t^c(x_1))\, \varphi^+(\vec{x}_2, t^c(x_2))\rangle = Sp\, \rho\, \varphi(\vec{x}_1, t^c(\vec{x}_1))\, \varphi(\vec{x}_2, t^c(\vec{x}_2)) \quad (1)$$

turn to zero if the distance $|\vec{x}_1 - \vec{x}_2|$ exceeds some typical heat length L_T. Here ρ is the locally equilibrium density operator which will be described in detail later on. The condition of hydrodynamic description validity for the system with the mean size a is $a \gg L_T$. Otherwise all the modes of the field $\varphi(x)$ would be collective for the whole systems volume and the thermodynamic description would be inapplicable. Such a system would radiate coherently by all its volume and the interference measurement based on the independent emission of particles by separate parts of the system wouldn't be possible.

Suppose the Caushy surface to be approximately plane within the heat length L_T and the temperature $T(x) = \beta^{-1}(x)$ and hydrodynamic 4-velocity $u^\mu(x)$ to be constant within the same limits. The corresponding parts of the Caushy surface will be numbered by capital index N. The two local reference frames are tied up to each part. The first system $(t^c, \vec{x}^c)$ with the time axis is directed along the surface normal and the second one, the local rest frame, with the time axis is parallel to the 4-vector $u^\mu_N = u^\mu(x_N)$, $(t^*, \vec{x}^*)$. The distinction between these systems causes the anisotropy of fluid element decay in its rest frame [8].

The local heat equilibrium in the predecay state of hydrodynamic system implies the possibility of the quantum field $\varphi(x)$ independent description within different fluid elements. We'll denote the field localized in the vicinity of point x_N (the center of the element) as $\varphi_N(x)$. The lack of long-range order is readily ensured by a direct cutting off the averages (1):

$$\langle \varphi_N(x)\, \varphi^+_L(x')\rangle = 0 \qquad \text{when} \qquad N \neq L \qquad (2)$$

and can be realized with the help of the following simple trick. Let's take

$$\begin{aligned} \varphi_N(x) &= \sum_{\vec{p}} a_{\vec{p}}(N)\, \phi_{\vec{p}N}(x), \\ \varphi^+_N(x) &= \sum_{\vec{p}} a^+_{\vec{p}}(N)\, \phi^*_{\vec{p}N}(x) \end{aligned} \qquad (3)$$

on each part of the surface. The description of localized locally-equilibrium field state on surface Σ_c is done by means of the following procedure [5,6]. The one-particle wave functions $\phi_{\vec{p}N}(x)$ used for the expansion of field operator are determined by periodic boundary conditions in the ($\overset{*}{t}_N$, $\overset{*}{x}_N$) - system with the periods a_1 , a_2 , a_3 .So in any frame [6] :

$$\phi_{\vec{p}N}(x) = [2(pu_N)\overset{*}{V}]^{-1/2} e^{-ipx} \tag{4}$$

where $\overset{*}{V} = a_1 a_2 a_3$ is the periodicity cell volume in the rest frame and the momentum $\vec{p}$ takes the known set of discrete values

$$\overset{*}{p}_i = 2 l_i \pi / a_i \; ; \quad l_i = 0, \pm 1, \pm 2, \dots \; ; \; p_0^2 - \vec{p}^{\,2} = m^2$$

in this system. Here $(pu_N) = \overset{*}{p}{}^{\,0}_N$ is the invariant form of particles energy in the rest frame of the N-th fluid element.

Let the creation and annihilation operators $a^+_{\vec{p}}$ and $a_{\vec{p}}$ obey the commutation relations

$$\begin{aligned} &[a_{\vec{p}}(N), a^+_{\vec{p}'}(L)] = \delta_{\vec{p}\vec{p}'}\, \delta_{NL} , \\ &[a_{\vec{p}}(N), a_{\vec{p}'}(L)] = 0. \end{aligned} \tag{5}$$

Using the operators $a^+_{\vec{p}}(N)$ the invariant space of the field state on hypersurface Σ_c is constructed. If there is no coherent part in the field $\varphi(x)$, i.e. $\langle\varphi(x)\rangle = Sp\, \rho\, \varphi(x) = 0$, so from (5) there immediately follows (2). The locally-equilibrium statistical operator possesses the very property. It can be represented in the form [5,6] :

$$\rho = \prod_N \rho_N \; ; \quad \rho_N = e^{-\beta_N H_N} / Sp\, e^{-\beta_N H_N} \tag{6}$$

on the Caushy surface $t = t^c(\vec{x})$. The pion field Hamiltonian in the N -th cell equals

$$H_N = \sum_{\vec{p}} (pu_N)\, a^+_{\vec{p}}(N)\, a_{\vec{p}}(N). \tag{7}$$

The space-like Caushy hypersurface where the initial data for the free propagation of the secondary particles are set should be complete and thus infinite in space directions. The hadronic matter occupies only a small domain in it. The condition $\varphi(x)=0$ outside this domain would mean the existence of the topological boundary which is not true. So the condition of the matter binding to the finite part of the initial data surface should be written as

$$T(x) = \begin{cases} T_c = \text{const, inside the matter} \\ 0 \qquad\quad \text{, outside the matter} \end{cases} \tag{8}$$

where $T_c \simeq m_\pi$ is the critical temperature at which the stage of free expansion begins.

The statistical averaging of the observables operators with respect to initial data on the Caushy surface is performed in the usual way with the help of thermodynamic Wick-Bloch-de Dominicis theorem [9]:

$$\begin{gathered} \langle a^+_{\vec{p}}(N)\, a_{\vec{p}'}(N')\rangle = \delta_{\vec{p}\vec{p}'}\,\delta_{NN'}\,\langle n_{\vec{p}}(x_N)\rangle \\ \langle n_{\vec{p}}(x_N)\rangle = [\exp(\beta_N(pu_N)) - 1]^{-1} \end{gathered} \tag{9}$$

The evolution of the free Heisenberg field $\varphi(x)$ with the initial data on the surface $t = t^c(\vec{x})$ is described by known equations

$$\begin{gathered} \varphi(x) = \sum_N \int_{V_N} d\Sigma^\mu(z)\, G_{ret}(x-z) \overleftrightarrow{\frac{\partial}{\partial z^\mu}} \varphi_N(z), \\ \varphi^+(x) = \sum_N \int_{V_N} d\Sigma^\mu(z)\, \varphi^+_N(z) \overleftrightarrow{\frac{\partial}{\partial z^\mu}} G_{adv}(z-x), \end{gathered} \tag{10}$$

where $G_{ret}(x)$ and $G_{adv}(x)$ are the retarded and adavnced Green functions of the free scalar field. Here and on we treat $\varphi(x)$ as the Heisenberg field operators. So Eqs. (10) give the solution of motion equations with initial data on the surface $t = t^c(\vec{x})$. The state space is motionless and is defined namely at this surface. The density operator (6) performs the averaging of Heisenberg observables over the chaotic heat distribution of the initial states. It had originated from the true systems dynamics and is freezen by switching off the interaction between the fields on attaining the critical temperature at the hypersurface $t = t^c(\vec{x})$.

The operator of local pion current density is defined as

$$j_\mu(x) = \varphi^+(x)\, i \frac{\overleftrightarrow{\partial}}{\partial x^\mu}\, \varphi(x) = \lim_{\xi\to 0} j_\mu(x,\xi),$$

$$j_\mu(x,\xi) = -2i \frac{\partial}{\partial \xi^\mu}\left[\varphi^+\left(x+\frac{\xi}{2}\right)\varphi\left(x-\frac{\xi}{2}\right)\right]. \tag{11}$$

Let's transform this operator to the phase space representation by applying the Wigner transformation to it:

$$j_\mu(x,k) = \int \frac{d^4\xi}{(2\pi)^4}\, j_\mu(x,\xi)\, e^{-ik\xi} \tag{12}$$

Making use of Eqs. (15)-(7) and (9)-(11) it is easy to find the thermodynamic averages for the phase densities of the particles current and the two currents product:

$$\langle j^\mu(x,k)\rangle = \sum_N \sum_{\vec p} \Lambda^\mu_{pp}(N,N|x,k)\,\langle n_p(x_N)\rangle \tag{13}$$

$$\langle j^\mu(x_1,k_1)\, j^\nu(x_2,k_2)\rangle = \langle j^\mu(x_1,k_1)\rangle\langle j^\nu(x_2,k_2)\rangle +$$

$$+\sum_{N,L}\sum_{p,q} \Lambda^\mu_{pq}(N,L|x_1,k_1)\Lambda^\nu_{qp}(L,N|x_2,k_2)\langle n_p(x_N)\rangle\langle 1+n_q(x_L)\rangle. \tag{14}$$

The quantities $\Lambda^\mu_{pq}(N,L|x,k)$ are of the form

$$\Lambda^\mu_{pq}(N,L|x,k) = 16k^\mu(\overset{*}{p}{}^0_N \overset{*}{q}{}^0_L \overset{*}{V}_N \overset{*}{V}_L)^{-1/2}\int d^4s\, G_{ret}(s)\, G_{adv}(2k-s)\times$$

$$\times(s^\nu+p^\nu)(2k^\sigma+q^\sigma-s^\sigma)\exp\{is(2x-x_N-x_L)-2ik(x-x_L)\}\times \tag{15}$$

$$\times\int_{V_N} d\Sigma_\nu(\xi)\, e^{i(p-s)\xi}\cdot\int_{V_L} d\Sigma_\sigma(\eta)\, e^{-i(q-2k+s)\eta},$$

where the internal integrals are evaluated over the volumes of elementary cells on the Caushy surface and the Fourier-transforms are introduced:

$$G_{\substack{ret\\adv}}(s) = -(2\pi)^{-4}\left[(s^0\pm i0)^2-\vec{s}^{\,2}-m^2\right]^{-1}.$$

The phase-space densities (13) and (14) being well defined in classical statistical mechanics are in general meaningless in quantum theory. They are not positively defined in general case, have no probability interpretation and are in line with none of measurable quantities. Nevertheless they present the convenient auxiliary material for deriving the observables distributions. If the total phase distribution in the space $\{x,k\} = \{x^0, \vec{x}\,;\, k^0, \vec{k}\}$ has been integrated with respect to one set of conjugate variables so the distribution over the second one will be well defined. Thus, the integration over all the momentum variables turns us back to the initial distribution (11) of observable density in the coordinate space. The integration over three space coordinates $\vec{x}$ and k^0 gives the observable's distribution over the particles momenta $\vec{k}$ as the function of time x^0. The other distributions corresponding to different experimental arrangements can be built also. The version fitting the geometry of electronic or track detecting the secondary particles will be analysed in detail below.

3. NUMBERS OF PARTICLES AND THEIR CORRELATIONS IN MOMENTUM SPACE

In order to derive the number of particles density and the correlation density in $\vec{k}$ -space the Eqs. (13) and (14) should be integrated over the space coordinates and the time-like components k^0 of 4-momenta k :

$$\langle n(x^0, \vec{k})\rangle = \int d^3\vec{x}\, dk^0 \quad \langle j^0(x,k)\rangle, \tag{16}$$

$$\langle n(x_1^0, \vec{k}_1)\, n(x_2^0, \vec{k}_2)\rangle = \int d^3\vec{x}_1 d^3\vec{x}_2\, dk_1^0 dk_2^0 \langle j^0(x_1,k_1)\, j^0(x_2,k_2)\rangle. \tag{17}$$

It can be easily seen that the integration immediately transfers to the functions $\Lambda_{pq}(N,L|x,k)$. The calculations are obvious. The integrations over $d^3\vec{x}$ and then $d^3\vec{s}$ are to be performed first. The rest integrals over k^0 and s^0 can be reduced to contour and evaluated by the residue method. This gives

$$\Lambda^0_{pq}(N,L|x^0,\vec{k}) = \frac{\theta(x^0-x_N^0)\,\theta(x^0-x_L^0)\,(k^\lambda+p^\lambda)(k^\nu+p^\nu)}{(2\pi)^3\cdot 4k^0\,(\overset{*}{V}_N\,\overset{*}{V}_L\,\overset{*}{p}{}^0_N\,\overset{*}{q}{}^0_L)^{1/2}}\cdot e^{ik(x_N-x_L)}\times$$

$$\times \int_{V_N} e^{i(p-k)\xi} d\Sigma_\lambda(\xi) \cdot \int_{V_L} e^{-i(q-k)\eta} d\Sigma_\nu(\eta) \qquad (18)$$

as the result. In what follows we denote $k = (k^0, \vec{k})$ with $k^0 = +\sqrt{\vec{k}^2 + m^2}$. As it can be seen from (18) the densities (16) and (17) depend upon the time until all the initial data on the Caushy surface will appear to be in the past of the observation time. Henceforth the momentum distribution will be completely formed and would not vary with time. So we assume the x^0 to be large enough and ommit it in what follow.

It is convenient to evaluate the integrals over the three-dimensional domains V_N of the Caushy hypersurafce in the local frame ($t^c_N, \vec{x}^c_N$) of each domain moving at a speed $\vec{v}^c = \mathrm{grad}\, t^c(\vec{x})$. The corresponding 4-velocity will be denoted as $u^c_N = u^c(x_N)$. Besides, that the 4-vector $\tilde{u}_N$ of the ($t^*_N, \vec{x}^*_N$) and ($t^c_N, \vec{x}^c_N$) -systems relative motion will be helpful. The 4-momenta p^μ are naturally given by their discrete values p^*_N in the rest frame and the 4-vectors k^μ are initially given in the ($t, \vec{x}$) -system. Let $p_N = p^c_N$ and $k_N = k^c_N$ be the same vectors but Lorentz-transformed to the reference system ($t^c_N, \vec{x}^c_N$). Then due to (18),(13) and (14) Eqs. (16) and (17) take the form:

$$\langle n(\vec{k})\rangle = \sum_N \sum_{\vec{p}} (4 k^0 \overset{*}{p}{}^0 \overset{*}{V}_N)^{-1} (k^0_N + p^0_N)^2 \langle n(p u_N)\rangle \times$$
$$\times \int_{V^c_N} e^{i(\vec{p}_N - \vec{k}_N)\vec{\xi}_N} d^3\vec{\xi}_N \cdot \int_{V^c_N} e^{-i(\vec{p}_N - \vec{k}_N)\vec{\eta}_N} d^3\vec{\eta}_N . \qquad (19)$$

$$R(k_1, k_2) = \langle n(\vec{k}_1)\, n(\vec{k}_2)\rangle - \langle n(\vec{k}_1)\rangle\langle n(\vec{k}_2)\rangle =$$

$$= \sum_{N,L} \sum_{\vec{p},\vec{q}} \frac{(k^0_{1N} + p^0_N)(k^0_{1L} + q^0_L)(k^0_{2L} + q^0_L)(k^0_{2N} + p^0_N)}{16\, k^0_1 k^0_2\, \overset{*}{p}{}^0_N\, \overset{*}{q}{}^0_L\, \overset{*}{V}_N \overset{*}{V}_L}\, e^{i(k_1 - k_2)(x_N - x_L)} \times$$
$$\times \langle n(p u_N)\rangle\langle 1 + n(q u_L)\rangle \cdot \int_{V^c_N} e^{i(\vec{p}_N - \vec{k}_{1N})\vec{\xi}_N} d^3\vec{\xi}_N \cdot \int_{V^c_L} e^{-i(\vec{q}_L - \vec{k}_{1L})\vec{\eta}_L} d^3\vec{\eta}_L \times \qquad (20)$$
$$\times \int_{V^c_L} e^{i(\vec{q}_L - \vec{k}_{2L})\vec{\xi}_L} d^3\vec{\xi}_L \cdot \int_{V^c_N} e^{-i(\vec{p}_N - \vec{k}_{2N})\vec{\eta}_N} d^3\vec{\eta}_N .$$

The evaluation of $\langle n(\vec{k})\rangle$ and $\langle n(\vec{k}_1)\, n(\vec{k}_2)\rangle$ is performed in the following approximation. We assume the volume V_N to be so large that the thermodynamic limit can be used and the sum over the discrete set of $\vec{p}^{\,*}_N$ values should be replaced by the integral according to the evident rule:

$$\sum_{\vec{p}^*}\ldots \rightarrow \frac{\overset{*}{V}_N}{(2\pi)^3}\int d^3\vec{p}^{\,*}\ldots = \frac{\overset{*}{V}_N}{(2\pi)^3}\int d^3\vec{p}^{\,c}_N\,\frac{\overset{*}{p}{}^{\,o}_N}{\overset{c}{p}{}^{\,o}_N}\ldots \qquad (21)$$

Following this idea one of the two integrals over the same volume V^c_N in (19) and (20) should be taken equal to the Dirack's delta. The one particle distribution function $\langle n(\vec{k})\rangle$ of the hydrodynamic model takes the known form [8] in this approximation. On the other hand we treat the volumes V_N as the unit ones of the continuous hydrodynamic description.

The elementary cell's size a_N is much less than the characteristic linear size L of the system. The rest integrals are to be calculated in the approximation $|\vec{k}_{1N}-\vec{k}_{2N}| \ll 1/a_N$, i.e.

$$\int_{V^c_N} exp\,[\,i\,(\vec{k}_{1N}-\vec{k}_{2N})\,\vec{\eta}\,]\,d^3\vec{\eta} = V^c_N$$

This is the very condition which is nesessary for the interference measurements of Hanbury Brown-Twiss type where the maximum of the measured momenta is restricted by $1/L$ [3] : $|\vec{k}_1-\vec{k}_2|_{max} \lesssim 1/L$.

The result reads as

$$\langle n(\vec{k})\rangle = \sum_N \frac{\overset{*}{V}_N}{(2\pi)^3\,\tilde{u}^o_N}\;\frac{k^o_N}{k^o}\;\langle n(k u_N)\rangle , \qquad (22)$$

$$R(\vec{k}_1,\vec{k}_2) = \sum_{N,L}\frac{\overset{*}{V}_N\,\overset{*}{V}_L}{(2\pi)^6\,\tilde{u}^o_N\,\tilde{u}^o_L}\cdot\frac{(k^o_{1N}+k^o_{2N})(k^o_{1L}+k^o_{2L})}{4\,k^o_1\,k^o_2}\,\times$$

$$\times\langle n(k_1 u)\rangle\langle 1+n(k_2 u)\rangle\, e^{-i(k_1-k_2)(x_N-x_L)} . \qquad (23)$$

Changing the summation of Eqs. (22) and (23) by the integration over the surface $t = t_c(\vec{x}\,)$ we rewrite the result in standard Lorentz-invariant form

$$k^o\,\frac{dN}{d\vec{k}} \equiv k^o\langle n(\vec{k})\rangle = (2\pi)^{-3}\int d\sigma^\mu(x)\,k_\mu\,\langle n(k\,u(x))\rangle, \qquad (24)$$

$$k_1^0 k_2^0 \frac{d^2N}{d\vec{k}_1 d\vec{k}_2} - k_1^0 k_2^0 \frac{dN}{d\vec{k}_1}\frac{dN}{d\vec{k}_2} \equiv k_1^0 k_2^0 \, R(\vec{k}_1, \vec{k}_2) =$$

$$= \int \frac{d\sigma_\mu(x)\, d\sigma_\nu(y)}{(2\pi)^6} \cdot \frac{(k_1^\mu + k_2^\mu)(k_1^\nu + k_2^\nu)}{4} \cdot e^{-i(k_1-k_2)(x-y)} \times \tag{25}$$

$$\times \langle n(k_1 u(x))\rangle \langle 1 + n(k_2 u(y))\rangle .$$

Using the above assumption of the Caushy surface infiniteness one of the integrations in Eq. (25) can be performed explicitly:

$$\int \frac{d\sigma_\nu(y)}{(2\pi)^3} \cdot \frac{k_1^\nu + k_2^\nu}{2} \, e^{i(k_1-k_2)y} = \delta(\vec{k}_1 - \vec{k}_2)\cdot k_1^0 \tag{26}$$

Taking advantage of the integrand symmetry we get finally

$$R(\vec{k}_1, \vec{k}_2) = \delta(\vec{k}_1 - \vec{k}_2)\langle n(\vec{k}_1)\rangle + R_{inc}(\vec{k}_1, \vec{k}_2), \tag{27}$$

where

$$R_{inc}(\vec{k}_1, \vec{k}_2) = \int \frac{d\sigma_\mu(x)\, d\sigma_\nu(y)}{(2\pi)^6} \cdot \frac{(k_1^\mu + k_2^\mu)(k_1^\nu + k_2^\nu)}{4 k_1^0 k_2^0} \times$$
$$\times \langle n(k_1 u(x))\rangle \langle n(k_2 u(y))\rangle \cos(k_1 - k_2)(x-y). \tag{28}$$

Combining (27) with the first of Eqs. (20) we get the number of particles pairs density averaged over the local heat distribution on the initial data surface:

$$\langle n(\vec{k}_1)[\, n(\vec{k}_2) - \delta(\vec{k}_1 - \vec{k}_2)]\rangle = \langle n(\vec{k}_1)\rangle\langle n(\vec{k}_2)\rangle + R_{inc}(\vec{k}_1, \vec{k}_2). \tag{29}$$

In Sec. 5 it will be shown that this quantity is closely allied to the measured in the experiment different inclusive cross-section. For two small but finite phase volumes $\Delta\vec{k}_1$ and $\Delta\vec{k}_2$

$$\sigma_T^{-1}\sigma_{inc}(\Delta\vec{k}_1, \Delta\vec{k}_2) = \langle n(\vec{k}_1)[\, n(\vec{k}_2) - \delta_{\vec{k}_1\vec{k}_2}]\rangle \, \Delta\vec{k}_1 \, \Delta\vec{k}_2$$

with σ_T being the total cross-section of the interaction. Namely, the number of particle pairs is averaging over the ensemble of events. The theoretical (numerical) evaluation of this function for the different kinds of initial data will be given in the next paper. Therein we'll analyse the contribution of the initial parameters to their explicit form.

4. THE PHASE SUBSPACE OF AN EXPERIMENT

The density functions $\langle n(\vec{k})\rangle$ and $\langle n(\vec{k}_1)\, n(\vec{k}_2)\rangle$ in the momentum space should be related to the quantities measured in experiment. The $\langle n(\vec{k})\rangle$ by its definition should equal the average number at particles with the momenta $\vec{k}$ arisen from the initial state decay. The usual experimental arrangement involves the target and the particles detectors placed at different angles on the sphere of large radius R. (The geometrical analysis of chamber tracks reproduces the same scheme).

Let the particle be registered at the time moment x^0 by the detector with coordinates $\vec{x} = (R, \theta, \varphi)$ and the particles' momenta be also given by spherical coordinates, $\vec{k} = (|\vec{k}|, \alpha, \beta)$. Then the counting of particles by a single detector will give the total number of particles passed in all the time in the radial direction through the front aperture of the detector at the angular coordinates θ, φ. Such a quantity is in line with the function

$$\langle n(|\vec{k}|, \theta, \varphi)\rangle = \lim_{R\to\infty} R^2 \int dx^0 dk^0 d\Omega(\vec{k}) \langle j^R(x^0, R, \theta, \varphi; k^0, |\vec{k}|, \alpha, \beta)\rangle \quad (30)$$

where j^R is the radial component of the current j^μ and $d\Omega(\vec{k}) = \sin\alpha\, d\alpha\, d\beta$. The total phase density of the radial current had been integrated with respect to surplus variables - the time, the particles energy and its momenta direction. The average of the two radial currents product is constructed in the same way:

$$\langle n(|\vec{k}_1|, \theta_1, \varphi_1)\, n(|\vec{k}_2|, \theta_2, \varphi_2)\rangle = \lim_{R\to\infty} R^4 \int dx_1^0 dx_2^0 dk_1^0 dk_2^0 \times$$
$$\int d\Omega(\vec{k}_1)\, d\Omega(\vec{k}_2) \langle j^R(x_1, k_1)\, j^R(x_2, k_2)\rangle . \quad (31)$$

In order to evaluate the function (30) one should turn back to Eqs. (13) and (15) taking $\vec{q} = \vec{p}$, $N = L$ in the last one. Having calculated the integrals over the Caushy surface in the thermodynamic limit described above we come to

$$\Lambda^{R}_{pp}(N,N|x,k) = 16\, k^{R} V_N\, p_0^{-1} \int d^4 s\; G_{adv}(s)\, G_{ret}(2k-s) \times$$

$$\times (s^0+p^0)(2k^0-s^0+p^0)\, e^{2i(k-s)(x-x_N)}\; \delta(\vec{p}-\vec{s}). \qquad (32)$$

(As we are interested now only in the new way of describing the final states, the distribution between the systems $(t,\vec{x})$, $(t^*_N, \vec{x}^*_N)$ and $(t^c_N, \vec{x}^c_N)$ shouldn't be made). After substituting (32) into (13) we shall perform in succession the integrations prescribed by Eq. (30). The integrals over x^0 , s^0 and $\vec{s}$ are easily taken, the integral with respect to k^0 can be evaluated by the residue method. Then the six-dimensional phase density of the radial current takes the form

$$\langle j^{R}(\vec{x},\vec{k}) = \frac{4ik^{R}}{(2\pi)^6} \sum_N V_N \int d^3\vec{p}\; \frac{e^{2i(\vec{p}-\vec{k})(\vec{x}-\vec{x}_N)}}{\vec{k}\vec{p}-\vec{k}^2+i0} \langle n(pu_N)\rangle. \qquad (33)$$

The vectors of the integrand may be preset in the spherical system with the polar axis along the vector $\vec{\Delta}_N = (\vec{x}-\vec{x}_N)$:

$$\vec{x} = (R, \delta_N, 0)\,; \quad \vec{k} = (k, \alpha, \beta)\,; \quad \vec{p} = (p, \theta, \varphi).$$

The radial component of the vector $\vec{k}$ equals to

$$k^{R} = k\cos\psi = k\,(\cos\alpha\cos\delta_N + \sin\alpha\sin\delta_N\cos\beta),$$

where ψ is the angle between the vectors $\vec{k}$ and $\vec{x}$.

Let γ be the angle between the vectors $\vec{k}$ and $\vec{p}$. It is clear that

$$\cos\gamma = \cos\alpha\cos\theta + \sin\alpha\sin\theta\cos(\varphi-\beta).$$

Considering that $\langle n(pu_N)\rangle$ depends only upon the length p of the vector $\vec{p}$, we get the following expression of desired function:

$$\langle j^{R}(\vec{x},|\vec{k}|)\rangle = \frac{4i|\vec{k}|}{(2\pi)^6} \sum_N V_N \int_0^{2\pi} d\beta \int_0^{\pi} \sin\alpha\, d\alpha \int_0^{\infty} p^2 dp \int_0^{2\pi} d\varphi \times$$

$$\times \int_0^{\pi} \sin\theta\, d\theta \cos\psi\; \frac{\exp\{-2i\Delta_N(p\cos\theta - k\cos\alpha)\}}{kp\cos\gamma - k^2+i0} \langle n(pu_N)\rangle \qquad (34)$$

The integrals over the angles θ and α are evaluated successively in the asymptotic limit of $\Delta_N \to \infty$ ($R \to \infty$) by means of integration by parts; the integrals over β and φ give the factor $4\pi^2$ in this approximation:

$$\langle j^R(\vec{x},|\vec{k}|)\rangle = \frac{2}{(2\pi)^4} \sum_N V_N \frac{\cos\delta_N}{|\vec{k}|\,\Delta_N^2} \int_0^\infty \rho\, d\rho \,\langle n(\rho u_N)\rangle \times$$
$$\times \left[\frac{\sin 2(\rho-k)\Delta_N}{\rho-k} - \frac{\sin 2(\rho+k)\Delta_N}{\rho+k} \right]. \qquad (35)$$

Multiplying by R^2 and going to the limit of $R \to \infty$ we get finally

$$\langle n(\vec{x},|\vec{k}|)\rangle \equiv \langle n(\theta,\varphi,|\vec{k}|)\rangle = \qquad (36)$$
$$= \sum_N \frac{V_N}{(2\pi)^3} \langle n(k u_N)\rangle = \langle n(\vec{k})\rangle .$$

As may be seen from close examination of integration procedure the only nonvanishing contribution to (36) gives the value $\alpha = 0$,i.e. the vector $\vec{k}$ directed to the point $\vec{x} = (R,\theta,\varphi)$ of detector position. Equation (36) appears to be a specific form of continuity equation for the phase distributions relating the distribution $\langle n(\vec{k})\rangle$ in the momentum subspace to the distribution $n(\theta,\varphi,|\vec{k}|)$ in the subspace of experimental variables. It justifies the identification of the detector angular position (or the chamber track direction) to the direction of particles momenta.

Similar relations are inherent in the correlation functions too. Indeed integrating over the Caushy surface and the phase variables x_1^0 , x_2^0 and k_1^0 , k_2^0 we'll reduce the correlation function of radial currents to the next form

$$\mathcal{R}(\vec{x}_1,\vec{k}_1;\vec{x}_2,\vec{k}_2) = \langle j^R(\vec{x}_1,\vec{k}_1)\, j^R(\vec{x}_2,\vec{k}_2)\rangle - \langle j^R(\vec{x}_1,\vec{k}_1)\rangle\langle j^R(\vec{x}_2,\vec{k}_2)\rangle =$$

$$= -2^4 k_1^R k_2^R \sum_{N,L} \frac{V_N V_L}{(2\pi)^{12}} \int d^3p\, d^3q \left\{ \theta(\tau) F_1 + \theta(-\tau) F_2 \right\} \times$$

$$\times \frac{(\omega_p+\omega_q)\langle n(pu_N)\rangle\langle 1+n(qu_L)\rangle}{2\,\omega_p\,\omega_q\,(\vec{k}_1\vec{p}-\vec{k}_1^{\,2}+i0)(\vec{k}_2\vec{q}-\vec{k}_2^{\,2}+i0)} \times$$

$$\times \exp\{-i\vec{p}\,(2\vec{x}_1-\vec{x}_N-\vec{x}_L)-i\vec{q}\,(2\vec{x}_2-\vec{x}_N-\vec{x}_L)+2i[\vec{k}_1(\vec{x}_1-\vec{x}_L)+\vec{k}_2(\vec{x}_2-\vec{x}_N)]\} \qquad (37)$$

where $\omega_p = \sqrt{\vec{p}^{\,2}+m^2}$,

$$F_1 = \omega_{2k_1-p}^{-1}\, e^{i\omega_q\tau}\,[(\omega_q+\omega_{2k_1-p})(\omega_p+\omega_{2k_1-p})\,e^{-i\omega_{2k_1-p}\tau} - (\omega_q-\omega_{2k_1-p})(\omega_p-\omega_{2k_1-p})\,e^{i\omega_{2k_1-p}\tau}],$$

and F_2 can be obtained from F_1 by means of replacing $\vec{p}\leftrightarrow\vec{q}$, $\vec{k}_1\rightarrow\vec{k}_2$, $\tau\rightarrow-\tau$. We have kept the dependence upon $\tau = x_N^0-x_L^0$ even though the difference between the reference systems was neglected. This allows us an easy passage to the common case.

Let's introduce the vectors $\vec{\Delta}_{L1}=\vec{x}_1-\vec{x}_L$, $\vec{\Delta}_{L2}=\vec{x}_2-\vec{x}_L$ and $\vec{\Delta}_{N1}=\vec{x}_1-\vec{x}_N$, $\Delta_{N2}=\vec{x}_2-\vec{x}_N$ running from the different points of the Caushy surface, $\vec{x}_N$ and $\vec{x}_L$, to the detectors, $\vec{x}_1$ and $\vec{x}_2$. It is convenient to perform the ensuing integration using two special spherical coordinate systems. One group of the vectors is given via the first system: $\vec{\Delta}_{L1}=(\Delta_{L1},0,0)$, $\vec{x}_1=(R,\delta_{L1},0)$, $\vec{k}_1=(|\vec{k}_1|,\alpha_1,\beta_1)$, $\vec{p}=(p,\theta_1,\varphi_1)$, $\vec{\Delta}_{N1}=(\Delta_{N1},\chi_1,\lambda_1)$, and another group in the second one: $\vec{\Delta}_{N2}=(\Delta_{N2},0,0)$, $\vec{x}_2=(R,\delta_{N2},0)$, $\vec{k}_2=(|\vec{k}_2|,\alpha_2,\beta_2)$, $\vec{q}=(q,\theta_2,\varphi_2)$, $\vec{\Delta}_{L2}=(\Delta_{L2},\chi_2,\lambda_2)$. Except the angles γ and ψ (a number of them is doubled now) the two new angles, η_1 and η_2 between the vectors $\vec{\Delta}_{N1}$ and $\vec{p}$ coupled with $\vec{\Delta}_{L2}$ and $\vec{q}$:

$$\cos\eta = \cos\chi\cos\theta + \sin\chi\sin\theta\cos(\varphi-\lambda)\,.$$

The calculations start from the integration over φ_1 and φ_2 . The exponent in the integrand has the internal stationary point at $\varphi=\lambda$ and the saddle point method is used when $\Delta\rightarrow\infty$ .Next we use the saddle points at $\theta=\chi/2$ and perform integration over θ_1 and θ_2. The integration over β_1 and β_2 gives the factor $4\pi^2$ and the rest integrals over α_1 and α_2 have no stationary points. Their asymptotic expansion at $\Delta \to \infty$ is evaluated by the integration by parts. This gives

$$\mathcal{R}(\vec{x}_1,|\vec{k}_1|;\vec{x}_2,|\vec{k}_2|) = -\sum_{N,L} \frac{V_N V_L \cos\delta_{L1}\cos\delta_{N2}\, e^{2i(|\vec{k}_1|\Delta_{L1}+|\vec{k}_2|\Delta_{N2})}\cdot}{(2\pi)^8 R\,(\Delta_{L1}\Delta_{N2})^{3/2}\cos\frac{\chi_1}{2}\cos\frac{\chi_2}{2}\cdot|\vec{k}_1|\cdot|\vec{k}_2|}\times$$

$$\times\int_0^\infty \frac{p\,dp\cdot q\,dq\,(\omega_p+\omega_q)\, e^{-ip(\Delta_{L1}-\Delta_{N1})\cos\frac{\chi_1}{2}-iq(\Delta_{L2}-\Delta_{N2})\cos\frac{\chi_2}{2}}}{\omega_p\,\omega_q\,(|\vec{k}_1|-p\cos\frac{\chi_1}{2}-i0)(|\vec{k}_2|-q\cos\frac{\chi_2}{2}-i0)}\times$$

$$\times\langle n(pu_N)\rangle\langle 1+n(qu_L)\rangle\left[\theta(\tau)F_1(\chi_1)+\theta(-\tau)F_2(\chi_2)\right].$$

Noting that,

$$\lim_{\Delta\to\infty}\frac{e^{i\Delta(k-p\cos\chi/2)}}{k-p\cos\frac{\chi}{2}-i\epsilon} = 2\pi i\,\theta(\epsilon)\,\delta(k_1-p\cos\tfrac{\chi}{2}).$$

and that at $|\vec{k}_1| = p\cos\frac{\chi_1}{2}$, $|\vec{k}_2| = q\cos\frac{\chi_2}{2}$ we have

$$\omega^2_{2k_1-p} = \omega_p^2 = m^2+(|\vec{k}_1|^2/\cos^2\tfrac{\chi_1}{2}) = \tilde{\omega}^2_{k_1},$$

$$\omega^2_{2k_2-q} = \omega_q^2 = m^2+(|\vec{k}_2|^2/\cos^2\tfrac{\chi_2}{2}) = \tilde{\omega}^2_{k_2}$$

we finally get

$$\mathcal{R}(\vec{x}_1,|\vec{k}_1|;\vec{x}_2,|\vec{k}_2|) = \sum_{N,L}\frac{V_N V_L\cos\delta_{L1}\cos\delta_{N2}}{R\,(\Delta_{L1}\Delta_{N2})^{3/2}}\cdot\frac{(\tilde{\omega}_{k_1}+\tilde{\omega}_{k_2})^2}{4\,\tilde{\omega}_{k_1}\tilde{\omega}_{k_2}}\times \tag{39}$$

$$\times\frac{\langle n(k_1u_N)\rangle\langle 1+n(k_2u_L)\rangle}{\cos^3\frac{\chi_1}{2}\,\cos^3\frac{\chi_2}{2}}\cdot e^{-i(\tilde{\omega}_{k_1}-\tilde{\omega}_{k_2})\tau}e^{i|\vec{k}_1|(\Delta_{L1}-\Delta_{N1})-i|\vec{k}_2|(\Delta_{L2}-\Delta_{N2})}$$

It is clearly seen now that the analysis of the distributions in phase space leads to the known shceme based on the interference of identical particles emitted independently by different parts of the system. In the initial quantum-mechanical approach we had to deal with the interference of two amplitudes. The first one, when the particle emitted at the point $\vec{x}_L$ was detected at the point $\vec{x}_1$ and that emitted at the point $\vec{x}_N$ was detected at the point $\vec{x}_2$. The second amplitude corresponds to detecting at the point $\vec{x}_1$ the particle emitted at the point $\vec{x}_N$, and at the point $\vec{x}_2$ - emitted at x_L [2,3].

In the present paper we treat not the transition probabilities but the observables. So the interferential exponent in Eq. (39) has another interpretation. Namely the first item of the equation

$$\langle n(\vec{k}_1) n(\vec{k}_2)\rangle = \langle n(\vec{k}_1)\rangle\langle n(\vec{k}_2)\rangle + R(\vec{k}_1,\vec{k}_2)$$

corresponds to the existing possibility of forming the response of every detector (or every chamber track) by the pion field emitted by one cell on the Caushy surface. The second item reflects the alternative possibility of forming the response (the track) by the fields initiated by different cells (or sources).

The simple geometric arguments show that

$$|\vec{k}_1|(\Delta_{L1}-\Delta_{N1}) = \vec{k}_1(\vec{\Delta}_{L1}-\vec{\Delta}_{N1}) - 2|\vec{k}_1|\Delta_{N1}\sin^2\frac{\chi_1}{2},$$

$$|\vec{k}_2|(\Delta_{L2}-\Delta_{N2}) = \vec{k}_2(\vec{\Delta}_{L2}-\vec{\Delta}_{N2}) - 2|\vec{k}_2|\Delta_{L2}\sin^2\frac{\chi_2}{2},$$

$$\vec{\Delta}_{L1}-\vec{\Delta}_{N1} = \vec{\Delta}_{L2}-\vec{\Delta}_{N2} = \vec{x}_N-\vec{x}_L.$$

As far as the angles χ_1 and χ_2, δ_{L1} and δ_{N2} have the order of the ratio of initial system's size to the distance to detectors we can (39), in powers of this small parameter, multiplying by the R^4, get

$$R^4\,\mathcal{R}(\vec{x}_1,|\vec{k}_1|;\vec{x}_2,|\vec{k}_2|) = \sum_{N,L}\frac{V_N V_L}{(2\pi)^6}\,\frac{(k_1^0+k_2^0)^2}{4k_1^0\,k_2^0}\times$$
$$\times\, e^{-i(k_1-k_2)(x_N-x_L)}\langle n(k_1u_N)\rangle\langle 1+n(k_2u_L)\rangle = R(\vec{k}_1,\vec{k}_2) \tag{40}$$

Equation (40) is just the "continuity equation" for the correlator which is similar to Eq. (36).

5. THE PION INTERFEROMETRY IN TERMS OF INCLUSIVE CROSS-SECTIONS

One more approach to the study of identical particles correlations is well-known. It is based on the research of the differential cross-sections of inclusive processes. In contrast to the initial papers of Kopylov and Podgoretzky [2] where the semi-classical calculations were used, this approach deals, from the start, with the quantized field of the secondary pions. The paper by Gulassy et all [4] where the pion interferometry of nucleon-nucleus collisions at low energies ($\sim$ 1 GeV/nucleon) was studied, proves to be a good example. Here the secondary pions are irradiated by the currents of nucleons being within the regime of cascade interactions with the nucleus matter. The nucleon currents are described phenomenologically and supposed to be classical, the emitted pion field is the quantum one. The classical currents were introduced actually in order to simulate the exact intermediate nucleon field of the total quantum field scattering problem.

Our problem allows also the consequent treatment in term of inclusive processes. In fact we change only the way of phenomenological description of the intermediate state bringing it in accordance with the new type of physical picture. The intermediate pion field is simulated now by the locally-equilibrium heat ensemble against the background of the hydrodynamic flow.

We expand the field $\varphi(x)$ of Eq. (10) at some moment after the decay's end in a series over the plane-wave solutions of the free Klein-Gordon equation.

$$\varphi(x) = \int d^3\vec{k}\; A_{\vec{k}}\;\; f_{\vec{k}}(x), \tag{41}$$

where

$$f_{\vec{k}}(x) = (2\pi)^{-3/2}\,(2k^0)^{-1/2}\, e^{-ikx}$$

In virtue of (10) the operator expantion coefficients are defined by

$$A_{\vec{k}} = \int d^3\vec{x}\;\; f^*_{\vec{k}}(x)\; i\overleftrightarrow{\frac{\partial}{\partial x^0}}\;\; \varphi(x)\,;\quad x^0 > t_c$$

$$=\sum_{N}\sum_{\vec{p}} a_{\vec{p}}(N)\int d^3\vec{x}\, f^*_{\vec{k}}(x)\, i\overleftrightarrow{\frac{\partial}{\partial x^0}} \int_{V_N} d\Sigma^\mu(z)\, G_{ret}(x-z)\overleftrightarrow{\frac{\partial}{\partial z^\mu}}\phi_p(z) \quad (42)$$

In terms of operators $A_{\vec{k}}$ and $A^+_{\vec{k}}$ the phase density operator for the number of particles with the momentum $\vec{k}$ at the moment x^0 is

$$j^0(x^0,\vec{k}) = \int d^3\vec{x}\int dk^0\, j^0(x,k) = A^+_{\vec{k}} A_{\vec{k}} . \quad (43)$$

So Eqs. (16) and (17) can be rewritten as

$$\langle n(\vec{k})\rangle = \langle A^+_{\vec{k}} A_{\vec{k}}\rangle , \quad (44)$$

$$\langle n(\vec{k}_1)\, n(\vec{k}_2)\rangle = \langle A^+_{\vec{k}_1} A_{\vec{k}_1} A^+_{\vec{k}_2} A_{\vec{k}_2}\rangle \quad (45)$$

with the averaging performed over the in-state of the system by means of the density operator (6).

The commutator of the operators $A_{\vec{k}_1}$ and $A^+_{\vec{k}_2}$ is calculated with the same formulae as the averages

$$[A_{\vec{k}_1}, A^+_{\vec{k}_2}] = (2\pi)^{-3}\int d\Sigma_\mu\, \frac{k_1^\mu + k_2^\mu}{2\sqrt{k_1^0 k_2^0}}\, e^{-i(k_1-k_2)x} , \quad (46)$$

where the integration is realized over the infinite space-like Caushy surface. In virtue of (26)

$$[A_{\vec{k}_1}, A^+_{\vec{k}_2}] = \delta(\vec{k}_1-\vec{k}_2). \quad (47)$$

So $A^+_{\vec{k}}$ and $A_{\vec{k}}$ appear to be the creation and annihilation operators for the particles with the momentum $\vec{k}$ (free particles in the out-states). Therefore they allow us to build the inclusive transition amplitude of the pion field from any initial state to the finite state with fixed one, two etc. quanta, $(A^+)^n|X\rangle$

$$\langle X|A_{\vec{k}}|in\rangle,\ \langle X|A_{\vec{k}_1}A_{\vec{k}_2}|in\rangle,\ \ldots$$

The squared moduli of these quantities are in direct proportion to the detecting probabilities. They are to be summed over the final uncontrolled states $|X\rangle$ of the pion field and averaged with respect to the initial one. In such a way there appears a sequence of inclusive cross-sections:

$$\frac{1}{\sigma_T}\frac{d\sigma_{inc}}{d\vec{k}} = \langle A^+_{\vec{k}} A_{\vec{k}}\rangle = \langle n_{\vec{k}}\rangle ,$$

$$\frac{1}{\sigma_T}\frac{d^2\sigma_{inc}}{d\vec{k}_1\, d\vec{k}_2} = \langle A^+_{\vec{k}_1} A^+_{\vec{k}_2} A_{\vec{k}_2} A_{\vec{k}_1}\rangle = \tag{48}$$

$$= \langle n(\vec{k}_1)\,[\,n(\vec{k}_2) - \delta(\vec{k}_1-\vec{k}_2)]\rangle , \ \ldots \tag{49}$$

This is just the above result (29). The Eqs. (28),(48) and (49) prescribe the method of deriving the correlation function $R_{inc}(\vec{k}_1,\vec{k}_2)$ if the experimental data handling is performed in terms of differential inclusive cross-sections.

6. CONCLUSION

Binding the inclusive correlation function with the size and shape of the secondary particle generation space-time region our results solve the problem settled above taking into account the following: i) the local thermodynamic equilibrium in the predecay state, ii) the interference of identical particles, iii) the relativistic motion of the matter created in the nuclear collision, iv) the relativistic character of the fields describing the nuclear matter. It can be seen by comparing the final expression for the inclusive cross-section written in the form of

$$\frac{1}{\sigma_T}\frac{d^2\sigma_{inc}}{d\vec{k}_1\, d\vec{k}_2} = \int d\sigma_\mu(x)\, d\sigma_\nu(y)\, \frac{\langle n(k_1 u(x))\rangle\langle n(k_2 u(y))\rangle}{(2\pi)^6\, k_1^0\, k_2^0} \times$$

$$\times \left[\, k_1^\mu k_2^\nu + \frac{(k_1^\mu + k_2^\mu)(k_1^\nu + k_2^\nu)}{4} \cos (k_1 - k_2)(x-y)\,\right]$$

with the formula (1) of the introduction that the conditions iii) and iv) lead to the structural but not to the quantitative changes only. The most remarkable seems to be the arising of the factor $(k_1^0 + k_2^0)^2/4\, k_1^0 k_2^0$ before the cosine describing the interference of the intensities. The explicit evaluation of the correlation function for the different types of predecay states of the system will make a subject of the separate paper.

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