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**ISAJET 5.30: A MONTE CARLO EVENT GENERATOR FOR  
*pp* and  $\bar{p}p$  INTERACTIONS**

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ISAJET is a Monte Carlo program which simulates  $pp$  and  $\bar{p}p$  interactions at high energy. It is based on perturbative QCD cross sections, leading order QCD radiative corrections for initial and final state partons, and phenomenological models for jet and beam jet fragmentation. This article describes ISAJET 5.30, which includes production of standard Higgs bosons and which will be released shortly.

### Introduction

Perturbative QCD provides a very good description of all available data on hard interactions, those with momentum transfers large compared to 1 GeV. But perturbative QCD is formulated in terms of quarks and gluons, not the observed hadrons. The hadrons are presumed to be formed by nonperturbative aspects of QCD characterized by small momentum transfers, implying the creation of a jet of hadrons with limited  $p_T$  from each quark or gluon. This nonperturbative hadronization is not understood in any fundamental way, but it can be described by a variety of phenomenological models. Since the pioneering work of Field and Feynman,<sup>1</sup> Monte Carlo programs based on a combination of perturbative QCD and nonperturbative models have played an increasingly important role in particle physics. They are now widely used to extract results on jets from experimental data, to study signatures and backgrounds for various processes, and to correct data for detector effects.

Any QCD Monte Carlo program must describe the complete range of  $Q^2$  from the initial hard scattering to the formation of hadrons at  $Q^2 \leq 1 \text{ GeV}^2$ . The largest values of  $Q^2$  are described by the appropriate cross section calculated in QCD perturbation theory. The small values are in the range for which confinement is important, so models must be used. For intermediate values higher order QCD processes must be taken into account. Partons (quarks and gluons) of a given  $Q^2$  will in general radiate additional partons at lower  $Q^2$ , giving rise to a cascade. This radiation is most important when the radiated partons are nearly collinear; in this limit the probability for each additional radiation is given by a factor,

$$\sigma = \sigma_0 \left[ \frac{\alpha_s(Q^2)}{2\pi Q^2} P(z) \right], \quad (1)$$

where  $z$  is the momentum fraction carried by one of the radiated partons and  $P(z)$  is an Altarelli-Parisi function.<sup>2</sup> A reasonable approximation is to keep this simple factorized form for the cross section but to use exact, noncollinear kinematics. This approximation was introduced some time ago

by Fox and Wolfram<sup>3</sup> for final state partons. Gottschalk<sup>4</sup> and Sjostrand<sup>5</sup> have extended the algorithm to initial state QCD radiation in a straightforward and efficient way. The essential idea is first to generate the primary hard scattering with the largest  $Q^2$  and then to work backwards to the initial protons. This approach has been used to incorporate initial state radiation in ISAJET. Then ISAJET gives a reasonably good description of the main features of hadronic reactions, including in particular the data on jet and  $W$  production from the SpS.

ISAJET simulates events in four distinct steps incorporating both perturbative QCD and nonperturbative models for hadronization:

- (1) A primary hard scattering is generated according to one of several available cross sections combined with nonscaling structure functions.
- (2) QCD radiation is added from both initial and final partons, thus producing scaling violations in jet fragmentation and also multiple jet events.
- (3) Hadrons are produced from each parton using the independent fragmentation ansatz. (This ansatz adequately describes the fast hadrons in a jet, although its treatment of the slow hadrons is not very good.)
- (4) Hadrons from the beam jets are added, with 'biased minimum bias' interactions of the spectators in hard scattering processes.

All four steps will be discussed in turn below. For further details see Ref. 6.

### Hard Scattering

The first step in simulating an event is to generate the primary hard scattering according to a cross section  $\hat{\sigma}$  calculated to leading order in QCD perturbation theory and convoluted with structure functions incorporating leading-log scaling violations. That is, the hard scattering cross section has the standard form for the QCD-improved parton model,

$$\sigma = \hat{\sigma} f(x_1, Q^2) f(x_2, Q^2), \quad (2)$$

where  $x_1$  and  $x_2$  are the usual momentum fractions. The default parameterization of the structure functions is Solution 1 of Eichten, Hinchliffe, Lane and Quigg (EHLQ).<sup>7</sup>

The following hard-scattering processes are included in ISAJET 5.30:

*Minimum Bias:* No hard scattering at all, so that the event consists only of beam jets. The term minimum bias suggests that these events are representative of the total non-diffractive cross section. At high energy this is not true, since the structure functions become large for  $x \rightarrow 0$ , causing the jet cross sections to become large. In fact, for  $\sqrt{s} = 40$  TeV, lowest order QCD gives an integrated jet cross section of about 200 mb. While this result is not quantitatively reliable, it seems<sup>8</sup> that the domain of validity of perturbation theory does extend down to rather small  $x$  and that events with jets will be common rather than rare at high energy. This may be quite important for estimating backgrounds from minimum bias interactions.

*QCD Jets:* All of the usual  $O(\alpha_s^2)$  processes for two-body QCD scattering, including

$$gg \rightarrow gg, \quad gq \rightarrow gq, \quad gq \rightarrow q\bar{q}, \dots \quad (3)$$

These processes all give two jets in lowest order, but they can produce multiple jets after initial-state and final-state QCD gluon radiation is included. Quark masses are included for  $c$  and heavier quarks. Note that the EHLQ structure functions include heavy quarks, so that processes like  $gt \rightarrow gt$  are also included. The algorithm for initial state radiation insures that the branching  $g \rightarrow t\bar{t}$  always occurs, so that heavy quark quantum numbers are conserved.

An optional fourth generation of quarks can be produced by gluon or quark fusion. The fourth generation is not included in the structure functions or in the QCD jet evolution.

*Drell-Yan:* Production and decay of a  $W$ , meaning any of  $\gamma$ ,  $W^+$ ,  $W^-$ , or  $Z^0$  in the standard model. The leading order process is

$$q\bar{q} \rightarrow W \rightarrow \ell\bar{\ell}, \quad q\bar{q}. \quad (4a)$$

While to lowest order this process gives a  $W$  with  $p_T = 0$ , transverse momentum will of course be generated by the initial state gluon radiation. Alternatively, the  $O(\alpha_s)$  processes

$$gq \rightarrow Wq, \quad q\bar{q} \rightarrow Wg \quad (4b)$$

can be simulated. These are the dominant processes at high  $p_T$ , but they give a  $1/p_T^2$  singularity as  $p_T \rightarrow 0$ . To obtain a cross section which gives sensible results for all  $p_T$ , a cutoff is introduced with a form roughly like that obtained<sup>9,10</sup> from calculating the  $p_T$  distribution by summing the leading double logarithms of QCD perturbation theory. The cutoff is adjusted to get an integrated cross section about equal to the standard Drell-Yan result. The initial-state QCD evolution is then started from the scale  $p_T$ , which is approximately correct both at high  $p_T$  and at low  $p_T$ , since the first radiation is explicitly generated. Thus the  $O(\alpha_s)$  cross section with the cutoff can be used for all  $p_T$ , although it does not give quite the right rapidity distribution.

*W Pairs:* Production of gauge boson pairs in the standard model. Only the processes

$$q\bar{q} \rightarrow W^+W^-, \quad Z^0Z^0, \quad W^\pm Z^0, \quad W^\pm\gamma \quad (5)$$

are included. The full matrix element for the decay of the  $W$  bosons is included in the narrow resonance approximation. Pairs of  $W$  bosons are expected to be one of the most important signatures for new physics in very high energy hadronic collisions.

*Higgs:* Production and decay of a standard model Higgs boson, including all of the important production mechanisms:

$$q\bar{q} \rightarrow H, \quad gg \rightarrow H, \quad WW \rightarrow H. \quad (6)$$

For high masses the dominant decay is into  $W^+W^-$  and  $Z^0Z^0$  pairs, and the full decay matrix elements for these is included. Also, for high masses the Higgs is very wide, so the narrow resonance approximation is not valid; it is necessary to include all the graphs for  $WW$  scattering in the effective  $W$  approximation.<sup>11,12</sup> This process is new in Version 5.30.

*Supersymmetry:* Production of pairs of supersymmetric particles in the simplest model with global supersymmetry. The scalar partners of left-handed and right-handed quarks are taken to be degenerate, the  $\tilde{\gamma} - \tilde{Z}^0$  and  $\gamma - Z^0$  mixings are assumed to be identical, and mixings of gauginos and Higgsinos are ignored. Then the cross sections for producing supersymmetric particles are completely determined by the masses and the standard model. Both the  $O(\alpha_s^2)$  processes

$$gg \rightarrow \tilde{g}\tilde{g}, \quad gq \rightarrow \tilde{g}\tilde{q}, \dots \quad (7a)$$

and the  $O(\alpha_s)$  processes,

$$gq \rightarrow \tilde{W}\tilde{q}, \quad q\bar{q} \rightarrow \tilde{W}\tilde{g} \quad (7b)$$

are included, where  $\tilde{W}$  can be  $\tilde{\gamma}$ ,  $\tilde{W}^\pm$ , or  $\tilde{Z}^0$ . This is essentially the supersymmetry model used in EHLQ.

## QCD Radiative Corrections

It is necessary to find an approximation for QCD radiation valid to all orders. Consider the radiation of one extra gluon from a quark line of momentum  $p$ . This radiation is most important in the collinear limit,  $p^2 \rightarrow 0$ , since it is this region which produces the leading-log scaling violations. From QCD perturbation theory, as  $p^2 \rightarrow 0$  the cross section is given by Eq. (1) where  $P(z)$  is an Altarelli-Parisi function,<sup>2</sup>

$$P_{q \rightarrow qg}(z) = \frac{4}{3} \left( \frac{1+z^2}{1-z} \right), \quad (8)$$

and  $z$  is the momentum fraction carried by the outgoing quark. Various choices for  $z$  are possible; these are equivalent for collinear radiation but give different results when continued to large angles. The same form with different functions  $P(z)$  holds for the other possible cases,

$g \rightarrow gg$  and  $g \rightarrow q\bar{q}$ . Thus the most important part of the QCD radiation can be expressed in terms of cross sections or probabilities rather than amplitudes. By using exact noncollinear kinematics multiple jet states can be included, at least approximately. This provides a natural basis for a Monte Carlo algorithm called the branching approximation.<sup>3</sup>

The branching approximation gives the correct leading-log scaling violations for the structure functions and for jet fragmentation. It also gives correctly the structure of jets in QCD perturbation theory, since the typical mass of a jet is small,  $M^2 = O(\alpha_s p_T^2) \ll p_T^2$ , so that nearly collinear radiation dominates. Finally, the branching approximation turns out to reproduce the leading order three-jet cross section within a factor of about two over all of phase space.<sup>13</sup> This is good enough for many purposes, although not for extracting a quantitative measurement of  $\alpha_s$ .

*Final State Radiation:* The branching approximation was introduced for final state radiation by Fox and Wolfram.<sup>3</sup> Consider for simplicity only gluon radiation from a quark line; the general case is essentially identical. The approximate cross section for the emission of  $n$  gluons is

$$\frac{d\sigma}{dp_1^2 dz_1 \dots dp_n^2 dz_n} = \sigma_0 \frac{1}{n!} \prod_i \left[ \frac{\alpha_s(p_i^2)}{2\pi p_i^2} P(z_i) \right], \quad (9)$$

where the masses are strongly ordered,

$$Q^2 > p_1^2 > p_2^2 > \dots > p_n^2. \quad (10)$$

Note that Eq. (9) contains both collinear singularities, the explicit  $1/p_i^2$ , and infrared singularities, the  $1/(1-z_i)$  in the  $P(z_i)$ .

The infrared singularities cancel in the usual way with those from the virtual graphs. This can be implemented simply by treating  $P(z)$  as a distribution containing a  $\delta(1-z)$  term such that its integral is zero, which is just the condition needed to ensure energy and momentum conservation in the Altarelli-Parisi equations. The collinear singularities do not cancel; they are needed to build up the leading-log QCD scaling violations. They are handled by introducing a cutoff  $p_i^2 = t_i > t_c$  and assuming that QCD below the cutoff is described by the nonperturbative hadronization model. If  $t_c$  is relatively large, then the model must explicitly produce jets, but if  $t_c \sim 1 \text{ GeV}^2$ , then it can be as simple as cluster decay with two body phase space. The choice made in ISAJET is to take  $t_c = (6 \text{ GeV})^2$  and to use independent fragmentation for the nonperturbative model.

The basic quantity needed to set up the Monte Carlo algorithm is the probability  $\Pi(t_0, t_1)$  for evolving from an initial mass  $t_0$  to a final mass  $t_1$  emitting no gluon radiation greater than the cutoff. That is,  $\Pi(t_0, t_1)$  is the probability for emitting no radiation which is to be treated explicitly rather than being included in the hadronization model. The formula is simple if the cutoff is taken to be not  $t > t_c$

but rather  $z_c < z < 1 - z_c$ . Then the  $t$  and  $z$  integrations separate, and the result is<sup>3</sup>

$$\Pi(t_0, t_1) = \left[ \frac{\alpha_s(t_0)}{\alpha_s(t_1)} \right]^{2\gamma(z_c)/b_0} \quad (11)$$

$$\gamma(z_c) = \int_{z_c}^{1-z_c} dz P(z).$$

Since the nonperturbative scale should be set by the mass at which QCD becomes strong, a cutoff on  $z$  is not very physical. But a  $t$  cutoff together with two-body kinematics for the decay implies a  $z$  cutoff.

For a fixed  $z$  cutoff the whole Monte Carlo algorithm is determined by Eq. (11). Since  $\Pi(t_0, t_1)$  is by definition the probability for no resolvable radiation, its derivative  $\Xi(t_1) = \partial\Pi(t_0, t_1)/\partial t_1$  gives the distribution for the mass  $t_1$  at which the first resolvable radiation occurs. In fact  $\Xi(t_1)$  is not needed, since  $t_1$  can be generated from the cumulative distribution  $\Pi(t_0, t_1)$ . After  $t_1$  for the first branching is chosen, a  $z$  for the branching is selected according to  $P(z)$ , and the masses of the two new partons are evolved starting from  $zt_1$  and  $(1-z)t_1$  respectively. Given the masses and  $z$ , the momenta can be calculated. The whole procedure is then iterated.

To obtain a fixed  $t$  cutoff rather than a fixed  $z$  cutoff, the minimum value of  $z_c$ , corresponding to the initial value of  $t$ , is calculated, and a branching is generated as described above. Then the value of  $z$  is compared with the  $z_c$  calculated for the new mass  $t_1$ . If  $z$  is outside the allowed range, the branching is discarded and the evolution is continued from the new mass. In this way the simple form of Eq. (11) can be used to generate events corresponding to the more complicated, but more reasonable, cutoff.<sup>3</sup>

*Initial State Radiation:* Gottschalk<sup>4</sup> and Sjostrand<sup>5</sup> have proposed efficient algorithms to extend the branching approximation to initial state QCD radiation. Both algorithms do the evolution backwards from the desired hard scattering, forcing the ordering of the virtual masses  $t_i$ . In Gottschalk's approach, the QCD radiation is used to produce the scaling violations for the structure functions. In Sjostrand's approach, the nonscaling structure functions are assumed to be known and are used to calculate the probabilities for radiation. This allows the initial hard scattering to be generated according to the cross section with nonscaling structure functions, so it has been adopted in ISAJET.

Consider the emission of one extra gluon from a process producing a state  $X$ . As for final state radiation, all definitions of the momentum fraction  $z$  are equivalent if the gluon is collinear, but a choice must be made for noncollinear gluons. The choice for which the branching approximation reproduces first order QCD is<sup>4</sup>

$$z = s_X / s_{X+g}. \quad (12)$$

Then the cutoff  $|t| > t_c$  plus two-body kinematics implies that the upper limit on  $z$  is

$$z_{\max} = \frac{1}{1 + t_c/s_X}. \quad (13)$$

while the lower limit is set by the available beam momentum.

The probability for one additional radiation is obtained from the Altarelli-Parisi equations,<sup>9</sup>

$$\frac{df_b(x,t)}{dt} = \frac{\alpha_s(t)}{2\pi t} \sum_a \int_x^1 \frac{dx'}{x'} f_a(x',t) P_{a \rightarrow bc} \left( \frac{x}{x'} \right). \quad (14)$$

Normally these equations are regarded as describing the evolution of the structure functions from small to large  $t$ . The key observation is that they can also be regarded as

$$S_b = \exp \left\{ - \int_{|t_1|}^{|t_0|} dt' \frac{\alpha_s(t')}{2\pi t'} \sum_a \int_{z_{\min}}^{z_{\max}} dz \left[ \frac{(x/z) f_a(x/z,t)}{x f_b(x,t)} \right] P_{a \rightarrow bc}(z) \right\}. \quad (15)$$

This formula is the basis for the initial state Monte Carlo algorithm. It is rather similar to the final state formula for  $\Pi(t_0, t_1)$  except for the ratio of structure functions in the square brackets. This ratio, which is derived from the Altarelli-Parisi equations, is physically very reasonable. It implies that a branching can occur only if the structure function  $(x/z) f_a(x/z, t)$  for the new initial parton  $a$  is large enough so that finding it in the incoming hadron is not improbable.

For a heavy quark the  $f_b(x, t)$  in Eq. (15) vanishes at the threshold  $t_q^* = 4m_q^2 x/(1-x)$ , making the ratio of structure functions infinite and so forcing the branching  $g \rightarrow q\bar{q}$  to occur. Hence a heavy quark in the initial state will be accompanied by an associated antiquark. In this case the evolution is restricted to be between  $|t_0|$  and  $\sqrt{|t_0 t_q^*|}$ , the ratio is evaluated at the lower limit, and the process is repeated if no branching occurs. A branching is forced if none occurs before  $t_q^* + t_c$ .

Once values of  $t$  and  $z$  have been found, a mass  $k^2$  for the radiated parton is generated by the final state evolution algorithm starting at the kinematic limit. Then given  $t$ ,  $z$ ,  $k^2$ , and the azimuthal angle  $\phi$ , which is generated uniformly, it is possible to solve for the four components of  $k$  with the new initial partons 2 and 3 having zero transverse momentum. The whole procedure is then iterated with the system  $X$  containing one additional parton.

## Jet Fragmentation

Once partons are generated by the hard scattering and the QCD jet evolution, hadrons must be formed. In ISAJET 5.30 this is done using the independent fragmentation ansatz originally proposed by Field and Feynman.<sup>1</sup> To fragment a quark  $q$  of momentum  $p$ , a  $q'\bar{q}'$  pair is generated with  $u : d : s = .43 : .43 : .14$ . (Baryons are incorporated by generating diquark pairs with a total probability of .10 and with the same mixture of flavors.) The  $q'$  and  $\bar{q}'$  are given transverse momenta  $\pm \vec{k}_T$ , with  $\langle k_T \rangle = .35 \text{ GeV}$ . Then the  $\bar{q}'$  is combined with the original  $q$  to form a  $0^-$  or  $1^-$  meson with equal probability for light mesons and with a 1 : 3 ratio as expected from spin counting for  $c$  and heavier mesons. The momentum  $p'_+ = E + p_L$  of the meson is taken

giving the probability that parton  $b$  disappears because of a branching

$$a \rightarrow bc$$

during a backwards evolution through an infinitesimal step  $dt$ . The probability for a finite  $\Delta t$  is just given by the exponential of the infinitesimal probability. With  $z = x/x'$  the probability for  $b$  to survive for evolution from  $t_0$  to  $t_1$  becomes<sup>5</sup>

to be  $p'_+ = zp_+$ , where  $z$  is generated according to

$$f(z) = 1 - a + a(b+1)(1-z)^b, \quad a = .96, b = 3 \quad (16)$$

for light quarks; the parameters are those obtained by the AFS Collaboration and also give reasonable agreement with  $e^+e^-$  data and with the preliminary UA1 data. For heavy quarks, the Petersson form

$$f(z) = \frac{1}{x[1 - 1/x - \epsilon/(1-x)]^2} \quad (17)$$

$$\epsilon = .8 \text{ GeV}^2/m_c^2 \quad \text{for } c$$

$$\epsilon = .5 \text{ GeV}^2/m_q^2 \quad \text{for } q = b, t$$

is used. Hadrons with negative  $p_L$  are discarded. The procedure is then iterated for the new quark  $q'$  with momentum  $(1-z)p$ . A gluon is fragmented like a light quark or antiquark with  $u : d : s = .43 : .43 : .14$ .

Independent fragmentation is very simple and correctly incorporates the most important features of jet fragmentation, particularly for the fast particles in the jet, but it has a number of obvious defects. Since a massless parton is fragmented into massive hadrons, energy and momentum cannot be conserved exactly. Energy and momentum conservation are enforced in ISAJET by boosting the system of fragmented jets to their rest frame, rescaling all the three-momenta by a factor, and recalculating all the energies. Similarly, flavor is not conserved, since hadrons with  $p_L < 0$  and the final quark are discarded. Finally, since jets are fragmented independently, a collinear branching of a quark into a quark and a gluon gives a larger multiplicity than a single quark, even if the quark-gluon mass is so small that they could not possibly be resolved. Branchings down to some fixed cutoff must be included in the QCD jet evolution to get the correct scaling violations. The problem is that the structure of the events depends on the cutoff. This problem is minimized in ISAJET by taking a relatively high cutoff,  $t_c = (6 \text{ GeV})^2$ , for which independent fragmentation is more reasonable. Nevertheless, the multiplicity of soft hadrons in very high  $p_T$  jets is too high.

Hadrons produced by the jet fragmentation are decayed uniformly in phase space, except that the  $V-A$  matrix element is used for semileptonic and quark decays. Standard branching ratios are used for the light hadrons. For  $D$

mesons the branching ratios are taken from the Mark III Collaboration;<sup>14</sup> unmeasured modes are estimated using statistical isospin weights.<sup>15</sup> Branching ratios for  $F$  mesons and charmed baryons are constructed by analogy. For  $b$  mesons and baryons the branching ratios are similar to those used in EUROJET.<sup>16</sup> These reproduce the main features, including the momentum spectrum for the secondary  $D$  mesons. Hadrons containing  $t$  or heavier quarks are decayed into quarks, which are allowed to radiate gluons and are then fragmented independently.

### Beam Jet Fragmentation

After a hard scattering event has been generated, something must be done with the remaining constituents of the proton. While factorization in QCD perturbation theory implies that high  $p_T$  jets must be treated like jets in  $e^+e^-$  reactions, there is very little theoretical guidance on what to do with the beam jets. The beam jets in hard interactions will receive contributions from mechanisms similar to those giving the total cross section, and the simplest assumption is that they are identical to a nondiffractive minimum bias event at the reduced energy. This now seems to be incorrect experimentally. Therefore, ISAJET 5.30 uses similar algorithms with different parameters for minimum bias events and for hard interaction beam jets.

All of the standard pictures of multiparticle production are based on the idea of pulling pairs of particles out of the vacuum, leading to only short-range rapidity correlations and essentially a Poisson multiplicity distribution. The minimum bias data<sup>17</sup> clearly shows long-range correlations and a broad multiplicity distribution. A very attractive resolution for this conflict was proposed by Abramovskii, Kanchelli, and Gribov (AKG).<sup>18</sup> Their idea is that the basic amplitude is a single chain, or cut Pomeron, having only short range correlations and giving an average multiplicity  $\bar{n}$  with Poisson fluctuations. But unitarity then requires that this amplitude be iterated, leading to graphs with several cut Pomerons giving multiplicity  $2\bar{n}$ ,  $3\bar{n}$ , ... A different discontinuity of the two Pomeron graph gives the leading contribution to the elastic cross section, so the probability for  $2\bar{n}$  should be of order  $\sigma_{el}/\sigma_{tot} \approx .20$ . Furthermore, events with high multiplicity in one region generally have several cut Pomerons and hence high multiplicity everywhere. All of this is in general agreement with experimental data on minimum bias interactions.

A simplified version of the AKG scheme modified to take account of leading particles has been implemented in ISAJET. A number  $K$  of cut Pomerons is selected with probabilities chosen to fit the data. Different distributions  $P_K$  are constructed to fit the data on minimum bias interactions and on hard scattering. It seems plausible that a hard interaction should bias the spectator interactions. For the left and right sides of each event an  $x_0$  for the leading baryon is selected with a distribution varying from flat for  $K = 1$  to like that for mesons for large  $K$ ; having more cut Pomerons should leave less energy for the

leading baryon. Single diffractive events with a  $1/(1-x_0)$  distribution for one of the baryons are included with a 15% probability for minimum bias interactions. The  $x_i$  for the Pomerons are generated uniformly between 0 and 1, and the sum  $x_1 + \dots + x_K$  is rescaled to  $1 - x_0$ . Then each cut Pomeron is hadronized in its own center of mass using a modified independent fragmentation model, taking the average transverse momentum to be .35 GeV for minimum bias events and .45 GeV for hard scatterings. To incorporate the observed increase<sup>17</sup> in  $dN/dy$ , the splitting function is made energy dependent:

$$\begin{aligned} f(x) &= 1 - a + a(b(s) + 1)(1 - x)^{b(s)}, \\ b(s) &= b_0 + b_1 \ln s. \end{aligned} \quad (18)$$

The probabilities  $P_K$  for  $K$  cut Pomerons are taken to be independent of energy. While it might appear more natural to make the  $P_K$  energy dependent, in the AKG analysis the single particle distribution is completely determined by the single chain graph. The energy dependence of  $f(x)$  corresponds in Regge language to a Pomeron which is more singular than a simple pole, perhaps because of Pomeron interactions.

The scheme just outlined gives a reasonable representation of the UA1 Collaboration<sup>19</sup> data for several processes over a wide range of  $Q^2$ . It also agrees reasonably well with data<sup>20</sup> at fixed target energies.

At SSC energies it is important to include low- $p_T$  QCD jets when studying the effects of minimum bias events. Note that ISAJET uses a different model for the beam jets in 'minimum bias' and 'jet' events, even for arbitrarily soft jets. It is clear that a mixture of the two types should be used, but the correct proportions are at present unknown.

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### Appendix: Implementation

ISAJET is maintained with PATCHY, the CERN Program Library code management system, for CDC 7600 and CYBER 175, DEC VAX, and IBM 370 and 30xx computers. It is written mainly in ANSI standard FORTRAN 77, but there is some machine dependent code, especially in the CDC version. The program is supplied with the FORTRAN code, the table of decay modes, a skeleton of an analysis job, and detailed instructions. ISAJET is available on request from the authors, and PATCHY and the instructions for using it are available from the DD Division of CERN.

The standard ISAJET package supplies subroutines for writing the labeled common blocks containing the event

information to files. On DEC VAX's there is also available a package ISAZEB for writing ISAJET events using the CERN data management system ZEBRA. ISAZEB provides some features which are not part of the usual ISAJET output. Momenta and masses of the partons are recalculated from the final particles so that they match exactly. Resonances decaying strongly are not written out; particles with heavy quarks decaying weakly are made part of a vertex list with a vertex position and with links to the particles belonging to that vertex. Thus there is no need to generate secondary vertices for charmed or other heavy quark particles, and it is easy to find which particles come from the decay of a heavy quark. The association of particles with parent partons is also made easy by reference links. For simple calorimeter studies ISAZEB also provides an option to write out only banks containing energy sums over calorimeter cells and banks containing the leptons. If only these banks are written, the resulting files are more compact and require considerably less CPU time to analyze, although the information is rather limited.

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