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Introduction

The cost of large high energy facilities is a matter of strong concern. Such facilities use very large amounts of electric power and a sizeable part of the total cost is directly related to that fact. It is therefore important to have a clear understanding of the factors that influence the power efficiency, i.e., the amount of power used per unit product, where product is the luminosity generated at a specified energy. The overall efficiency may be regarded as the product of the efficiencies with which the parts of the system perform their functions: the efficiency with which raw grid power is converted to r.f. power, the efficiency with which r.f. power can be transmitted from source to load, the efficiency of conversion of r.f. power into particle beam power and finally the efficiency with which beam power is converted to luminosity.

In this paper we address some factors that are relevant to the latter two. In the first part we investigate three possible ways of converting beam power into luminosity: two short bunches colliding with each other, two long ones doing so, and two pulses of bunch trains which interact. The last mode appears to be preferable for obtaining high efficiencies, but its possibilities are restricted by the limits imposed on the length of the interaction region by the users. These restrictions can be met if the wave

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length λ at which the accelerators work can be chosen small in comparison with that length, this leads to $\lambda < 1$mm. We consider therefore in a second part some of the implications of linacs for very high frequencies, emphasizing the factors that influence the efficiency of converting r.f. power into luminosity and assume that suitable power sources are or will be available. It appears that a resonant linac accelerates single bunches rather less efficiently than pulses of many bunches with the same total charge. The pulse requires a higher operating frequency however, this restricts the charge a single pulse of acceptable length can carry. This can be compensated for by increasing the pulse repetition rate. Operation at higher frequencies may, or may not, permit higher accelerating gradients. This would be an advantage in nearly all respects. Two processes pose limits to the gradient that can be achieved: the occurrence of field emission of electrons from the metal surfaces exposed to the field, and the possibility of physical damage due to mechanical stresses and shock waves generated by differential thermal expansion due to dissipation.

No suitable accelerator structures for very high frequencies are available at present. A third section describes some characteristics of structures that seem feasible.

1.1 Luminosity, beam power and mode of operation

Single pass colliders can be operated in various modes. In one mode a single bunch of electrons is made to interact with a single counter streaming bunch of positrons at a time. The process is repeated periodically with a, generally low, pulse repetition frequency. The expression for the luminosity produced in this mode is well known [1] and may be written as

$$\mathcal{L} = f \frac{N^2}{4\pi\sigma^2} = f \frac{N^2\gamma}{4\pi\varepsilon\beta^*} \qquad (1.1)$$

Here is

$\mathcal{L}$ = luminosity
f = number of bunch interactions per unit time
N = number of particles per bunch (assumed equal in each bunch of an interacting pair)
σ^2 = $\varepsilon\beta^*/\gamma$ square of r.m.s. bunch radius
ε = invariant emittance
γ = E/E_o = energy parameter
β^* = amplitude function in the interaction point.

The expression is valid if the bunch length σ is much smaller than β^*. The energy E_{sb} stored in each bunch is

$$E_{sb} = N\gamma E_o \qquad (1.2)$$

thus the power in the two beams is

$$P_b = 2\, fN\gamma E_o \qquad (1.3)$$

Calling the ratio of the luminosity and the beam power necessary to produce it η_B, one obtains

$$\eta_B = \frac{\mathcal{L}}{P_b} = \frac{N}{8\pi E_o\ \varepsilon\beta^*} \qquad (1.4)$$

The number of particles per bunch is limited either by the number of particles that the source can produce per unit time, i.e., by the source current i_b, or by the accelerators between sources and interaction point. In the former case:

$$N = i_b\, T/e = \frac{i_b}{e}\frac{\lambda}{c} \qquad (1.5)$$

where

e = charge per particle

$T = \frac{1}{\nu} = \frac{\lambda}{c}$ period, frequency and wavelength in free space of the accelerating field

Using this one finds for the efficiency

$$\eta_B = \frac{1}{8\pi e E_o c} \; \frac{i_b}{\varepsilon} \; \frac{\lambda}{\beta^*} \tag{1.6}$$

The first term in this expression is inviolate since it consists of constants of nature, the second one is a source parameter. Only the third term, λ/β^*, is available for adjustment. The requirement that the bunches be short compared to β^* implies a coupling between λ and β^*: $\lambda/\beta^* = (\lambda/\sigma_z)(\sigma_z/\beta^*) = \bar{B}\ \sigma_z/\beta^*$, where $\bar{B} = \lambda/\sigma_z$ is the bunching factor. If σ_z/β^* is regarded as restricted $\eta_B \propto \bar{B}$. The accelerator restricts the beam current to a value that is proportional to λG, where G represents the accelerating gradient. We conclude therefore that efficient single bunch operation requires bright sources, large bunching factors $\bar{B}$, large accelerating gradients G and a long wavelength, thus a low operating frequency.

1.2 Single long bunch.

If the colliding bunches are not short compared to β^* their local cross sections at any instant will be functions of position along the axis, because β varies with position. Assuming symmetry relative to the interaction point and disregarding the effects of the beam self fields, one has for $\beta = \beta(z)$:

$$\beta = \beta^*(1 + (z/\beta^*)^2) \tag{1.7}$$

where z is the distance to the interaction point.

The contribution to the instantaneous luminosity by a section of length dz at location z is then:

$$d\mathcal{L} = \frac{2\ i_b/e\ i_b/ec}{4\pi} \; \frac{\gamma}{2\beta^*} \; \frac{dz}{1 + (z/\beta^*)^2} \tag{1.8}$$

where i_b represents the instantaneous beam current. The contribution lasts as long as both colliding bunches are present at location z. Consider first bunch 1, which has length σ_z and moves in the direction of positive z. It will be present at location z during the interval $z/c \leq t \leq (z + \sigma_z)/c$. Bunch 2, also of length σ_z, but traveling in the opposite direction will be there while $-z/c \leq t \leq (\sigma_z - z)/c$; t=o represents the time at which each bunch reaches the interaction point. It follows that both bunches are present while

$$(\sigma_2 - z)/c \leq t \leq (\sigma_z + z)/c. \qquad (1.9)$$

The length of the interval of exposure is

$$\Delta t = (\sigma_z - 2|z|)/c$$

regardless of the sign of z because of the symmetry of the system. $\Delta t = 0$ for $|z| \geq \sigma_z/2$ because there the counter streaming bunches do not appear simultaneously. Using this and assuming for convenience that i_b is constant, i.e., that the bunch charge is distributed uniformly along its length, one obtains for the contribution to the time integrated luminosity:

$$\int \delta\mathcal{L}\, dt = \frac{2\, i_b^2}{4\pi e^2 c^2} \frac{\gamma}{\varepsilon\beta^*} \frac{\sigma_z - 2|z|}{1 + (\frac{z}{\beta^*})^2}\, dz$$

Integration over the full length $-\frac{1}{2}\sigma_z \leq z \leq \frac{1}{2}\sigma_z$ yields

$$\int \mathcal{L} dt = 2\frac{2 i_b^2}{4\pi e^2 c^2} \frac{\gamma}{\varepsilon\beta^*} \sigma_z\beta^*[\arctan(\sigma_z/2\beta^*) - \frac{\beta^*}{\sigma_z}\ln(1+(\sigma_z/2\beta^*)^2)] \qquad (1.10)$$

Repeating this process at a rate of f times per unit time one finds that the time averaged luminosity is

$$\mathcal{L} = f\,\frac{i_b^2}{\pi e^2 c^2}\,\frac{\gamma}{\varepsilon}\,\sigma_z\,F(\sigma_z/2\beta^*) \qquad (1.11)$$

where

$$F(x) = \arctan(x) - (1/2x)\ln(1 + x^2)$$

This luminosity requires a beam power P_b:

$$P_b = 2\,f\,\frac{i_b}{e}\,\frac{\sigma_z}{c}\,\gamma\,E_o \qquad (1.12)$$

so thus the efficiency becomes

$$\eta_B = \frac{\mathcal{L}}{P_b} = \frac{2}{4\pi e c E_o}\,\frac{i_b^2}{\varepsilon i_b}\;F(\sigma_z/2\beta^*)$$

$$= \frac{2}{4\pi e c E_o}\,\frac{i_b}{\varepsilon i_b}\,\bar{B}\,F(\sigma_z/2\beta^*)$$

$$= \left(\frac{1}{8\pi e E_o c}\,\frac{i_b}{\varepsilon}\,\frac{\gamma}{\beta^*}\right)2\left(\frac{2\beta^*}{\sigma_z}\right)F(\sigma_z/2\beta^*) \qquad (1.13)$$

The first term in parentheses in (1.13) corresponds to (1.6) which is valid for $\sigma_z \ll \beta^*$. In Table I, I tabulate some values for F(x) and 2F(x)/x as functions of x.

Table I: F(x) and 2/x F(x) as functions of α.

x	F(x)	$\frac{2}{x}$ F(x)
0.01	0.005	1
0.03	0.015	1
0.1	0.050	0.998
0.3	0.148	0.986
1	0.439	0.878
3	0.865	0.577
10	1.240	0.249
30	1.424	0.095

It may be seen that there is little point in demanding $\sigma_z \ll \beta^*$ as far as efficiency is concerned, $\sigma_z = 2\beta^*$ is certainly acceptable in that respect, as is $\sigma_z = 6\beta^*$, in all likelihood. However, the bunch length is limited by considerations of energy spread to a fairly small fraction of λ: for $|\Delta E/E| < 0.05\ \sigma_z/\lambda = 1/\bar{B} < 0.14$, thus $\bar{B} > 7.2$, where E stands for energy. This then sets an upper limit to β^*:

$$\beta^* < 0.1\lambda$$
$$\sigma_z < 0.14\lambda$$

for good efficiency.

1.3 Multibunch operation. (Fig. 1)

The coupling between β^* and λ can be broken by switching to multibunch operation. The basic idea is to subdivide the long bunch discussed above into N_B small ones, each with a charge N/N_B particles and with a center to center distance, thus a new wavelength Λ, of

$$\Lambda = L/(N_B - 1) \qquad (1.20)$$

Here is L the length the interaction region, which is equal to the length σ_z of the original long bunch. There are now N_B collision points, spaced at distances $1/2\ \Lambda$ along the system axis, in which a total of N_B^2 bunch-bunch collisions occur per pulse. In calculating the overall luminosity and efficiency one has to calculate those quantities separately for each interaction point because the β's are different in the various points, and because the number of interactions per pulse in a particular point decreases linearly with its distance to the central one. Computer simulation shows little difference between the exact result and the analytical one for the long bunch, particularly if N_B is large. In this way I find for the efficiency of this multibunch mode

$$\eta_B = (\mathcal{L}/P_b)_{mb} = \frac{1}{2\pi e E_o c} \frac{i_b}{\varepsilon} F(L/2\beta^*) \frac{N_B}{N_B-1} \qquad (1.21)$$

where I used (13) and (20) and where $N_B \geq 2$. This form is evidently independent of the choice of wavelength (although that variable may enter into the beam current i_b, as mentioned before), and nearly independent of that of N_B. It appears, therefore, that the efficiency in this mode is determined by the characteristics of the source, i.e., by i_b/ε and by the choice of $L/2\beta^*$, i.e., by the ratio of the length of the interaction region and β^*. It is evident from Table I that there is not much point in choosing $L/2\beta^* \gtrsim 10$.

Multiplication of any of the expressions for η_B, e.g., (21) with the beam power P_b yields the luminosity itself. Since

$$\begin{aligned} P_b &= 2\, i_b \gamma E_o T f \\ &= 2\, i_b E_t \frac{L}{c} f \end{aligned} \qquad (1.22)$$

where $E_t = \gamma E_o$ one finds that

$$\mathcal{L} \propto L\, i_b^2/\varepsilon$$

1.4 Comments.

It has been assumed, so far, that single particle optics is valid. In practice the beam densities will be sufficiently high that this is not true. The computer model referred to before incorporates a linear approximation of this beam-beam or beam disruption effect and shows clearly that things change with increasing density. It describes each bunch/bunch interaction as the application of a focussing lens that transforms the emittance ellipse of each of the two participating bunches. The strength of that lens is adjusted according to the local beam radius and is proportional to the sum of the charges in the two bunches involved. The initial conditions can be set up for each bunch individually so that Brian Montague's proposal for dynamic focusing [2] can be simulated, but this has not yet been tried. Another important factor that has been disregarded is the so-called beamstrahlung. Its effect is likely to be different in multibunch operation from what it is in short single bunch operation and will be studied.

2.1 Acceleration.

We now consider some aspects of accelerators that might be useable for our purposes. The principal factors of interest are the energy spent per unit energy in the final beam and the average accelerating gradient. The first factor enters immediately into the operating cost of the facility and indirectly into its capital cost, the second one directly into the capital cost. The characteristics of those accelerators will depend on the choice of operating mode: single bunch or multibunch. Developments of the induction linac, wakefield accelerator or switched power linac might be suitable for single bunch operation while the multibunch mode needs something similar to the conventional electron linac. The first three accelerators are all single pulse, wide band, non-resonant devices; the electron linac is a high Q, narrow band, resonant device. This latter accelerator can also be used for the acceleration of single bunches but then its energy efficiency is low. A convenient parameter for guiding the choice of r.f. parameters is the ratio

η_p of the luminosity and the r.f. power spent to produce it, or, equivalently, that of the integrated luminosity, integrated over a single beam-pulse and the r.f. energy invested in that pulse. We use the latter definition for convenience. An expression for the integrated luminosity was derived in the previous section, the present one addresses the determination of the r.f. energy. This requires a description of the accelerator installation. It appears that η_p, once an expression for it is available, can be maximized by proper choice of the r.f. frequency or wavelength and the coupling factor between the source of the r.f. power and the accelerator. This is a consequence of the fact that the system is pulsed to yield a beam-pulse length equal to the desired length of the interaction region. Each beam pulse is preceded by a filling period during which r.f. energy is spent in building up the fields in the accelerator to their nominal levels. This energy increases quickly with increasing wavelength and is lost after the last bunch of a pulse has passed if it is not recovered. The integrated luminosity due to a beam pulse of fixed duration increases also with wavelength because the tolerable beam current does. However, the energy expanded during the filling time increases faster than the integrated luminosity, so that an optimum wavelength, whose value depends on the length of the beampulse, must exist.

Consider as a prototype an accelerator that consists of a string of independent cavities, each supplied by its power source via an ideal, i.e., lossless and reflection free, power transmission system. The length of that system is such that the time it takes a wavefront to travel from one of its ends to the other is more than half the length τ_s of the powerpulse. The beam pulse begins only a filling time τ_F after the arrival of the power pulse front at the cavity and terminates simultaneously with the power pulse. The restriction on the length of the transmission guide ensures that the power source is turned off before reflections due to mismatches between guide and cavity or any signals from the beam can reach it. It is therefore always loaded by the characteristic impedance of the transmission line during the pulse, regardless of the conditions at the cavity end while the source impedance seen by cavity and beam is always the characteristic impedance of the transmission line at its cavity side. It is convenient to represent the cardinal features of this prototype with the equivalent circuit diagram of Fig. 2:

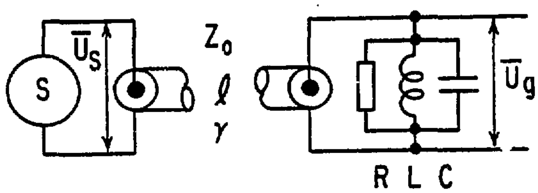

Fig. 2 Equivalent Circuit Diagram of Linac Cell

ℓ, γ, R_0 represent the length, propagation constant and characteristic impedance of the transmission system and L, C, R the magnetic, electric and dissipative parts of the cavity impedance, S is the power source, it produces a voltage $\bar{U}_s \cos \omega\tau$, U_g is the gap voltage and i_b is the beam current. The dissipative losses, which are primarily caused by eddy currents in the cavity walls, would be represented more accurately by a small resistor in series with the inductor than by the large parallel resistor shown, however, the difference is immaterial for the present purpose and the representation chosen is slightly more convenient. The beam consists of a stream of bunches whose lengths are short compared to the intra-bunch distances. The Fourier expansion of the beam current has therefore a dc component and many harmonics of the bunch repetition frequency, which we take to be identical to the r.f. frequency $\omega/2\pi$. We are only concerned about its fundamental component, which has an amplitude $\bar{i}_b$ that is twice the dc component i_b. The beam current is represented as coming from a current source, this is an acceptable approximation if the particle energy γE_o is so high that fractionally small changes cause negligible fractional change in the velocity βc, i.e., if

$$\Delta\beta/\beta = \frac{1}{\gamma^2-1}\,\frac{\Delta\gamma}{\gamma} = 0$$

Via this model one finds for the behaviour of the gap voltage amplitude $\bar{U}_g$ as a function of time in the absence of beam:

$$\bar{U}_g(t) = \hat{U}_g\,(1 - e^{-t/\tau}) \tag{2.1}$$

t = o represents the instant of arrival of the power pulse front at the cavity and the resonant frequency ω_r of the cavity is equal to the frequency of the power source:

$$\omega_r = \omega \tag{2.2}$$

$$\frac{2}{\tau} = \frac{1}{C}\left(\frac{1}{R_o} + \frac{1}{R}\right) \tag{2.3}$$

$$\omega_r^2 = 1/LC - 1/\tau^2 \tag{2.4}$$

ω_r is real if

$$\frac{LC}{\tau^2} = \frac{1}{2Q}\left(1 + \frac{R}{R_o}\right) < 1$$

thus if

$$R_o/R > 1/(2Q-1) \tag{2.5}$$

where $Q = R\sqrt{\frac{C}{L}}$ represents the quality factor of the unloaded resonator.

The asymptotic value U_g of the gap voltage amplitude is related to the powersource voltage (as measured at the cavity) according to

$$U_g = 2\,\bar{U}_s/(1 + R_o/R). \qquad (2.6)$$

It follows that the filling time τ_F necessary for a specific gap voltage amplitude $\bar{U}_g$ is

$$\tau_F = -\,\tau\,\ln(1 - \bar{U}_g/U_g) \qquad (2.7)$$

The effect of a beam current $\bar{i}_b$ may be described as a change in the amplitude of the effective power source voltage from $\bar{U}_s$ to $\bar{U}_s - 1/2\,\bar{i}_b\,R_o$. This expression is valid if the source voltage and the beam current are in phase; the situation becomes algebraically much more complex if they are not: phase factors have to be added, the phase between cavity voltage and drive will change and switching transients will have to be taken into account.

Using the simple model one obtains for the new asymptotic gap voltage U_{gb}:

$$\hat{U}_{gb} = \frac{2\bar{U}_s - \bar{i}_b\,R_o}{1 + R_o/R} \qquad (2.8)$$

A beam current $\bar{i}_b$ is such that

$$\bar{U}_g = \hat{U}_g(1 - e^{-\tau_F/\tau}) = \hat{U}_{gb} \qquad (2.9)$$

will therefore terminate the filling period and begin the accelerating period, during which the gap voltage amplitude remains $\bar{U}_g = U_{gb}$. This requires

$$\bar{i}_b = (\hat{U}_g - \bar{U}_g)(\frac{1}{R_o} + \frac{1}{R})\ . \qquad (2.10)$$

$\bar{U}_g$ will continue to change until this condition is satisfied if $\bar{i}_b$ deviates from this value. Successive bunches that pass while $\bar{U}_g$ is changing will receive different gains in energy. Such differences increase the energy spread in the ultimate beam and are undesirable. It is therefore important that the power source, filling time, beam current and phases be matched properly.

The number N_c of cavities to reach a specified final energy ΥE_o is simply:

$$N_c = \frac{\Upsilon E_o}{\Delta E_p} = \frac{\Upsilon E_o}{\bar{U}_g \, F_{tr}} \qquad (2.11)$$

where $\Delta E_p = \bar{U}_g \, F_{tr}$ represents the energy gain per particle per gap and F_{tr} the so-called transit time factor, introduced to account for the fact that a particle crosses a gap of non-zero length in a non-zero time, which may be as long as half an r.f. period. It corrects also for the non-uniformity of the gap field, due to end effects.

Before an expression can be written for the efficiency η_p a relation between beam current and emittance must be established. If we assume that the beam originates in a source of constant brightness, i.e., such that the density it produces is x, x', y, y' phase space is independent of the current, we may write

$$i_b = \tilde{B} \, \varepsilon^2$$

where $\tilde{B}$ is the brightness of the particle source.

η_p is now calculated by performing the division:

$$\eta_p = \frac{\int_{\tau_B} \mathcal{L}dt}{2N_c \int_{\tau_s} P_s dt} \tag{2.13}$$

$\tau_B = L_B/c$ is the length in time of the beam pulse, L_B its physical length = length of interaction area, $\tau_s = \tau_F + \tau_B$ is the length in time of the power pulse; the factor 2 keeps account of the fact that there are two, presumably identical, beams. Rewriting (1.10) and (1.11) we have for the time integrated luminosity:

$$\int_{\tau_B} \mathcal{L}dt = \frac{i_b^2}{4\pi e^2 c^2} \frac{\gamma}{\varepsilon} L_B F(L_B/2\beta^*) \tag{2.14}$$

The energy delivered by the power sources is:

$$N_c \int_{\tau_s} P_s dt = \frac{\gamma E_o}{\bar{U}_g F_{tr}} \frac{1}{2} \frac{\bar{U}_s{}^2}{R_o} (\tau_F + \tau_B)$$

$$= \frac{\gamma E_o}{\bar{U}_g F_{tr}} \frac{1}{2} i_b^2 R_o \tau_B \left[1 + \frac{\bar{U}_g}{2 i_b R_o} \left(1 + \frac{R_o}{R}\right)\right]^2 \left[1 + \frac{\tau}{\tau_B} \ln\left\{1 + \frac{\bar{U}_g}{2 i_b R_o}\left(1 + \frac{R_o}{R}\right)\right\}\right] \tag{2.15}$$

where we used $\bar{i}_b = 2\, i_b$, (2.6), (2.7) and (2.10).

The expression for η_p may now be derived.

$$\eta_p = \frac{\int_{\tau_B} \mathcal{L}dt}{2N_c \int_{\tau_s} P_s dt} =$$

$$= \frac{\frac{1}{4\pi e^2 c} i_b^2 \sqrt{\frac{\bar{B}}{\bar{i}_b}}\, \gamma\, \tau_B\, F(L_B/2\beta^*)}{2 \frac{\gamma E_o}{\bar{U}_g F_{tr}} \frac{1}{2} i_b^2 R_o \tau_B \left[1 + \frac{\bar{U}_g}{2 i_b R_o}\left(1 + \frac{R_o}{R}\right)\right]^2 \left[1 + \frac{\tau}{\tau_B} \ln\left\{1 + \frac{\bar{U}_g}{2 i_b R_o}\left(1 + \frac{R_o}{R}\right)\right\}\right]}$$

$$= \frac{F(L_B/2\beta^*)F_{tr}\sqrt{\bar{B}}}{4\pi\ e^2c\ E_o} \frac{\sqrt{i_b}\quad \frac{\bar{U}_g}{i_b R_o}}{[1 + \frac{\bar{U}_g}{2i_b R_o}(1+\frac{R_o}{R})]^2\ [1+ \frac{\tau}{\tau_B}\ln\{1+ \frac{\bar{U}_g}{2i_b R_o}(1+ \frac{R_o}{R})\}]} \tag{2.16}$$

(2.16) may be represented by

$$\eta_p = A \frac{x\sqrt{\lambda}}{(1+x)^2\ (1+a\lambda)} \tag{2.17}$$

Here is

$$x = \frac{\bar{U}_g}{2i_b R_o}\ (1+ \frac{R_o}{R})$$

$$= \frac{2\pi F_{tr}}{\bar{\alpha}}\ \frac{1}{R_o}\sqrt{\frac{L}{C}} \frac{(1 + \frac{R_o}{R})}{\sqrt{1 - (\frac{1}{2Q})^2\ (1+ \frac{R}{R_o})^2}}$$

$$\approx \frac{2\pi F_{tr}}{\bar{\alpha}}\ \frac{1}{R_o}\sqrt{\frac{L}{C}} \qquad (R \gg R_o,\ QR_o/R \gg 1)$$

$$a = \frac{2}{L_B}\ \frac{F_{tr}}{\bar{\alpha}}\ \frac{\ln(1+x)}{x}$$

$$A = \frac{F(L_B/2\beta^*)F_{tr}\ \tilde{B}}{4\pi\ e^2cE_o} \sqrt{\frac{\bar{g}\ \bar{G}\ \bar{\alpha}}{4\pi\ F_{tr}}\sqrt{\frac{C}{L}}\ [1-(\frac{1}{2Q})^2\ (1+\frac{R}{Ro})^2\ ]\frac{2}{1+R_o/R}}$$

$$= 2\ \frac{F(L_B/2\beta^*)}{4\pi\ e^2cE_o}\sqrt{\frac{g\ \bar{B}\ \bar{G}\ \bar{\alpha}\ F_{tr}}{4\pi}\sqrt{\frac{C}{L}}}$$

$$\bar{\alpha} = \frac{\text{energy gain per bunch}}{\text{energy stored}}$$

with $g\lambda$ the length of the acceleration gap and $\bar{G}$ the field in it. At this level of approximation x, a, A, g and $\bar{\alpha}$ are all more or less independent of the choice of the wavelength λ, particularly if $R_o/R \ll 1$ and if $(1 + R/R_o) \ll 2Q$, with Q the quality factor of the unloaded cavities. We note that $L/C \leq 200\ \Omega$, $g \leq 0.5$, $F_{tr} < 1$ are all geometrical form factors whose exact values are set by the geometry of the accelerating cavities and the choice of operating mode (π, $2\pi/3$ or other). The source brightness $\tilde{B}$ and the accelerating field G should be maximized. The design of the accelerator cannot affect $\tilde{B}$, presumably, while $G \stackrel{?}{=} G(\lambda, T_B)$ is restricted by surface effects on the cavity walls: field emission of electrons, sparking, physical damage due to dissipation, etc. L_B, the length of the beam pulse, but also the length of the interaction region, is restricted by the users, as is $\bar{\alpha} \stackrel{?}{=} \bar{\alpha}(\lambda) < 0.05$, since the momentum spread in the beam tends to increase with $\bar{\alpha}$ while the beam becomes less stable with increasing $\bar{\alpha}$.

Returning to (2.17) one finds that η_p is maximized by choosing $\lambda \approx 1/a$ and $x \approx 1.4$. For those values

$$\eta_p \approx 0.12\ A/\sqrt{a}$$

x can be manipulated via the product $\bar{\alpha}\ R_o$, thus

$$R_o \approx \frac{1}{x}\ \frac{2\pi\ F_{tr}}{\bar{\alpha}}\sqrt{\frac{L}{C}} \approx 0.71\ \frac{2\pi}{\bar{\alpha}}\sqrt{\frac{L}{C}} \tag{2.18}$$

Substitution in (2.16) yields

$$\eta_p \approx 0.217\ \frac{\bar{\alpha}\ F(L_B/2\beta^*)}{4\pi\ ec\ E_o}\ \ \frac{g\ \bar{G}\ \tilde{B}\ L_B}{4\pi}\sqrt{\frac{C}{L}} \tag{2.19}$$

$$\propto \bar{\alpha}\sqrt{\bar{G}\bar{B}L_B}$$

$$\lambda_{opt} = 0.8\,\frac{\bar{\alpha}}{F_{tr}}\,L_B$$

$$R_o \doteq 0.71\,\frac{2\pi\,F_{tr}}{\bar{\alpha}}\sqrt{\frac{L}{C}}$$

It is clear that for maximum η_p, $\bar{\alpha}$, L_B, $\bar{B}$ and G should all be as large as possible and that the optimum wavelength and source impedance are directly tied to the choice of α and L_B.

At the optimum wavelength the filling time and beam pulse length (in time) are equal, thus the source pulse length is then twice the beam pulse length. Since the length of the interaction region L_B will not be more than a few cm, the length of the power transmission system has to exceed those few cm, in order to validate our initial assumption. In practise it will be difficult to violate this condition, short of integrating the power source directly with the accelerating cavity. It seems that for optimum efficiency η_p one has to operate as close to the permissible limits in beam current (via $\bar{\alpha}$) and accelerating gradient as possible. The instantaneous luminosity can then only be controlled via the final focus, i.e., β^*. The average luminosity can always be changed via the pulse repetition rate.

The results assembled in Table II illustrate the behaviour of systems of this type. We took

$\tilde{B} = 8 \times 10^{10}\ (A/(\text{rad-m})^2)$

$\tilde{G} = 10^9\ V/m$

π mode, i.e.

$g = 0.5$

$F_{tr} = 2/\pi$

$\sqrt{L/C} = 200\Omega$

$R/\sqrt{\lambda} = 5.3 M\Omega/m^{1/2}$

$F(L_B/2\beta^*) = 0.44$

$\beta^* = 1/2\ L_B.$

We chose two interaction lengths:

$$L_B = 1\text{ cm},\ 10\text{ cm}$$

corresponding with $\tau_s = \tau_F + \tau_B = 2\tau_B = 64$ psec, resp 640 psec, and five $\tilde{\alpha}$ values in the interval $0.01 \leq \alpha \leq 0.05$.

In this table $$\eta_p = \frac{\langle \mathcal{L} \rangle}{\langle P \rangle} = \frac{\int \mathcal{L} dt}{\int P_s dt} \quad [\text{cm}^{-2}\ \text{sec}^{-1}\ W^{-1}]$$

N_B number of bunches per pulse.

P_s r.f. power/cavity during the pulse

$\partial P_s/\partial l = P_s/g\lambda$ r.f. power per unit length during the pulse.

N_{33} Pulse repetition rate for $L=10^{33}$ cm^{-2} sec^{-1} at final energy γE_o.

Table II.

L_B (cm)	α	n_p (10^{22})	λ_{opt} (mm)	R_o (kΩ)	R (kΩ)	i_b (A)	P_s (MW)	$\partial P_s/\partial l$ (GW/m)	γN_{33}	N_B
1 10	0.01	2.5 8	0.12 1.25	57	59 188	0.4 4	0.025 2.5	0.4 4.0	4.8×10^{14} 1.5×10^{12}	80
1 10	0.02	5 16	0.25 2.51	28	84 265	1.6 16	0.201 20.1	16 8.0	6.0×10^{13} 1.9×10^{11}	41
1 10	0.03	7.5 24	0.38 3.77	19	103 325	3.5 35	0.68 68.0	3.6 36	1.8×10^{13} 5.6×10^{10}	28
1 10	0.04	10 31	0.50 5.02	14	119 375	6.3 63	1.60 160	6.4 64	7.4×10^{12} 2.4×10^{10}	21
1 10	0.05	12 39	0.63 6.28	11	133 420	9.8 98	3.15 315	100 100	3.8×10^{12} 1.2×10^{10}	17

It is noted that for L_B and $\bar{\alpha}$ small our approximation $R_o \ll R$ breaks down. The associated values should be recalculated, taking this effect into account. It appears that heavy loading ($\bar{\alpha}$ large) and long interaction lengths are favourable for efficiency, but also that that leads to very high peak powers P_s and $\partial P_s/\partial \ell$, for which there is, presumably, an upper bound. The pulse repetition rates are very high as a consequence of our very modest estimate of the achievable source brightness $\tilde{B}$, they decrease as $(\tilde{B})^{-1/2}$.

An important restriction is imposed by the beamstrahlung, i.e., by the changes in the energies of individual particles due to synchrotron radiation caused by the beam self field. So far this complex subject seems to have been studied only for single bunch interactions, but not yet for the multibunch mode discussed here [3]. There is a critical energy

$$E_{cr} = 3\,\hbar\, c\, r_e\, \gamma^2\, N/(r\sigma_z)$$
$$= 3\,\frac{\hbar}{e}\, r_e\;\; i_b^{3/4}\gamma^{5/2}\; \bar{B}\; \tilde{B}^{1/4}/\sqrt{\beta}$$
$$= 2 \times 10^{-11}\; i_b^{3/4}\gamma^{5/2}\; \bar{B}\; \tilde{B}^{1/4}/\sqrt{\beta} \quad [\mathrm{eV}]$$

where $\hbar$ = $h/2\pi$ Planck's constant

r_e classical radius of electron

$\bar{B}$ bunching factor (peak/average)

One finds, by simple scaling, that, in the crudest approximation, the relative energy change δ due to beamstrahlung should behave as $\bar{B}\lambda^{1/2}$ for $\Upsilon < \Upsilon_{cr}$ and as $\bar{B}^{-1/3}\lambda^{-1/2}$ for $\Upsilon > \Upsilon_{cr}$. Since $\Upsilon_{cr} \propto \gamma^{5/2}$ one is forced to operate in the second regime if the energy is sufficiently high. Using the earlier numerical assumptions one expects the changeover to occur somewhere in the 1-10 TeV range.

3.1 Accelerating structures.

A number of structures that support accelerating modes have been described [4] and model studies, which demonstrated the existence of such modes, have been performed on some of them. However, the support of accelerating modes, though necessary, is not sufficient. A second condition is that there be no beam deflection since such deflection would be the cause of energy loss due to synchrotron radiation. Such loss is proportional to γ^4/ρ^2 with $\gamma = E/E_0$ and ρ the local instantaneous radius of orbit curvature. Any curvature leads always, regardless of its sign, to a loss because of the quadratic relationship, and the loss increases sharply with increasing energy. The radiation is directed along the beam and will have practically the same velocity as the electrons so that each bunch is a mixture of electrons (or positrons) and photons. Some of it will hit the accelerator structure, and cause emission of electrons and γ rays. Minimization of the synchrotron radiation requires evidently minimization of orbit curvature. The net field in the accelerator, basically a standing wave, may be described as a superposition of many modes. Only a few of these, if any, are accelerating modes, but nearly all can contribute to beam deflection. Requiring virtual absence of deflection presents therefore a severe restriction, which, however, can be met by imposing certain symmetry conditions on the transverse geometry, as demonstrated by existing linacs. These machines have circular cylindrical symmetry about the machine axis. Though their structures still support certain deflecting modes, such modes occur only due to asymmetry in the excitation, e.g., in the connection to the power source, or, in the case of collective effects, to beam-axis misalignments; they are generally weak. Circular cylindrical symmetry becomes increasingly difficult to arrange if the wavelength becomes smaller, but structures with two-fold and higher symmetry offer similar characteristics relative to deflection: although deflecting modes are possible, the nominal net field has no transverse dipole component in the vicinity of the axis.

Lately the emphasis of our studies has been on two structures which might be acceptable. One, the foxhole structure, can be seen as extrapolation from the conventional linac structure, the other, the colonnade, invented by Palmer, may be seen as a development of the gratings with which this enterprise began [4]. Both seem feasible down to and including λ = 10 μm on the basis of the experience with micro machining by, e.g., ion etching, we have gained so far. Model studies are in progress for each.

3.2 Foxhole structure (2π mode).

Let me consider the foxhole structure first because it is relatively simple and closest to present practice. A sketch for its simplest version is given in Fig. 3. It shows a base plate in which rectangular holes, the foxholes, have been formed. The cross section of each hole is of order 1/2 λ x 1/2 λ, depth an integer multiple of 1/4 Λ and the distance between centers is λ. Here is Λ the local wavelength in the foxhole. They are interconnected by slots through which the beam passes, each slot has a width of order 0.1λ and reaches from top to bottom. Geometries of this type can be realized, presumably with sufficient accuracy, by means of techniques with which we have some experience at BNL. The structure is excited by a travelling wave of e.m. radiation that propagates perpendicularly to the base plate with its E vector directed along the slots. Each foxhole acts as a resonator, I shall neglect the coupling between them. The incident radiation generates a standing wave in each foxhole with a node for the magnetic field and a maximum for the electric one in the midplane at 1/4 Λ from the bottom. The beam axis is located at that height. The fields in all resonators are in phase and the acceleration process is reminiscent of that in an Alvarez proton linac, with the slots acting as drift tubes and the resonators as accelerating gaps. Regarding each resonator as a cavity with dimensions h x w x d, as indicated in Fig. 3, and assuming a transverse electric mode one finds for the resonant wavelength in free space

$$\lambda = \left[\left(\frac{1}{2w}\right)^2 + \left(\frac{n}{4h}\right)^2 \right]^{-1/2}$$

when n is the number of quarter wavelengths along the height. Choosing n = 2 the electric field across the open end of the resonator will be zero if there is no energy loss in the cavity, for n = 3 the magnetic field will be zero at that location. In both cases the incident radiation will reflect totally, the Poynting vector will be zero everywhere in this end surface and the amplitudes of the magnetic and electric fields in the cavity will be twice those in the incident wave. In actuality some energy is lost in the cavity to dissipation in the resistivity of the walls and to the beam. The lost energy is replenished by the incident wave, resulting in a small in phase component of the electric, resp. magnetic field in the open end plane of the cavity. There will be no reflection if the E/H ratio of the incident wave matches that at the mouth of the cavities and all its energy is absorbed. The first case represents a low impedance match with a small value for E/H, the second with its large electric field and low magnetic field represents a high impedance match. The value of the impedance $Z = E/H$ is easily calculable from the physical constants of the cavity, the matching conditions can be realized by proper arrangement of the source and of the optical system between it in the cavity orifice.

The power source would produce a beam which is focussed on the apertures of the resonators. Its cross section in that aperture plane would be a pulse length, i.e., $c\tau_s$, long in the direction of motion of the beam, and its width would cover the resonator apertures. The beam spot might be made to move synchronously with the train of bunches it is accelerating, or the accelerator could be built in sections, each with its own power source, as is standard practice for conventional accelerators. There would have to be many short sections for reasons of energy efficiency. The length of the radiation pulse would have to be longer than the beam pulse by $2N_s$ r.f. periods plus a filling time if a section is N_s wave lengths long: the last cavity is excited during N_s periods before the head of the beam reaches it while the first one is driven during N_s periods after the last bunch of a bunch train has left it.

I have disregarded so far the perturbation introduced by the slots. Slots in the boundary walls of wave guides and cavity resonators have been used for a long time and for various reasons. In this particular case their effect is thought to be small for the desired mode of operation, since there are no wall currents that have to cross them. The slots themselves act as wave guides that are driven in a higher mode, their characteristic impedance is low and they are close to a half wavelength long. Our model studies show the existence of the desired mode in a single cavity with the appropriate slots, which, however, are only a small fraction of a wave length long in the model used.

The presence of the slots, which divide the structure into two mirror symmetric halves suggests the possibility of constructing it in two halves. Doing so adds important flexibility to the design of the resonators and slots, they have no longer to be cylindrical and fabrication may be easier since the depressions to be generated are less deep by factors of 2 to 3.

3.3 Foxhole structure (π and $2\pi/3$ modes).

The foxhole structure described above operates in the 2π mode, like most Alvarez linacs; its effective accelerating gradient is therefore only $F_{tr} \lessapprox 1/\pi = 0.31$ of the amplitude of the resonator field. Operation in the π, resp $2\pi/3$ modes would yield factors of $F_{tr} \lessapprox 0.62$ and $\lessapprox 0.86$, because they use the available space and field more efficiently than the 2π mode. Their realization requires the use of two or three resonators per wavelength and a large reduction in the lengths of the slots between successive resonators. The resonators would no longer run in phase but with phase differences of π rad (π mode) or $2\pi/3$ rad ($2\pi/3$ mode) between them. Although the latter is standard practice in conventional electron linacs it produces problems if the power source excites the resonators in parallel. These angles are too large to be obtained by simple detuning of the resonators relative to the frequency of the power source: the relative loss in field amplitude due to the detuning is larger than the gain derived from this mode of operation.

One solution for a π mode structure could be to dimension alternate resonators differently: both types would resonate with the source frequency but their heights would have the ratio $h_1/h_2 = 5/6$. As indicated in Fig. 4 the shorter one would be 3/4 of a local wavelength long, the longer 5/4 of a different local wavelength. The field maximum in the longer one would occur at the same depth as the second field maximum in the shorter one, so that the beam sees a phase reversal. This trick cannot be used for the $2\pi/3$ mode since in essence the cavity fields are still in phase. Other solutions may be achieved by driving the resonators in groups of two (π mode) or three ($2\pi/3$ mode) while coupling the resonators within a group in a suitable manner. This is likely to require more complex geometries for the resonators than the simple cylindrical ones considered so far.

3.4 Colonnade (2π mode).

A second structure under study is Palmer's colonnade. This structure is a truly "open" one, in contrast with the foxholes, which are only semi-open at best. As shown in Fig. 5 it consists of a base plate on which two parallel rows of cylinders have been placed. The cylinders are not necessarily circular in cross section. The beam axis is located in the midplane between the cylinders at some distance from the base plate. In the simplest version, which operates in the 2π mode, the distance between successive cylinders along the axis is exactly λ, the distance between the rows about $1/2\ \lambda$. Fig. 6 shows an experimental realization of a colonnade intended for 10 μm radiation. The structure is illuminated from above by a long source with its axis parallel to the beam path axis, the e.m. beam is focussed down to a narrow strip which covers the tops of the cylinders. The beam is polarized with its electric vector along the system axis. All cylinders oscillate in phase, and the distance between the rows is adjusted to prevent any net radiation perpendicular to the system axis. The system may be regarded as an antenna array and it is easy to see that it supports waves that travel in both directions along the axis [5] The absence of radiation in the transverse plane depends upon the mutual cancellation of the elementary waves from each element and requires high dimensional accuracy. Misalignments from the design cause radiation which shows up as a reduction of the Q of the system. Such a depression has nothing to do with the dissipative losses in the cylinders.

The cancellation of transverse waves and the presence of longitudinal waves suggest strong coupling between the cylinders. Local e.m. energy will be redistributed along the length of the structure with group velocity as in conventional linacs. This is wasteful since the beam is no more than a few mm long along the axis. Local energy concentrations become possible if the group velocity is sufficiently small. A convenient measure is that the power loss due to radiation from a cell (formed by 4 adjacent half cylinders) should be small compared to the dissipation in that cell.

Let me assume that a sufficiently long, if need be infinitely long, section of the colonnade is uniformly illuminated. The incident beam wave fronts will continue to travel towards the base plate after they have arrived at the tops of the cylinders. They will be reflected there and return, forming a standing wave with planes of nodes and maxima that are parallel to the surface of the base plate and separated from it by multiple 1/4 Λ's, with Λ the local wavelength, as modified from the free space λ by the presence of the cylinders. The system axis is defined by the intersection of the midplane with the first magnetic nodal plane.

The magnetic dipole field is zero in the vicinity of this axis while the amplitude of longitudinal electric field is maximum. The fields are periodic in time with the frequency of the incident radiation and periodic in space with the periodicity of the structure. The field distribution in the vicinity of the axis is similar to the one in the foxhole structure, each pair of cylinders (one on each side of the midplane) acting as one of the slot sections, longitudinal space between pairs as a foxhole. It is suitable for the acceleration of particles.

Neither the foxhole structure nor the colonnade depend for their operation on a resonance with the power source, but resonating them is essential for energy efficiency. Whenever there is a mismatch between the impedances of source and load, e.g., through lack of resonance, reflections occur and the energy in the rejected beam is wasted if it cannot be recovered. The colonnade can be tuned by adjustment of the length or height of the cylinders and the impedance it presents to the load can be made either low (cylinder height = 1/2Λ) or high (cylinder height = 3/4 Λ). The actual value of the impedance is determined by the physical characteristics, among them the rate of energy loss to dissipation and radiation.

3.5 Colonnade (π mode).

A π mode colonnade is much more attractive than a 2π mode one for the same reasons that a π mode foxhole structure is to be preferred above a 2π mode one: a potential gain in effective accelerating gradient of close to two without serious loss of energy efficiency. It is thus well worth pursuing. It differs geometrically from the 2π colonnade in the longitudinal distance between successive cylinder pair centers, which is 1/2 λ rather than λ. The electrical difference is that the accelerating field along the axis alternates between successive pairs. Even a single row colonnade will not radiate transversely when operated in the π mode, thus the transverse geometry of a two row colonnade is less critical in this respect for the π mode than it is for the 2π mode. The equivalent of an infinitely long π mode single row colonnade with circular cylinders has been measured and the existence of an accelerating mode has been demonstrated. For this simulation a metallic circular cylinder was placed between two parallel metallic mirror sheets with its axis in the midplane between the mirrors. Field measurements were made as function of frequency, length and diameter of the cylinder. Lack of time has so far prevented similar measurements from being made for the 2π mode. These would require a more complex and less flexible model: identical half cylinders have to be attached to each mirror, properly located and oriented, facing each other.

The π mode structure requires that spacially alternating fields in the vicinity of the axis be obtained from a single power source. Palmer predicted that nearly any superperiodic perturbation with a super period of two periods, i.e., treating neighbours differently but next neighbours identically, will drive the π mode. Computer simulations of π mode structures of point dipoles have substantiated this.[6] However, the coupling factors must be sufficiently large to produce, from the same power source, and in a real structure, a mean accelerating gradient that is substantially larger than what can be achieved with the simple 2π mode. This is still a challenge.

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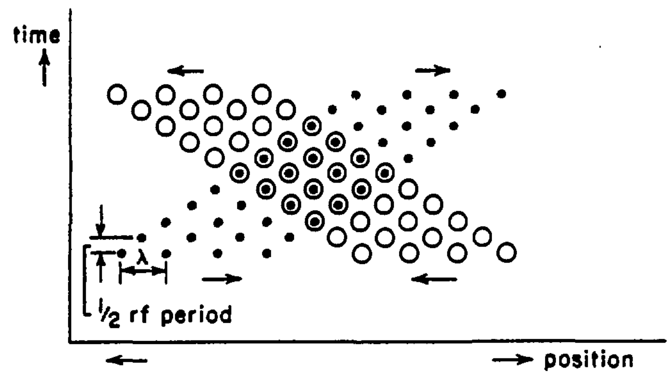


Fig. 1   Multibunch Interaction

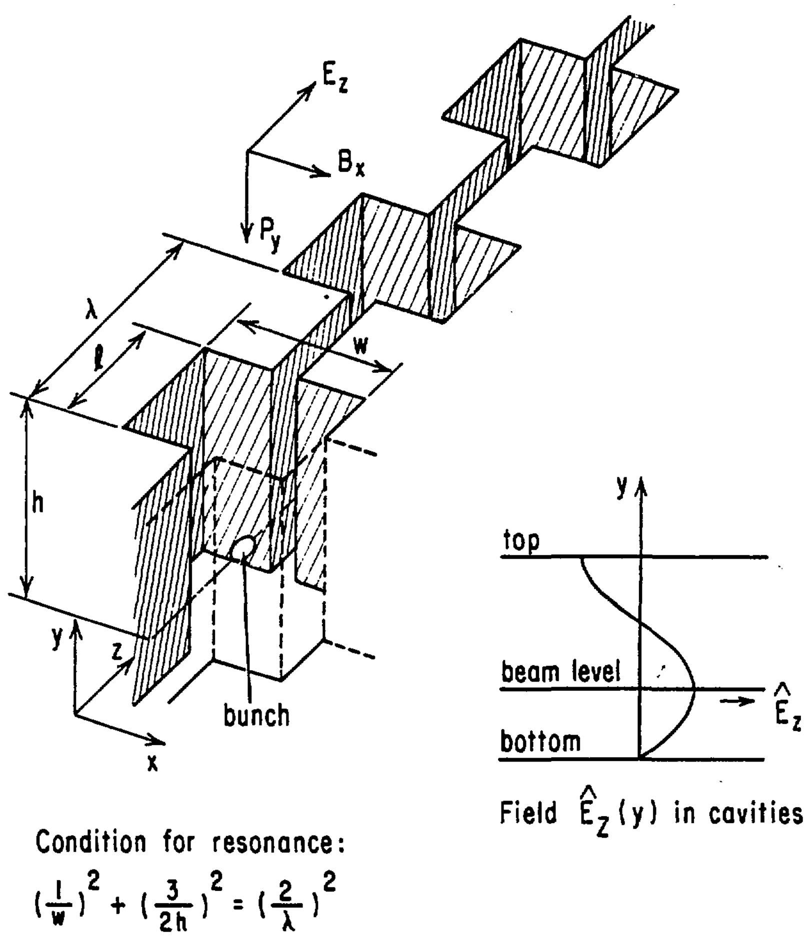
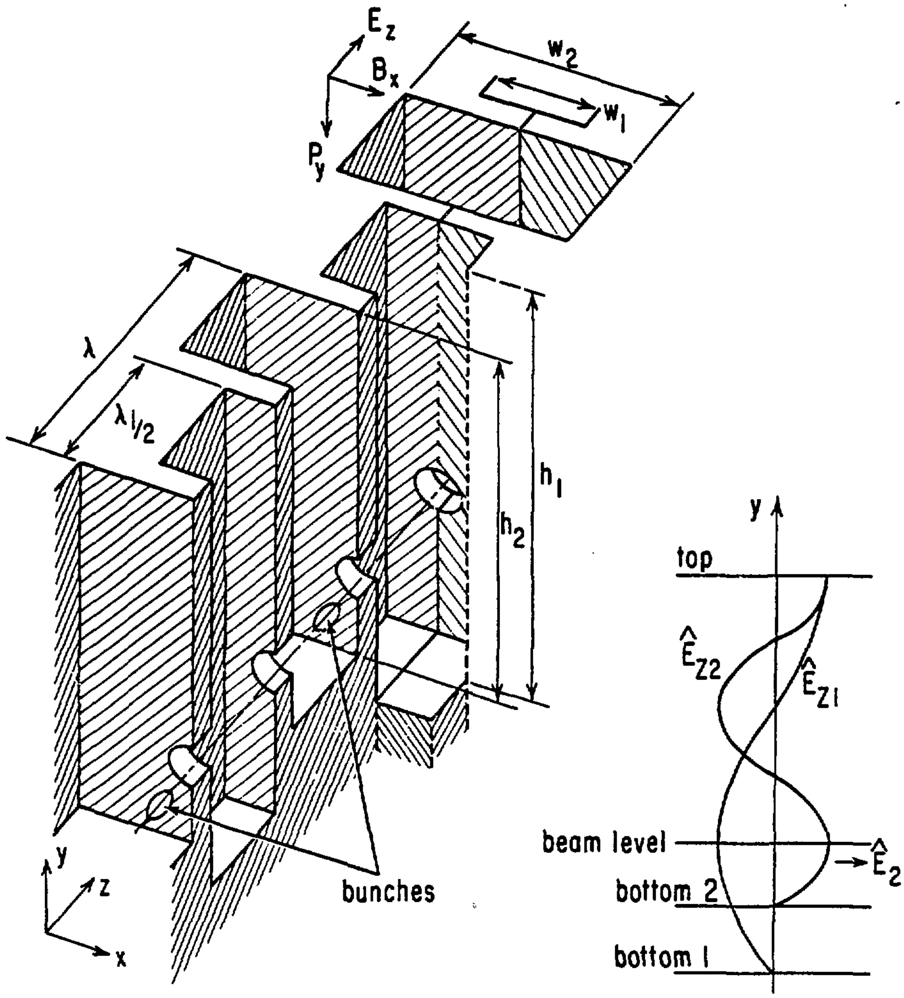


Fig. 3   Foxhole Structure (2π mode)



Fields in cavities

$\hat{E}_{Z1}(y)$ in cavities 1

$\hat{E}_{Z2}(y)$ in cavities 2

Conditions for resonance

$$\left(\frac{1}{w_1}\right)^2 + \left(\frac{3}{2h_1}\right)^2 = \left(\frac{2}{\lambda}\right)^2$$

$$\left(\frac{1}{w_2}\right)^2 + \left(\frac{5}{2h_2}\right)^2 = \left(\frac{2}{\lambda}\right)^2$$

Condition for colinearity

$$\tfrac{2}{3} h_1 = \tfrac{4}{5} h_2$$

Fig. 4 Foxhole Structure (π mode)

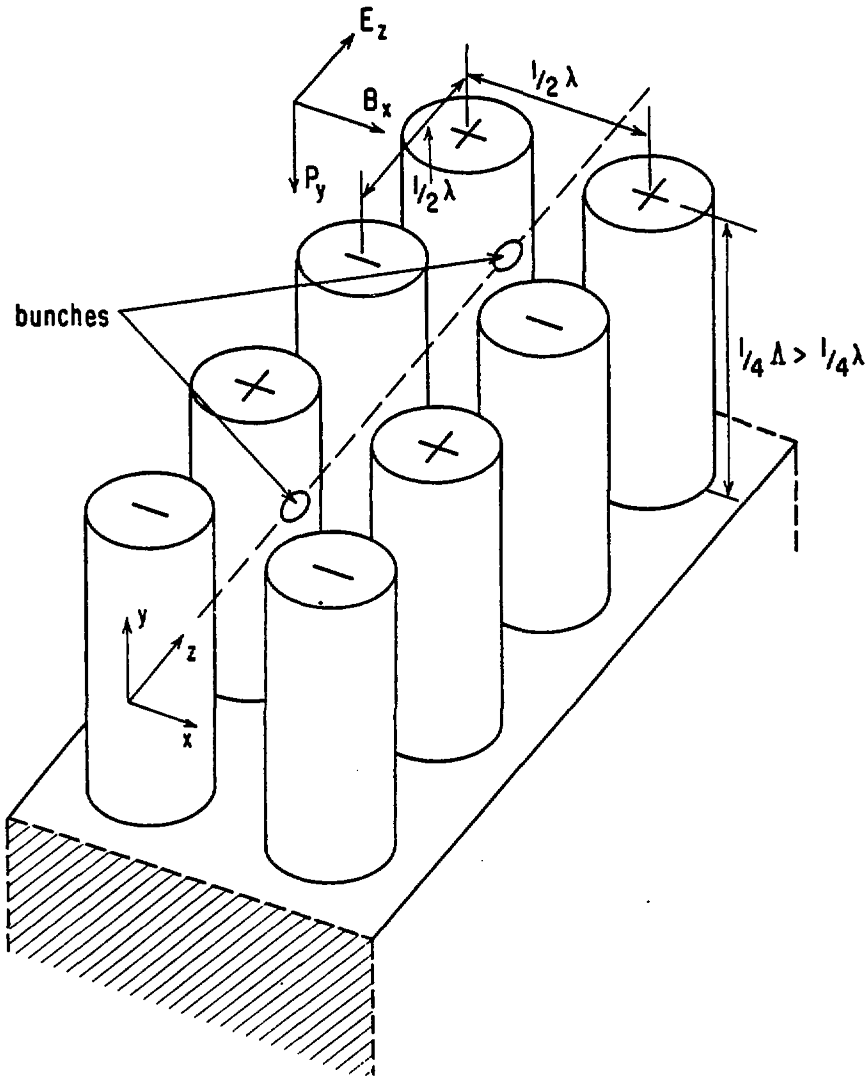


Fig. 5 Colonnade (π mode)

Fig. 6 Experimental Realization of Colonnade for Y = 10 μm

(Courtesy J. Warren, BNL)