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TITLE: Dispersive Effects of Transverse Displacements of SLC Arc Magnets

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Summary

The SLC Arc magnets are subject to random displacements and field errors resulting in unpredictable transverse displacement of the central trajectory from that of the design. The chosen method of correcting this perturbed trajectory in the SLC Arcs utilizes mechanical movement of the combined function magnets which compose the Arc transport lines. Here we present the results of a recent investigation substantiating the earlier results which led to the adoption of this method.

Introduction

During the design phase of the SLC Arcs various correction methods were suggested and studied. Two of them were considered as most promising:

- use of Backleg Windings (BLW) and
- transverse translation of the combined function magnets, i. e. use of Magnet Movers (MM).

In the course of these studies one of the authors (JJM) first observed that trajectory correction by MM introduced significantly less anomalous dispersion than that introduced by BLW. This important observation was subsequently corroborated by others working on Arc Beam Dynamics[1] using both analytic methods and computer simulations. Most of these efforts are documented in minutes of different meetings only and are not readily available.

However, the decision to use MM was based upon that earlier collaborative effort. What is reported here are the results of a recent review of the dispersive effects of the MM including some original approaches not previously used.

The FODO cells of the SLC Arcs are put together to form second-order achromats.[2] Each cell is composed of two combined function magnets each having superimposed dipole, quadrupole and sextupole fields. The strength of the dipole field is the same for both magnets. The field gradients in both magnets are nearly the same but the signs are opposite. The sextupole components are different both in the signs and in the strengths.

In this lattice the dispersion function, defining the deviation of the trajectory for an off-momentum particle with momentum p from that for the on-momentum particle with momentum p_o, has been matched in the sense that the dispersion function and its derivative with respect to the path length s are both periodic with a period equal to the cell length for an unperturbed (i. e. no misalignment or field errors) system. This matched dispersion function and its derivative are called the η and η' functions.

This statement must be qualified somewhat by pointing out that deviations from this perfectly "matched" condition are created by necessity of rolling some achromats as a whole to allow vertical deflections of the Arcs for following the site terrain. The effect of including these rolls into the design has been studied but will not be discussed here.

Dipole field errors and transverse displacements of multipole fields will in general produce dispersive angular "kicks" (momentum dependent changes of direction) in a particle beam trajectory. The kicks are linear with displacement of quadrupoles and quadratic with displacement of sextupoles. The momentum dependence of a kick can be characterized by a change $\Delta\eta'$ of the slope of the matched dispersion function. This change will propagate through the downstream system as anomalies of the functions η and η'.

Non-Dispersive Condition

Consider a differential slice of an Arc magnet with length ds. The transverse field components can be expressed as follows,

$$B_y = B_o + B_o' x + \frac{1}{2} B_o''(x^2 - y^2), \tag{1}$$

$$B_x = B_o' y + B_o'' xy, \tag{2}$$

where the coefficients B_o, B_o' and B_o'' represent the dipole, quadrupole and sextupole strengths of the combined function magnet, respectively.

Suppose at this point that there is no dispersion in the y plane and that the horizontal dispersion is equal to η. Suppose further that the elemental magnet is displaced from the reference axis by Δx and Δy. Then, with $p = p_o(1 + \delta)$,

$$x = \eta\delta + \Delta x, \tag{3}$$

$$y = \Delta y. \qquad (4)$$

With these values inserted in Eqs. (1,2), the angular "kicks" $d\theta$ in horizontal and $d\phi$ in vertical planes of a ray passing through the elemental magnet may be written as follows,

$$d\theta = \frac{ds}{p_o(1+\delta)}[(B_o + B_o'\eta\delta + \frac{1}{2}B_o''\eta^2\delta^2) + (B_o' + B_o''\eta\delta)\Delta x + \frac{1}{2}B_o''(\Delta x^2 - \Delta y^2)], \qquad (5)$$

$$d\phi = \frac{ds}{p_o(1+\delta)}[(B_o' + B_o''\eta\delta)\Delta y + B_o''\Delta x\Delta y]. \qquad (6)$$

The first term in $d\theta$ is simply the deflection produced by the unperturbed magnet and is of no interest here.

Let us consider the linear approximation in Δx and Δy and consequently neglect all quadratic terms in these quantities. Then we have

$$d\theta \approx \frac{ds}{p_o(1+\delta)}(B_o' + B_o''\eta\delta)\Delta x, \qquad (7)$$

$$d\phi \approx \frac{ds}{p_o(1+\delta)}(B_o' + B_o''\eta\delta)\Delta y. \qquad (8)$$

Clearly, both $d\theta$ and $d\phi$ will be approximately independent of momentum if the following condition is satisfied,

$$B_o''\eta \approx B_o'. \qquad (9)$$

This condition cannot be satisfied at all points in an arc magnet since within a magnet η varies while B_o' and B_o'' do not. To make a system non-dispersive we should use $\overline{\eta}$, i. e. an η averaged over the magnet segment actually being moved:

$$B_o''\overline{\eta} = B_o'. \qquad (10)$$

Eq. (10) is the non-dispersive condition.

Neglecting the short drift between magnets, the matched η-function in the FODO lattice can be expressed as follows,

$$\eta_F = \eta_{max} + A_F(\cos\frac{s}{\lambda} - 1), \tag{11}$$

$$\eta_D = \eta_{min} + A_D(\cosh\frac{s}{\lambda} - 1), \tag{12}$$

where s is distance from the center of a magnet measured along the reference axis and

$$\lambda = |\frac{p_o}{B_o'}|^{\frac{1}{2}}. \tag{13}$$

A_F and A_D are determined by the requirement that η_F and η_D join smoothly at the junction between magnets where $s = L/4$ and L is the length of a cell. The subscripts $F(D)$ refer to focussing (defocussing) magnets.

In MURTLE simulations, translations, both random and deliberate, are applied to half magnets. In that case the average η's for s between 0 and $L/4$ are given by

$$\overline{\eta_F} = \eta_{max} + A_F(\frac{\sin\alpha}{\alpha} - 1), \tag{13}$$

$$\overline{\eta_D} = \eta_{min} + A_D(\frac{\sinh\alpha}{\alpha} - 1), \tag{14}$$

where $\alpha = L/4\lambda$.

The critical quantity, $B_o''\overline{\eta}/B_o'$, has been evaluated in four cases, namely, for F and D versions of SLC Arc magnets (108° phase shift per cell) and for F and D versions of the magnets of an achromat similar to the SLC achromat but with 135° phase shift per cell.

Key parameters at 50 GeV in these cases are listed in Table 1.

Table 1

Magnet	B_o'	B_o''	$\eta_{max,min}$	$\bar{\eta}$	$B_o''\bar{\eta}/B_o'$
	kg/cm	*kg/cm²*	*cm*	*cm*	
phase/cell	108°				
FFOC108	7.014	1.621	4.743	4.296	0.993
DFOC108	-7.014	-2.710	2.273	2.651	1.024
phase/cell	135°				
FFOC135	8.034	2.367	3.757	3.363	0.991
DFOC135	-8.034	-4.310	1.600	1.926	1.034

As demonstrated in this Table, Eq. (10) is satisfied within a few percent for the approximation used to evaluate $\bar{\eta}$ for these two and probably all second-order achromats.

Simulations Results

We have shown by Eqs. (7,8 and 10) that the direction of the beam can be changed non-dispersively by magnet movement in the linear approximation in its displacement. The suppression of the linear effects and the onset of non-linear behavior has been studied for a single Arc achromat using the code MURTLE.

The results of these calculations are illustrated in Figures. 1 through 6. The model used in these simulations is that of a single Arc achromat starting from the symmetry point of a focusing magnet. Each of the 20 magnets has been divided into two equal halves. These half-magnets are numbered sequentially from 1 to 40. Those halves designated as 4,8,12,...,40 then represent horizontal magnet movers, whereas numbers 2,6,10,...,38 correspond to vertical movers. Particular movers 4, 6, 8 and 40 were chosen for these illustrations and in each case the effect of a horizontal or vertical translation was measured by calculating the change of η and η' as an expansion about the perturbed orbit at the end of the achromat. Two series of calculations were performed with and without the sextupole field component in the half-magnet that was moved to generate the non-dispersive and dispersive kicks, respectively. In general, with the exception of the last half-magnet *No.* 40, these kicks initiate a free betatron oscillation in the downstream magnets. Thus the changes of η and η' are the cummulative result of the initial and subsequent kicks due to orbit displacement in these magnets. Whether the effect is most noticeable as a change in η or η' as measured at the end of the achromat depends upon the relative phase shift of the moved magnet.

In Fig. 1 the changes in η_x and η_x' produced by *horizontal* displacement of the mover *No.* 4 are shown. The phase shift for this mover is close to the multiple of $\pi/2$. As expected it is η which demonstrates the linear effect when the sextupole term of this mover is turned off. Fig. 2 illustrates the effect of the *horizontal* displacement of the mover *No.* 8. For this mover the phase shift is approximately a multiple of π so the linear effect shows up predominantly in $\Delta\eta'$.

In Figs. 3, 4 and 5 the effects of the vertical magnet mover *No.* 6 are shown for both *vertical* and *horizontal* displacements. Fig. 3 illustrates the broad range of *vertical* movement without large second-order effects on η_y and η_y' as can be seen from eq. (6) when Δx is put to zero. On the other hand Fig. 4 shows the second-order effects on η_x and η_x' with *vertical* displacement as predicted by eq. (5). Note especially, that the behavior is similar for the sextupole either turned on or off. This is because the perturbed orbit has a vertical offset in all downstream magnets.

Fig. 5 illustrates the effect of a *horizontal* offset in half-magnet *No.* 6 where β_x and η_x are at their respective minima. The resultant downstream trajectory is $\approx$ 3 times smaller in amplitude than that caused by mover *No.* 8 in Fig. 2, which is at the maximum of β_x. The effect on η_y and η_y' of moving this half-magnet *No.* 6 *horizontally* is negligible and is not illustrated.

Fig. 6 is plotted for the last magnet mover *No.* 40 so there are no contributions to $\Delta\eta$ and $\Delta\eta'$ stemming from the perturbed orbit in the downstream magnets. This figure most clearly illustrates that the non-dispersive feature is the consequence of the non-dispersive condition (10). Such a combined function magnet can be used as a non-dispersive orbit corrector in any arbitrary transport system if positioned at a point of non-zero dispersion.

In view of nonlinearities exhibited by the simple system modeled above, one might speculate that in a complete Arc with all magnets subject to random translations their cumulative effect might contribute significantly to the onset of uncorrectable nonlinear effects observed to occur with rms displacements in the 200 to 300 micron range.

Conclusions

Based on these observations we conclude that the magnets in the SLC achromats satisfy the non-dispersive condition (10) with respect to translations and that magnets in any η matched, second-order achromat probably do so also. But it is clear that for any η matched, periodic lattice not necessarily composed of second-order achromats, values of B_o'' can always be found such that the non-dispersive conditions for translation are satisfied in both F and D magnets.

When the non-dispersive condition (10) is satisfied the linear dispersive effects in magnet displacements are suppressed but the higher-order terms are still present and distort the dispersion of the system. It is these and other nonlinear effects that have limited the tolerance for displacements in the SLC Arcs.

REFERENCES

1. Private communications of the Beam Dynamics Task Force (BDTF). At the time of the decision to use MM the group consisted of *K. Brown, A. Chao, J. Jaeger, S. Kheifets, J. Murray, R. Servranckx and H. Shoaee.*

2. K. L. Brown, "A Second-Order Magnetic Optical Achromat", SLAC-PUB-2257, February 1979

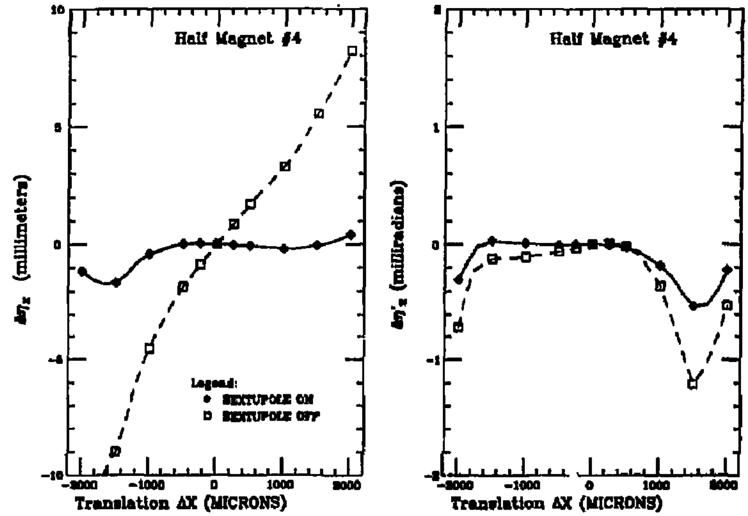

Fig. 1. Effect on η_x, η'_x for horiz. displacement of half magnet 4 (F).

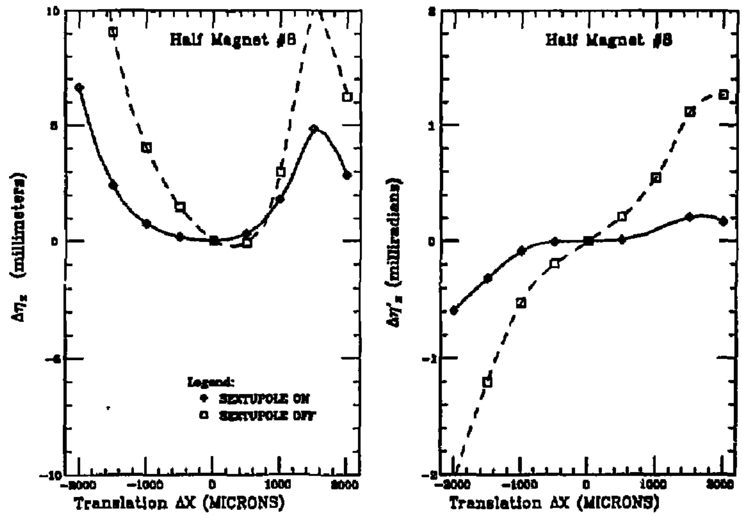

Fig. 2. Effect on η_x, η'_x for horiz. displacement of half magnet 8 (F).

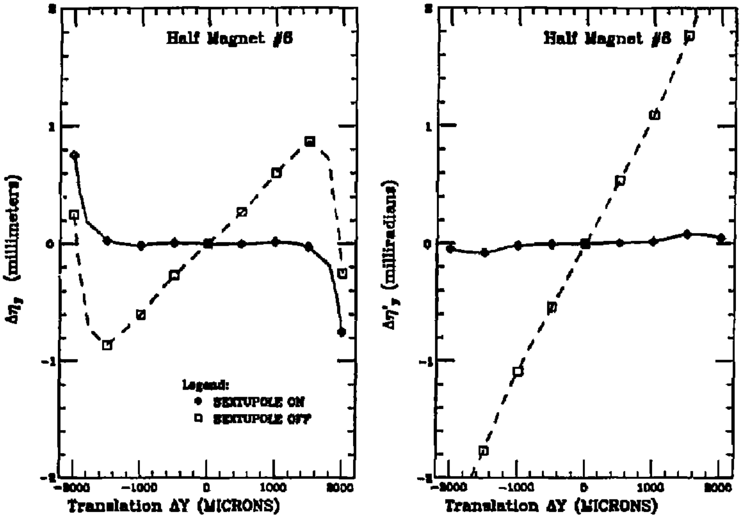

Fig. 3. Effect on η_y, η_y' for vert. displacement of half magnet 6 (D).

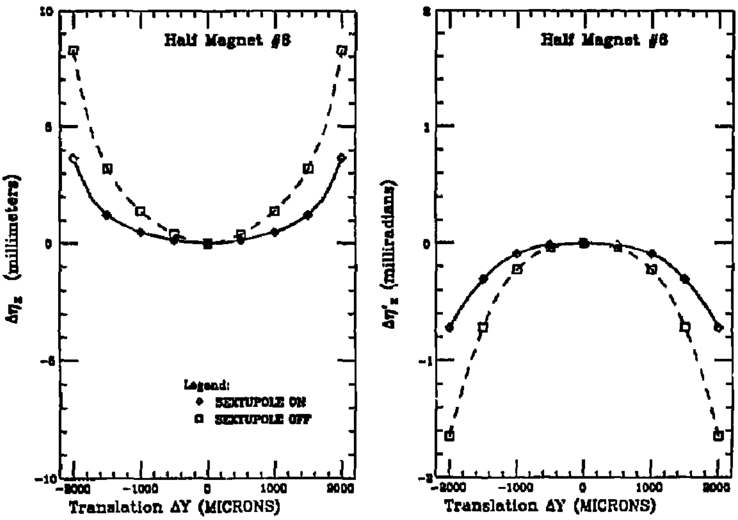

Fig. 4. Effect on η_x, η_x' for vert. displacement of half magnet 6 (D).

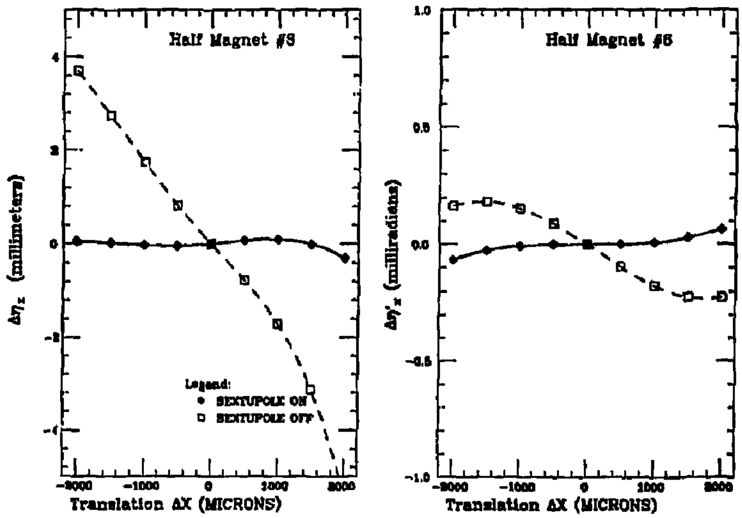

Fig. 5. Effect on η_x, η_x' for horiz. displacement of half magnet 6 (D).

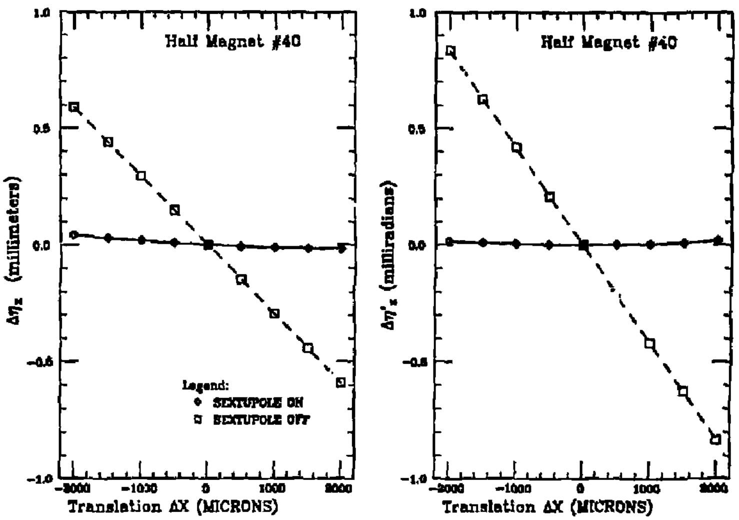

Fig. 6. Effect on η_x, η_x' for horiz. displacement of half magnet 40 (F).