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ЕРЕВАНСКИЙ ФИЗИЧЕСКИЙ ИНСТИТУТ

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ON THE ACCELERATION OF CHARGED PARTICLES BY STRONG LONGITUDINAL PLASMA WAKE FIELDS EXCITED BY ELECTRON BUNCHES

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ОБ УСКОРЕНИИ ЗАРЯЖЕННЫХ ЧАСТИЦ СИЛЬНЫМИ ПРОДОЛЬНЫМИ ВОЛНАМИ В ПЛАЗМЕ, ВОЗБУЖДАЕМЫМИ ЭЛЕКТРОННЫМИ СГУСТКАМИ

[illegible] исследуется возможно[illegible] [illegible] возбуж[illegible] [illegible] в плазме электронными [illegible] [illegible] ускорения заряженных част[illegible] [illegible] что наибо[illegible] [illegible] значени[illegible] напряженности [illegible] случае, [illegible] [illegible] частицы [illegible] [illegible] половине равно[illegible] [illegible] плотности электронов плазм[illegible].

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ON THE ACCELERATION OF CHARGED PARTICLES BY [illegible] LONGITUDINAL PLASMA WAKE FIELD [illegible] BY ELECTRON BUNCHES

[illegible] shown [illegible] proportional to the [illegible] density approaches [illegible] equilibrium density.

Yerevan Physics Institute
Yerevan 1985

The interest to the methods of particle acceleration in plasma recently has grown significantly [1,2], though some of them were discussed already in late fifties and sixties (see the review article [3] and references therein). Always the possibility of production of strong longitudinal electric fields in plasma is the important factor attracting attention. On the other hand, after the appearance of powerful lasers many new ideas have been suggested for using them in charged particle acceleration [1,2]. The combination of the both two methods for production of strong accelerating fields is one of the promising directions of the investigations. In the case of beat wave accelerators (BWA), for instance, the longitudinal accelerating field in plasma is created by the superposition of two parallel laser beams of frequencies ω_1, ω_2 and wave vectors K_1, K_2, satisfying to the conditions $\omega_1-\omega_2=\omega_p$, $K_1-K_2=\omega_p/v_p=K_p$, where ω_p is the plasma frequency, v_p is the longitudinal wave phase velocity equal to $v_p=c(1-\omega_p^2/2\omega_o^2)$ and it is assumed $\omega_1\sim\omega_2\approx\omega_o\gg\omega_p$, $K_1, K_2\gg K_p$. According to the estimates [1,2] in BWA one may achieve accelerating field gradients $\sim$ 100GeV/m.

[illegible] these [illegible] the laser technology, namely, [illegible] to have powerful lasers providing very short pulses during which the plasma instabilities would not have time to be developed, the necessity to achieve an accurate choice of the frequency [illegible] ference equal to the plasma frequency etc.

The use of the longitudinal waves excited in plasma by electron beams for charged particle acceleration (see [4]) [illegible] to be not less promising.

As it has been shown experimentally in the works [5,6] the non-linear waves, excited in plasma by preliminary shaped [illegible] beams, may travel long [illegible] without [illegible] amplitude increase, while the particles [illegible] in [illegible] with the excited wave [illegible] the [illegible].

[illegible] stationary [illegible] wave [illegible] by a charged plane and by sequence of [illegible] series of [illegible] [illegible] of plasma [illegible] the same lines as in [illegible] longitudinal waves [illegible] however, [illegible] to [illegible] [illegible]

[illegible] note we discuss in more detail the [illegible] [illegible] in [7,8] having in mind [illegible]

[illegible] the electron density n_B [illegible] along the z axis with relative [illegible] v_0 through [illegible] homogeneous, collisionless plasma the ions of which are in rest. The dimensions of the bunch along the x and y - axes are assumed to be infinite. The complete system of the hydrogynamics equations and Maxwell's equations describing the interaction of the bunch with plasma has the form:

$$\frac{\partial \vec{P}}{\partial t} + (\vec{u}\,\vec{\nabla})\vec{p} = -e\vec{E} - \frac{e}{c}[\vec{u}\,\vec{B}],$$

$$rot\,\vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t},$$

$$rot\,\vec{B} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t} - \frac{4\pi}{c} e n \vec{u} -$$

$$-\frac{4\pi}{c} e n_B \vec{v}_0 \left\{\theta\left(z - v_0 t + \frac{a}{2}\right) - \theta\left(z - v_0 t - \frac{a}{2}\right)\right\}, \quad (1)$$

$$div\,\vec{E} = 4\pi e(n_0 - n) - 4\pi e n_B \left\{\theta\left(z - v_0 t + \frac{a}{2}\right) -\right.$$

$$\left. -\theta\left(z - v_0 t - \frac{a}{2}\right)\right\},$$

$$div\,\vec{B} = 0,$$

where $\vec{p} = m\vec{u}/\sqrt{1-u^2/c^2}$, $\vec{u}$ is the plasma electron velocity, n_0 is the equilibrium plasma electron density equal to the ion density, e is the absolute magnitude of the electron charge, and $\theta(\xi)$ is the Heaviside function.

We shall look for the solution of the system (1) in the form of travelling plane periodic waves, for which all the unknown quantities are functions of one single variable $\tilde{z} = z - v_0 t$ where v_0 is the wave phase velocity coinciding with the bunch velocity. Then the following system of equations for the components of the dimensionless momentum $\vec{\rho} = \vec{p}/mc$ [13] will emerge

$$\frac{d^2 \rho_x}{d\tilde{z}^2} - \frac{\omega_p^2 \beta}{(1-\beta^2)c^2} \cdot \frac{\rho_x}{\beta\sqrt{1+\rho^2} - \rho_z} = 0$$

$$\frac{d^2 \rho_y}{d\tilde{z}^2} - \frac{\omega_p^2 \beta}{(1-\beta^2)c^2} \cdot \frac{\rho_y}{\beta\sqrt{1+\rho^2} - \rho_z}$$

$$\frac{d^2}{d\tilde{z}^2}\left(\beta\rho_z - \sqrt{1+\rho^2}\right) + \frac{\omega_p^2}{c^2} \cdot \frac{\rho_z}{\beta\sqrt{1+\rho^2} - \rho_z}$$

$$\frac{[illegible]\, n_e}{mc^2}\left[\theta\left(\tilde{z} + \frac{a}{2}\right) - \theta\left(\tilde{z} - \frac{a}{2}\right)\right] = 0$$

where $\omega_p = \sqrt{4\pi e^2 n_0/m}$ is the plasma frequency, $\beta = v_0/c$ and $\rho^2 = \rho_x^2 + \rho_y^2 + \rho_z^2$.

The excited wave will spread with a constant amplitude (for the given parameters), if the electron bunch moves without deformation which is possible only in the case when all the components of the electric field $\vec{E}$ inside the bunch are equal to zero (see sect.7 [4] and reference therein). For such conditions from (1) and (2) one obtains for the region inside the bunch:

$$\rho_x = 0 \; ; \quad \rho_y = 0 \; , \quad \rho_z = -\frac{n_b}{n_0}\beta \Big/ \sqrt{1 - 2\frac{n_b}{n_0} + (1-\beta^2)\frac{n_b^2}{n_0^2}} \; ,$$

$$n = n_0 - n_b \geq 0 \; , \quad \frac{d\rho_z}{d\tilde{z}} = 0 \; , \quad \beta = v_0/c \; , \tag{3}$$

i.e. total charge and current neutralization of the bunch takes place.

To derive the solutions behind the bunch $(\tilde{z} < -a/2)$ we assume in (2) $n_b = 0$ and use continuity conditions for $\rho_{x,y,z}(\tilde{z})$ and $d\rho_{x,y,z}(\tilde{z})/d\tilde{z}$ on the bunch boundary. These conditions result in the fact that for $\beta < 1$ the first two equations of the system (2) have only zero solutions, $\rho_x = \rho_y = 0$. Assuming $\rho_x = \rho_y = 0$ in the third equation of (2) and integrating it we obtain the following implicit expression for the dimensionless momentum ρ_z taking into account the continuity conditions:

$$\frac{\sqrt{2}\,\omega_p}{c}\left(\tilde{z} + \frac{a}{2}\right) = \pm \int_{-\rho_{z\,max}}^{\rho_z} \frac{\left(\beta\sqrt{1+\rho_z^2} - \rho_z\right)\, d\rho_z}{\sqrt{1+\rho_z^2}\left[\sqrt{1+\rho_{z\,max}^2} - \sqrt{1+\rho_z^2}\,\right]^{1/2}} \; , \tag{4}$$

where

$$\rho_{z\,max} = \frac{n_b}{n_0}\beta \Big/ \sqrt{1 - 2\frac{n_b}{n_0} + (1-\beta^2)\frac{n_b^2}{n_0^2}} \; . \tag{5}$$

The wave length $\tilde{z}_\lambda$ of the nonlinear oscillations excited by the bunch is given by the expression:

[illegible] wave length $\tilde{z}_\lambda$ [illegible]

$$a < \frac{n_b}{n_0 - n_b} \tilde{z}_\lambda \qquad (10)$$

Consider first the case $n_b / n_0 \ll 1$. The maximal value of the excited longitudinal field can be given by the [illegible]

$$\left| E_{z\,max} \right| \simeq \frac{m\omega_p c}{e} \frac{n_b}{n_0} \qquad (11)$$

[illegible] n_b/n_0 times less than the maximal [illegible] wave field [illegible] laser beams. However, as the analysis of the [illegible] formulae for velocity, density and electric field [illegible] the field amplitude begins to grow sharply when the bunch [illegible] n_b approaches to the value $n_0/2$. When the following [illegible] are fulfilled

$$n_b \lesssim \frac{n_0}{2}, \quad 1 \ll \gamma^2 \ll \frac{n_b^2}{n_0^2 \left(1 - 2\frac{n_b}{n_0}\right)}, \quad \gamma^2 = \frac{1}{1-\beta^2} \qquad (12)$$

[illegible] $\tilde{z}_\lambda$ become dependent on γ-factor [illegible] and they are determined by the expres-

$$E_z = \pm \frac{mc}{e} \omega_p \sqrt{2} \left\{ \gamma - \tilde{\gamma} \right\}^{1/2},$$

$$\tilde{z}_\lambda = \frac{+\sqrt{2}\, v_0}{\omega_p} \left\{ \gamma + 1 \right\}^{1/2}. \qquad (13)$$

The maximal field value (at $\rho_z = 0$, $\tilde{\gamma} = 1$) depends on γ-factor of the relativistic bunch as $\sim \gamma^{1/2}$ and may be high enough.

The value of the bunch density $n_b = n_0/2$ is critical at which the wave front becomes steeper and then overturns. The useful high field value $E_{z max} = E_z(\rho_z = 0)$ is determined by the condition (12), i.e. by such values of γ at which the magnitude $1 - 2n_b/n_0$ is sufficiently small (high values of γ), but the wave still does not break down.

Let us note that the maximal energy density of the excited wave equals to $E^2_{z max}/8\pi$ when $n_b \approx n_0/2$ and coincides to the bunch kinetic energy density $n_0 mc^2(\gamma - 1)$.

In the limitting case $a \ll \tilde{z}_\lambda$

$$\rho_{z max} = \rho_\sigma = \sqrt{\left(1 + \frac{\pi\sigma^2}{2n_0 mc^2}\right)^2 - 1}, \tag{14}$$

$$\gamma = \sqrt{1 + \rho^2_{z max}} = 1 + \frac{\pi\sigma^2}{2n_0 mc^2},$$

where $\sigma = \lim_{a \to 0,\, n_b \to \frac{n_0}{2}} n_b\, a/2$ and one obtains the solution for the charged plane [7,9]. It is necessary to mention that in general case the electric field in front of the bunch ($\tilde{z} > a/2$) is zero.

In the case of N relativistic periodic bunches, in order to provide stationary solution it is necessary to keep the distance between the bunches equal to an integer number of wave length while the accelerated particles must be injected during the time interval between the bunches in a phase corresponding to acceleration. In such a case there is no gain in the accelerating field compared with the case of a single bunch, and it is provided only the required repetition of the acceleration processes.

In conclusion of this note we will give one numerical estimate: for $n_g \approx n_o/2 \approx 10^{13}$ the maximal acceleration rate is equal to $eE_{max} \approx 2.4 \cdot 10^{-1}\ \gamma^{1/2}$ GeV/m, which for $\gamma \approx 10^4$ gives $eE_{max} \approx 24$ GeV/m.

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