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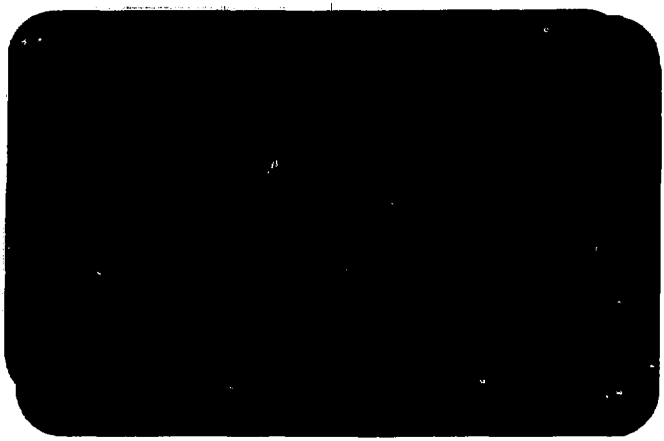

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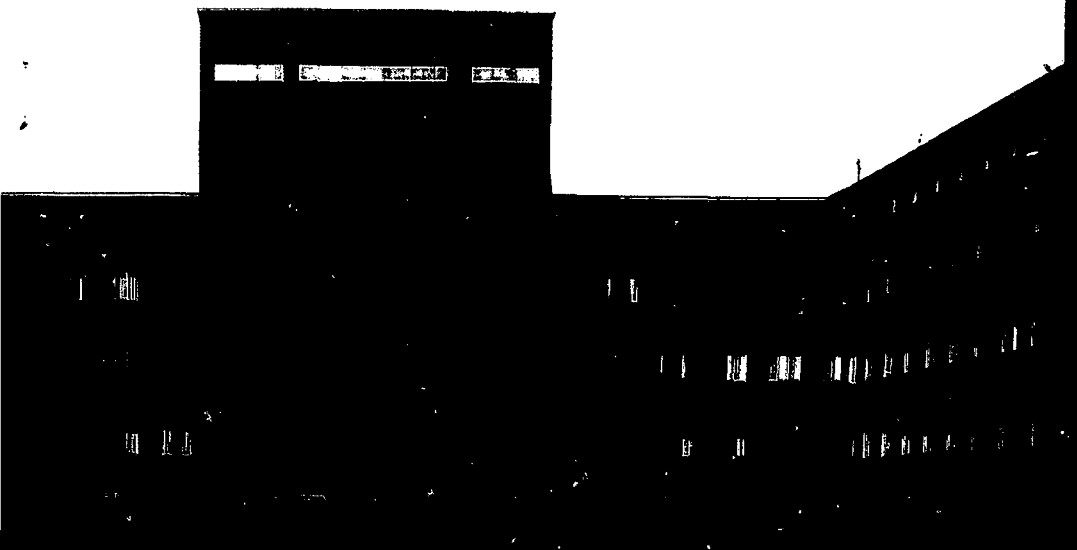

Recoil Corrected Bag Model Calculations for Semileptonic Weak Decays

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Abstract

We develop recoil corrections to various bag model results for strangeness changing weak decay amplitudes. It is shown that the spurious reference frame dependence of earlier calculations is reduced. The second class currents are generally less important than obtained by calculations in the static approximation. Finally, theoretical results are compared to observations. The agreement is quite good, although the values for the Cabibbo angle obtained by fits to the decay rates are somewhat too large.

1 Introduction

There is today no doubt that the fundamental interactions in nature should be formulated at the level of quarks and leptons. The coupling of the field quanta of the electroweak interaction is to these pointlike particles. Their interaction with the hadrons that are quark-composite systems has to be calculated by taking this compositeness into account. The treatment of bound qqq and $q\bar{q}$ systems automatically necessitates inclusion of long distance effects. This makes exact calculations impossible today, and one is forced to use models for bound quarks. Bag models are obvious candidates, and there have been considerable theoretical efforts in computing weak interaction form factors in baryon decays from these [1–5]. In this paper we shall limit ourselves to problems where only three flavours enter.

Cabibbo theory predates quarks, and has since its creation been used to relate different processes by three parameters, the Cabibbo angle θ_C and the values of F and D that characterize the weak currents at the baryon and meson level, assuming flavour symmetry. When one uses a particular quark model for particles, F and D are calculable and only the Cabibbo angle (or the generalization to the quark mixing matrix elements of the Kobayashi-Maskawa type) is a parameter.

Our present interest in strangeness changing weak amplitudes has been inspired by the CERN WA2 experiment [6] on hyperon semileptonic decays, and high statistics experiments at Fermilab [7,8]. In the CERN experiment no indications of flavour symmetry breaking effects were found, and it thus agreed excellently with the Cabibbo model. This result was somewhat surprising to us, since flavour symmetry is broken by the order of 20 % in the masses and magnetic moments of the baryon octet.

In bag models, flavour symmetry is dynamically broken by assigning different masses to the strange quark s and the u and d quarks. By reasonable choices for the quark masses (and other bag model parameters), bag models are generally capable of reproducing the static properties of the baryons, such as masses and magnetic moments. As mentioned above, several attempts have already been made to calculate the form factors which describe the semileptonic transitions of the baryons, using static bag model quark wave functions. However, the baryon mass difference in $\Delta S = 1$ semileptonic decays is appreciable, and one would therefore expect that recoil effects might not be negligible in these decays. Omitting such effects (by using static quark wave functions in the calculation) leads to frame dependence of the final results, i.e. they will depend on the particular Lorentz frame used in the calculation. In [5] we showed that this was indeed the case, and we also found that the frame dependence of the so-called weak electric form factor g_2 — which plays an important role in the fit to experimental data — turned out to be unacceptably high.

In the hope of diminishing this frame dependence, we shall in this paper apply boosted quark wave functions in the calculation of the relevant matrix elements. We should point out that to our knowledge no completely rigorous formalism exists for boosting a confined three-quark hadronic system in a way that strictly respects Lorentz-invariance. The straightforward prescription for including recoil effects that we have used (introduced by Guichon [9]), therefore cannot be expected to restore full Lorentz-invariance. Nevertheless we shall see that the frame dependence will be reduced to an acceptable level, taking into account the accuracy of the bag model itself, as long as we do not have bigger mass differences than we have with three flavours.

The outline of this paper is as follows: Section 2 defines the form factors which describe the semileptonic transitions, and gives the relations between these covariant form factors and the Sachs form factors which are directly calculable in the bag model. Section 3 describes the quark sector of the bag models that we use, and gives the prescription for boosting the quark spinors. Finally we write down the analytic expressions for the quark contribution to the form factors. Section 4 presents the pseudoscalar fields coupled to the quark currents of the previous section, and gives the analytic expressions for the pseudoscalar contributions to the axial form factors. Finally, section 5 contains all the numerical results and comparisons with experimental data.

2 Form Factors

From Lorentz covariance and parity invariance of the strong interactions, the matrix element of the vector current between spin-1/2 baryon states must have the form

$$\begin{aligned} \langle B'(p')|\, V_\mu(k)\, |B(p)\rangle &= \begin{bmatrix} \cos\theta_C \\ \sin\theta_C \end{bmatrix} \overline{u}_{B'}(p') \Big\{ f_1(k^2)\gamma_\mu \\ &\quad - \frac{f_2(k^2)}{m+m'}\sigma_{\mu\nu} i k^\nu + \frac{f_3(k^2)}{m+m'} k_\mu \Big\} u_B(p) \end{aligned} \tag{1}$$

and likewise for the axial current:

$$\begin{aligned} \langle B'(p')|\, A_\mu(k)\, |B(p)\rangle &= \begin{bmatrix} \cos\theta_C \\ \sin\theta_C \end{bmatrix} \overline{u}_{B'}(p') \Big\{ g_1(k^2)\gamma_\mu\gamma_5 \\ &\quad - \frac{g_2(k^2)}{m+m'}\sigma_{\mu\nu}\gamma_5 i k^\nu + \frac{g_3(k^2)}{m+m'} k_\mu\gamma_5 \Big\} u_B(p) \end{aligned} \tag{2}$$

Here $|B\rangle, |B'\rangle, m, m', p, p'$ are the initial and final baryon states, masses and momenta, respectively. $k = p - p'$ is the four-momentum transfer and θ_C the Cabibbo-angle. The Dirac spinors can be written

$$u_B(p) = \begin{pmatrix} \cosh\frac{\omega_B}{2}\chi_B \\ \sinh\frac{\omega_B}{2}\,\vec{\sigma}\cdot\hat{\vec{p}}\,\chi_B \end{pmatrix} \tag{3}$$

where $\tanh\omega_B = |\vec{v}_B|$, $\vec{v}_B$ is the velocity of the baryon. Similar definitions hold for the final baryon.

The six form factors are Lorentz scalars. Time-reversal invariance requires that all six form factors are real. f_1 and g_1 are the well-known vector and axial-vector form factors ($g_1 = g_A$, the axial charge, in neutron β-decay). f_2 and g_2 are the weak magnetic and weak electric dipole form factors. f_3 and g_3 are the induced scalar and pseudoscalar form factors, respectively. f_3 and g_2 are so-called second-class form factors, which must vanish in the SU(3) symmetric limit. However, since we intend to study SU(3) breaking effects, we must expect nonzero f_3 and g_2. Indeed, g_2 will play an important role in the comparison with experimental data, as we shall see. f_3 and g_3 are rather unimportant here since these are multiplied by a factor $(m_l/m)^2$ (m_l is the lepton mass) in the expression for the differential decay rate, and therefore can be safely ignored in *electronic* hyperon decays. We shall therefore not give any calculated values for them.

To perform explicit calculations, it is necessary to rewrite the matrix elements in (1) and (2) in terms of so-called Sachs form factors. The vector current may then, equally general, be written as

$$J_0^V = G_0(k^2)\chi_{B'}^\dagger\chi_B \quad (4)$$

$$\vec{J_V} = \frac{G_1(k^2)}{m+m'}\chi_{B'}^\dagger i\vec{k}\times\vec{\sigma}\,\chi_B + \frac{G_2(k^2)}{m+m'}\vec{k}\,\chi_{B'}^\dagger\chi_B \quad (5)$$

Similarly, the axial current may be expressed as

$$J_0^A = \frac{H_0(k^2)}{m+m'}\chi_{B'}^\dagger\vec{k}\cdot\vec{\sigma}\,\chi_B \quad (6)$$

$$\vec{J_A} = H_1(k^2)\chi_{B'}^\dagger\vec{\sigma}\chi_B + \frac{H_2(k^2)}{(m+m')^2}\chi_{B'}^\dagger\vec{k}\,(\vec{k}\cdot\vec{\sigma})\,\chi_B \quad (7)$$

It is important to keep in mind that the Sachs form factors are frame dependent; that is, they depend on the Lorentz frame in which they are calculated. In principle, this frame dependence should be such that the covariant form factors $f_1 - g_3$, expressed as functions of the Sachs form factors, remained true scalars. But as already pointed out, even with boost effects included in the bag model, Lorentz invariance will not be completely satisfied. To estimate this frame dependence, we have performed all our calculations in two different frames. As in [5] we have chosen the center-of-mass (CM) frame in which the initial baryon is at rest, and a modified version of Breit-frame (Br) in which the two baryons, instead of having opposite momenta, have opposite velocities. In this Breit-frame the hyperbolic angles are the same ($\omega_B = \omega_{B'}$), and $\vec{p} = \vec{k}m/(m+m')$, $\vec{p'} = -\vec{k}m'/(m+m')$. Expanding (1) and (2) in this frame using the spinors (3), and comparing with (4–5) and (6–7), we may write the Sachs form factors in terms of the covariant form factors. Inverting these linear equations, one obtains for the vector form factors

$$f_1^{Br} = \frac{1}{1-\delta^2}\left[G_0^{Br} - \delta^2 G_1^{Br} - \delta G_2^{Br}\right] + \frac{k^2}{(m+m')^2}\frac{1}{1-\delta^2}\left[-G_0^{Br} + G_1^{Br} + \frac{1}{2}\delta G_2^{Br}\right] + \partial(k^3) \quad (8)$$

$$f_2^{Br} = \frac{1}{1-\delta^2}\left[-G_0^{Br} + G_1^{Br} + \delta G_2^{Br}\right] - -\frac{k^2}{(m+m')^2}\frac{1}{1-\delta^2}\left[-G_0^{Br} + G_1^{Br} + \frac{1}{2}\delta G_2^{Br}\right] + \partial(k^3) \quad (9)$$

where $\delta \equiv (m-m')/(m+m')$ and $k \equiv |\vec{k}|$. Similarly, we find for the axial form factors:

$$g_1^{Br} = \frac{1}{1-\delta^2}\left[-\delta H_0^{Br} + (1-\frac{1}{2}\delta^2)H_1^{Br} + \delta^2 H_2^{Br}\right] +$$

Table 2

Optical Calibration, Lysebu, August 1985

	Brightness (R/Å)						
Source	3918	4280	4866	5573	5882	6299	6562
Filament Lamps							
McEwen	1.72	5.7	22.6	71.9	110.5	179.9	228.8
DMI-1	0.05	0.27	2.0	15.3	32.0	65.0	180.0
DMI-2	0.05	0.27	2.6	19.0	38.0	85.0	200.0
Sodankylä (tungsten)	0.12	0.71	5.4	31.8	62.8	125.8	137.0
Sodankylä (glow discharge)	2.0	6.0	10.1	9.9	15.3	4.0	0.66
Phosphors							
Uppsala Y275	0.03	0.3	3.8	251.0	378.0	217.0	113.0
Uppsala 920B	5.1	126.0	61.5	18.6	10.5	6.7	8.1
Uppsala L1614	0.07	0.73	32.5	27.7	8.7	2.5	4.3
Oslo R411	0.04	0.07	0.79	20.7	68.2	98.4	151.3
Oslo 920B	15.3	198.0	88.0	14.2	6.8	6.8	4.1
Oslo L1614	0.79	11.9	185.0	245.0	73.1	10.4	11.5
Oslo Y275	1.2	23.0	26.0	204.0	300.0	262.0	195.0
Oslo 920B/ KR#2	108.0	583.0	462.0	216.0	163.0	216.0	144.0
Andenes (yellow)	0.78	15.7	17.2	129.1	181.6	182.0	261.2
Andenes (blue)	5.1	90.1	47.7	9.7	6.9	10.3	20.0
MPI 2	0.02	0.16	2.43	193.7	290.7	191.8	98.2

In all models, the single quark Hamiltonian inside the bag is given by

$$H_Q = \vec{\alpha} \cdot \vec{p} + \beta m_Q + V(r) \tag{16}$$

In models A–C, $V(r) = 0$. In the LAPP model (D) the residual interaction among the quarks is approximated by a Coulomb-like potential

$$V(r) = -\beta_s \left(\frac{1}{r} - \frac{1}{R}\right) \tag{17}$$

The strength of the potential β_s — though in principle related to the strong coupling α_s — is treated as a free parameter in the model.

The static, ground state quark spinors, which satisfy $H_Q \psi = E_Q \psi$, can be written

$$\psi_q^0(\vec{r}, t) = \begin{bmatrix} iF_q(r)\chi_q \\ G_q(r)\,\vec{\sigma} \cdot \hat{\vec{r}}\,\chi_q \end{bmatrix} e^{-iE_q t}\theta(R - r) \tag{18}$$

where the superscript 0 indicates that these are *static* spinors. χ_q is a Pauli spinor and F and G are proportional to the spherical Bessel functions $j_0(kr)$ and $j_1(kr)$ in models A–C, and to relativistic Coulomb wave functions in model D.

In models A,B and D the quark energy is as usual quantized by the Bogoliubov boundary condition

$$i\not{n}\psi_q = \psi_q \qquad (r = R) \tag{19}$$

where n^μ is the unit outward normal of the bag. In the bag rest frame $n^\mu = (0, \hat{\vec{r}})$. The chiral invariant boundary condition of model C will be discussed in the next section.

To include recoil effects, we need boosted quark wave functions. Let q and q' denote incoming and outgoing quarks participating in the weak interaction. Then the boosted quark spinors can be written

$$\psi_q(x) = S(\vec{v}_B)\psi_q^0(\vec{r}_B^0, t_B^0) \tag{20}$$

$$\psi_{q'}(x) = S(\vec{v}_{B'})\psi_{q'}^0(\vec{r}_{B'}^0, t_{B'}^0) \tag{21}$$

where $(\vec{r}_B^0, t_B^0)$ are the rest frame coordinates of baryon B, $S(\vec{v}_B)$ is the Dirac boost matrix from the rest frame of B to the frame in which we perform our calculations (Breit or CM frame). Similarly for B'. Note that the rest frames of B and B' are in general different.

In models A–C, the static spinors (18) satisfy the free Dirac equation inside the bag. Therefore, the boosted spinors (20) and (21) will also satisfy the free Dirac equation. The LAPP model boosted spinors will also satisfy the Dirac equation in the moving frame, but then one must keep in mind that the potential $V(r)$ is not a Lorentz scalar, but the zeroth component of a four-vector.

At quark level, the weak hadronic current operator in semileptonic decays has the usual $V-A$ structure

$$J_\mu^{W,Q} = \begin{bmatrix} \cos\theta_C \\ \sin\theta_C \end{bmatrix} (V_\mu^Q - A_\mu^Q) \tag{22}$$

with

$$\begin{aligned} V_\mu^Q &= \bar{\psi}_{q'}(x)\gamma_\mu\psi_q(x) && (23)\\ A_\mu^Q &= \bar{\psi}_{q'}(x)\gamma_\mu\gamma_5\psi_q(x) && (24)\end{aligned}$$

In addition, in models B–D the axial current will also have a contribution from the pseudoscalar field, which we shall come back to in the next section.

To make contact with the current at baryon level (1,2) we shall basically use the prescription given by Betz and Goldflam [15] and identify the matrix element in (1,2) with the Fourier-transform of $J_\mu^{W,Q}$, using the boosted quark spinors (20,21), at time $t=0$. Written out, we then have

$$\begin{aligned}\langle B'|\,J_\mu(k)\,|B\rangle &= \begin{bmatrix} \cos\theta_C \\ \sin\theta_C \end{bmatrix} \langle B'| \int_{Bag} d^3\vec{r}\, e^{-i\vec{k}\cdot\vec{r}}\bar{\psi}^0_{q'}(\vec{r}^{\,0}_{B'},t^0_{B'})S(-\vec{v}_{B'})\gamma_\mu \times \\ &\quad \times(1-\gamma_5)S(\vec{v}_B)\psi^0_q(\vec{r}^{\,0}_B,t^0_B)\,|B\rangle_{t=0}\prod_{q_s} O_{q_s} \end{aligned}\tag{25}$$

with

$$O_{q_s} = \int_{Bag} d^3\vec{r}\,\bar{\psi}'_{q_s}(\vec{r}^{\,0}_{B'},t^0_{B'})\gamma_0\psi_{q_s}(\vec{r}^{\,0}_B,t^0_B)|_{t=0} \tag{26}$$

$|B\rangle$ and $|B'\rangle$ are properly symmetrized SU(6) spin-flavour baryon states. The product $\prod_{q_s}$ runs over the (two) spectator quarks not participating in the interaction. ψ_{q_s} and ψ'_{q_s} are the wave functions of the spectator quark q_s in baryons B and B' respectively. If the quark mass, bag radii and boundary conditions are the same, $\psi_{q_s} = \psi'_{q_s}$. Then, to first order in boost velocity $\vec{v}$, O_{q_s} is just the normalization integral: $O_{q_s} = 1 + \partial(\vec{v}^2 \sim \vec{k}^2)$. Apart from this term from the spectator quarks, (25) is identical to the boost prescription given by Guichon [9]. In (25) observe that $t=0$ does not necessarily imply that $t^0_B = t^0_{B'} = 0$. This fact gives rise to the so-called retardation term, which plays an important role in calculations of recoil effects to magnetic moments [9].

The remaining calculations are now well defined. We choose some particular Lorentz frame (Breit or CM frame in our case), expand (25) in powers of $|\vec{v}| \sim |\vec{k}|$, and compare with eqs. (4–7). Thus we obtain the Sachs form factors expanded in powers of $\vec{k}^2$.

It should be emphasized that this is an expansion in the 3-momentum $\vec{k}$. In the analysis of experimental data [6], the k^2-dependence of the form factors was taken into account, so that the experimental values are given for $k_\mu k^\mu = 0$. It would therefore be desirable for us to compute the form factors at $k_\mu k^\mu = 0$. To do this however, we must compute the form factors to order $\vec{k}^2$. From eqs. (4–7) we see that to find the vector form factors to order $\vec{k}^2$ implies boosting to order $|\vec{k}|^3$, and for the axial form factors boosting to order $|\vec{k}|^4$. As we shall see, attempting to include such higher order terms leads to serious difficulties with the bag boundary. This also applies to the pseudoscalar field. Moreover, such higher order terms would become very complex, especially terms coming from the pseudoscalar field.

To estimate this $\vec{k}^2$ dependence of the form factors, we have nevertheless calculated the vector form factors to order $\vec{k}^2$. For the quark contribution to the axial form factors, we have boosted to order $|\vec{k}|^4$ in Breit frame. To avoid some of the difficulties, we have boosted to order $|\vec{k}|^3$ only in CM frame, so that H_2^{CM} is given at $\vec{k} = 0$ only. For the pseudoscalar field (see section 4) boost effects are included to order $\vec{k}^2$, so that only the $\vec{k}^2$-dependence of H_1^Φ can be calculated.

Let us then be more specific, and expand (25) in the two frames we have chosen.

3.1 Breit Frame (Equal Speeds)

In this frame $\vec{v}_B = -\vec{v}_{B'} = \vec{v}$, and the coordinate transformation from the rest frames of B and B' to Breit frame can be written

$$\vec{r}^0_{\binom{B}{B'}} = \vec{r} + (\gamma - 1)\hat{\vec{v}}(\vec{r}\cdot\hat{\vec{v}}) \mp \gamma\vec{v}\,t \tag{27}$$

$$t^0_{\binom{B}{B'}} = \gamma(t \mp \vec{v}\cdot\vec{r}) \tag{28}$$

with

$$\gamma\vec{v} = \frac{\vec{k}}{m + m'} \tag{29}$$

$$\gamma = \sqrt{1 + \frac{\vec{k}^2}{(m + m')^2}} \tag{30}$$

The boost matrices can be written

$$S(\vec{v}) = S(\vec{v}_B) = S(-\vec{v}_{B'}) = \cosh\frac{\omega}{2} + \sinh\frac{\omega}{2}\vec{\alpha}\cdot\hat{\vec{k}} \tag{31}$$

where $\tanh\omega = |\vec{v}|$.

It is now straightforward to expand (25) in powers of $\vec{k}$ at $t = 0$. Observe from (18) and (27) that the deformation of the bags (due to Lorentz contraction) is of order $\vec{k}^2$ and the same for both baryons:

$$\vec{r}^0_B(t = 0) = \vec{r}^0_{B'}(t = 0) = \vec{r} + \mathcal{O}(\vec{k}^2) \tag{32}$$

which is one of the advantages of using this modified version of Breit frame. After some algebra we find [1]

$$
\begin{aligned}
G_0^{Br} &= \hat{N}\hat{N}_{uu}^2 f_1^{SU(6)} \\
&\quad - \frac{\vec{k}^2}{(m+m')^2}\left[\frac{1}{2}\hat{N} + \frac{1}{6}\beta^2\hat{N}_2\right]\hat{N}_{uu}^2 f_1^{SU(6)} + G_0^{Br,BG}
\end{aligned} \tag{33}
$$

where

$$
G_0^{Br,BG} = -\frac{\vec{k}^2}{(m+m')^2}\left[\hat{N}\hat{N}_{uu}^2 + \frac{1}{3}(\beta_{q_s}-1)^2\hat{N}\hat{N}_{uu}\hat{N}_{2uu}\right] f_1^{SU(6)} \tag{34}
$$

$$
\begin{aligned}
G_1^{Br} &= (\hat{g}_A + \beta\hat{\mu})\hat{N}_{uu}^2 g_1^{SU(6)} + \\
&\quad \frac{\vec{k}^2}{(m+m')^2}\Big[-\frac{1}{10}\beta^3\hat{\mu}_3 + \beta^2\left(\frac{4}{15}\hat{B} - \frac{1}{6}\hat{N}_2\right) - \\
&\quad -\frac{1}{2}(\beta\hat{\mu} + \hat{g}_A)\Big]\hat{N}_{uu}^2 g_1^{SU(6)} + G_1^{Br,BG}
\end{aligned} \tag{35}
$$

$$
G_1^{Br,BG} = -\frac{\vec{k}^2}{(m+m')^2}(\beta\hat{\mu} + \hat{g}_A)[\hat{N}_{uu}^2 + \frac{1}{3}(\beta_{q_s}-1)^2\hat{N}_{uu}\hat{N}_{2uu}] g_1^{SU(6)} \tag{36}
$$

$$
\begin{aligned}
G_2^{Br} &= -\beta\hat{d}\hat{N}_{uu}^2 f_1^{SU(6)} + \\
&\quad \frac{\vec{k}^2}{(m+m')^2}\left[\frac{1}{10}\beta^3\hat{d}_3 + \beta\hat{d}\right]\hat{N}_{uu}^2 f_1^{SU(6)} + G_2^{Br,BG}
\end{aligned} \tag{37}
$$

$$
G_2^{Br,BG} = \frac{\vec{k}^2}{(m+m')^2} f_1^{SU(6)}\beta\hat{d}\left[\hat{N}_{uu}^2 + \frac{1}{3}(\beta_{q_s}-1)^2\hat{N}_{uu}\hat{N}_{2uu}\right] \tag{38}
$$

Similarly we find for the axial form factors

$$
\begin{aligned}
H_0^{Br} &= -\beta\hat{d}\hat{N}_{uu}^2 g_1^{SU(6)} + \\
&\quad \frac{\vec{k}^2}{(m+m')^2}\left[\beta\hat{d} + \frac{1}{10}\beta^3\hat{d}_3\right]\hat{N}_{uu}^2 g_1^{SU(6)} + H_0^{Br,BG}
\end{aligned} \tag{39}
$$

$$
H_0^{Br,BG} = \frac{\vec{k}^2}{(m+m')^2}\beta\hat{d}\hat{N}_{uu}\left[\hat{N}_{uu} + \frac{1}{3}(\beta_{q_s}-1)^2\hat{N}_{2uu}\right] g_1^{SU(6)} \tag{40}
$$

$$
\begin{aligned}
H_1^{Br} &= \hat{g}_A\hat{N}_{uu}^2 g_1^{SU(6)} + \\
&\quad \frac{\vec{k}^2}{2(m+m')^2}\hat{N}_{uu}^2 g_1^{SU(6)}\left[2\beta\hat{\mu} + \frac{1}{15}\beta^2(8\hat{B} - 5\hat{N}_2)\right] \\
&\quad + H_1^{Br,BG}
\end{aligned} \tag{41}
$$

$$
H_1^{Br,BG} = -\frac{\vec{k}^2}{(m+m')^2} g_1^{SU(6)}\hat{g}_A\hat{N}_{uu}\left[\hat{N}_{uu} + \frac{1}{3}(\beta_{q_s}-1)^2\hat{N}_{2uu}\right] \tag{42}
$$

---

[1]Strictly speaking, the terms from the spectator quarks in the following expressions, are valid only when both spectator quarks are nonstrange (as in $\Lambda \to p$ and $\Sigma^- \to n$). For Ξ decays, there will be minor (and numerically unimportant) modifications.

$$H_2^{Br} = -g_1^{SU(6)}\hat{N}_{uu}^2\left[\frac{1}{2}\hat{g}_A + \beta\hat{\mu} + \frac{2}{15}\beta^2\hat{B}\right] +$$
$$g_1^{SU(6)}\frac{\vec{k}^2}{840(m+m')^2}\hat{N}_{uu}^2\left[579\hat{g}_A - 264\hat{N} + 840\beta\hat{\mu} +\right.$$
$$\left. +\beta^2(56\hat{B} + 70\hat{N}_2) + 84\beta^3\hat{\mu}_3 + 8\beta^4\hat{B}_4\right] + H_2^{Br,BG} \tag{43}$$

$$H_2^{Br,BG} = g_1^{SU(6)}\hat{N}_{uu}^2\frac{\vec{k}^2}{(m+m')^2}\left(\frac{1}{2}\hat{g}_A + \beta\hat{\mu} + \frac{2}{15}\beta^2\hat{B}\right) \times$$
$$\times\left[\hat{N}_{uu}^2 + \frac{1}{3}(\beta_{q_s} - 1)^2\hat{N}_{2uu}\hat{N}_{uu}\right] \tag{44}$$

Here, $f_1^{SU(6)}$ and $g_1^{SU(6)}$ are SU(6) spin-flavour factors defined as follows:

$$f_1^{SU(6)} \equiv \langle B'|\,\lambda_{q'q}\,|B\rangle \tag{45}$$
$$g_1^{SU(6)} \equiv \langle B'|\,\lambda_{q'q}\otimes\sigma_3\,|B\rangle \tag{46}$$

where $|B\rangle$ ($|B'\rangle$) are SU(6)-symmetric spin-flavour baryon states, and $\lambda_{q'q}$ the appropriate combination of Gell-Mann matrices for the quark transition $q \rightarrow q'$. Values of $f_1^{SU(6)}$ and $g_1^{SU(6)}$ can be found in standard textbooks, e.g. Okun'[16]. The retardation factor β is defined as

$$\beta = 1 - \frac{E_q + E_{q'}}{m + m'} \tag{47}$$

where E_q, $E_{q'}$ are the eigenenergies of the quarks participating in the interaction. Correspondingly, the retardation term from the spectator quarks reads:

$$\beta_{q_s} = 1 - \frac{E_{q_s} + E'_{q_s}}{m + m'} \tag{48}$$

where E_{q_s} and E'_{q_s} are the eigenenergies of the spectator quark q_s in baryons B and B', respectively. In addition, we have introduced the following bag integrals:

$$\hat{N} = \int d^3\vec{r}\,[F_{q'}^*F_q + G_{q'}^*G_q] \tag{49}$$
$$\hat{N}_{uu} = \int d^3\vec{r}\,[F_{q_s}'^*F_{q_s} + G_{q_s}'^*G_{q_s}] \tag{50}$$
$$\hat{N}_2 = (m+m')^2\int d^3\vec{r}\,r^2[F_{q'}^*F_q + G_{q'}^*G_q] \tag{51}$$
$$\hat{N}_{2uu} = (m+m')^2\int d^3\vec{r}\,r^2[F_{q_s}'^*F_{q_s} + G_{q_s}'^*G_{q_s}] \tag{52}$$
$$\hat{g}_A = \int d^3\vec{r}\,[F_{q'}^*F_q - \frac{1}{3}G_{q'}^*G_q] \tag{53}$$
$$\hat{\mu} = -\frac{1}{3}(m+m')\int d^3\vec{r}\,r[F_{q'}^*G_q + G_{q'}^*F_q] \tag{54}$$
$$\hat{d} = \frac{1}{3}(m+m')\int d^3\vec{r}\,r[F_{q'}^*G_q - G_{q'}^*F_q] \tag{55}$$

$$
\begin{aligned}
\hat{B} &= (m+m')^2 \int d^3r\, r^2 G_{q'}^* G_q & (56)\\
\hat{\mu}_3 &= -\frac{1}{3}(m+m')^3 \int d^3r\, r^3 [F_{q'}^* G_q + G_{q'}^* F_q] & (57)\\
\hat{d}_3 &= \frac{1}{3}(m+m')^3 \int d^3r\, r^3 [F_{q'}^* G_q - G_{q'}^* F_q] & (58)\\
\hat{B}_4 &= (m+m')^4 \int d^3r\, r^4 G_{q'}^* G_q & (59)
\end{aligned}
$$

$\hat{N}$ is the so-called wave function mismatch integral, which is unity in the flavour symmetric limit when $q = q'$. When the quark masses are unequal, $\hat{N}$ is less than unity. Similarly, $\hat{N}_{uu}$ will be the normalization integral for the spectator quark q_s as long as the two bags have the same radius, and the boundary condition is the same (always true in models A,B,D). $\hat{g}_A$ gives the main contribution to the axial charge in neutron β-decay. $\hat{\mu}$ and $\hat{d}$ contribute to the magnetic (transition) moment and electric dipole (transition) moment, respectively. $\hat{d}$ vanishes in the flavour symmetric limit. The terms marked 'BG' in the Sachs form factors are due to the O_{q_s}-terms in (25), coming from the spectator quarks.

3.2 CM Frame

Here $\vec{v}_B = 0$, $\vec{v}_{B'} \equiv -\vec{v}$, where

$$
\begin{aligned}
\gamma\vec{v} &= \frac{\vec{k}}{m'}\\
\gamma &= \sqrt{1+\frac{\vec{k}^2}{m'^2}} & (60)\\
\tanh\omega &= |\vec{v}|
\end{aligned}
$$

The coordinate transformation is given as

$$
\vec{r}_B^{\,0} = \vec{r}, \quad t_B^0 = t \tag{61}
$$

since the rest frame of B coincides with the CM frame, and

$$
\begin{aligned}
\vec{r}_{B'}^{\,0} &= \vec{r} + (\gamma-1)\hat{\vec{v}}(\vec{r}\cdot\hat{\vec{v}}) + \gamma\vec{v}t\\
t_{B'}^0 &= \gamma(t+\vec{v}\cdot\vec{r}) & (62)\\
S(\vec{v})\,(=S(-\vec{v}_{B'})) &= \cosh\frac{\omega}{2} + \sinh\frac{\omega}{2}\vec{\alpha}\cdot\hat{\vec{k}} & (63)
\end{aligned}
$$

Note that the deformation of the bags is still of order $\vec{k}^2$, but the deformation is no longer the same for the initial and final baryons. We shall see below that this will lead to some difficulties. Expanding (25) in this frame at $t = 0$, we find

$$
\begin{aligned}
G_0^{CM} &= \hat{N}\hat{N}_{uu}^2 f_1^{SU(6)} + \\
&\quad f_1^{SU(6)}\hat{N}_{uu}^2\frac{\vec{k}^2}{6m'^2}\left[\frac{3}{4}\hat{N} + 3\beta\frac{m'}{m+m'}\hat{d} - \right. \\
&\quad \left. -\beta^2\frac{m'^2}{(m+m')^2}\hat{N}_2 - \Theta\tilde{N} + \hat{N}_D\right] + G_0^{CM,BG} \qquad (64) \\
G_0^{CM,BG} &= -f_1^{SU(6)}\hat{N}\hat{N}_{uu}\frac{\vec{k}^2}{3m'^2}\times \\
&\quad \times\left[-\hat{N}_{Duu} + (\beta_{q_s}-1)^2\frac{m'^2}{(m+m')^2}\hat{N}_{2uu} + \Theta\tilde{N}_{uu}\right] \qquad (65) \\
G_1^{CM} &= g_1^{SU(6)}\hat{N}_{uu}^2\left\{\frac{m+m'}{2m'}\hat{g}_A + \beta\hat{\mu} - \right. \\
&\quad -\frac{\vec{k}^2}{10m'^2}\left[\frac{m+m'}{24m'}(6\hat{N} + 9\hat{g}_A - \right. \\
&\quad -20\hat{N}_D + 32\hat{G}_D + 4\Theta(5\tilde{N} - 8\tilde{G})) - \\
&\quad -\beta\left(\hat{d} + \frac{9}{4}\hat{\mu} + 3\hat{\mu}_D - (m+m')\,\Theta R_{B'}\tilde{S}^+\right) \\
&\quad \left.\left. +\frac{1}{6}\beta^2\frac{m'}{m+m'}(5\hat{N}_2 - 8\hat{B}) + \beta^3\frac{m'^2}{(m+m')^2}\hat{\mu}_3\right]\right\} + \\
&\quad G_1^{CM,BG} \qquad (66) \\
G_1^{CM,BG} &= -\frac{\vec{k}^2}{3m'^2}g_1^{SU(6)}\hat{N}_{uu}\left(\frac{m+m'}{2m'}\hat{g}_A + \beta\hat{\mu}\right)\times \\
&\quad \times\left[(\beta_{q_s}-1)^2\frac{m'^2}{(m+m')^2}\hat{N}_{2uu} - \hat{N}_{Duu} + \Theta\tilde{N}_{uu}\right] \qquad (67) \\
G_2^{CM} &= f_1^{SU(6)}\hat{N}_{uu}^2\left\{-\beta\hat{d} - \frac{m+m'}{2m'}\hat{N}\right. \\
&\quad +\frac{\vec{k}^2}{240m'^3}(m+m')\left[15\hat{N} - 20\hat{N}_D + 20\,\Theta\tilde{N}\right. \\
&\quad +6\beta\frac{m'}{m+m'}\left(4(m+m')\,\Theta R_{B'}\tilde{S}^- - 4\hat{\mu} - 9\hat{d} - 12\hat{d}_D\right) \\
&\quad \left.\left. +20\beta^2\frac{m'^2}{(m+m')^2}\hat{N}_2 + 24\beta^3\frac{m'^3}{(m+m')^3}\hat{d}_3\right]\right\} + G_2^{CM,BG} \qquad (68) \\
G_2^{CM,BG} &= \frac{\vec{k}^2}{3m'^2}f_1^{SU(6)}\hat{N}_{uu}\left(\beta\hat{d} + \frac{m+m'}{2m'}\hat{N}\right)\times \\
&\quad \times\left[(\beta_{q_s}-1)^2\frac{m'^2}{(m+m')^2}\hat{N}_{2uu} - \hat{N}_{Duu} + \Theta\tilde{N}_{uu}\right] \qquad (69)
\end{aligned}
$$

and for the axial Sachs form factors:

$$
\begin{aligned}
H_0^{CM} &= g_1^{SU(6)}\hat{N}_{uu}^2\Big\{-\frac{m+m'}{2m'}\hat{g}_A-\beta\hat{d}+\\
&\frac{\vec{k}^2}{240m'^3}(m+m')\Big[-12\hat{N}+27\hat{g}_A-\\
&-20\hat{N}_D+16\hat{G}_D+\Theta(20\tilde{N}-16\tilde{G})+\\
&6\beta\frac{m'}{m+m'}\left(4(m+m')\Theta R_{B'}\tilde{S}^- -4\hat{\mu}-9\hat{d}-12\hat{d}_D\right)+\\
&\beta^2\frac{m'^2}{(m+m')^2}(20\hat{N}_2-16\hat{B})+\\
&24\beta^3\frac{m'^3}{(m+m')^3}\hat{d}_3\Big]\Big\}+H_0^{CM,BG} \qquad (70)\\
H_0^{CM,BG} &= \frac{\vec{k}^2}{3m'^2}g_1^{SU(6)}\hat{N}_{uu}\left(\beta\hat{d}+\frac{m+m'}{2m'}\hat{g}_A\right)\times\\
&\times\left[(\beta_{q_e}-1)^2\frac{m'^2}{(m+m')^2}\hat{N}_{2uu}-\hat{N}_{Duu}+\Theta\tilde{N}_{uu}\right] \qquad (71)\\
H_1^{CM} &= g_1^{SU(6)}\hat{N}_{uu}^2\Big\{\hat{g}_A+\frac{\vec{k}^2}{120m'^2}\Big[-6\hat{N}+21\hat{g}_A+20\hat{N}_D-32\hat{G}_D-\\
&-\Theta(20\tilde{N}-32\tilde{G})+60\beta\frac{m'}{m+m'}\hat{\mu}-\\
&-\beta^2\frac{m'^2}{(m+m')^2}(20\hat{N}_2-32\hat{B})\Big]\Big\}+H_1^{CM,BG} \qquad (72)\\
H_1^{CM,BG} &= -\frac{\vec{k}^2}{3m'^2}g_1^{SU(6)}\hat{N}_{uu}\hat{g}_A\times\\
&\times\left[(\beta_{q_e}-1)^2\frac{m'^2}{(m+m')^2}\hat{N}_{2uu}-\hat{N}_{Duu}+\Theta\tilde{N}_{uu}\right] \qquad (73)\\
H_2^{CM} &= -g_1^{SU(6)}\hat{N}_{uu}^2\frac{(m+m')^2}{60m'^2}\Big[9(\hat{g}_A-\hat{N})-8\hat{G}_D+8\,\Theta\tilde{G}+\\
&+30\beta\frac{m'}{m+m'}(\hat{\mu}-\hat{d})+8\beta^2\frac{m'^2}{(m+m')^2}\hat{B}\Big] \qquad (74)
\end{aligned}
$$

In CM frame the retardation factors read

$$
\begin{aligned}
\beta &= 1-\frac{E_{q'}}{m'} \qquad (75)\\
\beta_{q_e} &= 1-\frac{E'_{q_e}}{m'} \qquad (76)
\end{aligned}
$$

New bag integrals entering the above expressions are defined as

$$
\hat{N}_D = \int d^3r\, r[\dot{F}^*_{q'}F_q+\dot{G}^*_{q'}G_q] \qquad (77)
$$

$$\hat{N}_{Duu} = \int d^3r\, r [\dot{F}'^*_{q_e} F_{q_e} + \dot{G}'^*_{q_e} G_{q_e}] \quad (78)$$

$$\hat{G}_D = \int d^3r\, r \dot{G}^*_{q'} G_q \quad (79)$$

$$\hat{\mu}_D = -\frac{1}{3}(m + m') \int d^3r\, r^2 [\dot{F}^*_{q'} G_q + \dot{G}^*_{q'} F_q] \quad (80)$$

$$\hat{d}_D = \frac{1}{3}(m + m') \int d^3r\, r^2 [\dot{F}^*_{q'} G_q - \dot{G}^*_{q'} F_q] \quad (81)$$

where $\dot{F} \equiv \frac{dF}{dr}$ etc. In addition we have introduced the surface terms

$$\tilde{N} = 4\pi R^3 [F^*_{q'} F_q + G^*_{q'} G_q] \quad (82)$$

$$\tilde{N}_{uu} = 4\pi R^3 [F'^*_{q_e} F_{q_e} + G'^*_{q_e} G_{q_e}] \quad (83)$$

$$\tilde{S}^+ = -4\pi R^3 [F^*_{q'} G_q + G^*_{q'} F_q] \quad (84)$$

$$\tilde{S}^- = 4\pi R^3 [F^*_{q'} G_q - G^*_{q'} F_q] \quad (85)$$

$$\tilde{G} = 4\pi R^3 G^*_{q'} G_q \quad (86)$$

R_B and $R_{B'}$ are the bag radii of baryons B and B' respectively, and $R = \min(R_B, R_{B'})$. In the last expression the wave functions have their value at $r = R$. Observe that in the form factors these surface terms are multiplied by Θ, which is the step function on the bag surface: $\Theta \equiv \theta(R_B - R_{B'})$. This strange factor, coming from the derivatives of the wave functions (18), is related to the sharp boundary of the bag. If we had introduced a softer bag surface, all surface terms would have disappeared, and we could have expressed the form factors entirely in terms of bag integrals. However, we have not considered such modified bag models here. The surface terms, multiplied by $\theta(R_B - R_{B'})$, have little or no practical importance however. This was borne out by all our numerical calculations (in a restricted k^2 interval), where we gave Θ values ranging from zero to one.

In Breit frame such surface terms do not occur, due to the symmetric treatment of the two baryons in that case. In this frame all derivatives of the wave functions may be eliminated through integration by parts.

4 The Pseudoscalar Field

To lowest order in chiral perturbation theory the pseudoscalar field of models B–D satisfies the free Klein-Gordon equation

$$(\Box + \mu^2)\, \Phi(\vec{r}, t) = 0 \quad (87)$$

In $\Delta S = 0$ decays, Φ corresponds to the pion triplet (with $\mu = m_\pi = 138$ MeV), and in $\Delta S = 1$ decays to the kaon fields ($\mu = m_K = 494$ MeV). The contribution to the axial current reads

$$A^\Phi_\mu(\vec{r}, t) = -f_\Phi \partial_\mu \Phi(\vec{r}, t) \quad (88)$$

with $f_\Phi(= f_\pi, f_K)$ the pseudoscalar decay constant. The strength of the field is determined by requiring that the axial current is continuous through the bag surface, and depends on whether the field is continuous through the bag surface (models B,C) or excluded from the bag interior (model D). In both cases the requirement can be written

$$n \cdot A^Q = n^\mu (A_\mu^{\Phi^{out}} - A_\mu^{\Phi^{in}}) \qquad (r = R) \tag{89}$$

where $\Phi^{out(in)}$ denotes the pseudoscalar field outside (inside) the bag ($\Phi^{in} = 0$ in model D). $A_\mu^Q(\vec{r}, t)$ is the quark axial (transition) current calculated in the previous section, and n_μ some bag normal yet to be determined.

In models B and D the only effect of the pseudoscalar field is to give a contribution to the axial current, so that PCAC will be satisfied through (89). In the chiral bag model (C) the pseudoscalar field has an additional effect in that it modifies the quark boundary condition (19), and thereby the quark wave functions. Assuming $SU(2) \otimes SU(2)$ chiral invariance, the chiral invariant boundary condition of model C reads

$$- i\vec{\gamma} \cdot \hat{\vec{r}}\, \psi_q = \exp[i\, \vec{\pi} \cdot \vec{\tau}\gamma_5 / f_\pi] \psi_q \qquad (r = R) \tag{90}$$

in the rest frame of the bag. For bag radii ≥ 1 fermi it is a good approximation to expand (90) to first order in the pion field only:

$$- i\vec{\gamma} \cdot \hat{\vec{r}}\, \psi_q \approx [1 + i\, \vec{\pi} \cdot \vec{\tau}\gamma_5 / f_\pi] \psi_q \qquad (r = R) \tag{91}$$

Using the well-known static solution to the Klein-Gordon equation (87), the boundary condition for a single non-strange quark in a hadron H can be written in the form

$$F_q(R) + G_q(R)(1 + \kappa/n) = 0 \tag{92}$$

with

$$\kappa = \frac{g}{8\pi M} \frac{1 + m_\pi R}{f_\pi R^2} e^{-m_\pi R} \frac{1}{3} \Sigma \tag{93}$$

$$\Sigma = \sum_{i,j} \langle H | \sigma_i \tau_i \sigma_j \tau_j | H \rangle \tag{94}$$

n is the number of non-strange quarks in H. g is the quark-pion coupling constant, determined from the boundary condition (89), and M the nucleon mass. In the numerical calculations, the values of Σ will be taken from [17] where baryon mass differences in the intermediate state have been taken into account.

In the chiral bag model, the two boundary conditions (89) and (92) are interrelated. We have followed the iterative procedure described in [13] to obtain self-consistent solutions to these equations.

Since we shall study strangeness-changing processes, it would be desirable to use the SU(3) extention of the boundary condition (90). However, by explicit calculations we find that κ decreases with increasing pion mass. As in [5] we shall therefore quantize the strange quark also by the Bogoliubov boundary condition (19). As a consequence, the wave function of the strange quark depends only on the bag radius and (in the LAPP model) on the value of $V(r)$. It is of some importance to realize that the strange quark therefore has very similar wave functions in Λ and in Σ.

Let us now determine the pseudoscalar (kaon) field in strangeness changing decays. Since we have used boosted quark spinors in the quark current of (89), we can no longer regard the field as static. On the other hand, we do not need the full time-dependence of Φ since, eventually, we shall take the Fourier-transform of A_μ^Φ at $t = 0$. As in [18] it suffices to expand Φ to second order in t:

$$\Phi(\vec{r}, t) = \phi(\vec{r}) + \xi(\vec{r})\, t + \frac{1}{2}\eta(\vec{r})\, t^2 + \partial(t^3) \tag{95}$$

Inserted into (87) we then have:

$$(\nabla^2 - \mu^2)\phi(\vec{r}) = \eta(\vec{r}) \tag{96}$$
$$(\nabla^2 - \mu^2)\xi(\vec{r}) = 0 \tag{97}$$
$$(\nabla^2 - \mu^2)\eta(\vec{r}) = 0 \tag{98}$$

Since the form factors will be evaluated at $t = 0$, the η-field will have only an indirect influence on the form factors through the inhomogeneous equation (96). $\xi(\vec{r})$ determines the time-component of the current A_0^Φ, and thereby contributes to the Sachs form factor H_0. In models with a static pseudoscalar field only, the field therefore does not contribute to the time-component of the axial current.

If we had included higher order terms in the expansion (95), this would have led to nonzero terms on the right-hand side of (97,98) as well. However, by explicit calculations using the boundary condition (89), it turns out that these terms are proportional to $(\Delta E_q)^3$ and higher powers in $\Delta E_q \equiv E_q - E_{q'}$. Numerically, these higher order terms are of little importance in the decays of the non-charmed baryons, and they are neglected in the following calculations.

Without doing any calculations, we can say something about the angular structure of the pseudoscalar field. We construct pseudoscalars out of the only three kinematical variables we have at our disposal; $\vec{r}, \vec{k}$ and $\vec{\sigma}$. The only possibilities are:

To order $\lvert\vec{k}\rvert^0$	$\vec{\sigma} \cdot \vec{r}$
To order $\lvert\vec{k}\rvert$	$\vec{\sigma} \cdot \vec{k}$, $(\vec{k} \cdot \vec{r})(\vec{\sigma} \cdot \vec{r})$
To order $\lvert\vec{k}\rvert^2$	$\vec{k}^2 \vec{\sigma} \cdot \vec{r}$, $\vec{\sigma} \cdot \vec{k}\, \vec{k} \cdot \vec{r}$, $(\vec{k} \cdot \vec{r})^2 \vec{\sigma} \cdot \vec{r}$

We can therefore write the ϕ-field in the form

$$\phi(\vec{r}) = \phi^{(0)}(\vec{r}) + \phi^{(1)}(\vec{r}) + \phi^{(2)}(\vec{r}) + \partial(|\vec{k}|^3) \tag{99}$$

with

$$\phi^{(0)}(\vec{r}) = \alpha_\phi^{(0)}(r)\vec{\sigma}\cdot\hat{\vec{r}} \tag{100}$$

$$\phi^{(1)}(\vec{r}) = \frac{k}{m+m'}\left[\beta_\phi(r)\hat{\vec{k}}\cdot\hat{\vec{r}}\vec{\sigma}\cdot\hat{\vec{r}} + \gamma_\phi(r)\vec{\sigma}\cdot\hat{\vec{k}}\right] \tag{101}$$

$$\phi^{(2)}(\vec{r}) = \frac{k^2}{(m+m')^2}\left[\alpha_\phi^{(2)}(r)\vec{\sigma}\cdot\hat{\vec{r}} + \rho_\phi(r)\vec{\sigma}\cdot\hat{\vec{k}}\hat{\vec{k}}\cdot\hat{\vec{r}} + \nu_\phi(r)(\hat{\vec{k}}\cdot\hat{\vec{r}})^2\vec{\sigma}\cdot\hat{\vec{r}}\right] \tag{102}$$

(where $k \equiv |\vec{k}|$), and similarly for the ξ and η fields. Inserting these expressions into (96–98), it is now straightforward to solve the resulting ordinary differential equations under the constraints that the field outside the bag should vanish at infinity, and the field inside the bag should be regular at the origin. The solutions are given by spherical Bessel and Hankel functions inside and outside the bag respectively, and have been written out in the appendix.

It then remains to determine the coefficients of the pseudoscalar field using the boundary condition (89). We must therefore decide which bag normal to use in (89). Since we now have two different bags boosted in different directions, the bag normal is no longer unique; $n_\mu^B \neq n_\mu^{B'}$. But using the fact that the boosted quark spinors (20) and (21) satisfy the Dirac equation and the Bogoliubov boundary condition (19), it is easy to verify that

$$(n_\mu^B + n_\mu^{B'})\bar{\psi}_{q'}\gamma^\mu\psi_q = 0 \tag{103}$$

Insisting that there should be no vector current through the bag surface, we shall therefore follow [18] and assume the appropriate bag normal to be

$$n_\mu(\vec{r},t) = c(n_\mu^B(\vec{r},t) + n_\mu^{B'}(\vec{r},t)) \tag{104}$$

where c is a normalization constant ensuring that $n_\mu n^\mu = -1$.

Expanding (104) in powers of k we find in Breit frame:

$$n_0^{Br} = \frac{k^2}{(m+m')^2}\frac{t}{r}\left[(\hat{\vec{k}}\cdot\hat{\vec{r}})^2 - 1\right] + \partial(k^3) \tag{105}$$

$$\vec{n}^{Br} = \hat{\vec{r}} + \frac{k^2}{(m+m')^2}\left[\hat{\vec{k}}\cdot\hat{\vec{r}}\hat{\vec{k}} - (\hat{\vec{k}}\cdot\hat{\vec{r}})^2\hat{\vec{r}} + \frac{t^2}{r^2}\left((\hat{\vec{k}}\cdot\hat{\vec{r}})^2\hat{\vec{r}} - \hat{\vec{k}}\cdot\hat{\vec{r}}\hat{\vec{k}}\right)\right] + \partial(k^3) \tag{106}$$

and in CM frame

$$n_0^{CM} = -\frac{k}{2m'}\left[\hat{\vec{k}}\cdot\hat{\vec{r}} - \frac{k}{m'r}t\left((\hat{\vec{k}}\cdot\hat{\vec{r}})^2 - 1\right)\right] + \partial(k^3) \quad (107)$$

$$\begin{aligned} \vec{n}^{CM} &= \hat{\vec{r}} - \frac{k}{2m'r}t\left[\hat{\vec{r}}\,\hat{\vec{k}}\cdot\hat{\vec{r}} - \hat{\vec{k}}\right] \\ &+\frac{k^2}{4m'^2r^2}\left\{r^2\left[2\hat{\vec{k}}\,\hat{\vec{k}}\cdot\hat{\vec{r}} - \frac{3}{2}\hat{\vec{r}}(\hat{\vec{k}}\cdot\hat{\vec{r}})^2\right] + \right. \\ &\left. t^2\left[\frac{5}{2}\hat{\vec{r}}(\hat{\vec{k}}\cdot\hat{\vec{r}})^2 - 2\hat{\vec{k}}\,\hat{\vec{k}}\cdot\hat{\vec{r}} - \frac{1}{2}\hat{\vec{r}}\right]\right\} + \partial(k^3) \end{aligned} \quad (108)$$

Note that the deviation from a static, spherical bag is of order k in CM frame, and of order k^2 in Breit frame. Again, the difference is due to the symmetric treatment of the two bags in Breit frame.

With the bag normal properly defined, we can expand both sides of (89) in powers of t and k up to order t^2, k^2. Equating terms with the same angular structure, we then determine the unknown coefficients of the ϕ, ξ and η fields. These coefficients are listed in Appendix B.

In the expansion of the left-hand side of (89) we encounter problems at the bag surface again. The k^2 terms will include derivatives of the quark wave functions at the bag surface, and from (18) we see that this leads to Dirac δ-functions at the surface. In models B and C, where the pseudoscalar field is continuous, it turns out that we don't need the k^2 part of the field (if we calculate the form factors at $\vec{k} = 0$), and the problem disappears. We have estimated the effects of these terms in HCB-type models (CM frame), and found them negligible. In the numerical computations, terms containing this δ-function have therefore been omitted altogether.

The pseudoscalar contribution to the axial form factors is now given by the Fourier-transform of the axial current at $t = 0$:

$$\langle B'|\, J_\mu^{A,\Phi}(k)\,|B\rangle = \begin{bmatrix}\cos\theta_C \\ \sin\theta_C\end{bmatrix}\langle B'|\int d^3\vec{r}\, e^{-i\vec{k}\cdot\vec{r}}\, A_\mu^\Phi(\vec{r}, t=0)\,|B\rangle \quad (109)$$

From (88) and (95) we have

$$A_0^\Phi(\vec{r}, t=0) = -f_\Phi\,\xi(\vec{r}) \quad (110)$$

$$\vec{A}^\Phi(\vec{r}, t=0) = f_\Phi\vec{\nabla}\phi(\vec{r}) \quad (111)$$

Since f_Φ also enters the boundary condition (89), it will drop out of the final expressions. We therefore set $f_\Phi = 1$ from now on. Then

$$J_0^{A,\Phi} = -\int d^3\vec{r}\, e^{-i\vec{k}\cdot\vec{r}}\,\xi(\vec{r}) \quad (112)$$

Writing $\xi(\vec{r})$ in the same form as (100–102), and comparing with (6) we find

$$\begin{aligned} H_0^\Phi(\vec{k}=0) &= 4\pi g_1^{SU(6)}\left\{\frac{i}{3}(m+m')\int dr\, r^3\alpha_\xi^{(0)}(r) \right. \\ &\left. -\int dr\, r^2\left[\frac{1}{3}\beta_\xi(r) + \gamma_\xi(r)\right]\right\} \end{aligned} \quad (113)$$

where the integration should go from zero to infinity in CBM type models (B,C) and from R to infinity in model D.

For the spatial part we have likewise

$$\vec{J}^{A,\Phi} = \int d^3\vec{r}\, e^{-i\vec{k}\cdot\vec{r}}\vec{\nabla}\phi(\vec{r}) \tag{114}$$

which may be integrated by parts:

$$\vec{J}^{A,\Phi} = \int d\Omega\, r^2\hat{\vec{r}} e^{-i\vec{k}\cdot\vec{r}}\phi(\vec{r})\Big|_{r=0,R}^{r=\infty} + i\vec{k}\int_{r=0,R}^{\infty} d^3\vec{r}\, e^{-i\vec{k}\cdot\vec{r}}\phi(\vec{r}) \tag{115}$$

Since the pseudoscalar field is not massless, $\phi(\vec{r})$ vanishes at infinity. Comparing with (7) we then find

$$\begin{aligned} H_1^{\Phi} &= -\frac{4\pi}{3} g_1^{SU(6)} r^2 \alpha_\phi^{(0)}(r)\Big|_{r=0,R} - \\ &\quad -\frac{4\pi}{3} g_1^{SU(6)} \frac{\vec{k}^2}{(m+m')^2}\left\{\frac{1}{10}(m+m')^2 r^4 \alpha_\phi^{(0)}(r) + \frac{i}{5}(m+m')r^3\beta_\phi(r) - \right. \\ &\quad \left. -r^2\left[\alpha_\phi^{(2)}(r) + \frac{1}{5}\nu_\phi(r)\right]\right\}\Big|_{r=0,R} \end{aligned} \tag{116}$$

and

$$\begin{aligned} H_2^{\Phi}(\vec{k}=0) &= 4\pi g_1^{SU(6)}\left\{\frac{1}{15}(m+m')^2 r^4\alpha_\phi^{(0)}(r) + \right. \\ &\quad \left. \frac{i}{3}(m+m')r^3\left[\frac{2}{5}\beta_\phi(r) + \gamma_\phi(r)\right] - \frac{1}{3}r^2\left[\rho_\phi(r) + \frac{2}{5}\nu_\phi(r)\right]\right\}\Big|_{r=0,R} \\ &\quad +4\pi g_1^{SU(6)}\left\{\frac{1}{3}(m+m')^2\int_{0,R}^{\infty} dr\, r^3\alpha_\phi^{(0)}(r) + \right. \\ &\quad \left. i(m+m')\int_{0,R}^{\infty} dr\, r^2\left[\frac{1}{3}\beta_\phi(r) + \gamma_\phi(r)\right]\right\} \end{aligned} \tag{117}$$

Since the pseudoscalar field must be regular at the origin, only the integrals of (117) survive in CBM models. That is,

$$H_1^{\Phi,CBM} = 0 \tag{118}$$

$$\begin{aligned} H_2^{\Phi,CBM}(\vec{k}=0) &= 4\pi g_1^{SU(6)}\left\{\frac{1}{3}(m+m')^2\int_0^{\infty} dr\, r^3\alpha_\phi^{(0)}(r) + \right. \\ &\quad \left. i(m+m')\int_0^{\infty} dr\, r^2\left[\frac{1}{3}\beta_\phi(r) + \gamma_\phi(r)\right]\right\} \end{aligned} \tag{119}$$

Hence, the k^2 terms in the fields $(\alpha_\phi^{(2)}, \rho_\phi, \nu_\phi)$ are not needed in CBM models (at $\vec{k} = 0$ at least). Moreover, since H_2 is multiplied by δ^2 and H_0 by δ in g_1 (see (10) and (14)), g_1^{Φ} will be very small compared to the quark contribution g_1^Q in CBM-type models. That is why CBM-type models tend to underestimate the axial charge in neutron β-decay.

Table 1: The ratio $f_1/f_1^{SU(6)}$ for various models and processes computed at $\vec{k}=0$.

Process	MIT		CHIRAL		LAPP	
	Static	Recoil	Static	Recoil	Static	Recoil
$n \to p$, Breit	1.00	1.00	1.00	1.00	1.00	1.00
CM	1.00	1.00	1.00	1.00	1.00	1.00
$\Lambda \to p$, Breit	0.993	0.987	0.961	0.964	0.929	0.924
CM	0.907	0.987	0.878	0.963	0.848	0.924
$\Sigma^- \to n$, Breit	1.030	1.020	0.991	0.995	0.967	0.961
CM	0.906	1.018	0.871	0.995	0.851	0.960
$\Xi^- \to \Lambda$, Breit	1.004	0.997	0.970	0.972	0.950	0.946
CM	0.919	0.997	0.888	0.972	0.870	0.945
$\Xi^- \to \Sigma^0$, Breit	0.988	0.985	0.973	0.973	0.934	0.932
CM	0.938	0.985	0.923	0.973	0.886	0.932

We end this chapter by making a comment on the (single) pseudoscalar particle contribution to the covariant form factors g_1, g_2 and g_3. As is well known, dispersion relations for g_1 and g_2 do not contain the pseudoscalar (π or K) pole. From eqs. (10), (11), (14) and (15) it can be seen that H_2, which contains the pole term, is multiplied by powers of δ, which is zero in the flavour symmetric limit where all baryon masses are equal. For finite mass differences there are small pole contributions to g_1 and g_2, poles which are a pathological property of our model calculations. In the only process where they matter numerically ($\Sigma^- \to \Lambda e^- \bar{\nu}$), the unwanted pion pole contributions to g_1 and g_2 can easily be regularized away (see the next section).

5 Numerical Results

We then turn to the numerical computation of our expressions. In these computations we have used the same values for the bag model parameters as in [5] except for the LAPP bag model where we have modified the parameters somewhat in order to give the right value for the axial charge g_A in neutron β-decay [19].

We first turn to the obvious theoretical question whether our labour has contributed to the desired reference frame independence of the form factors which should be Lorentz invariant.

Table 1 shows the ratio $f_1/f_1^{SU(6)}$ calculated in two different reference frames in the cases with and without recoil corrections. As we can see, the unflattering 10% differences we find between the Breit and CM frames

Table 2: The ratio g_1/f_1 for various models and processes, computed at $\vec{k}=0$.

	MIT		CBM		CHIRAL		LAPP	
Process	Static	Recoil	Static	Recoil	Static	Recoil	Static	Recoil
$n \to p$, Breit	1.09	1.09	1.09	1.09	1.26	1.26	1.25	1.25
CM	1.09	1.09	1.09	1.09	1.26	1.26	1.25	1.25
$\Sigma^- \to \Lambda^a$, Breit	0.53	0.53	0.34	0.38	0.39	0.45	0.40	0.41
CM	0.51	0.53	0.34	0.39	0.38	0.44	0.39	0.41
$\Lambda \to p$, Breit	0.72	0.71	0.66	0.70	0.69	0.75	0.63	0.66
CM	0.72	0.72	0.66	0.71	0.70	0.75	0.64	0.66
$\Sigma^- \to n$, Breit	-0.23	-0.23	-0.19	-0.21	-0.19	-0.22	-0.18	-0.19
CM	-0.23	-0.23	-0.19	-0.22	-0.20	-0.23	-0.18	-0.20
$\Xi^- \to \Lambda$, Breit	0.24	0.24	0.21	0.23	0.22	0.24	0.20	0.21
CM	0.24	0.24	0.21	0.23	0.22	0.24	0.20	0.21
$\Xi^- \to \Sigma^0$, Breit	1.20	1.20	1.15	1.20	1.20	1.26	1.11	1.15
CM	1.20	1.20	1.15	1.21	1.20	1.26	1.11	1.14

[a] g_1 quoted in this case.

Table 3: The ratio g_2/g_1 computed at $\vec{k}=0$.

	MIT		CBM		CHIRAL		LAPP	
Process	Static	Recoil	Static	Recoil	Static	Recoil	Static	Recoil
$\Sigma^- \to \Lambda$, Breit	-0.15	-0.15	-15.7	-11.7	-15.6	-9.8	-15.6	-14.3
CM	-1.17	-0.14	-16.4	-10.5	-16.3	-10.0	-16.3	-14.7
$\Lambda \to p$, Breit	0.04	-0.06	-1.05	-0.33	-1.49	-0.48	-1.11	-0.59
CM	-1.0	-0.05	-2.07	-0.18	-2.50	-0.41	-2.13	-0.57
$\Sigma^- \to n$, Breit	-0.05	-0.15	-1.95	-0.87	-2.57	-1.17	-2.09	-1.34
CM	-1.11	-0.13	-2.93	-0.61	-3.49	-0.97	-3.05	-1.14
$\Xi^- \to \Lambda$, Breit	0.00	-0.10	-1.57	-0.58	-2.13	-0.82	-1.66	-0.96
CM	-1.04	-0.10	-2.58	-0.41	-3.12	-0.72	-2.67	-0.90
$\Xi^- \to \Sigma^0$, Breit	0.10	0.00	-0.82	-0.03	-1.11	-0.13	-0.85	-0.27
CM	-0.92	0.00	-1.81	0.10	-2.13	-0.06	-1.87	-0.30

without recoil corrections, have been virtually eliminated by the recoil corrections.

In table 2 we show the ratio g_1/f_1 where results always show little difference between the two frames. The reason is that f_1 and g_1 separately have approximately the same frame dependence in the static case. (If we had calculated only this ratio, we could have been misled into thinking that it never mattered much which frame was used for calculations, even when boost effects are ignored.)

The most spectacular improvement is seen in table 3 where the second class current g_2 is shown. The strong frame dependence of the static results has been significantly reduced when recoil effects have been included, which clearly shows the necessity of recoil effects in the calculation of g_2, as pointed out in [5]. Note also that g_2 has roughly been reduced by a factor of 3 when

Table 4: g_1/f_1 regularized, computed in Breit frame at $\vec{k}=0$.

Process	CBM	CHIRAL	LAPP
$n \to p$	1.09	1.26	1.25
$\Sigma^- \to \Lambda^a$	0.53	0.59	0.61
$\Lambda \to p$	0.71	0.76	0.68
$\Sigma^- \to n$	-0.23	-0.24	-0.21
$\Xi^- \to \Lambda$	0.23	0.25	0.22
$\Xi^- \to \Sigma^0$	1.20	1.25	1.14

[a] g_1.

Table 5: g_2/g_1 regularized, computed in Breit frame at $\vec{k}=0$.

Process	CBM	CHIRAL	LAPP
$\Sigma^- \to \Lambda$	-0.22	-6.6	-0.12
$\Lambda \to p$	-0.06	-0.35	-0.32
$\Sigma^- \to n$	-0.15	-0.51	-0.55
$\Xi^- \to \Lambda$	-0.15	-0.47	-0.47
$\Xi^- \to \Sigma^0$	0.00	-0.22	-0.25

recoil effects are included, which means that the g_2-dependence of g_1 will not be so important now as in the static approximation.

At this point it makes sense to return to the problem of the pion pole which was touched upon at the end of the previous section. In order to remove the spurious terms in g_1 and g_2, we have carried out a regularization procedure for g_1 and g_2. That is, we write g_1 (at $k_\mu k^\mu = 0$) as

$$g_1 = g_1^{reg.} + \frac{c_1}{\mu^2}$$

where $g_1^{reg.}$ remains finite when $\mu \to 0$. Taking this limit, we isolate the residue:

$$c_1 = \lim_{\mu \to 0} \mu^2 g_1$$

so that

$$g_1^{reg.} = g_1 - \frac{c_1}{\mu^2}$$

This procedure is carried out numerically, both for g_1 and g_2. To simplify the procedure, we removed the (numerically unimportant) inhomogeneous terms in the equations for the pseudoscalar fields. No new parameters are introduced in doing the regularization; the Cabibbo angle will serve as the unique parameter relating theory and observations. The effect on the form factors is seen in tables 4 and 5. We give results only in Breit frame as we have already shown that the frame dependence is now negligible. There

Table 6: The ratio f_2/f_1 computed in Breit frame at $\vec{k} = 0$. CVC and experimental results from [6], corrected for different definition of f_2.

Process	MIT		CHIRAL		LAPP		CVC	Exp.
	Static	Recoil	Static	Recoil	Static	Recoil		
$n \to p$	2.17	1.88	2.59	2.81	3.08	2.95		
$\Sigma^- \to \Lambda^a$	1.91	1.76	2.22	2.27	2.40	2.33	2.34	3.53±3.53
$\Lambda \to p$	0.84	0.69	1.19	1.17	1.17	1.14	1.79	2.43±1.49
$\Sigma^- \to n$	-1.61	-1.57	-1.74	-1.73	-1.72	-1.72	-2.03	-1.78±0.61
$\Xi^- \to \Lambda$	-0.28	-0.33	-0.14	-0.15	-0.17	-0.17	-0.13	-0.44±0.46
$\Xi^- \to \Sigma^0$	2.77	2.51	3.52	3.39	3.38	3.33		

[a] f_2 ($f_1^{SU(6)} = 0$ for this process).

Table 7: The ratio $g_1^{reg.}/f_1$ for various models and processes with recoil effects included. The results have been computed at $k_\mu k^\mu = 0$ in Breit frame. Numbers in parentheses are experimental values from [6] corrected for nonzero g_2 using table 5.

Process	MIT		CBM		CHIRAL		LAPP	
$n \to p$	1.09	(1.24)	1.09	(1.24)	1.26	(1.24)	1.25	(1.24)
$\Sigma^- \to \Lambda^a$	.53		.53		.59		.61	
$\Lambda \to p$	.72	$(.70 \pm .03)$	.72	$(.70 \pm .03)$	.77	$(.67 \pm .03)$	.70	$(.68 \pm .03)$
$\Sigma^- \to n$	-.24	$(-.34 \pm .05)$	-.24	$(-.34 \pm .05)$	-.26	$(-.32 \pm .05)$	-.24	$(-.32 \pm .05)$
$\Xi^- \to \Lambda$	.24	$(.25 \pm .05)$	.24	$(.25 \pm .05)$	.26	$(.24 \pm .05)$	.23	$(.24 \pm .05)$
$\Xi^- \to \Sigma^0$	1.20		1.20		1.25		1.16	

[a] g_1

is of course no similar modification in the MIT model, as there is no pion or kaon field contributing to any form factor. As we had hoped for, the regularized g_1 values for $n \to p$ and the $\Delta S = 1$ processes remain virtually unchanged, whereas in $\Sigma^- \to \Lambda$ g_1 is substantially increased. The effect on g_2 is much bigger, and $|g_2|$ is typically reduced by 50%. This shows that the spurious poles in g_2 are, to some extent, responsible for the large values of g_2 found earlier.

We now turn to the confrontation of our model calculations with experiments on semileptonic decays.

Table 6 lists the values of f_2/f_1 computed in Breit frame. The recoil effects are not very significant. The ratio f_2/f_1 is not sensitive to changes in k^2 since f_1 and f_2 have roughly the same k^2-dependence. Note also that the experimental error-bars are so large for f_2 that it is not possible to get clues to any symmetry breaking effects from experiment here.

More interesting is our calculation of the weak axial charge g_1 that we present in table 7. As can be seen, all the models do quite well for the strangeness changing transitions with the exception of the value in $\Sigma^- \to n e^- \bar{\nu}$, and for the chiral model in $\Lambda \to p$.

Table 8: $g_1^{reg.}/f_1$ with corrections from one gluon tree diagrams [4] added.

Process	MIT	CBM	CHIRAL	LAPP
$n \to p$	1.12	1.12	1.30	1.29
$\Sigma^- \to \Lambda$[a]	^ 57	0.57	0.63	0.65
$\Lambda \to p$	0.73	0.73	0.78	0.71
$\Sigma^- \to n$	-0.31	-0.31	-0.33	-0.31
$\Xi^- \to \Lambda$	0.22	0.22	0.24	0.21
$\Xi^- \to \Sigma^0$	1.23	1.23	1.28	1.19

[a] g_1.

The recent high statistics Fermilab experiment [7,8] on $\Sigma^- \to n$ gave results compatible with the CERN experiment, and also determined the sign of g_1 in this decay, in agreement with Cabibbo theory.

In the Fermilab experiment they also tried to detect the second-class current g_2 [8]. Inside experimental errors g_2 could be very large, in fact so large that through its influence on the fitted value of g_1, g_1 would come out compatible with the values we have calculated. The bag models are unable to reproduce such large second class currents however, and although our calculated values of g_2 influence the extracted values of g_1, that change is only on the 10% level. We note however that the sign of g_2 in $\Sigma^- \to n$ that all the models give, is in agreement with the sign found in the analysis of the Fermilab data.

There is one important dynamical effect that we have not included in these calculations, and that is the effect of the colour (magnetic) interaction on quark wave functions. Ushio and Konashi [4] have shown that the same tree diagrams with one-gluon exchange that give the colour-magnetic part of the Delta-nucleon mass difference, also lead to correction terms in the axial charge for some transitions. We stress that the most important parts of these corrections are not radiative corrections due to loop diagrams, and that they are as legitimate and natural to use in magnetic moment and axial charge calculations as in mass calculations. Ushio and Konashi calculated the axial charge corrections in the MIT model (model A) only, and found them rather small in most processes, except for $\Sigma^- \to n$. As the other models are improvements upon model A, we expect that gluon corrections in these models will be of roughly the same size. As a good approximation, we have therefore added Ushio and Konashi's corrections to our values for g_1 in all models. The results are shown in table 8. As can be seen from this table, the axial charges for strangeness changing weak decays are now quite satisfactory in all the models. The small deviations from the experimental value that we have in neutron decay in models C and D can certainly be eliminated by small changes in model parameters, without upsetting the

overall agreement between theory and experiment for other observables.

On the whole we are almost surprised by the fact that we reproduce observations for the axial charges as well as we do, and this also gives us some confidence in our calculation of the second class currents.

Before we leave the subject of axial charges, we should make some comments on the momentum transfer dependence of the invariant form factors. In their analysis of experimental data, the CERN group [6] parametrized the k^2-dependence of f_1 and g_1 as

$$f_1(k^2) = f_1(0)\left(1 + 2\frac{k^2}{m_V^2}\right) + \partial(k^4)$$

and

$$g_1(k^2) = g_1(0)\left(1 + 2\frac{k^2}{m_A^2}\right) + \partial(k^4)$$

where now $k^2 \equiv k_\mu k^\mu$. In $\Delta S = 1$ transitions they used $m_V = 0.98$ GeV and $m_A = 1.25$ GeV as input masses (taken from electroproduction and neutrino experiments data, which were renormalized using the m_ρ/m_{K^*} mass ratio). From our expressions for the k^2-dependence of f_1 we are able to calculate m_V. Depending on frame and model we find for $\Lambda \to p$ and $\Sigma^- \to n$:

$$m_V \sim 0.74 - 1.03\,\mathrm{GeV}$$

Keeping in mind the considerations mentioned in section 3, concerning the k^2-dependence of g_1, we may estimate m_A as well. For the same processes we find that it lies in the range

$$m_A \sim 0.93 - 1.30\,\mathrm{GeV}$$

depending on process and model. Thus our estimates for the k^2-dependence seem to be in fairly good agreement with the values assumed in the experimental analysis, in spite of the ambiguities associated with the theoretical calculation. In addition, the experimental analysis is not very sensitive to the precise values of m_V and m_A: A 0.15 GeV shift in m_V or m_A would cause only a 2% shift in g_1/f_1 [6].

We now turn to the last and most delicate part of our discussion concerning the comparison between theory and observation. This is the prediction of the absolute values of the form factors as they are measured in the decay rates. We recall that Cabibbo theory has three parameters: θ_C, F and D, whereas quark theory has only the Cabibbo angle θ_C. Table 9 shows the computed branching ratios, compared with experiment. For each model we chose the value of θ_C that minimized the χ^2 in a fit to all the measured branching ratios (of [6]) except $n \to p$. In the computation of the branching ratios we have included the same radiative corrections of Tóth et al. [20] that were used in the analysis of the CERN data.

Table 9: The branching ratio (×10³), using regularized values of g_1 from table 8. Experimental values from [6].

Process	MIT	CBM	CHIRAL	LAPP	Exp.
$\Sigma^- \to \Lambda$	0.051	0.051	0.062	0.065	0.0561±0.0031
$\Lambda \to p$	0.928	0.926	0.973	0.922	0.857±0.036
$\Sigma^- \to n$	0.881	0.880	0.808	0.890	0.96±0.05
$\Xi^- \to \Lambda$	0.518	0.517	0.495	0.531	0.564±0.031
$\Xi^- \to \Sigma^0$	0.093	0.093	0.100	0.093	0.087±0.017
$V_{us} \sim \sin\theta_C$	0.251	0.251	0.257	0.273	

It is obvious that if we could choose the Cabibbo angle (or equivalently the element V_{us} from the Kobayashi-Maskawa matrix) freely, we would have reason to be completely satisfied by our model calculations. However, it is evident that such big values of V_{us} will bring us in conflict with what is known about neutron decay. By comparing muon decay and superallowed Fermi transitions in nuclei, the most recent analysis gives $V_{ud} \approx 0.975 \pm 0.001$ [21], which is about 1% bigger than the upper limit given by the values of V_{us} in table 9. We note however that this way of determining V_{ud} assumes that there is no flavour mixing in the lepton sector, or equal neutrino masses.

6 Conclusions

The motivations for writing this article were twofold. On the one hand it was purely theoretical — to convince ourselves that we could carry out the first calculational steps necessary to reestablish Lorentz invariance in bag models of hadrons. On the other hand we wanted to see how well different bag models could account for experimental observations of semileptonic weak decays of baryons. Concerning the first problem, we think we have shown in a convincing manner that for momentum transfers of the magnitude one encounters in well measured β-decays, the problem of Lorentz invariance of bag calculations has been eliminated in practice. In particular we feel that for the first time bag models have furnished values of second-class currents which are bag model dependent only, being Lorentz-frame dependent only to a slight degree. Although these values of g_2 are shown to affect the experimental determination of the axial charge form factors in some models, they are not so large as to change radically the values of g_1 that one obtains from experimental data by assuming that there are no second-class currents.

On the whole, we think that agreements with experimental data are quite satisfactory. To obtain overall agreement between data and theory

for the ratio g_1/f_1 it was essential to include Ushio and Konashi's [4] colour-magnetic correction to Σ^- decay.

We are left with the problem of the big value of the Cabibbo angle (or the V_{us} element of the Kobayashi-Maskawa matrix) necessary to obtain the correct $\Delta S = 1$ decay rates. The easiest thing we could say was that this is a reflection of the approximate nature of the models we have worked with.

A more exciting interpretation of our results however, would be to assume that there is a mixing in the lepton sector of the electroweak interaction. We do not know of anything that excludes a rather heavy τ neutrino mass, and sizable mixing between the μ and τ neutrinos. If nature behaves like that, the cosine of the Cabibbo angle extracted from comparing neutron decay and muon decay would be overestimated. In that case the bag models could work better than we expected them to do.

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Appendix A

In this appendix we give explicit solutions to the Klein-Gordon equation (87). Let us start with the η-field which satisfies the homogeneous equation (98). The η-field must have the same angular structure as the ϕ-field. Replacing ϕ with η in eqs. (100)–(102), and inserting into (98), we obtain six ordinary differential equations. Solving these under the constraint that the pseudoscalar field must vanish at infinity, we find for the η-field outside the bag:

$$\alpha_\eta^{(0)}(r) = \frac{a^{(0)}}{r^2}[1+\mu r]e^{-\mu r} \qquad (r > R) \qquad \text{(A1)}$$

$$\beta_\eta(r) = \frac{a^{(1)}}{r^3}\left[1+\mu r+\frac{\mu^2 r^2}{3}\right]e^{-\mu r} \qquad \text{(A2)}$$

$$\gamma_\eta(r) = \frac{1}{r^3}\left[-\frac{a^{(1)}}{3}(1+\mu r)+b^{(1)}\mu^2 r^2\right]e^{-\mu r} \qquad \text{(A3)}$$

$$\nu_\eta(r) = \frac{a^{(2)}}{r^4}\left[1+\mu r+\frac{2}{5}\mu^2 r^2+\frac{1}{15}\mu^3 r^3\right]e^{-\mu r} \qquad \text{(A4)}$$

$$\rho_\eta(r) = \frac{1}{r^4}\left[-\frac{2}{5}a^{(2)}\left(1+\mu r-\frac{1}{3}\mu^3 r^3\right)+b^{(2)}\mu^2 r^2(1+\mu r)\right]e^{-\mu r} \qquad \text{(A5)}$$

$$\alpha_\eta^{(2)}(r) = \frac{1}{r^4}\left[-\frac{1}{5}a^{(2)}\left(1+\mu r-\frac{1}{3}\mu^3 r^3\right)+c^{(2)}\mu^2 r^2(1+\mu r)\right]e^{-\mu r} \qquad \text{(A6)}$$

Solving the same D.E.'s under the constraint that the field should be regular at the origin, we find for the field inside the bag:

$$\alpha_\eta^{(0)}(r) = A^{(0)}\left[\frac{\cosh\mu r}{\mu r} - \frac{\sinh\mu r}{\mu^2 r^2}\right] \qquad (r < R) \qquad \text{(A7)}$$

$$\beta_\eta(r) = A^{(1)}\left[3\frac{\cosh\mu r}{\mu^2 r^2} - \left(1+\frac{3}{\mu^2 r^2}\right)\frac{\sinh\mu r}{\mu r}\right] \qquad \text{(A8)}$$

$$\gamma_\eta(r) = B^{(1)}\frac{\sinh\mu r}{\mu r} - A^{(1)}\frac{1}{\mu r}\left[\frac{\cosh\mu r}{\mu r} - \frac{\sinh\mu r}{\mu^2 r^2}\right] \qquad \text{(A9)}$$

We have omitted the second order part of the η-field inside the bag since, as mentioned in Section 4, we only need the first order part of the field in CBM-type models.

The ξ-field will have exactly the same solution as the η-field since it has the same angular structure and satisfies the same homogeneous differential equation (97) — the only differer e being the constants of integration.

The ϕ-field is somewhat mo : complicated since it satisfies the inhomogeneous equation (96). In addition to the general solution to the corresponding homogeneous equation given above, it will also contain a part from the η-field. The solution outside the bag reads:

$$\alpha_\phi^{(0)}(r) = \left[-\frac{a^{(0)}}{2} + \frac{g^{(0)}}{r^2}(1+\mu r)\right]e^{-\mu r} \qquad (r > R) \text{(A10)}$$

$$\beta_\phi(r) = \left[-\frac{a^{(1)}}{6}\frac{1+\mu r}{r} + \frac{g^{(1)}}{r^3}\left(1+\mu r+\frac{\mu^2 r^2}{3}\right)\right]e^{-\mu r} \qquad \text{(A11)}$$

$$\gamma_\phi(r) = \left[-\frac{b^{(1)}}{2}\mu + \frac{1}{r^3}\left(-\frac{g^{(1)}}{3}(1+\mu r) + h^{(1)}\mu^2 r^2\right)\right]e^{-\mu r} \qquad \text{(A12)}$$

$$\begin{aligned}\nu_\phi(r) = &\left[-\frac{a^{(2)}}{10}\frac{1}{r^2}\left(1+\mu r+\frac{1}{3}\mu^2 r^2\right) + \right.\\ &\left. +\frac{g^{(2)}}{r^4}\left(1+\mu r+\frac{2}{5}\mu^2 r^2 + \frac{1}{15}\mu^3 r^3\right)\right]e^{-\mu r} \qquad \text{(A13)}\end{aligned}$$

$$\begin{aligned}\rho_\phi(r) = &\ e^{-\mu r}\left[-\left(\frac{2}{15}a^{(2)} + b^{(2)}\right)\frac{\mu^2}{2} + \right.\\ &\left.\frac{1}{r^4}\left(-\frac{2}{5}g^{(2)}\left(1+\mu r-\frac{1}{3}\mu^3 r^3\right) + h^{(2)}\mu^2 r^2(1+\mu r)\right)\right] \text{(A14)}\end{aligned}$$

$$\begin{aligned}\alpha_\phi^{(2)}(r) = &\ e^{-\mu r}\left[-\left(\frac{a^{(2)}}{15} + c^{(2)}\right)\frac{\mu^2}{2} + \right.\\ &\left.\frac{1}{r^4}\left(-\frac{1}{5}g^{(2)}\left(1+\mu r-\frac{1}{3}\mu^3 r^3\right) + j^{(2)}\mu^2 r^2(1+\mu r)\right)\right] \text{(A15)}\end{aligned}$$

At last we find for the ϕ-field inside the bag:

$$\alpha_\phi^{(0)}(r) = \frac{A^{(0)}}{2\mu^2}\sinh\mu r + G^{(0)}\left[\frac{\cosh\mu r}{\mu r} - \frac{\sinh\mu r}{\mu^2 r^2}\right] \qquad (r < R) \text{ (A16)}$$

$$\beta_\phi(r) = \frac{A^{(1)}}{2\mu^2}\left[\frac{\sinh\mu r}{\mu r} - \cosh\mu r\right] + G^{(1)}\left[3\frac{\cosh\mu r}{\mu^2 r^2} - \left(1+\frac{3}{\mu^2 r^2}\right)\frac{\sinh\mu r}{\mu r}\right] \tag{A17}$$

$$\gamma_\phi(r) = \frac{B^{(1)}}{2\mu^2}\cosh\mu r + H^{(1)}\frac{\sinh\mu r}{\mu r} - \frac{G^{(1)}}{\mu r}\left[\frac{\cosh\mu r}{\mu r} - \frac{\sinh\mu r}{\mu^2 r^2}\right] \tag{A18}$$

In HCB-type models where the pseudoscalar field is excluded from the bag interior, all the upper case constants are zero. In CBM-type models (models B and C in our case), the upper and lower case constants are related by requiring that the pseudoscalar field itself is continuous through the bag surface. The remaining unknown constants are then uniquely determined in terms of quark wave functions at the bag surface using the boundary condition (89). The constants are listed in Appendix B.

Appendix B

Here we shall give complete (Fortran) expressions for the various constants of the pseudoscalar field, expressed in terms of quark wave functions at the bag surface. The following naming conventions are used: Combinations of quark wave functions at the surface are denoted by `ff`, `fg`, `gf` and `gg`, where

$$\texttt{ff} \equiv F^*_{q'}F_q,\ \texttt{fg} \equiv F^*_{q'}G_q$$

etc. Similarly, derivatives of the wave functions are written as

$$\texttt{dff} \equiv \frac{dF^*_{q'}}{dr}F_q,\ \texttt{gdf} \equiv G^*_{q'}\frac{dF_q}{dr},\ \texttt{d2ff} \equiv \frac{d^2F^*_{q'}}{dr^2}F_q,\ \texttt{gd2g} \equiv G^*_{q'}\frac{d^2G_q}{dr^2}$$

and so on. `eu` $= E_{q'}$, `es` $= E_q$, `r` $= R = \min(R_B, R_{B'})$ and `m` $= \mu$. The lower case constants of Appendix A, $a^{(0)}$, $g^{(0)}$, $a^{(1)}, \ldots$, are written as `a0`, `g0`, `a1` etc., while the upper case constants, $A^{(0)}$, $G^{(0)}, \ldots$, are written as `a0c`, `g0c` etc. The ξ-field constants begin with letters `d` and `e`, so that `d0` corresponds to `a0` in the η-field, `d0c` to `a0c`, `e1` to `b1`, `e1c` to `b1c` etc. (recall that the ξ- and η-fields have exactly the same form).

Breit Frame

First we write down the necessary combinations of quark wave functions at the bag surface. These expressions are valid in both CBM and HCB type models. `(0.0,1.0)` denotes the imaginary number i.

```
bOO  = (eu-es)*(fg-gf)
avOa = ff-gg
avOb = 2*gg
av1a = (0.0,1.0)*(eu+es)*(ff-gg)*r-(0.0,1.0)*(gf+fg)
av1b = 2*(0.0,1.0)*(eu+es)*gg*r
av1c = 0
av1d = (0.0,1.0)*(gf+fg)
av2a = -(eu+es)**2*(ff-gg)*r**2/2.0+(eu+es)*(gf+fg)*r+(-gdg+fdf-dg
1   g+dff)*r/2.0
av2b = (ff-gg)/2.0
av2c = -(eu+es)**2*gg*r**2+(gdg+dgg)*r-2*gg
av2d = gg
av2e = 0
av2f = gg-(eu+es)*(gf+fg)*r
av2g = -(ff-gg)/2.0
bvOa = (0.0,1.0)*(eu-es)*(ff-gg)
bvOb = 2*(0.0,1.0)*(eu-es)*gg
bv1a = (0.0,1.0)*(eu-es)*((0.0,1.0)*(eu+es)*(ff-gg)*r-(0.0,1.0)*(g
1   f+fg))+gdg-fdf-dgg+dff
bv1b = 2*(dgg-gdg)-2*(eu-es)*(eu+es)*gg*r
bv1c = 0
bv1d = -(eu-es)*(gf+fg)
evOa = -(eu-es)**2*(ff-gg)
evOb = -2*(eu-es)**2*gg
ev1a = 2*(0.0,1.0)*(eu-es)*(gdg-fdf-dgg+dff)-(eu-es)**2*((0.0,1.0)
1   *(eu+es)*(ff-gg)*r-(0.0,1.0)*(gf+fg))
ev1b = 4*(0.0,1.0)*(eu-es)*(dgg-gdg)-2*(0.0,1.0)*(eu-es)**2*(eu+es
1   )*gg*r
ev1c = 0
ev1d = -(0.0,1.0)*(eu-es)**2*(gf+fg)
ev2a = -(eu-es)**2*(-(eu+es)**2*(ff-gg)*r**2/2.0+(eu+es)*(gf+fg)*r
1   +(-gdg+fdf-dgg+dff)*r/2.0)-2*(eu-es)*(eu+es)*(gdg-fdf-dgg+dff)*
2   r+2*(eu-es)*((fg-gf)/r-gdf-fdg+dgf+dfg)+(gdg+dgg)/r-(fdf+dff)/r
3   -4*gg/r**2-gd2g+fd2f+2*dgdg-2*dfdf-d2gg+d2ff
ev2b = -2*(eu-es)*(fg-gf)/r+(-gdg+fdf-dgg+dff)/r+4*gg/r**2-(eu-es)
1   **2*(-gg+(ff-gg)/2.0+ff)
ev2c = -(eu-es)**2*(-(eu+es)**2*gg*r**2+(gdg+dgg)*r-2*gg)-4*(eu-es
1   )*(eu+es)*(dgg-gdg)*r+2*(-(gdg+dgg)/r+gd2g-2*dgdg+d2gg)+8*gg/r*
2   *2
ev2d = 2*(gdg+dgg)/r-4*gg/r**2-3*(eu-es)**2*gg
ev2e = 0
ev2f = 2*(eu-es)*(-(fg-gf)/r+gdf+fdg-dgf-dfg)-(eu-es)**2*(gg-(eu+e
1   s)*(gf+fg)*r)
ev2g = 2*(eu-es)*(fg-gf)/r-4*gg/r**2+(eu-es)**2*(ff-gg)/2.0
```

Below we distinguish between HCB (D) and CBM (B,C) type models. In the following expressions (both here and in CM frame) ua=ub=uc=1.

HCB Models

```
      qa1 = ev1c+ev1b+ev1a
      qdd1 = bv1c+bv1b+bv1a
      qg1 = av1c+av1b+av1a
      qa2 = 2*av0a*uc/r**2-ev0a*ub-2*b00*ua/r+ev2e+ev2c+ev2a
      qb2 = -2*av0a*uc/r**2+ev0a*ub+ev2g+ev2f
      qcc2 = 2*b00*ua/r+ev2d+ev2b
      qh2 = av0a*ub+av2g+av2f

      a0 = -(ev0b+ev0a)*r**3*exp(m*r)/(m**2*r**2+2*m*r+2)
      d0 = -(bv0b+bv0a)*r**3*exp(m*r)/(m**2*r**2+2*m*r+2)
      g0 = -((2*av0b+2*av0a)*r**3*exp(m*r)-a0*m*r**3)/(2*m**2*r**2+4*m*r
     1   +4)
      a1 = -3*qa1*r**4*exp(m*r)/(m**3*r**3+4*m**2*r**2+9*m*r+9)
      b1 = -(3*ev1d*r**4*exp(m*r)-a1*m**2*r**2-3*a1*m*r-3*a1)/(3*m**3*r*
     1   *3+3*m**2*r**2)
      dd1 = -3*qdd1*r**4*exp(m*r)/(m**3*r**3+4*m**2*r**2+9*m*r+9)
      e1 = -(3*bv1d*r**4*exp(m*r)-dd1*m**2*r**2-3*dd1*m*r-3*dd1)/(3*m**3
     1   *r**3+3*m**2*r**2)
      g1 = -(6*qg1*r**4*exp(m*r)-a1*m**2*r**4-a1*m*r**3-a1*r**2)/(2*m**3
     1   *r**3+8*m**2*r**2+18*m*r+18)
      h1 = -(6*av1d*r**4*exp(m*r)-3*b1*m**2*r**4-2*g1*m**2*r**2-6*g1*m*r
     1   -6*g1)/(6*m**3*r**3+6*m**2*r**2)
      a2 = -((15*a0*r**2-30*g0*m*r-30*g0)*uc+(15*a0*m*r**3+15*a0*r**2)*u
     1   b+(-30*a0*m*r**3-30*a0*r**2)*ua+15*qa2*r**5*exp(m*r))/(m**4*r**
     2   4+7*m**3*r**3+27*m**2*r**2+60*m*r+60)
      b2 = ((15*a0*r**2-30*g0*m*r-30*g0)*uc+(15*a0*m*r**3+15*a0*r**2)*ub
     1   -15*qb2*r**5*exp(m*r)-2*a2*m**4*r**4-2*a2*m**3*r**3+6*a2*m**2*r
     2   **2+24*a2*m*r+24*a2)/(15*m**4*r**4+30*m**3*r**3+30*m**2*r**2)
      cc2 = -((30*a0*m*r**3+30*a0*r**2)*ua+15*qcc2*r**5*exp(m*r)+a2*m**4
     1   *r**4+a2*m**3*r**3-3*a2*m**2*r**2-12*a2*m*r-12*a2)/(15*m**4*r**
     2   4+30*m**3*r**3+30*m**2*r**2)
      g2 = ((30*av0a*r**5*exp(m*r)+15*a0*r**4-30*g0*m*r**3-30*g0*r**2)*u
     1   b+(-30*av2e-30*av2c-30*av2a)*r**5*exp(m*r)+a2*m**3*r**5+3*a2*m*
     2   *2*r**4+6*a2*m*r**3+6*a2*r**2)/(2*m**4*r**4+14*m**3*r**3+54*m**
     3   2*r**2+120*m*r+120)
      h2 = -((15*a0*r**4-30*g0*m*r**3-30*g0*r**2)*ub+30*qh2*r**5*exp(m*r
     1   )+(-15*b2-2*a2)*m**3*r**5+4*g2*m**4*r**4+4*g2*m**3*r**3-12*g2*m
     2   **2*r**2-48*g2*m*r-48*g2)/(30*m**4*r**4+60*m**3*r**3+60*m**2*r*
     3   *2)
      j2 = -((30*av2d+30*av2b)*r**5*exp(m*r)+(-15*cc2-a2)*m**3*r**5+2*g2
     1   *m**4*r**4+2*g2*m**3*r**3-6*g2*m**2*r**2-24*g2*m*r-24*g2)/(30*m
     2   **4*r**4+60*m**3*r**3+60*m**2*r**2)
```

CBM Models

```
      qa1 = ev1c+ev1b+ev1a
```

```
      qdd1 = bv1c+bv1b+bv1a
      qg1 = av1c+av1b+av1a

      a0 = ((ev0b+ev0a)*exp(m*r)*sinh(m*r)+(-ev0b-ev0a)*m*r*exp(m*r)*cos
     1   h(m*r))/(m**3*sinh(m*r)+m**3*cosh(m*r))
      a0c = -a0*m**2*(m*r+1)*exp(-m*r)/(sinh(m*r)-m*r*cosh(m*r))
      d0 = ((bv0b+bv0a)*exp(m*r)*sinh(m*r)+(-bv0b-bv0a)*m*r*exp(m*r)*cos
     1   h(m*r))/(m**3*sinh(m*r)+m**3*cosh(m*r))
      d0c = -d0*m**2*(m*r+1)*exp(-m*r)/(sinh(m*r)-m*r*cosh(m*r))
      g0 = ((a0c*m**2*r**2+2*a0c)*exp(m*r)*sinh(m*r)**2+(-a0c*m*r*exp(m*
     1   r)*cosh(m*r)+(2*av0b+2*av0a)*m**2*r*exp(m*r)+a0*m**4*r**2-a0*m*
     2   *3*r+2*a0*m**2)*sinh(m*r)-a0c*m**2*r**2*exp(m*r)*cosh(m*r)**2+(
     3   (-2*av0b-2*av0a)*m**3*r**2*exp(m*r)+a0*m**4*r**2-2*a0*m**3*r)*c
     4   osh(m*r))/(2*m**5*r*sinh(m*r)+2*m**5*r*cosh(m*r))
      g0c = (a0c*r**2*exp(m*r)*sinh(m*r)+a0*m**2*r**2-2*g0*m**3*r-2*g0*m
     1   **2)/(2*exp(m*r)*sinh(m*r)-2*m*r*exp(m*r)*cosh(m*r))
      a1 = -((3*m**2*qa1*r**2+9*qa1)*exp(m*r)*sinh(m*r)-9*m*qa1*r*exp(m*
     1   r)*cosh(m*r))/(m**5*r*sinh(m*r)+m**5*r*cosh(m*r))
      a1c = -a1*m**3*(m**2*r**2+3*m*r+3)*exp(-m*r)/(m**2*r**2*sinh(m*r)+
     1   3*sinh(m*r)-3*m*r*cosh(m*r))/3.0
      b1 = ((3*a1c*m**2*r**2+6*a1c)*exp(m*r)*sinh(m*r)**2+(-3*a1c*m*r*ex
     1   p(m*r)*cosh(m*r)-3*ev1d*m**3*r**4*exp(m*r)+a1*m**5*r**2+2*a1*m*
     2   *4*r+2*a1*m**3)*sinh(m*r)-3*a1c*m**2*r**2*exp(m*r)*cosh(m*r)**2
     3   +(a1*m**5*r**2+a1*m**4*r)*cosh(m*r))/(3*m**6*r**3*sinh(m*r)+3*m
     4   **6*r**3*cosh(m*r))
      b1c = -exp(-m*r)*(3*a1c*exp(m*r)*sinh(m*r)-3*a1c*m*r*exp(m*r)*cosh
     1   (m*r)-3*b1*m**5*r**2+a1*m**4*r+a1*m**3)/(m**2*r**2*sinh(m*r))/3
     2   .0

      dd1 = -((3*m**2*qdd1*r**2+9*qdd1)*exp(m*r)*sinh(m*r)-9*m*qdd1*r*ex
     1   p(m*r)*cosh(m*r))/(m**5*r*sinh(m*r)+m**5*r*cosh(m*r))
      d1c = -dd1*m**3*(m**2*r**2+3*m*r+3)*exp(-m*r)/(m**2*r**2*sinh(m*r)
     1   +3*sinh(m*r)-3*m*r*cosh(m*r))/3.0
      e1 = ((3*d1c*m**2*r**2+6*d1c)*exp(m*r)*sinh(m*r)**2+(-3*d1c*m*r*ex
     1   p(m*r)*cosh(m*r)-3*bv1d*m**3*r**4*exp(m*r)+dd1*m**5*r**2+2*dd1*
     2   m**4*r+2*dd1*m**3)*sinh(m*r)-3*d1c*m**2*r**2*exp(m*r)*cosh(m*r)
     3   **2+(dd1*m**5*r**2+dd1*m**4*r)*cosh(m*r))/(3*m**6*r**3*sinh(m*r
     4   )+3*m**6*r**3*cosh(m*r))
      e1c = -exp(-m*r)*(3*d1c*exp(m*r)*sinh(m*r)-3*d1c*m*r*exp(m*r)*cosh
     1   (m*r)-3*e1*m**5*r**2+dd1*m**4*r+dd1*m**3)/(m**2*r**2*sinh(m*r))
     2   /3.0
      g1 = ((3*a1c*m**4*r**4-18*a1c)*exp(m*r)*sinh(m*r)**2+((3*a1c*m**3*
     1   r**3+36*a1c*m*r)*exp(m*r)*cosh(m*r)+(-6*m**5*qg1*r**4-18*m**3*q
     2   g1*r**2)*exp(m*r)+a1*m**7*r**4-3*a1*m**6*r**3-6*a1*m**4*r-6*a1*
     3   m**3)*sinh(m*r)+(-3*a1c*m**4*r**4-18*a1c*m**2*r**2)*exp(m*r)*co
     4   sh(m*r)**2+(18*m**4*qg1*r**3*exp(m*r)+a1*m**7*r**4-2*a1*m**6*r*
     5   *3+6*a1*m**5*r**2+6*a1*m**4*r)*cosh(m*r))/(2*m**8*r**3*sinh(m*r
     6   )+2*m**8*r**3*cosh(m*r))
```

```
 g1c = (3*a1c*r**2*exp(m*r)*sinh(m*r)-3*a1c*m*r**3*exp(m*r)*cosh(m*
1    r)+a1*m**4*r**3+(a1*m**3-2*g1*m**5)*r**2-6*g1*m**4*r-6*g1*m**3)
2    /((6*m**2*r**2+18)*exp(m*r)*sinh(m*r)-18*m*r*exp(m*r)*cosh(m*r)
3    )
 h1 = -((3*b1c*m**2*r**4-6*g1c*m**2*r**2-12*g1c)*exp(m*r)*sinh(m*r)
1    **2+((3*b1c*m*r**3+6*g1c*m*r)*exp(m*r)*cosh(m*r)+6*av1d*m**3*r*
2    *4*exp(m*r)-3*b1*m**5*r**4+3*b1*m**4*r**3-2*g1*m**5*r**2-4*g1*m
3    **4*r-4*g1*m**3)*sinh(m*r)+(6*g1c*m**2*r**2-3*b1c*m**2*r**4)*ex
4    p(m*r)*cosh(m*r)**2+(-3*b1*m**5*r**4-2*g1*m**5*r**2-2*g1*m**4*r
5    )*cosh(m*r))/(6*m**6*r**3*sinh(m*r)+6*m**6*r**3*cosh(m*r))
 h1c = -exp(-m*r)*(6*g1c*exp(m*r)*sinh(m*r)+(3*b1c*m*r**3-6*g1c*m*r
1    )*exp(m*r)*cosh(m*r)+3*b1*m**4*r**3-6*h1*m**5*r**2+2*g1*m**4*r+
2    2*g1*m**3)/(m**2*r**2*sinh(m*r))/6.0
```

CM Frame

Below, delta denotes the Dirac δ-function at the surface of the bag. As mentioned in section 4, we have set delta (and steprir) equal to zero in all the numerical computations.

```
 a00 = -(0.0,1.0)*(fg-gf)
 a01a = gg-ff
 a01b = 2*eu*(fg-gf)*r+2*gg
 b00 = (eu-es)*(fg-gf)
 e00 = (0.0,1.0)*(eu-es)**2*(fg-gf)
 e01a = -4*(eu-es)*gf/r-(eu-es)**2*(gg-ff)
 e01b = 4*(eu-es)*(gf/r-dgf+dfg)-(eu-es)**2*(2*eu*(fg-gf)*r+2*gg)
 av0a = ff-gg
 av0b = 2*gg
 av1a = (0.0,1.0)*(2*eu*(ff-gg)*r-gf-fg)
 av1b = 4*(0.0,1.0)*eu*gg*r
 av1c = (0.0,1.0)*(fg-gf)
 av1d = (0.0,1.0)*(gf+fg)
 av2a = -delta*(ff-gg)*r*steprir-eu**2*(ff-gg)*r**2+eu*(gf+fg)*r+(d
1    ff-dgg)*r
 av2b = (ff-gg)/4.0
 av2c = -2*(delta*gg*r*steprir+eu**2*gg*r**2-dgg*r+gg)
 av2d = gg/2.0
 av2e = gg-eu*(fg-gf)*r
 av2f = gg-eu*(gf+fg)*r
 bv0a = (0.0,1.0)*(eu-es)*(ff-gg)
 bv0b = 2*(0.0,1.0)*(eu-es)*gg
 bv1a = 2*(dff-dgg)-(eu-es)*(2*eu*(ff-gg)*r-gf-fg)
 bv1b = 4*(dgg-gg/r)-4*eu*(eu-es)*gg*r
 bv1c = 2*gg/r-(eu-es)*(fg-gf)
 bv1d = 2*gg/r-(eu-es)*(gf+fg)

 ev0a = -(eu-es)**2*(ff-gg)
```

```
 ev0b = -2*(eu-es)**2*gg
 ev1a = 4*(0.0,1.0)*(dff-dgg)*(eu-es)-(0.0,1.0)*(eu-es)**2*(2*eu*(f
1    f-gg)*r-gf-fg)
 ev1b = 8*(0.0,1.0)*(eu-es)*(dgg-gg/r)-4*(0.0,1.0)*eu*(eu-es)**2*gg
1    *r
 ev1c = 4*(0.0,1.0)*(eu-es)*gg/r-(0.0,1.0)*(eu-es)**2*(fg-gf)
 ev1d = 4*(0.0,1.0)*(eu-es)*gg/r-(0.0,1.0)*(eu-es)**2*(gf+fg)
 ev2a = -(eu-es)**2*(-delta*(ff-gg)*r*steprir-eu**2*(ff-gg)*r**2+eu
1    *(gf+fg)*r+(dff-dgg)*r)-4*(dff-dgg)*eu*(eu-es)*r+2*(eu-es)*(-gf
2    /r+dgf+dfg)+2*(d2ff-dff/r)-2*(d2gg-gg/r**2)
 ev2b = 2*(eu-es)*gf/r+2*dff/r-2*gg/r**2-(eu-es)**2*(ff-gg)/4.0+2*e
1    s*eu*(ff-gg)
 ev2c = 2*(eu-es)**2*(delta*gg*r*steprir+eu**2*gg*r**2-dgg*r+gg)-8*
1    eu*(eu-es)*(dgg-gg/r)*r+4*(-2*dgg/r+3*gg/r**2+d2gg)
 ev2d = 4*gg/r**2-(eu-es)**2*gg/2.0+4*es*eu*gg
 ev2e = -(eu-es)**2*(gg-eu*(fg-gf)*r)-2*(eu-es)*(gf/r-dgf+dfg)+4*(d
1    gg-gg/r)/r-4*eu*(eu-es)*gg
 ev2f = -(eu-es)**2*(gg-eu*(gf+fg)*r)-2*(eu-es)*(-gf/r+dgf+dfg)+4*(
1    dgg-gg/r)/r-4*eu*(eu-es)*gg
 ev2g = -4*(eu-es)*gf/r
```

Below, `m1` $= m$, `m2` $= m'$ and `sm` $= m + m'$.

HCB Models

```
 qa1 = (e00*ua+ev1c+ev1b+ev1a)/2.0-bv0a*ub/r
 qdd1 = -av0a*ub/r+b00*ua+bv1c+bv1b+bv1a
 qg1 = a00*ua+av1c+av1b+av1a
 qa2 = (av0b+5*av0a)*uc/r**2/2.0+ev0b*uc/4.0+(-3.0)*ev0a*uc/4.0-(bv
1    1c+bv1a)*ub/r-2*b00*ua/r+e01b*ua/4.0+ev2e+ev2c+ev2a
 qb2 = -2*av0a*uc/r**2+ev0a*uc+bv1a*ub/r+e01a*ua/4.0+ev2g+ev2f
 qcc2 = -(av0b+av0a)*uc/r**2/2.0+bv1c*ub/r+2*b00*ua/r+ev2d+ev2b
 qh2 = 2*av0a*uc+a01a*ua+2*av2f

 a0 = -(ev0b+ev0a)*r**3*exp(m*r)/(m**2*r**2+2*m*r+2)
 d0 = -(bv0b+bv0a)*r**3*exp(m*r)/(m**2*r**2+2*m*r+2)
 g0 = -((2*av0b+2*av0a)*r**3*exp(m*r)-a0*m*r**3)/(2*m**2*r**2+4*m*r
1    +4)
 a1 = -((3*d0*m*r+3*d0)*sm*ub+3*qa1*r**4*exp(m*r)*sm)/(m**3*m2*r**3
1    +4*m**2*m2*r**2+9*m*m2*r+9*m2)
 b1 = -((6*bv0a*r**3*exp(m*r)-6*d0*m*r-6*d0)*sm*ub+3*ev1d*r**4*exp(
1    m*r)*sm-2*a1*m**2*m2*r**2-6*a1*m*m2*r-6*a1*m2)/(6*m**3*m2*r**3+
2    6*m**2*m2*r**2)
 dd1 = ((3*a0*r**2-6*g0*m*r-6*g0)*sm*ub+(-6*a0*m*r**3-6*a0*r**2)*sm
1    *ua-6*qdd1*r**4*exp(m*r)*sm)/(4*m**3*m2*r**3+16*m**2*m2*r**2+36
2    *m*m2*r+36*m2)
 e1 = -((6*av0a*r**3*exp(m*r)+3*a0*r**2-6*g0*m*r-6*g0)*sm*ub+6*bv1d
```

```
     1    *r**4*exp(m*r)*sm-4*dd1*m**2*m2*r**2-12*dd1*m*m2*r-12*dd1*m2)/(
     2    12*m**3*m2*r**3+12*m**2*m2*r**2)
      g1 = -((3*d0*m*r**3+3*d0*r**2)*sm*ua+3*qg1*r**4*exp(m*r)*sm-a1*m**
     1    2*m2*r**4-a1*m*m2*r**3-a1*m2*r**2)/(2*m**3*m2*r**3+8*m**2*m2*r*
     2    *2+18*m*m2*r+18*m2)
      h1 = -(3*av1d*r**4*exp(m*r)*sm-3*b1*m**2*m2*r**4-2*g1*m**2*m2*r**2
     1    -6*g1*m*m2*r-6*g1*m2)/(6*m**3*m2*r**3+6*m**2*m2*r**2)

      a2 = -((15*a0*m**2*r**4+75*a0*m*r**3+(30*g0*m**2+150*a0)*r**2-60*g
     1    0*m*r-60*g0)*sm**2*uc+(80*dd1*m**2*m2*r**2+240*dd1*m*m2*r+240*d
     2    d1*m2)*sm*ub+(-120*a0*m*r**3-120*a0*r**2)*sm**2*ua+60*qa2*r**5*
     3    exp(m*r)*sm**2)/(8*m**4*m2**2*r**4+56*m**3*m2**2*r**3+216*m**2*
     4    m2**2*r**2+480*m*m2**2*r+480*m2**2)
      b2 = ((15*a0*m*r**3+30*a0*r**2-30*g0*m*r-30*g0)*sm**2*uc+(10*dd1*m
     1    **2*m2*r**2+30*dd1*m*m2*r+30*dd1*m2)*sm*ub-15*qb2*r**5*exp(m*r)
     2    *sm**2-4*a2*m**4*m2**2*r**4-4*a2*m**3*m2**2*r**3+12*a2*m**2*m2*
     3    *2*r**2+48*a2*m*m2**2*r+48*a2*m2**2)/(30*m**4*m2**2*r**4+60*m**
     4    3*m2**2*r**3+60*m**2*m2**2*r**2)
      cc2 = -((15*a0*m*r**3-30*g0*m**2*r**2-60*g0*m*r-60*g0)*sm**2*uc+(-
     1    40*dd1*m**2*m2*r**2-120*dd1*m*m2*r-120*dd1*m2)*sm*ub+(120*a0*m*
     2    r**3+120*a0*r**2)*sm**2*ua+60*qcc2*r**5*exp(m*r)*sm**2+8*a2*m**
     3    4*m2**2*r**4+8*a2*m**3*m2**2*r**3-24*a2*m**2*m2**2*r**2-96*a2*m
     4    *m2**2*r-96*a2*m2**2)/(120*m**4*m2**2*r**4+240*m**3*m2**2*r**3+
     5    240*m**2*m2**2*r**2)
      g2 = -(((30*av0b-90*av0a)*r**5*exp(m*r)-15*a0*m*r**5+(30*g0*m**2-6
     1    0*a0)*r**4+180*g0*m*r**3+180*g0*r**2)*sm**2*uc+(60*a01b*r**5*ex
     2    p(m*r)*sm**2+(40*dd1*m**2*m2*r**4+120*dd1*m*m2*r**3+120*dd1*m2*
     3    r**2)*sm)*ua+(120*av2e+120*av2c+120*av2a)*r**5*exp(m*r)*sm**2-8
     4    *a2*m**3*m2**2*r**5-24*a2*m**2*m2**2*r**4-48*a2*m*m2**2*r**3-48
     5    *a2*m2**2*r**2)/(16*m**4*m2**2*r**4+112*m**3*m2**2*r**3+432*m**
     6    2*m2**2*r**2+960*m*m2**2*r+960*m2**2)
      h2 = -((15*a0*r**4-30*g0*m*r**3-30*g0*r**2)*sm**2*uc+(30*e1*m**2*m
     1    2*r**4-10*dd1*m*m2*r**3-10*dd1*m2*r**2)*sm*ua+15*qh2*r**5*exp(m
     2    *r)*sm**2+(-30*b2-4*a2)*m**3*m2**2*r**5+8*g2*m**4*m2**2*r**4+8*
     3    g2*m**3*m2**2*r**3-24*g2*m**2*m2**2*r**2-96*g2*m*m2**2*r-96*g2*
     4    m2**2)/(60*m**4*m2**2*r**4+120*m**3*m2**2*r**3+120*m**2*m2**2*r
     5    **2)
      j2 = -((15*av2d+15*av2b)*r**5*exp(m*r)*sm**2+(-15*cc2-a2)*m**3*m2*
     1    *2*r**5+2*g2*m**4*m2**2*r**4+2*g2*m**3*m2**2*r**3-6*g2*m**2*m2*
     2    *2*r**2-24*g2*m*m2**2*r-24*g2*m2**2)/(30*m**4*m2**2*r**4+60*m**
     3    3*m2**2*r**3+60*m**2*m2**2*r**2)
```

CBM Models

```
      qa1 = (e00*ua+ev1c+ev1b+ev1a)/2.0-bv0a*ub/r
      qb1 = bv0a*ub/r+ev1d/2.0
      qdd1 = -av0a*ub/r+b00*ua+bv1c+bv1b+bv1a
```

```
      qe1 = av0a*ub/r+bv1d
      qg1 = a00*ua+av1c+av1b+av1a

      a0 = ((ev0b+ev0a)*exp(m*r)*sinh(m*r)+(-ev0b-ev0a)*m*r*exp(m*r)*cos
     1   h(m*r))/(m**3*sinh(m*r)+m**3*cosh(m*r))
      a0c = -a0*m**2*(m*r+1)*exp(-m*r)/(sinh(m*r)-m*r*cosh(m*r))
      ae0h = a0*(m*r+1)*exp(-m*r)/r**2
      ae0c = a0c*(cosh(m*r)/(m*r)-sinh(m*r)/(m**2*r**2))
      d0 = ((bv0b+bv0a)*exp(m*r)*sinh(m*r)+(-bv0b-bv0a)*m*r*exp(m*r)*cos
     1   h(m*r))/(m**3*sinh(m*r)+m**3*cosh(m*r))
      d0c = -d0*m**2*(m*r+1)*exp(-m*r)/(sinh(m*r)-m*r*cosh(m*r))
      ax0h = d0*(m*r+1)*exp(-m*r)/r**2
      ax0c = d0c*(cosh(m*r)/(m*r)-sinh(m*r)/(m**2*r**2))
      g0 = ((a0c*m**2*r**2+2*a0c)*exp(m*r)*sinh(m*r)**2+(-a0c*m*r*exp(m*
     1   r)*cosh(m*r)+(2*av0b+2*av0a)*m**2*r*exp(m*r)+a0*m**4*r**2-a0*m*
     2   *3*r+2*a0*m**2)*sinh(m*r)-a0c*m**2*r**2*exp(m*r)*cosh(m*r)**2+(
     3   (-2*av0b-2*av0a)*m**3*r**2*exp(m*r)+a0*m**4*r**2-2*a0*m**3*r)*c
     4   osh(m*r))/(2*m**5*r*sinh(m*r)+2*m**5*r*cosh(m*r))
      g0c = (a0c*r**2*exp(m*r)*sinh(m*r)+a0*m**2*r**2-2*g0*m**3*r-2*g0*m
     1   **2)/(2*exp(m*r)*sinh(m*r)-2*m*r*exp(m*r)*cosh(m*r))
      ap0 = (g0*(m*r+1)/r**2-a0/2.0)*exp(-m*r)
      ap0c = g0c*(cosh(m*r)/(m*r)-sinh(m*r)/(m**2*r**2))+a0c*sinh(m*r)/m
     1   **2/2.0

      psa1 = -(ax0h-ax0c)*ub/r**2
      a1 = -(((3*m**2*qa1-3*m**2*psa1)*r**2+9*qa1-9*psa1)*exp(m*r)*sinh(
     1   m*r)+(9*m*psa1-9*m*qa1)*r*exp(m*r)*cosh(m*r))*sm/(m**5*m2*r*sin
     2   h(m*r)+m**5*m2*r*cosh(m*r))
      a1c = -a1*m**3*(m**2*r**2+3*m*r+3)*exp(-m*r)/(m**2*r**2*sinh(m*r)+
     1   3*sinh(m*r)-3*m*r*cosh(m*r))/3.0
      psb1 = (ax0h-ax0c)*ub/r**2
      b1 = -((3*m**3*qb1-3*m**3*psb1)*r**4*exp(m*r)*sinh(m*r)*sm+(-3*a1c
     1   *m**2*m2*r**2-6*a1c*m2)*exp(m*r)*sinh(m*r)**2+(3*a1c*m*m2*r*exp
     2   (m*r)*cosh(m*r)-a1*m**5*m2*r**2-2*a1*m**4*m2*r-2*a1*m**3*m2)*si
     3   nh(m*r)+3*a1c*m**2*m2*r**2*exp(m*r)*cosh(m*r)**2+(-a1*m**5*m2*r
     4   **2-a1*m**4*m2*r)*cosh(m*r))/(3*m**6*m2*r**3*sinh(m*r)+3*m**6*m
     5   2*r**3*cosh(m*r))
      b1c = -exp(-m*r)*(3*a1c*exp(m*r)*sinh(m*r)-3*a1c*m*r*exp(m*r)*cosh
     1   (m*r)-3*b1*m**5*r**2+a1*m**4*r+a1*m**3)/(m**2*r**2*sinh(m*r))/3
     2   .0
      psdd1 = -(ap0-ap0c)*ub/r**2-(ae0h-ae0c)*ua
      dd1 = -(((3*m**2*qdd1-3*m**2*psdd1)*r**2+9*qdd1-9*psdd1)*exp(m*r)*
     1   sinh(m*r)+(9*m*psdd1-9*m*qdd1)*r*exp(m*r)*cosh(m*r))*sm/(2*m**5
     2   *m2*r*sinh(m*r)+2*m**5*m2*r*cosh(m*r))
      d1c = -dd1*m**3*(m**2*r**2+3*m*r+3)*exp(-m*r)/(m**2*r**2*sinh(m*r)
     1   +3*sinh(m*r)-3*m*r*cosh(m*r))/3.0
      pse1 = (ap0-ap0c)*ub/r**2
      e1 = -((3*m**3*qe1-3*m**3*pse1)*r**4*exp(m*r)*sinh(m*r)*sm+(-6*d1c
```

```
1    *m**2*m2*r**2-12*d1c*m2)*exp(m*r)*sinh(m*r)**2+(6*d1c*m*m2*r*ex
2    p(m*r)*cosh(m*r)-2*dd1*m**5*m2*r**2-4*dd1*m**4*m2*r-4*dd1*m**3*
3    m2)*sinh(m*r)+6*d1c*m**2*m2*r**2*exp(m*r)*cosh(m*r)**2+(-2*dd1*
4    m**5*m2*r**2-2*dd1*m**4*m2*r)*cosh(m*r))/(6*m**6*m2*r**3*sinh(m
5    *r)+6*m**6*m2*r**3*cosh(m*r))
 e1c = -exp(-m*r)*(3*d1c*exp(m*r)*sinh(m*r)-3*d1c*m*r*exp(m*r)*cosh
1    (m*r)-3*e1*m**5*r**2+dd1*m**4*r+dd1*m**3)/(m**2*r**2*sinh(m*r))
2    /3.0

 psg1 = -(ax0h-ax0c)*ua
 g1 = -((((3*m**5*qg1-3*m**5*psg1)*r**4+(9*m**3*qg1-9*m**3*psg1)*r*
1    *2)*exp(m*r)*sinh(m*r)+(9*m**4*psg1-9*m**4*qg1)*r**3*exp(m*r)*c
2    osh(m*r))*sm+(18*a1c*m2-3*a1c*m**4*m2*r**4)*exp(m*r)*sinh(m*r)*
3    *2+((-3*a1c*m**3*m2*r**3-36*a1c*m*m2*r)*exp(m*r)*cosh(m*r)-a1*m
4    **7*m2*r**4+3*a1*m**6*m2*r**3+6*a1*m**4*m2*r+6*a1*m**3*m2)*sinh
5    (m*r)+(3*a1c*m**4*m2*r**4+18*a1c*m**2*m2*r**2)*exp(m*r)*cosh(m*
6    r)**2+(-a1*m**7*m2*r**4+2*a1*m**6*m2*r**3-6*a1*m**5*m2*r**2-6*a
7    1*m**4*m2*r)*cosh(m*r))/(2*m**8*m2*r**3*sinh(m*r)+2*m**8*m2*r**
8    3*cosh(m*r))
 g1c = (3*a1c*r**2*exp(m*r)*sinh(m*r)-3*a1c*m*r**3*exp(m*r)*cosh(m*
1    r)+a1*m**4*r**3+(a1*m**3-2*g1*m**5)*r**2-6*g1*m**4*r-6*g1*m**3)
2    /((6*m**2*r**2+18)*exp(m*r)*sinh(m*r)-18*m*r*exp(m*r)*cosh(m*r)
3    )
 h1 = -(3*av1d*m**3*r**4*exp(m*r)*sinh(m*r)*sm+(3*b1c*m**2*m2*r**4-
1    6*g1c*m**2*m2*r**2-12*g1c*m2)*exp(m*r)*sinh(m*r)**2+((3*b1c*m*m
2    2*r**3+6*g1c*m*m2*r)*exp(m*r)*cosh(m*r)-3*b1*m**5*m2*r**4+3*b1*
3    m**4*m2*r**3-2*g1*m**5*m2*r**2-4*g1*m**4*m2*r-4*g1*m**3*m2)*sin
4    h(m*r)+(6*g1c*m**2*m2*r**2-3*b1c*m**2*m2*r**4)*exp(m*r)*cosh(m*
5    r)**2+(-3*b1*m**5*m2*r**4-2*g1*m**5*m2*r**2-2*g1*m**4*m2*r)*cos
6    h(m*r))/(6*m**6*m2*r**3*sinh(m*r)+6*m**6*m2*r**3*cosh(m*r))
 h1c = -exp(-m*r)*(6*g1c*exp(m*r)*sinh(m*r)+(3*b1c*m*r**3-6*g1c*m*r
1    )*exp(m*r)*cosh(m*r)+3*b1*m**4*r**3-6*h1*m**5*r**2+2*g1*m**4*r+
2    2*g1*m**3)/(m**2*r**2*sinh(m*r))/6.0
```

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