IC/86/159

# INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

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# MODEL FOR NUCLEUS-NUCLEUS, HADRON-NUCLEUS AND HADRON-PROTON MULTIPLICITY DISTRIBUTIONS *

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ABSTRACT

A model relating hadron-proton, hadron-nucleus and nucleus-nucleus multiplicity distributions is proposed and some interesting consequences are derived. The values of the parameters are the same for all the processes and are given by the QCD hypothesis of "universal" hadronic multiplicities which are found to be asymptotically independent of target and beam in hadronic and current induced reactions in particle physics.

MIRAMARE - TRIESTE
July 1986

* To be submitted for publication.

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## I. INTRODUCTION

The study of hadron-nucleus and nucleus-nucleus collisions has witnessed a fast increase of interest in recent years. It does not only throw light on the nature of hadronic interactions at very short time/distance, but also on the internal structure of hadrons and on the deconfining phase transition from the hadronic gas to quark gluon plasma (QGP). Nuclear targets allow to analyze the hadron-nucleon system by a second scattering shortly after the primary one and hence the average shower particle multiplicity hints at the production mechanism. These targets also make the studies possible in case of some rare nuclear channels which are not accessible in the laboratory. Recently we proposed [1] a new parametrization for the hadron-nucleus multiplicity distributions and derived [2] some interesting consequences including scaling laws and we found that these features are well supported by available experimental data. The most important feature of our parametrization is that it relates the hadron-nucleus interactions to hadron-proton data with the values of the parameters appearing unchanged from those given by QCD hypothesis of "universal" hadronic multiplicities [3] in electron-positron, lepton-proton and hadron-proton collisions. Therefore, we thought that these parameters have their origin in basic quark gluon interactions and this possibility was verified by evaluating the value of one of the parameters by using [4] Low and Nussinov model in QCD. In this paper, our central motivation is to extend this model to nucleus-nucleus collisions so that we have a consistent description of LN, LA and A-B interactions. This result becomes significant when we try to utilize it to explore the formation of QGP.

In QCD oriented models, the interaction mechanism can be thought as follows. A projectile constituent exchanges a gluon with a target constituent and colour forces act between them and other costituents as well because they try to restore the colour singlet states. When two constituents separate, the colour force builds up a field in the form of a "colour flux tube" between them. As the energy in the colour field increases quark pairs are created and the colour tube or chain or string breaks up into hadrons. The different models such as additive quark model [5] (AQM), dual parton model [6] (DPM) or colour neutralization model [7] (CNM) differ mainly in their assumption about the potential number of constituents which can interact independently. In AQM, this number is limited by the number of valence quarks. These quarks are assumed to be "dressed" hadron-like objects as opposed to bare, point-like quarks and the reinteraction of produced particles limited by the formulation time. In DPM or CNM, the number of point-like quarks or sea quarks which can interact independently is unlimited. Also these two models do not have a constraint on formation time and they neglect the reinteraction of the secondaries. By means of the present data one cannot decide between the potential number of interacting constituents but the data suggest that more colour strings between projectile hadron and target are formed in hadron-nucleus than in hadron-proton interactions and the weak energy dependence of $R = \langle n_S\rangle_{nA}/\langle n_{ch}\rangle_{np}$ clearly favours a model which incorporates a formation time concept. Recently we proposed [1),2)] a model for hadron-nucleus interactions which incorporates the essential features of AQM including the formation time concept and in addition it takes into consideration the multiple collision effect in target nuclei. We start with a simple model for the multiplicity distribution in hadron-nucleon scattering which we then relate to hadron nucleus process. In this paper, we want to extend the applicability of this model to nucleus-nucleus case without changing the values of the parameters and we show that the calculated results fit the experimental data [8)] on $\alpha\alpha$ scattering at CERN-ISR energy.

## II. MODEL

Let us first examine the question as to how hadron-proton, and hadron-nucleus interactions are related. The main differences between these two interactions are the following. More than one quark of the initial hadronic beam may interact in nucleus and multiple collisions with quarks belonging to different nucleons of the same nucleus may occur. Similarly average number of constituent quarks Nq participating in the interaction increases as the size of the target increases. So Shabelsky et al. [5)] proposed that the hadron nucleus charged particles multiplicity will in general be Nq times hadron-proton multiplicity, i.e.,

$$\langle n_S\rangle_{nA} = N_q \langle n_{ch}\rangle_{np} \,, \tag{1}$$

where

$$N_q = \frac{N_c\, \sigma_{qA}^{in}}{\sigma_{nA}^{in}} \,. \tag{2}$$

Thome et al. [9)] have parametrized the energy dependence of the hadron-proton multiplicity distribution:



$$\langle n_{ch}\rangle_{np} = a + b\ln s + c(\ln s)^2 \,, \tag{3}$$

where the values of the parameters are different for pp ($a = 1.17$, $b = 0.30$, $c = 0.13$) and $\pi p$ ($a = 0.02$, $b = 1.07$ and $c = 0.05$) interactions.
The recent emulsion data illustrate that the curves given by Eq. (1) do not fit well with other available parametrizations.

We proposed that the mean multiplicity for the produced charged particles (mainly pions) in hadron-nucleus interactions can be modified for multiple collision effect in nucleus as follows:

$$\langle n_S\rangle_{nA} = N_q\left[a' + b'\ln(\sqrt{s_A}/N_q) + c'\ln^2(\sqrt{s_A}/N_q)\right] - \alpha \,, \tag{4}$$

where the values of the parameters are unchanged from those given by the QCD hypothesis of universal hadronic multiplicities in $e^+e^-$, lepton proton and hadron-proton collisions. We have used [10)] $a' = 2.50$, $b' = 0.28$ and $c' = 0.53$ to fit the experimental data. Here $\alpha$ is the leading particle contribution ($\alpha \sim 0.85$ as determined experimentally) and should be subtracted. An important feature of Eq. (3) is that the values of a', b' and c' do not change for different hadron nucleus processes. We define $s_A$ as the total available centre-of-mass energy in hA collisions and is given by the relation $s_A = \nu_q s_a$, where $\sqrt{s_a}$ is the available centre-of-mass energy in hN collisions:

$$\sqrt{s_a} = \sqrt{s} - m_b - m_t \,, \tag{5}$$

where $m_b$ and $m_t$ are the rest masses of the beam and target hadrons. The quantity $\nu_q$ is the mean number of collisions of quarks with target nucleus:

$$\nu_q = \frac{A\,\sigma_{qN}^{in}}{\sigma_{qA}^{in}} \,. \tag{6}$$

Here A is the atomic number of target nucleus and quark nucleus inelastic cross-section $\sigma_{qA}^{in}$ is obtained from $\sigma_{qN}^{in}(\approx \frac{1}{3}\sigma_{NN}^{in})$ by using Glauber's approximation [5)]:

$$\sigma_{qA}^{in} = \int d^2b\left[1 - \{(1 - \sigma_{qN}^{in} D_A(b)\}^A\right] \tag{7}$$

and profile function $D_A(b)$ is related to nuclear density by the relation:

$$D_A(b) = \int_{-\infty}^{\infty} \rho(b, z)\, dz \quad . \tag{8}$$

So we essentially assume that the number of constituent quarks which become wounded in hadron-nucleus collisions share the total available centre-of-mass energy $\sqrt{S_A}$ and thus the energy available to each interacting quark becomes $\sqrt{S_A}/N_q$ in hA collisions. Similarly $s_a$ is the available centre-of-mass energy in hadron-proton collision for one effective collision of quarks of hadronic beam in the target. Therefore, total available centre-of-mass energy $S_A$ in hadron nucleus case becomes $\nu_q$ times $s_a$ provided each quark suffers $\nu_q$ collisions. From this, we can infer that the mean multiplicity for hadron-proton interactions is obviously given by the following relation ($n_q = 1$, $\nu_q = 1$)

$$\langle n_{ch} \rangle_{np} = (a' + b' \ln \sqrt{S_a} + c' \ln^2 \sqrt{S_a}) - \alpha \; . \tag{9}$$

We define the relative multiplicity R as the ratio of average number of the relativistic particles produced on a nuclear target to that produced on a proton target, i.e. $R_{hA} = \langle n_s \rangle_{hA} / \langle n_{ch} \rangle_{hp}$. Previously we have shown the energy dependence of R shows that it onstant and $R = 1.53$ for p - Em and $R = 1.32$ for $\pi^-$ - Em interactions. In Fig. 1, we have shown the dependence of $R_{hA}$ on $\bar{\nu}_{hA}$ or mean number of inelastically scattering nucleous with nuclear target and defined as follows:

$$\bar{\nu}_{hA} = A \sigma_{hp}^{in} / \sigma_{hA}^{in} \quad . \tag{10}$$

We find that our theoretical curves give quite close agreement with the experimental data for $\pi^-$ - A and p - A data at 200 GeV/e.

We define a new quantity [11]:

$$D = \frac{N_q \left[ \langle n_{ch} \rangle_{np} + \alpha \right] - \left[ \langle n_s \rangle_{hA} + \alpha \right]}{N_q} \tag{11}$$

which measures the difference in hadron-nucleus charged particle multiplicities from Nq times h-p multiplicities per mean number of wounded quarks. Thus we can get:



$$D = A \ln \nu_q + B \ln^2 \nu_q \quad , \tag{12}$$

where

$$A = -\gamma (b' + 2c' \ln \sqrt{S_a})$$
$$B = -c' \gamma^2$$
$$\gamma = \frac{1}{2} - \frac{\ln N_q}{\ln \nu_q} \quad .$$

In Fig. 2, we have shown the variation of D with $\nu_q$ for proton-nucleus collisions at 200 GeV/c together with the available experimental data. Similarly in Fig. 3, we have shown the variation of D with $\nu_q$ for $\pi^-$ - A interactions at 200 GeV/c. In both cases, our theoretical curves agree with the experimental data. Similarly we derive the following interesting property:

$$\frac{D}{\ln \sqrt{S_a}} = 2c' \ln\left( \frac{N_q}{\sqrt{\nu_q}} \right) \qquad \text{as} \quad S_a \to \infty \tag{13}$$

In Fig. 4, we show the variation of $D/\ln\sqrt{s_a}$ with $\ln N_q/\sqrt{\nu_q}$ for p - A and $\pi^-$ - A collisions at 200 GeV/c. We again find that the theoretical curve from Eq. (13) agrees well with the available experimental data.

We now extend our model to nucleus-nucleus interactions. We define the available total centre-of-mass energy per nucleon $\sqrt{s'}_{AB}$ as follows:

$$\sqrt{S'_{AB}} = \frac{\sqrt{S_{AB}}}{A} = \sqrt{\nu'_q S_a} \tag{14}$$

where

$$\nu'_q = \frac{B\, \sigma_{qN}^{in}}{\sigma_{qB}^{in}} \; , \quad \sqrt{S_a} = \sqrt{S} - m_b - m_t$$

Similarly we define

$$N'_q = \frac{\sigma_{qA}^{in} \, \sigma_{qB}^{in}}{\sigma_{qq}^{in} \, \sigma_{AB}^{in}} \tag{15}$$

where $\sigma_{qq}^{in} \approx \frac{1}{9}\,\sigma_{NN}^{in} \approx 3.67$ mb. has been used in the calculation. Also [12] $\sigma_{AB}^{in} = \pi\gamma^2\left(A^{1/3} + B^{1/3} - \frac{c}{A^{1/3} + B^{1/3}}\right)^2$ fm$^2$ with $\gamma = 1.31 \pm 0.01$ fm and $c = 4.45 \pm 0.15$. Thus we find that the average multiplicity in A-B collisions can be written as:

$$\langle n_s \rangle_{AB} = N_q \left[ a' + b' \ln\left(\frac{\sqrt{s'_{AB}}}{N_q}\right) - c' \ln^2\left(\frac{\sqrt{s'_{AB}}}{N_q'}\right)\right]^{-\alpha} . \qquad (16)$$

## III. RESULTS AND CONCLUSIONS

In Fig. 5, we have shown the variation of $\langle n_s \rangle_{AB}$ with atomic number B at $\sqrt{s}_{NN} = 31.2$ GeV and 26.3 GeV respectively. We have also shown the results from DPM and CNM at $\sqrt{s}_{NN} = 31.2$ GeV. We find that our results agrees closely with CERN-ISR experimental data for $\alpha\alpha$ interactions. We have not shown comparison with cosmic ray data because the experimental figures are not quite consistent with each other. For example, the collision A = 11, B = 108 at 0.3 TeV/N produces more shower particles than the same collision at 1.5 TeV/N and 1.7 GeV/N. Similarly the number of particles produced in collisions between A = 27 and B = 108 and A = 28, B = 108 differ from each other by a factor larger than 2. In our opinion, the experimental comparison does not show any worthwhile purpose in such situations.

We can now define the quantity $D_{AB}$ for nucleus-nucleus collision as was done in Eq. (11). In Fig. 6, we have shown the variation of $D_{AB}$ with $\nu_q$ for $\alpha$-A collisions at $\sqrt{s}_{NN} = 31.2$ GeV and $\sqrt{s}_{NN} = 26.3$ GeV, respectively. Similarly in Fig. 7, we have shown that the scaling rules (13) derived for h - A case, are valid for nucleus-nucleus case as well. In Fig. 8, we have shown the variation of the relative multiplicity $R_{\alpha A} = \langle n_s \rangle_{\alpha A} / \langle n_{ch} \rangle_{hp}$ with energy. In Fig. 9, we have demonstrated that the quantity $\langle n_s \rangle_{AB} \Big/ \left[\sigma_{AB}^{in} \ln \frac{\sqrt{s}_{AB}}{N_q'}\right]$ reduces to a constant for large $\sqrt{s}_{AB}$. In all these cases, the experimental data for $\alpha\alpha$ collision from CERN-ISR support our model. Similarly we can predict the mean multiplicity $\langle n_s \rangle_{AB} = 30.40$ for $A = O^{16}$ and B = Em collisions at 225 GeV/N. We hope that the proposed experiment at CERN will soon verify our result.

In conclusion, we have formulated a model which describes consistently the various features of hadron-proton, hadron-nucleus and nucleus-nucleus collisions.



Although much detailed information has still to be obtained about the soft collision processes, it looks quite clear from the present available experimental data that the main trends of observable like $\langle n \rangle$ are similar for hadron-hadron, hadron-nucleus and nucleus-nucleus collisions. We have drawn a line of similarity between these collisions and we have further made an attempt to correlate all these interactions in terms of basic QCD processes. However, much more experimental information is needed in order to verify the predictions made in this paper. We further believe that the scaling laws and other consequences derived for hadron-nucleus and nucleus-nucleus collisions and their simple relations with the hadron-proton multiplicities as indicated in our parametrizatiopn permit some optimism so that a better understanding in terms of QCD may emerge soon.

## ACKNOWLEDGMENTS

One of the authors (C.P.S.) would like to thank Professor Abdus Salam, the International Atomic Energy Agency and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste. He is also grateful to Professors M. Jacob, H.J. Pirner, J. Hufner and C. Caso for very useful discussions.

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## FIGURE CAPTIONS

Fig. 1 Variation of relative multiplicity $R_{hA}$ with mean number of inelastically scattering nucleons $\bar{v}_{hA}$ for $\pi^- - A$ (dashed line) and p - A (solid line) scattering at 200 GeV/c.

Fig. 2 Variation of D with mean number of collisions of quark with target nucleus, $v_q$ for p - A collisions at 200 GeV/c.

Fig. 3 Variaiton of D with mean number of collisions of quarks with target nucleus $v_q$ for $\pi^- - A$ collisions at 200 GeV/c.

Fig. 4 Variation of $D/\ln \sqrt{s_a}$ with $\ln N_q/\sqrt{v_q}$ for p - A and $\pi^- - A$ collisions at 200 GeV/c.

Fig. 5 Variation of mean multiplicity $\langle n_s \rangle_{AB}$ in the nucleus-nucleus collisions with the atomic number B of target nucleus at $\sqrt{s_{NN}}$ = 31.2 GeV and 26.3 GeV, respectively. The dashed line represents the calculation of Brodsky et al., and the dashed dotted line represents the calculation of Capella et al.

Fig. 6 Variation of $D_{AB}$ for α-Nucleus collisions with mean number of collisions of quarks with target nucleus $v'_q$ at $\sqrt{s_{NN}}$ = 31.2 GeV, and 26.3 GeV, respectively.

Fig. 7 Variation of $D_{AB}/\ln\sqrt{s_a}$ with $\ln(N'_q/\sqrt{v'_q})$ for nucleus-nucleus collision at $\sqrt{s_{NN}}$ = 31.2 GeV.

Fig. 8 Variation of relative multiplicity for α - A collisions $R_{\alpha A}$ with $\sqrt{s}$, the centre-of-mass energy.

Fig. 9 Variation of $\langle n_s \rangle_{AB}/\sigma^{in}_{AB}$ for α - A collisions with $\ln(\sqrt{s_{AB}}/N'_q)$.

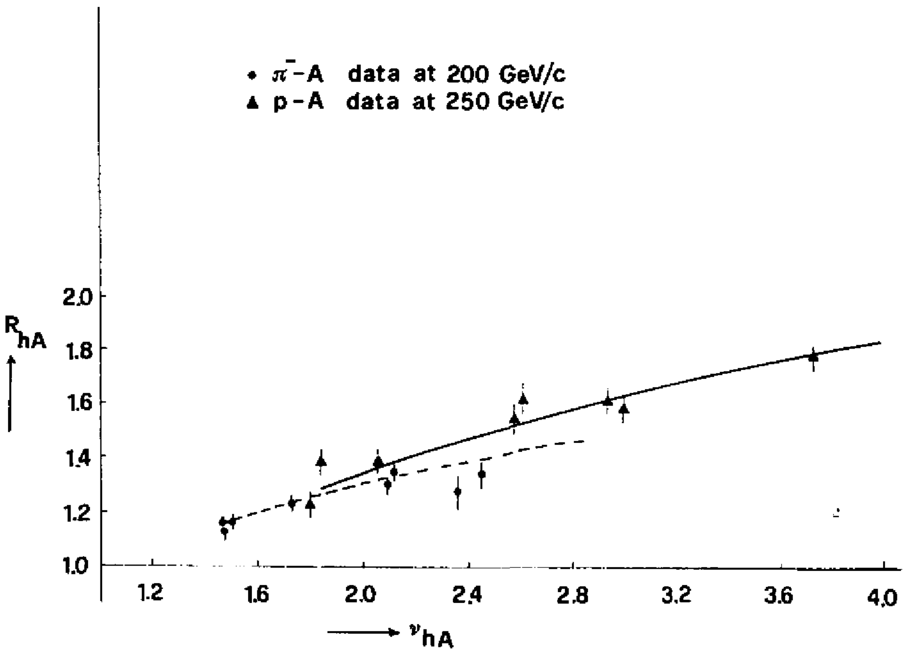



Fig. 1



p-A Collision

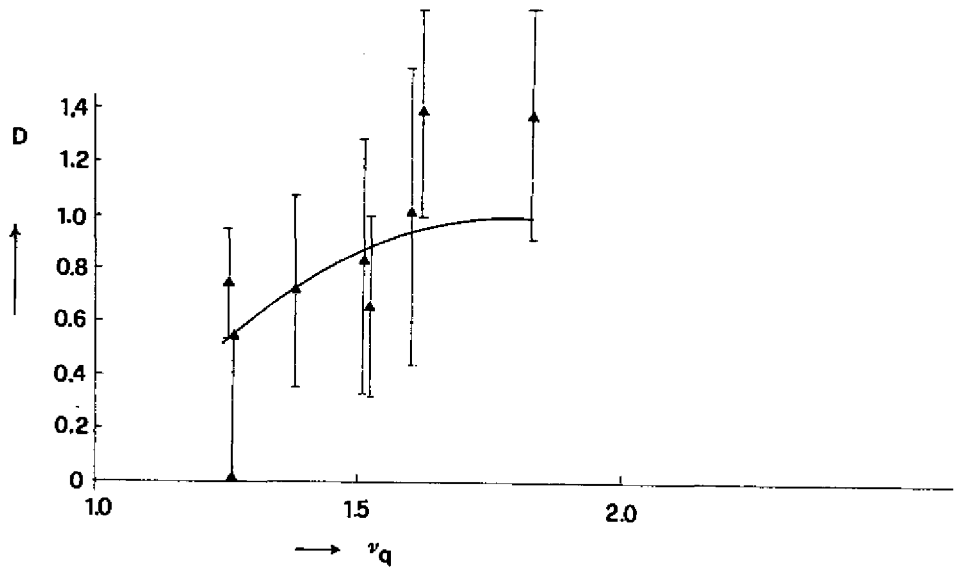
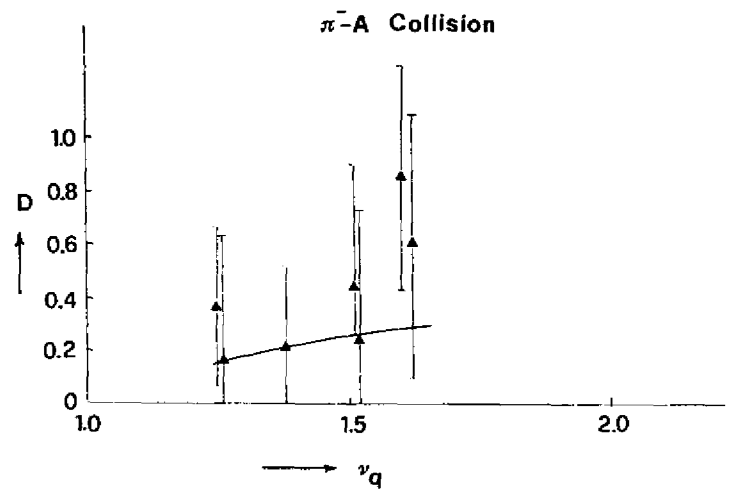
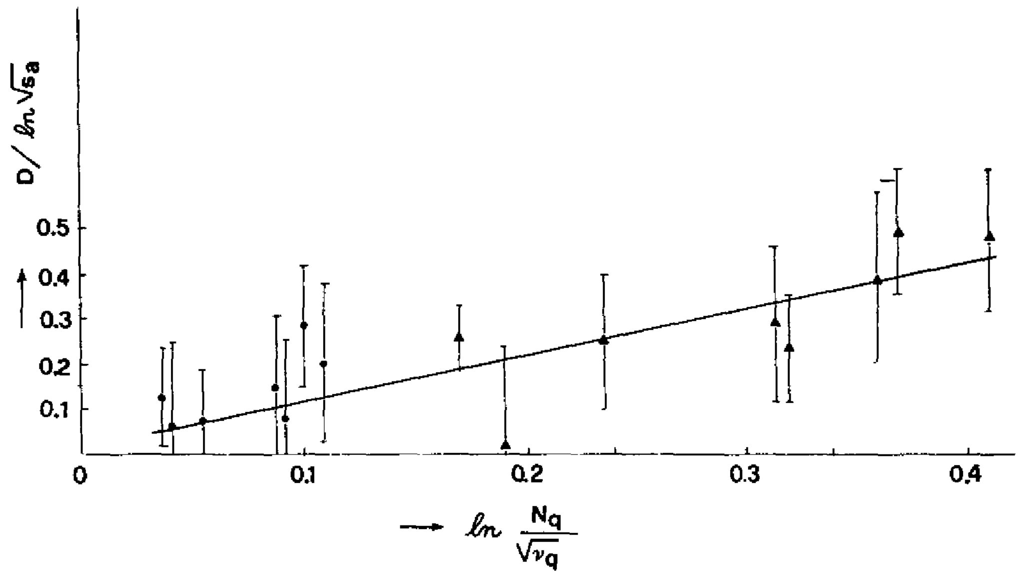



Fig. 2



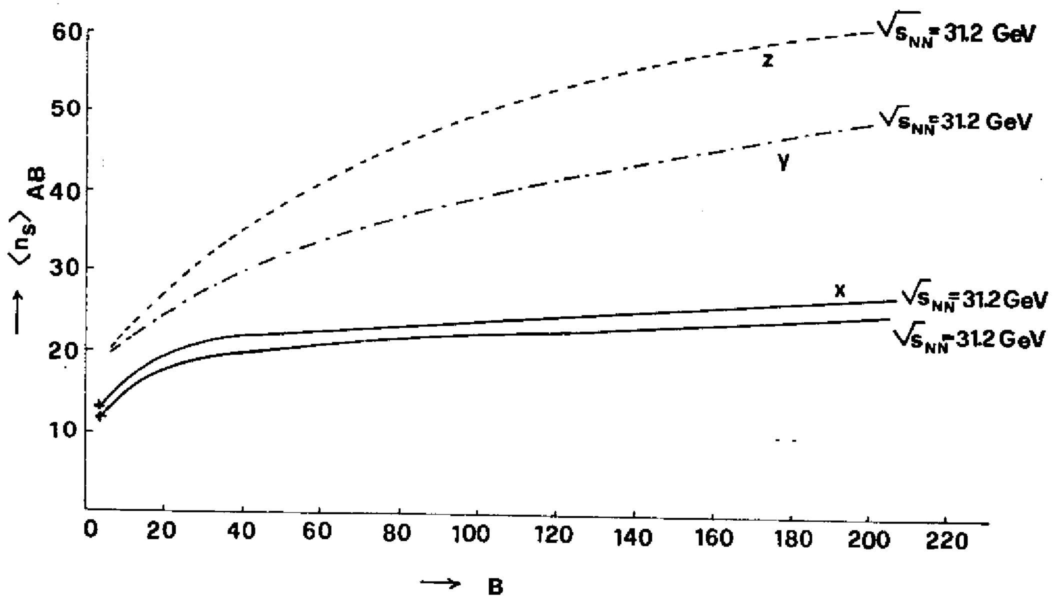
$\pi^-$-A Collision
D
1.0
0.8
0.6
0.4
0.2
0
1.0
1.5
2.0
$\nu_q$


Fig. 3



$D/\ln\sqrt{s_a}$
0.5
0.4
0.3
0.2
0.1
0
0.1
0.2
0.3
0.4
$\ln \frac{N_q}{\sqrt{\nu_q}}$

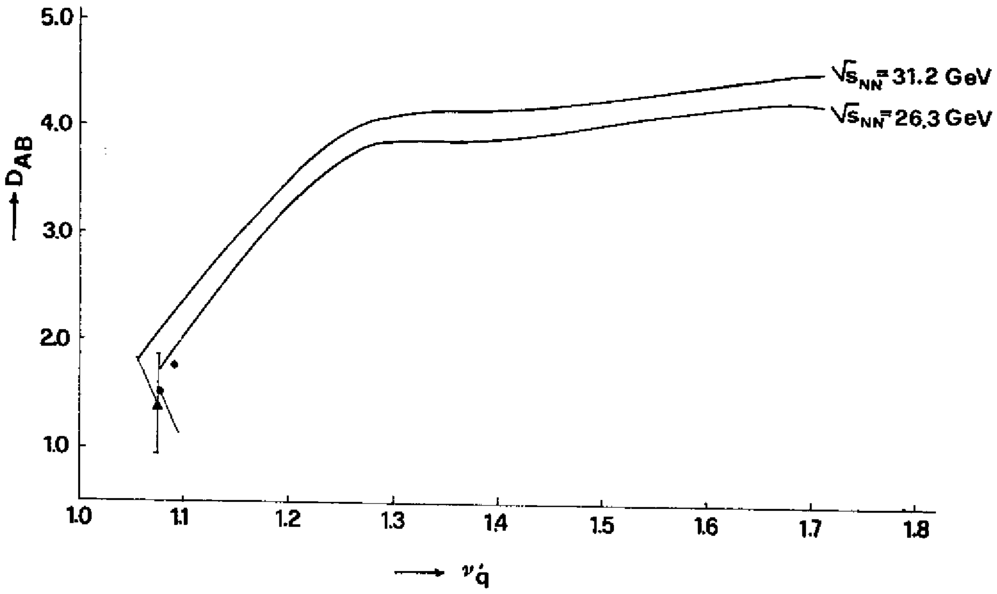

Fig. 4



$\sqrt{s_{NN}}$=31.2 GeV
z
$\sqrt{s_{NN}}$=31.2 GeV
y
x
$\sqrt{s_{NN}}$=31.2GeV
$\sqrt{s_{NN}}$=31.2GeV
$\langle n_s \rangle_{AB}$
60
50
40
30
20
10
0
20
40
60
80
100
120
140
160
180
200
220
B

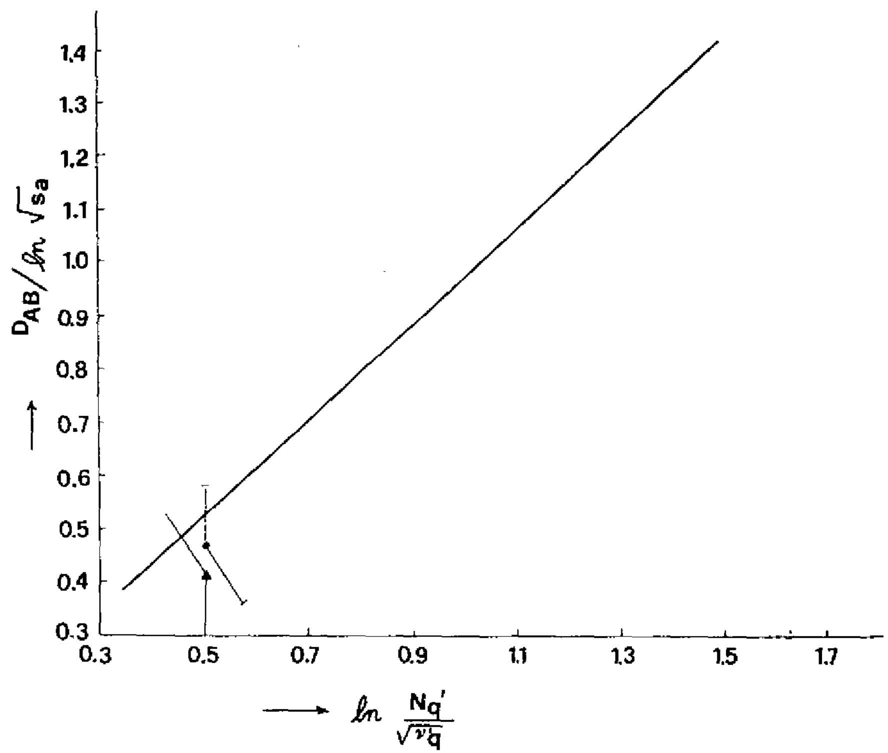

Fig. 5





$\sqrt{s_{NN}}$=31.2 GeV
$\sqrt{s_{NN}}$=26.3 GeV
$D_{AB}$
5.0
4.0
3.0
2.0
1.0
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
$\nu_q'$

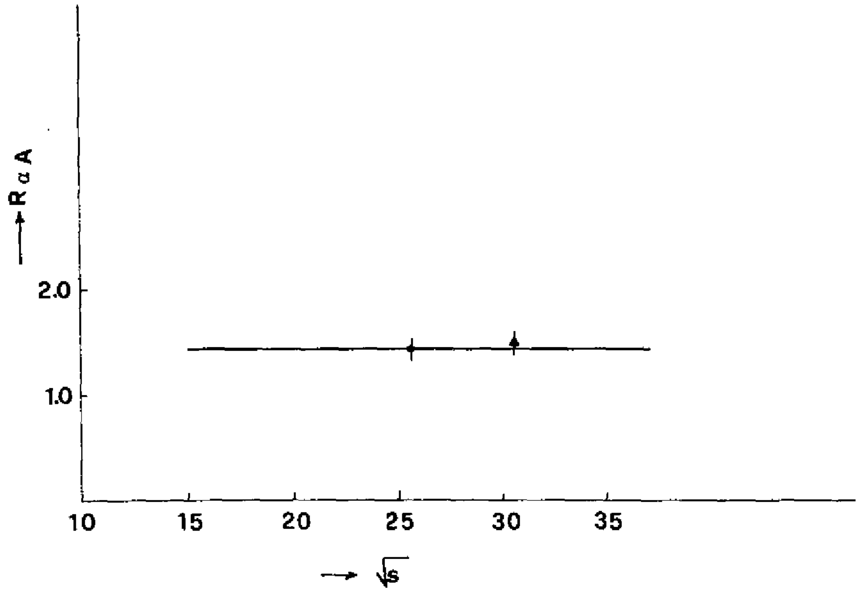

Fig. 6



1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.3
0.5
0.7
0.9
1.1
1.3
1.5
1.7

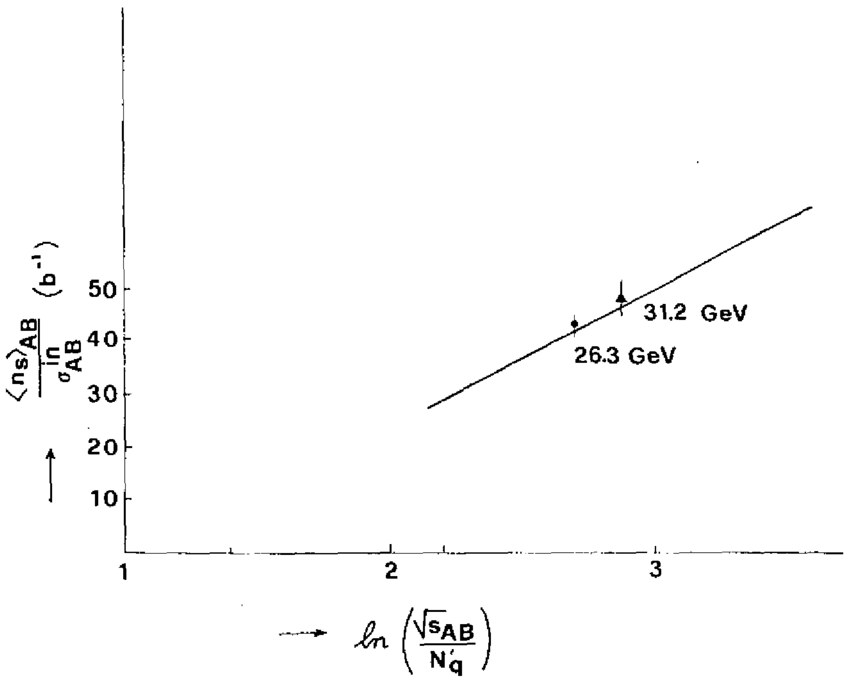

Fig. 7





2.0
1.0
10
15
20
25
30
35


Fig. 8



50
40
30
20
10
1
2
3
31.2 GeV
26.3 GeV
$\frac{\langle n_s \rangle_{AB}}{\sigma^{in}_{AB}}$ ($b^{-1}$)
$\ell n \left( \frac{\sqrt{s_{AB}}}{N'_q} \right)$


Fig. 9