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DOUBLY GRADED SIGMA MODEL WITH TORSION

J. Kowalski-Glikman†

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Using the Hull-Witten construction we show how to introduce torsion to the doubly graded sigma model. This construction enables us to find a link between this model and the ten-dimensional supergravity theory in superspace.

† On leave of absence from the Institute of Theoretical Physics, University of Warsaw.

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Using the Hull-Witten construction we show how to introduce torsion to the doubly graded sigma model. This construction enables us to find a link between this model and the ten-dimensional supergravity theory in superspace.

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1. Introduction

In the last two years a remarkable development in superstring theory has taken place [1]. There are however still many open questions, including the most important one how to obtain a realistic, four-dimensional theory from superstrings that can be consistently defined only in ten space-time dimensions. In other words we must know the theory of superstrings propagating on generic curved ten-dimensional manifold and to understand why the latter takes a form $K^6 \times M^4$, where K^6 is a compact euclidean manifold and M^4 one of Minkowski signature. Such a theory is not known yet, so it is claimed that instead we should look for the consistent (i.e. finite and anomaly-free) two-dimensional supersymmetric sigma-model, which target space is a manifold of interest [2]. In the latter approach it has been shown that the condition for the conformal anomaly freedom can be expressed as a vanishing of certain β-functions which is equivalent to the equations of motion of the bosonic part of D=10, N=1 supergravity (Callan et al. in [2]).

It is clear therefore that since we are interested in supersymmetric theory we should look for the supersymmetric sigma-model with a superspace as a target (super)manifold. Such a model, which I shall call a doubly graded sigma-model (DGSM) was recently proposed by J.W. van Holten and the present author [3]. It possesses a large class of invariances including the two-dimensional superconformal supersymmetry and the target superspace general co-ordinate invariance, the fermionic part of which can be considered as a supersymmetry in a usual way. As before we can expect to obtain superspace field equations of the ten-dimensional supergravity from the condition that the theory is a superconformal anomaly-free one. However, since by the construction a DGSM is forced to have a Riemannian superspace as a target manifold we should have used an Arnowitt-Nath approach to superspace [4] rather than the more common Wess-Zumino one [5]. This may not be a very big problem in principle, but it would be nice to have a theory formulated in more direct Wess-Zumino superspace language. In this paper I shall present such a formulation using the generalization of the Hull and Witten construction [6].

2. Structure of DGSM

The scalar multiplet of DGSM is a representation of two-dimensional superconformal algebra [7] (we use conventions of ref. [3]) which fermionic part looks as follows

$$[\delta_Q(\varepsilon_2), \delta_Q(\varepsilon_1)] = \delta_{GCT}{}^{cov}(2\bar{\varepsilon}_2\sigma^\mu\varepsilon_1),$$
$$[\delta_Q(\varepsilon), \delta_S(\eta)] = \delta_D(2\bar{\eta}\varepsilon) + \delta_M(2\bar{\eta}\sigma_3\varepsilon),$$
$$[\delta_S(\varepsilon_2), \delta_S(\eta_1)] = \delta_K(-2\bar{\eta}_2\sigma_a\eta_1). \tag{2.1}$$

The transformation rules for the multiplet $\Sigma^M = (X^M, \lambda^M, F^M)$, where we use the convention that X^M is commuting for M=m and anticommuting for M=α; F^M has the same statistics as X^M and λ^M an opposite one, are

$$\begin{aligned}
\delta X^M &= \bar\varepsilon\lambda^M + w\xi_D X^M, \\
\delta\lambda^M &= (-1)^{M+1}(\mathscr{D}X^M + F^M)\varepsilon + (-1)^{M+1}2wX^M\eta - \tfrac{1}{2}\xi_M\sigma^3\lambda^M + (w+\tfrac{1}{2})1\xi_D\lambda^M, \\
\delta F^M &= \bar\varepsilon\,\mathscr{D}\lambda^M - 2w\bar\eta\lambda^M + (w+1)\xi_D F^M
\end{aligned} \tag{2.2}$$

(we use the notation $(-1)^M=1$ for M=m and -1 for M=α) where

$$\begin{aligned}
\mathscr{D}_\mu X^M &= (\partial_\mu - wb_\mu)X^M - \bar\psi_\mu\lambda^M, \\
\mathscr{D}_\mu\lambda^M &= (\partial_\mu + \tfrac{1}{2}\omega_\mu\sigma_3 - (w+\tfrac{1}{2})b_\mu)\lambda^M + (-1)^M(\mathscr{D}X^M + F^M)\psi_\mu + (-1)^M wX^M\phi_\mu.
\end{aligned} \tag{2.3}$$

Having two multiplets $\Sigma^M = (X^M, \lambda^M, F^M)$ and $\Sigma^N = (X^N, \lambda^N, F^N)$, we can construct a product multiplet

$$\Sigma^M\cdot\Sigma^N = (X^M X^N, \lambda^M X^N + (-1)^{MN}\lambda^N X^N, F^M X^N + X^M F^N + (-1)^M\bar\lambda^M\lambda^N) \tag{2.4}$$

and also a composite one

$$W(\Sigma) = (W(X), \lambda^M W_{,N}, F^M W_{,M} + \tfrac{1}{2}(-1)^{M(N+1)}\bar\lambda^M\lambda^N W_{,NM} \tag{2.5}$$

(all derivatives are the left ones).

Next we define the spinor multiplet $\mathfrak{Z}^M = (\psi^M, \Omega^M, \chi^M)$ with transformation rules (ψ^M, χ^M are Majorana spinors; Ω^M is a 2×2 matrix)

$$\begin{aligned}
\delta\psi^M &= \Omega^M\varepsilon + w\xi_D\psi^M - \tfrac{1}{2}\xi_M\sigma_3\psi^M, \\
\delta\Omega^M &= \mathscr{D}_\mu\psi^M\bar\varepsilon\sigma^\mu + \chi^M\bar\varepsilon - 2w\psi^M\bar\eta + \sigma_3\psi^M\sigma_3 - \tfrac{1}{2}\xi_M[\sigma_3,\Omega^M] + (w+\tfrac{1}{2})\xi_D\Omega^M, \\
\delta\chi^M &= \mathscr{D}_\mu\Omega^M\sigma^\mu\varepsilon + (2w\Omega^M + \sigma^3\Omega^M\sigma^3)\eta - \tfrac{1}{2}\xi_M\sigma^3\chi^M + (w+1)\xi_{,D}\chi^M,
\end{aligned} \tag{2.6}$$

where

$$\begin{aligned}
\mathscr{D}_\mu\psi^M &= (\partial_\mu + \tfrac{1}{2}\omega_\mu\sigma_3 - wb_\mu)\psi^M - \Omega^M\psi_\mu, \\
\mathscr{D}_\mu\Omega^M &= (\partial_\mu - (w+\tfrac{1}{2})b_\mu)\Omega^M + \tfrac{1}{2}\omega_\mu[\sigma_3,\Omega^M] - \chi^M\bar\psi_\mu - D_\nu\psi^M\bar\psi_\mu\sigma^\nu - 2w\psi^M\bar\phi_\mu \\
&\quad - \sigma_3\psi^M\bar\phi_\mu\sigma_3.
\end{aligned} \tag{2.7}$$

Using the notion of the spinor multiplet we can construct a spinorial derivative of the scalar multiplet $\Sigma^M = (X^M, \lambda^M, F^M)$ as a following spinor multiplet

$$D\Sigma^M = (\lambda^M, (-1)^{M+1}(\mathscr{D}X^M + F^M), \mathscr{D}\lambda^M). \tag{2.8}$$

One can also define two products of spinor multiplets, which are scalar multiplets themselves:

$$\Sigma_0{}^{MN} = \bar{\mathfrak{Z}}^M\mathfrak{Z}^N = \big(\bar\psi^M\psi^N, (-1)^{M+1}\Omega^M\psi^N - (-1)^{M(N+1)}\Omega^N\psi^M, (-1)^N \mathrm{Tr}(\bar\Omega^M\Omega^N) - \bar\chi^M\psi^N - \bar\psi^M\chi^N\big) \tag{2.9a}$$

and

$$\Sigma_3{}^{MN} = \bar{\mathfrak{Z}}^M\sigma_3\mathfrak{Z}^N = \big(\bar\psi^M\sigma_3\psi^N; (-1)^{M+1}\Omega^M\sigma_3\psi^N + (-1)^{M(1+N)}\bar\Omega^N\sigma_3\psi^M; (-1)^N \mathrm{Tr}(\bar\Omega^M\sigma_3\Lambda^N) - \bar\chi^M\sigma^3\psi^N - \bar\psi^M\sigma^3\chi^N\big). \tag{2.9b}$$

The construction of invariant action consists of two steps: firstly by using algebraic operations defined above we construct a scalar multiplet of weight w=1 (thus the last component F has w=2), and then we define the lagrange density

$$e^{-1}\mathscr{L} = F + \bar\psi\cdot\sigma\lambda - \frac{i}{e}\varepsilon_{\mu\nu}\bar\psi_\mu\sigma^2\psi_\nu, \tag{2.10}$$

which transforms under supersymmetry into total divergence.

The superconformal action for DGSM is constructed from the multiplet

$$\tfrac{1}{2}\bar{D}\bar\Sigma^M G_{MN} D\Sigma^N + \frac{\beta}{2}\bar{D}\bar\Sigma^M A_{MN}\sigma^3 D\Sigma^N \tag{2.11}$$

and, after eliminating the auxiliary field F^M, reads

$$\begin{aligned} e^{-1}\mathscr{L} = &-\partial_\mu X^M\partial^\mu X^N G_{NM} + i\beta\, {}^1\!/_e\, \varepsilon^{\mu\nu}\partial_\mu X^M\partial_\nu X^N A_{NM} + \bar\lambda^M \mathring{\not{D}}\lambda_M + 2\bar\psi_\mu \not\partial X^M\sigma^\mu\lambda_M \\ &+ \tfrac{1}{2}\bar\psi_\mu\sigma^\nu\sigma^\mu\bar\lambda^M\lambda_M - (-1)^M\, {}^\beta\!/_3\, \bar\psi_\mu\sigma^\nu\sigma^\mu\sigma_3\lambda^K\bar\lambda^M\sigma_\nu\lambda^N H_{NMK} + \\ &+ {}^1\!/_6 (-1)^{(1+K)(1+M)+MN}\bar\lambda^M\lambda^N\bar\lambda^K\lambda^L R_{LMKN} + {}^\beta\!/_4 (-1)^{K+M+MN}\bar\lambda^M\sigma^3\lambda^N\bar\lambda^K\lambda^L H_{KHN;L} \\ &- {}^{\beta^2}\!/_4 (-1)^{K(N+L)}\bar\lambda^M\sigma^3 H_{MN}{}^K\lambda^N\bar\lambda^Q H_{QLK}\lambda^L, \end{aligned} \tag{2.12}$$

where

$$\begin{aligned} &\lambda_M = G_{MN}\lambda^N, \\ &\mathring{\not{D}}\lambda_M = \not\partial\lambda_M - \not\partial X^K(\mathring\Gamma_{KM}{}^N + \beta H_{KM}{}^N\sigma_3)\lambda_N, \\ &\mathring\Gamma_{MNK} = \mathring\Gamma_{MN}{}^L G_{LK} = \tfrac{1}{2}(G_{NK,M} + (-1)^{MN}G_{MK,N} - (-1)^{K(M+N)}G_{MN,K}), \\ &H_{KMN} = \tfrac{1}{2}(A_{MN,K} + (-1)^{K(M+N)}A_{NK,M} + (-1)^{N(K+M)}A_{KM,N}), \\ &\mathring R_{NK}{}^L = \mathring\Gamma_{MK}{}^L{}_{,M} - (-1)^{MN}\mathring\Gamma_{MK}{}^L{}_{,N} - (-1)^{N(K+L)}\mathring\Gamma_{MK}{}^Q\mathring\Gamma_{NQ}{}^L + (-1)^{M(N+K+L)}\mathring\Gamma_{NK}{}^Q\mathring\Gamma_{MQ}{}^L. \end{aligned} \tag{2.13}$$

The model above is usually called a (1,1) supersymmetric one since it is reducible to the chiral model with Majorana-Weyl supersymmetry parameter ε_+. In this case we have two irreducible multiplets $(X^M, \lambda^M{}_-)$ and $(\lambda^M{}_+, F^N)$ with $\lambda^M{}_\pm = \pm\sigma_3\lambda^M{}_\pm$. The reduction of Lagrangian (2.12) to the first multiplet, which I shall call a $(1,0)^-$ model, reads

$$e^{-1}\mathcal{L} = -\partial_\mu X^M \partial^\mu X^N G_{NM} + i\beta \frac{1}{e}\varepsilon^{\mu\nu}\partial_\mu X^M \partial_\nu X^N A_{NM} + \bar{\lambda}_-{}^M \mathcal{D}\lambda_{-M} + 2\bar{\psi}_{+\mu}\partial\!\!\!/ X^M \sigma^\mu \lambda_{-M} + \frac{\beta}{3}(-1)^M \bar{\psi}_{+\mu}\sigma^\nu\sigma^\mu\lambda_-{}^K \bar{\lambda}^M \sigma_\nu \lambda_-{}^N H_{NMK}, \tag{2.14}$$

where

$$\mathcal{D}\lambda_{-M} = \partial\!\!\!/\lambda_{-M} - \partial\!\!\!/ X^K (\mathring{\Gamma}_{KM}{}^N - \beta H_{KM}{}^N)\lambda_{-N}. \tag{2.15}$$

The lagrangian (2.13) is invariant under local (1,0) supersymmetry

$$\delta X^M = \varepsilon_+ \lambda_-{}^M,$$
$$\delta\lambda_-{}^M = (-1)^{1+M}\mathcal{D}X^M \varepsilon_+ + \tfrac{1}{2}(\xi_M + \xi_D)\lambda_-{}^M. \tag{2.16}$$

Observe, that by the construction, the model presented above has a Riemannian, i.e. supertorsion-free supermanifold as a target space. This fact makes a theory a bit inconvenient and to overcome this difficulty we shall start with the formulation of the DGSM in language of supervielbeins and superconnections.

3. DGSM in supervielbein formalism

We define a (super)vielbein $e^A{}_M(x)$ by the assumption that

$$\partial_\mu X^A \partial^\mu X^B \eta_{BA} = \partial_\mu X^M \partial_\nu X^N G_{NM}, \quad \partial_\mu X^A = \partial_\mu X^M e_M{}^A, \tag{3.1}$$

from which we find

$$G_{MN} = e^A{}_M e^B{}_N \eta_{AB} (-1)^{A(N+B)}. \tag{3.2}$$

Defining the inverse vielbein by the equation $e_M{}^A e_A{}^N = \delta_M{}^N$ we can also find a number of useful identities:

$$e_B{}^M e^A{}_M = \delta^A{}_B, \quad e_B{}^M = (-1)^{PB} e^A{}_P \eta_{AB} G^{PM},$$
$$(-1)^{N(B+N)} e_A{}^N e_B{}^N G_{MN} = \eta_{AB}. \tag{3.3}$$

In order to define a covariant derivative in vielbein formalism we use the equality

$$\bar{\lambda}^M \overset{o}{\not{D}}\lambda_M = \bar{\lambda}^A \overset{o}{\not{D}}\lambda_A, \tag{3.4}$$

where $\overset{o}{\not{D}}$ is a covariant derivative (2.13) with respect only to the metric connection $\overset{o}{\Gamma}$,

$$\lambda^A = (-1)^{A(M+1)} e^A{}_M \lambda^M,\ \lambda_A = \eta_{AB}\lambda^B, \tag{3.5}$$

$$\overset{o}{\not{D}}\lambda_A = \not{\partial}\lambda_A - \not{\partial}X^M\ \omega_{MA}{}^B\lambda_B. \tag{3.6}$$

Substituting (2.5,6) into (3.4) we obtain a condition

$$e_{AK,P}(-1)^{PA} - \omega_{PAB}\, e^B{}_K(-1)^{K(B+1)+PA} - (-1)^{M(P+K)} e_A{}^M\ \overset{o}{\Gamma}_{PKM} = 0, \tag{3.7}$$

which can be solved for ω:

$$\begin{aligned}
\overset{o}{\omega}_{ABC} &= -\tfrac{1}{2}\left(\Omega_{ABC}(-1)^{(A+B)(C+1)+C} - \Omega_{CBA}(-1)^{B(A+1)+CA} - \Omega_{BAC}(-1)^{C}\right),\\
\Omega_{ABC} &= e^P{}_B\left(e_{AK,P}(-1)^{CK+P} - e_{AP,K}(-1)^{K(A+C)+P(A+K+1)}\right) e^K{}_C,\\
\overset{o}{\omega}_{ABC} &= (-1)^K e_A{}^K \overset{o}{\omega}_{KBC}.
\end{aligned} \tag{3.8}$$

Since H_{MNP} is a tensor it is sufficient to put it to the covariant derivative changing appropriate indices into flat ones. Thus the full $(1,0)^-$ Lagrangian reads

$$\begin{aligned}
e^{-1}\mathcal{L} &= \partial_\mu X^A\, \partial^\mu X^B \eta_{BA} + \frac{i\beta}{e}\, \varepsilon^{\mu\nu}\, \partial_\mu X^A\, \partial_\nu X^B\, A_{BA} + \bar{\lambda}_-{}^A \not{D}\lambda_{-A}\\
&\quad + 2\bar{\psi}_{\mu+}\not{\partial}X^A\, \sigma^\mu\lambda_{-A} + \frac{\beta}{3}(-1)^M\, \bar{\psi}_{+\mu}\, \sigma^\nu\sigma^\mu\lambda_-{}^C \bar{\lambda}_-{}^B{}_\nu \lambda_-^M\, H_{MBC},\\
\not{D}\lambda_{-A} &= \not{\partial}\lambda_A - \not{\partial}X^M\left(\omega_{MA}{}^B - \beta H_{MA}{}^B\right)\lambda_{-B},\\
H_{MAB} &= (-1)^{M(A+P)+Q(B+1)} e_A{}^P H_{MPQ} e_B{}^Q.
\end{aligned}$$

Since we are mainly interested in the (1,0) model we shall not present extension of these results to the (1,1) case, which can easily be done.

4. The Hull-Witten construction and the origin of torsion

As we have seen the (1,1) model is reducible and we have constructed an irreducible $(1,0)^-$ model above. Now we shall try to construct a model with spinors of opposite chirality, the $(1,0)^+$ model coupled to the external gauge field. Since the positive chirality multiplet consists of one spinor λ_+ and the auxiliary field F, after eliminating the latter by using its algebraic field equation we obtain a supersymmetric theory with only one fermionic field.

The main entries of our construction are four multiplets

$$\Sigma_I{}^{Ma} = (\bar{\lambda}_-{}^M\lambda_+{}^a, \not{D}X^M\lambda_+{}^a - (-1)^M\lambda^M F^a, 0), \tag{4.1}$$

$$\Sigma_{II\,Ma} = (0, A_{Mab}(x)\lambda_+{}^b, (-1)^{M+a+b}A_{Mab}F^b + \bar{\lambda}^N A_{Mab,N}\,\lambda_+{}^b), \tag{4.2}$$

$$\Sigma_{III\,ab} = (G_{ab}(x), \lambda^M G_{ab,M}, 0), \tag{4.3}$$

$$\Sigma_{IV}{}^{ab} = (0, F^a\lambda_+{}^b, (-1)^a F^a F^b + (-1)^{(a+1)(b+1)+1}\bar{\lambda}_+{}^b\not{D}\lambda_+{}^a). \tag{4.4}$$

Using formulas of multiplets algebra presented in §2, we construct the Lagrangian of the form

$$\mathcal{L}_+ = \mathcal{L}_{II\cdot I} + \alpha\mathcal{L}_{IV\cdot III} \tag{4.5}$$

where $\mathcal{L}_{II\cdot I}$ is a lagrangian for $\Sigma_{II}{}^{Ma}\Sigma_{I\,Ma}$. After eliminating F^a's by their field equations

$$F^a = -\tfrac{1}{2}(-1)^c\,\bar{\lambda}_+{}^c\lambda_-{}^N(G_{cb,N} + 2A_{Ncb})\,G^{ba} \tag{4.6}$$

the final lagrangian reads:

$$e^{-1}\mathcal{L}_+ = \bar{\lambda}_+{}^a\not{D}\lambda_{+a} + \tfrac{1}{4}(-1)^{a+M+1}\alpha\,\bar{\lambda}_-{}^M\sigma^\mu\lambda_-{}^N\;\hat{F}_{MNab}\,\bar{\lambda}^b\sigma_\mu\lambda^a, \tag{4.7}$$

where

$$\hat{A}_{Mab} = A_{Mab} + \frac{1}{\alpha}G_{ab,M},$$

$$\hat{F}_{MNab} = \hat{A}_{Mab,N} + (-1)^{(a+c)M}\alpha\;\hat{A}_{Na}{}^c\;\hat{A}_{Ncb} - (-1)^{MN}(M\leftrightarrow N), \tag{4.8}$$

$$\not{D}\lambda_+{}^a = \not{\partial}\lambda_+{}^a - \alpha\,\not{\partial}X^M\;\hat{A}_{Mab}\,\lambda_+{}^b. \tag{4.9}$$

Since by the construction $A_{Mab} = -(-1)^{ab}A_{Mba}$, we see that the gauge group must be a subgroup of the group OSp. For example we can take A_{Mab} to be a connection in a tangent bundle to the target supermanifold and take a.. to be indices in tangent space (we shall first use capital letters as in §3). We can also introduce gauge fields A_{Mab} with group index a, b. Therefore, the most general lagrangian of the form $(1,0)^-\oplus(1,0)^+$ is of the form:

$$\begin{aligned} e^{-1}\mathcal{L} = &-\partial_\mu X^M \partial^\mu X^N G_{NM} + i\beta\,\varepsilon^{\mu\nu}/e\ \partial_\mu X^M \partial_\nu X^N A_{NM} + \bar\lambda^M{}_- \not{D}\lambda_{-M} \\ &+ \bar\lambda^A{}_+ \not{D}\lambda_{+A} + \bar\lambda^{\mathfrak{a}}{}_+ \not\lambda_{\mathfrak{a}} + 2\psi_{\mu+} \not\partial X^M \sigma^\mu \lambda_{-M} \\ &+ \beta/_3\,(-1)^M \bar\psi_{\mu+} \sigma^\nu \sigma^\mu \lambda_-{}^K \bar\lambda_-{}^M \sigma_\nu \lambda_-{}^N H_{NMK} + \\ &+ \tfrac{1}{2}(-1)^{A+M+1} \bar\lambda_-{}^M \sigma^\mu \lambda_-{}^N R_{MN}{}^{AB} \bar\lambda_+{}^A \sigma_\mu \lambda_+{}^B \\ &+ \alpha/_4\,(-1)^{\mathfrak{a}+M+1} \bar\lambda_-{}^M \sigma^\mu \lambda_-{}^N\ \hat F_{MN\mathfrak{a}\mathfrak{b}} \bar\lambda_+{}^{\mathfrak{a}} \sigma_\mu \lambda_+{}^{\mathfrak{b}}, \end{aligned} \tag{4.10}$$

where we use R_{MNAB} to denote a field strength of connection ω_{MAB}. This lagrangian can be put into even more symmetrical a form if we decompose ω_{MAB} into a vielbein part and the contorsion: $\omega_{MAB} = \mathring{\omega}_{MAB} + K_{MAB}$. In this case we have:

$$\begin{aligned} e^{-1}\mathcal{L} = &-\partial_\mu X^M \partial^\mu X^N G_{NM} + i\,\varepsilon^{\mu\nu}/e\ \partial_\mu X^M \partial_\nu X^N A_{NM} \\ &+ \bar\lambda^A \not{D}\lambda_A + \bar\lambda^{\mathfrak{a}}{}_+ \not{D}\lambda_{\mathfrak{a}} + \bar\psi_{\mu+} \not\partial X^M \sigma^\mu \lambda_M \\ &+ \beta/_3\,(-1)^M \bar\psi_\mu \sigma^\nu \sigma^\mu \lambda^K \bar\lambda^M \sigma_\nu (1-\sigma_3) \lambda^N H_{NMK} + \\ &+ 1/_{18}\,(-1)^{A+M+1} \bar\lambda^M \sigma^\mu (1-\sigma^3) \lambda^N R_{MNAB} \bar\lambda^A \sigma_\mu (1+\sigma^3) \lambda^B \\ &+ \alpha/_8\,(-1)^{\mathfrak{a}+M+1} \bar\lambda^M \sigma^\mu (1-\sigma^3) \lambda^N\ \hat F_{MN\mathfrak{a}\mathfrak{b}} \bar\lambda_+{}^{\mathfrak{a}} \sigma_\mu \lambda_+{}^{\mathfrak{b}}, \end{aligned} \tag{4.11}$$

where

$$\not{D}\lambda_A = \not\partial\lambda_A - \not\partial X^M \left(\mathring{\omega}_{MAB} - \beta/_2 (1-\sigma^3) H_{MAB} - \tfrac{1}{2} K_{MAB} (1+\sigma^3) \right) \lambda^B. \tag{4.12}$$

Thus we can interpret H_{MAB} as a part of torsion that couples to the negative-chirality spinors and K_{MAB} as a part that couples to positive-chirality ones.

There is a straightforward link between the model of (4.12) and the ten-dimensional supergravity. One can use a Wess-Zumino approach to superspace and use the result of ref. [7] to obtain the nonvanishing components of connection ω_{MAB} and H_{MAB}.

5. Conclusions

The main motivation for this paper was to find a link between a DGSM and the ten-dimensional supergravity. It was done by introducing an additional positive-chirality multiplet coupled to the general (not torsion-free) connection in tangent bundle.

We have also found how to couple our model to any OSp gauge field, which may be important from the point of view of anomaly cancellation, and which can lead to the ten-dimensional supergravity-Yang-Mills system [8]. However it may not be necessary to introduce an additional super Yang-Mills field to our theory. The reason is, that in order to cancel the superconformal anomaly, the theory must live in a critical number of dimensions, the condition for which will involve the number of commuting and anticommuting fields X^M and X^α with opposite signs. Therefore there is no reason to expect that the number of fermionic dimensions will be of the form $2^{[D/2]}$, where D is a number of bosonic dimensions. In fact the naive calculation [9] shows that in the (1,1) case the critical dimension formula looks like D–D'=10 where D' is a number of fermionic dimensions. Observe that D=26, D'=16 (which is a number of components of a Majorana-Weyl spinor in ten dimensions) is a solution of this equation. Thus we can expect that some of the bosonic dimensions are compact and it is possible to obtain the Yang-Mills fields by dimensional reduction as in the heterotic string case.

To check this possibility a β-function of the model must be calculated. It will also give an ultimate consistency proof of our model, since it will show if the equations resulting from vanishing of β-functions are those of ten-dimensional supergravity. Work in this direction is in progress.

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