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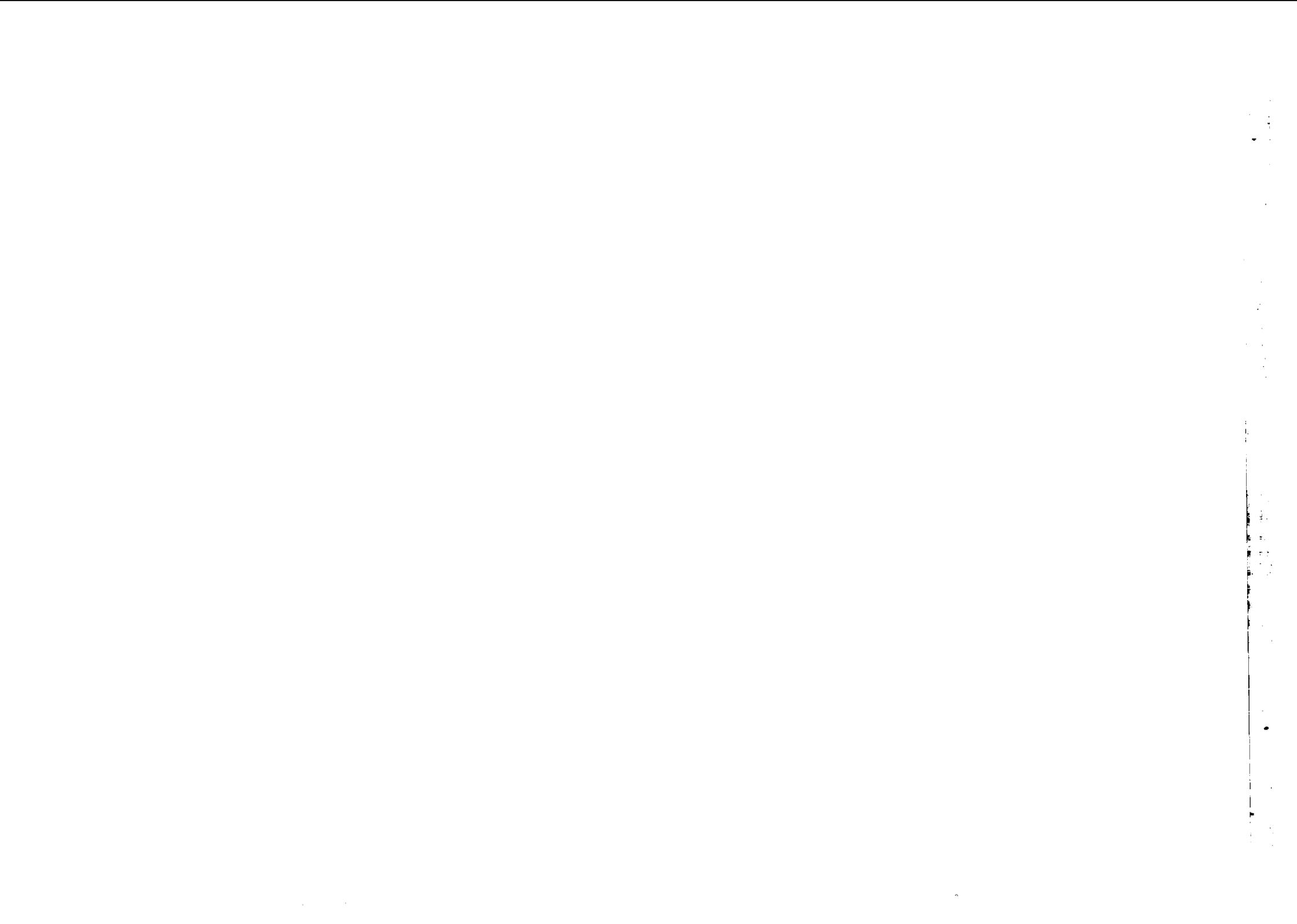


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## HIGHER DERIVATIVE SUPER YANG-MILLS THEORIES \*

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## ABSTRACT

The most general higher derivative Yang-Mills actions of the type  $(F^2 + \alpha^2 F^4)$  which are globally supersymmetric up to order  $\alpha^2$  in six and ten dimensional spacetimes are given. The  $F^4$ -terms turn out to occur in the combination  $\alpha^2 (\text{tr } F^4 - 1/4 (\text{tr } F^2)^2)$ , where the trace is over the Lorentz indices. This result agrees with the low energy limit of the open superstring in ten dimensions, where  $\alpha$  is the string tension. Surprisingly, the transformation rules of the Yang-Mills multiplet receive order  $\alpha^2$  corrections even in the *off-shell* formulation. For the case of Abelian Yang-Mills group, the action is expressed in Born-Infeld form with a metric generically given by  $(1 + \alpha^2 F^2 + \dots)$ .

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## 1. Introduction

Higher derivative Yang-Mills actions are known to occur in the low energy limit of superstrings [1]. However not much is known about the explicit form of such actions. Recently Gross and Witten [2] have derived all nonderivative  $F^4$  couplings (i.e. no terms of the type  $F^2 (\partial F)^2$ ) for the Type I superstring by studying the four point scattering amplitude. Using path integral technique Fradkin and Tseytlin [3] have found a closed expression (to all order in the string tension parameter  $\alpha$ ) for constant Abelian field strength. Remarkably, the result is an appropriately modified Born-Infeld action [4].

In this letter we give the explicit form of a higher derivative Yang-Mills action of the type  $(F^2 + \alpha^2 F^4)$  which is globally supersymmetric up to order  $\alpha^2$  in 6 and 10 dimensional spacetimes. Here,  $\alpha$  is a parameter of dimension  $(\text{mass})^{-2}$ , which we later interpret as the string tension parameter. The requirement of supersymmetry fixes the  $F^4$ -terms to be the combination

$$\mathcal{L} = -\frac{1}{4} \alpha^2 \left\{ \text{Tr } F^4 - \frac{1}{4} (\text{Tr } F^2)^2 \right\} \quad (1)$$

where the trace is over the Lorentz indices, and there is a suppressed totally symmetric trace of the Yang-Mills generators. It is gratifying to note that this result agrees with that of Gross and Witten [2] mentioned above.

In the case of six dimensional spacetime, we exhibit our result for both on-shell and off-shell super-Yang-Mills theory. A surprising feature of our results is that the transformation rules of the Yang-Mills multiplet receive order  $\alpha^2$  corrections even in the *off-shell* case. We can show that this is compatible with the superspace formulation of the theory. The reason for this is that the spinor-spinor component of the Yang-Mills field strength can be modified as

$$F_{\alpha\beta}^{ij} = \frac{1}{72} \alpha^2 \gamma_{\alpha\beta}^{(5)} \bar{\chi}^{(i)} \gamma^a \gamma_{(3)} \gamma^b \chi^{(j)} F_{ab} \quad (2)$$

and that the Bianchi identities can be solved to order  $\alpha^2$  without going on-shell. In (2) a group theory factor  $\text{Str}(T^a T^b T^c T^d)$  is suppressed, and  $\chi, F_{ab}$  are superfields whose  $\theta=0$  components are the Yang-Mills spinor and field strength, respectively. For the on-shell super-Yang-Mills theory in 10 dimensions, we find that the analog of (2) is given by

$$F_{\alpha\beta} = \frac{1}{16 \times 360} \alpha^2 \gamma_{\alpha\beta}^{(5)} \bar{\chi} \gamma^a \gamma_{(3)} \gamma^b \chi F_{ab} \quad (3)$$

It is natural to try to interpret our results in terms of a Born-Infeld type geometry where the metric is a function of the Yang-Mills field strength. In general it is not clear how to do this. However, in the Abelian case we find that the supersymmetry transformation rules can be cast into the following geometric form

$$\delta A_\mu = -\bar{\eta} \gamma_a \chi e_\mu^a, \quad (4)$$

$$\delta \chi = \frac{1}{8} \gamma^{ab} F_{\mu\nu} \eta e_a^\mu e_b^\nu,$$

where the vielbein, and the supersymmetry parameter are given by

$$e_\mu^a = \delta_\mu^a + \frac{2}{\sqrt{6}} \alpha F_\mu^a + \alpha^2 (F^2)_\mu^a + \dots \quad (5)$$

$$\eta = \epsilon - \frac{1}{2\sqrt{6}} \alpha \gamma \cdot F \epsilon - \frac{1}{24} \alpha^2 (\text{Tr} F^2) \epsilon + \frac{1}{48} \alpha^2 \gamma^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} \epsilon + \dots$$

where  $(F^2)_{\mu\nu} = F_{\mu\rho} F_{\rho\nu}$ . In this case the bosonic part of the action is simply

$$\mathcal{L}(\text{bosonic}) = \frac{1}{2\alpha^2} (\sqrt{g} - 1) \quad (6)$$

The fermionic terms, however, do not seem to lend themselves to a simple geometrical interpretation. In the text, we shall allow certain explicit order  $\alpha^2$  terms in the transformation rules, in which case the full action can be considerably simplified.

We do not attempt to construct  $F^n$ -actions for  $n>4$  here. Clearly a general algorithm is needed in order to do this. Such an algorithm may very well arise from a better understanding of the Born-Infeld type geometry (and the existence of Killing spinors in such geometries) which is expected to emerge in these theories.

Finally we emphasize that the higher derivative actions presented in this letter are very likely to play a role as low energy limits of yet to be constructed superstring theories in 6 dimensions as well. Such string theories could conceivably arise from heterotic constructions on Lorentzian [5] orbifolds [6].

## 2. Higher Derivative Super-Yang-Mills Actions in Components

(a) **d=10**: We first present the result for the ten dimensional super-Yang-Mills theory.

By the usual Noether procedure, we find the following action:

$$\begin{aligned} \mathcal{L} = & \frac{1}{4} \text{Tr} F^2 - 2 \bar{\chi} \not{D} \chi - \frac{1}{4} \alpha^2 \{ \text{Tr} F^4 - \frac{1}{4} (\text{Tr} F^2)^2 - 8 (F^2)^{\mu\nu} \bar{\chi} \gamma_\mu \not{D}_\nu \chi \\ & - 2 F_\mu^\lambda (\not{D}_\lambda F_{\nu\rho}) \bar{\chi} \gamma^{\mu\nu\rho} \chi + c_1 (\text{Tr} F^2) \bar{\chi} \not{D} \chi \\ & + c_2 F_{\mu\nu} (\not{D}_\lambda F^\lambda{}_\rho) \bar{\chi} \gamma^{\mu\nu\rho} \chi + c_3 F_{\mu\nu} F_{\rho\sigma} \bar{\chi} \gamma^{\mu\nu\rho\sigma} \not{D} \chi \} + \text{quartic fermions} . \end{aligned} \quad (7)$$

The notation is self explanatory. In particular, note that the traces are over the Lorentz

indices, and  $\cdot$  Suppressed are the group theory factors, which are the totally symmetric traces  $\text{Str}(T^a T^b T^c T^d)$ . Here, a,...,d label the adjoint representation of the Yang-Mills group. The parameters  $c_1, c_2$  and  $c_3$  are arbitrary, because the terms they multiply can be induced by the following field redefinitions:

$$\begin{aligned} A_\mu & \mapsto A_\mu - \frac{1}{4} \alpha^2 c_2 F^{\nu\rho} \bar{\chi} \gamma_{\mu\nu\rho} \chi \\ \chi & \mapsto \chi + \frac{1}{16} \alpha^2 c_1 (\text{Tr} F^2) \chi + \frac{1}{16} \alpha^2 c_3 F_{\mu\nu} F_{\rho\sigma} \gamma^{\mu\nu\rho\sigma} \chi \end{aligned} \quad (8)$$

Also here, the group theoretical factor  $\text{Str}(T^a T^b T^c T^d)$  is suppressed. Of course, one could

allow different type of group theoretical factors in the field redefinitions. For simplicity in notation we shall not do so.

There is only one more arbitrary parameter one can introduce, when one considers the action upto order  $\alpha^2$ , and quartic fermions. It is associated with the redefinition

$$\chi^{\text{old}} = (\chi + c_4 \gamma \cdot F \chi)^{\text{new}} \quad (9)$$

On the right hand side, the structure constant for the Yang-Mills group is suppressed (Of course, such a factor is absent in the Abelian case). For uniformity in the notation for group theoretical factors in the action, we have not performed this redefinition. (See next section, however).

The action of the Lagrangian (7) is invariant up to order  $\alpha^2$  under the following supersymmetry transformations:

$$\begin{aligned} \delta A_\mu &= -\bar{\epsilon} \gamma_\mu \chi - \frac{\alpha^2}{16} (c_1 + 2c_2 - 6) (\text{Tr} F^2) \bar{\epsilon} \gamma_\mu \chi + \frac{\alpha^2}{4} (c_2 - 4) (F^2)_\mu{}^\nu \bar{\epsilon} \gamma_\nu \chi \\ &\quad - \frac{\alpha^2}{8} (-c_2 + 2c_3 + 2) F_{\mu\nu} F_{\rho\sigma} \bar{\epsilon} \gamma^{\nu\rho\sigma} \chi - \frac{\alpha^2}{16} (c_2 + c_3 - 1) F^{\rho\sigma} F^{\lambda\tau} \bar{\epsilon} \gamma_{\rho\sigma\lambda\tau} \chi \\ &\quad + \text{trilinear fermions} \\ \delta \chi &= \frac{1}{8} \gamma \cdot F \epsilon - \frac{\alpha^2}{128} (c_1 + 4c_3 - 2) (\text{Tr} F^2) \gamma \cdot F \epsilon + \frac{\alpha^2}{16} (c_3 - 1) (F^2)_{\mu\nu} \gamma^{\mu\nu} \epsilon \\ &\quad - \frac{\alpha^2}{384} (3c_3 + 1) \gamma^{\mu\nu\rho\sigma\lambda\tau} \epsilon F_{\mu\nu} F_{\rho\sigma} F_{\lambda\tau} + \text{bilinear fermions} \end{aligned} \quad (10)$$

On the right hand side of (10), again the group theory factor  $\text{Str}(T^a T^b T^c T^d)$  is suppressed.

Two remarks are in order. First, we note that the combination of  $F^4$  terms is precisely the one found by Gross and Witten [2] from the study of low energy limit of the four-point amplitudes in the Type I superstring. From the Noether procedure point of view, the relative factor  $-1/4$  in the  $F^4$  terms is determined from the cancellation of the  $(\bar{\epsilon} \gamma_\mu \chi) F_{\mu\rho} \partial_\rho \text{tr} F^2$  terms, which, remarkably, arise only from the  $F^4$  terms alone. Second, the commutator algebra is not modified by order  $\alpha^2$ , with the exception of the Yang-Mills transformation.

(Incidentally, the latter modification arises only if the quartic fermions are taken into account).

**(b)  $d=6$** : By straightforward computation, one can verify that in  $d=6$  the on-shell  $N=2$  supersymmetric  $(F^2 + \alpha^2 F^4)$ -action and the transformation rules are formally the same as those given in (7) and (10). The spinors are now  $\text{Sp}(1)$  symplectic-Majorana, with the  $\text{Sp}(1)$  doublet index suppressed everywhere.

In six dimensions an off-shell formulation of the  $N=2$  super-Yang-Mills theory is known [7]. It contains the auxiliary field  $Y_{ij} = Y_{ji}$  ( $i,j=1,2$ ). After some algebra, we find that the  $(F^2 + \alpha^2 F^4)$  action in  $d=6$  can be made off-shell supersymmetric upto order  $\alpha^2$  by adding to the action given in (7) for  $c_1=-2$ ,  $c_2=4$  and  $c_3=1$ , the following  $Y$ -dependent terms:

$$\begin{aligned} \mathcal{L}(Y_{ij}) &= Y^2 - \alpha^2 \left\{ (Y^2)^2 + \frac{1}{2} Y^2 (\text{Tr} F^2) - 6 Y^2 \bar{\chi} \not{D} \chi \right. \\ &\quad - 2 Y^{ij} (\partial_\mu F^{\mu\nu}) \bar{\chi}_i \gamma_\nu \chi_j + 2 (\partial_\mu Y^{ij}) F^{\mu\nu} \bar{\chi}_i \gamma_\nu \chi_j \\ &\quad \left. - 2 Y^{ij} F_{\mu\nu} \bar{\chi}_i \gamma^{\mu\nu\lambda} \partial_\lambda \chi_j + 4 Y_{ij} (\partial_\mu Y^i{}_k) \bar{\chi}^j \gamma^k \chi^k \right\} \\ &\quad + \text{quartic fermions} \end{aligned} \quad (11)$$

where  $Y^2 = Y_{ij} Y^{ij}$ . The following remarks are useful:

(1) The transformation rule of  $\chi$  now contains the usual term  $-1/2 Y^{ij} \epsilon_j$ , and  $Y^{ij}$  transforms as  $\delta Y^{ij} = -\bar{\epsilon}^{(i} \gamma^{\mu} D_\mu \chi^{j)}$ . Apart from these terms there are no further modifications in the transformation rules (10).

(2) The particular values of the redefinition parameters  $c_1$ ,  $c_2$  and  $c_3$  is a matter of convenience. They can be reintroduced by the redefinitions (8).

(3) Since the lowest order field equation for  $Y^{ij}$  is simply  $Y^{ij} = 0$ , all the terms in (11) can be eliminated by redefining  $Y^{ij}$ . However, this will introduce extra  $Y$ -dependent terms in the transformation rules (10).

(4) It is most remarkable that, even in the off-shell formulation of the higher derivative super-Yang-Mills action, order  $\alpha^2$  modifications are needed in the transformation rules. (They can not be eliminated for any choice of  $c_1, c_2$  and  $c_3$ ). To our knowledge, this is the first example of an off-shell theory in which the transformation rules have to be modified for the construction of an action. Note, in particular that the  $\alpha^2 F^3$ -terms in the  $\chi$ -transformation rule can not be eliminated by a redefinition of  $Y^i$ .

(5) The field equation for  $Y^i$  is not algebraic. Thus, although  $Y^i$  does not propagate, to go on-shell, it is difficult to solve for  $Y^i$ . Even if one solves for it iteratively, it does not seem to yield the transformation rules of the on-shell theory (i.e. the theory described by (7) and (10), before the introduction of  $Y^i$ ).

### 3. A Born-Infeld Type Geometry

In the present context, by Born-Infeld type geometry we mean a geometry characterized by a metric of the form

$$g_{\mu\nu} = \eta_{\mu\nu} + X_{\mu\nu}(F) \quad (12)$$

where  $X_{\mu\nu}(F)$  is an arbitrary function of the Maxwell field strength  $F_{\mu\nu}$ . The Lagrangian  $\det(\eta_{\mu\nu} + F_{\mu\nu})^{1/2}$  was first proposed by Born and Infeld in 1934 [4] as a model for nonlinear electrodynamics in four dimensions. Remarkably, this Lagrangian does give the combination (1) in any dimension, to order  $F^4$ . In fact, Fradkin and Tseytlin [3] have found that for constant  $F_{\mu\nu}$  the effective action in an open bosonic string is exactly given by the Born-Infeld action. In the case of Type I superstring, for the bosonic part of the effective action they find the result  $\det[(\eta_{\mu\nu} + F_{\mu\nu})/(1 + \gamma \cdot F)]^{1/2}$ .

In the presence of fermions, presumably one needs to introduce an  $F$ -dependent vielbein. It is not clear how this can be done with the choice  $X_{\mu\nu}(F) = F_{\mu\nu}$ . With the choice  $X_{\mu\nu}(F) = iF_{\mu\nu}$ , one can introduce a complex vielbein. However, in this case it is not clear to us how to incorporate this vielbein into the super-Yang-Mills transformation rules, at least if we insist on the closure of the superalgebra in the usual sense. One more criterium which we have chosen in determining  $X_{\mu\nu}(F)$  is that  $(\det(g))^{1/2} = (1 + 2\alpha^2)$  yields (1), both, in 6 and 10 dimensions. Thus we are led to consider a metric of the form (12) with

$$X_{\mu\nu}(F) = \alpha^2 (F^2)_{\mu\nu} - \frac{1}{2} \alpha^4 (F^4)_{\mu\nu} \quad (13)$$

For a geometric interpretation of the supersymmetry transformation rules we must also allow an  $F$ -dependent supersymmetry parameter.

Using (13), we find that with the choices  $c_1=4, c_2=4/3, c_3=0$ , and after making the field redefinition (9) with  $C_4 = (24)^{-1/2}$ , the transformation rules (10) can be casted into the most minimal elegant form given in (4), with the  $F$ -dependent vielbein and supersymmetry parameter given by (5). The purely bosonic part of the action is also simple, and is given by (6). However, the remaining fermionic terms do not simplify. More specifically, they are not accounted for by the covariant kinetic term  $\bar{\chi} \gamma^\mu D_\mu \chi$ , with the exception of the  $F \partial F \chi^2$ -terms, which can be interpreted as torsion.

There is one more intriguing possibility for geometrization. Namely, in the order  $\alpha^2$  action and transformation rules, if we allow only those terms which contain wedge products of the Maxwell field strength, then we can cast (7) and (10) into the following form

$$\begin{aligned} \mathcal{L} = & \frac{1}{2\alpha^2} (\sqrt{g} - 1) - 2\sqrt{g} \bar{\chi} \gamma^\mu \partial_\mu (\omega) \chi e_\alpha^\mu \\ & - \frac{\alpha^2}{16} F_{\mu\nu} F_{\rho\sigma} \bar{\chi} \gamma^{\mu\nu\rho\sigma} \partial_\lambda \chi + \text{quartic fermions} \end{aligned} \quad (14)$$

$$\delta A_\mu = -\bar{\eta} \gamma_a \chi e_\mu^a - \frac{5\alpha^2}{16} F_{\mu\nu} F_{\rho\sigma} \bar{\eta} \gamma^{\nu\rho} \chi + \frac{3\alpha^2}{64} F^{\nu\rho} F^{\sigma\tau} \bar{\eta} \gamma_{\mu\nu\rho\sigma} \chi$$

+ trilinear fermions

$$\delta \chi = \frac{1}{8} \gamma^{ab} F_{\mu\nu} \eta e_\mu^a e_\nu^b - \frac{7\alpha^2}{32\sqrt{2}} \gamma^{\mu\nu\rho\sigma\lambda\tau} \eta F_{\mu\nu} F_{\rho\sigma} F_{\lambda\tau}$$

+ bilinear fermions .

In obtaining the above result we had to choose  $c_1=17/2$ ,  $c_2=0$ ,  $c_3=1/4$  in (7) and (10), and perform the redefinition (9) with  $c_4=(32)^{-1/2}$ . In (14) and (15) the vielbein, the torsionful spin connection and the supersymmetry parameter are defined by

$$e_\mu^a = \delta_\mu^a + \frac{\alpha}{\sqrt{2}} F_\mu^a + \frac{5\alpha^2}{4} (F^2)_\mu^a, \quad (16)$$

$$\omega_\mu^{ab} = \omega_\mu^{ab}(e) + \frac{7\alpha^2}{8} F_{[\mu}{}^\lambda{}_{\nu]} \Delta^\lambda F_{ab\lambda}, \quad (17)$$

$$\eta = \varepsilon - \frac{\alpha}{4\sqrt{2}} \gamma \cdot F \varepsilon - \frac{5\alpha^2}{32} (\text{Tr} F^2) \varepsilon - \frac{7\alpha^2}{64} \gamma^{\mu\nu\rho\sigma} \varepsilon F_{\mu\nu} F_{\rho\sigma}. \quad (18)$$

We emphasize that the results (14) and (15) are valid both in 6 and 10 dimensions. Moreover, there is a nontrivial consistency check in the calculation which leads to this result. We do not know whether in higher order in  $\alpha^2$  this still holds. If one allows dimension dependence in the geometrical action, then, in particular in  $d=6$ , one may have the possibility of further simplifications, for example, by allowing terms of the form  $\eta_{\mu\nu} \text{tr} F^2$  and  $\varepsilon_{\mu\nu\rho\sigma\lambda\tau} F_{\nu\kappa} F_{\rho\sigma} F_{\lambda\tau}$  in the vielbein.

#### 4. Comments

1. The supersymmetry transformation rules for the higher derivative super-Yang-Mills theories presented in Sec. 2 can be derived in superspace. This is most naturally done by modifying the usual constraint  $F_{\alpha\beta}=0$  by order  $\alpha^2$  terms as given in (2) and (3). It is straightforward to solve the Yang-Mills Bianchi identities to order  $\alpha^2$  in the presence of these constraints. In order to compare the resulting supersymmetry transformation rules for the Yang-Mills multiplet with the component results (10), we use the standard relations

$$\delta A_\mu = \varepsilon^\alpha F_{\alpha\mu} \Big|_{\theta=0}, \quad \delta \chi = \varepsilon^\alpha \mathcal{D}_\alpha \chi \Big|_{\theta=0}. \quad (19)$$

In the case of  $d=10$ , the left hand side of these equations agree with (10) for  $c_1=26/15$ ,  $c_2=5/3$  and  $c_3=7/15$ . In the case of the on-shell  $d=6$  theory, again an agreement is found with (10) for  $c_1=2/3$ ,  $c_2=2$  and  $c_3=-1/3$ . In the case of off-shell  $d=6$  theory, after an appropriate field dependent redefinition of the auxiliary field  $Y_{ij}$ , again, the result agrees with (10).

2. An action formula for  $N=2$ ,  $d=6$  super-Yang-Mills is known in superspace [7]. It is given by

$$I = \int d^6x \mathcal{D}_\alpha^i \mathcal{D}_\beta^j (X_i^\alpha X_j^\beta). \quad (20)$$

We have checked that, using (19) in this action formula does not yield the component result (7), although the combination  $\alpha^2 (\text{tr} F^4 - 1/4 (\text{tr} F^2)^2)$  does emerge. For a full agreement with the component result, one should probably add terms of the type  $\alpha^2 \chi^2 (F_{ab})^2$  to the superspace action (20).

3. It is not clear to us how to couple the Green-Schwarz superstring action [8] to a Yang-Mills background with  $F_{\alpha\beta} \neq 0$  [9]. One possibility which remains to be explored is to set  $F_{\alpha\beta} = 0$  and modify appropriate components of the superspace torsion by order  $\alpha^2$   $F$ -dependent terms. In that case the supervielbein would, of course, depend on the Yang-Mills curvature, and it would be interesting to see whether the action could be written as the superdeterminant of this vielbein.

4. Consistent truncations of the super-Yang-Mills theories in  $d=6$  and 10 to  $d=4$  will give rise to  $N=2$  and 4 super-Yang-Mills theories, respectively. As usual this procedure corresponds to compactification on a torus and discarding of the massive modes. In  $d=4$ , a further consistent truncation to  $N=1$  is possible. For example, the off-shell  $d=6$  theory can be truncated in the following way

$$A_5 = A_6 = 0, \quad \varepsilon^i = i \varepsilon^2, \quad \chi^i = i \chi^2, \quad \gamma^{ij} = i \delta^{ij} D. \quad (21)$$

The resulting action and transformation rules in  $d=4$  have the same form as given in (7) and (10). Note that with a suitable choice of  $c_1, c_2$  and  $c_3$  one can arrange that all order  $\alpha^2$  terms in (10) arise only in the bilinear and trilinear fermion terms. Note also that, in  $d=4$  the action (14) takes a rather simple form given by

$$\mathcal{L}(\alpha=4) = \frac{1}{2\alpha^2} (\sqrt{g} - 1) - 2\sqrt{g} \bar{\chi} \not{D} \chi. \quad (22)$$

The vielbein and the spin connection are defined in (16) and (17), respectively. The supersymmetry transformation rules are those given in (15), with only one order  $\alpha^2$  term occurring in the variation of the Yang-Mills vector field.

Although the higher derivative actions in 6 and 10 described above are the most general such actions, we expect that their dimensional reduction to  $d=4$  selects a particular higher derivative action, which need not be the most general one. By using tensor calculus in  $d=4$ , one can probably construct more general higher derivative actions which may even be exactly supersymmetric.

5. It would be interesting to determine whether the supersymmetry parameter  $\eta$  defined in (18) is a Killing spinor with respect to the torsionful connection (17).

6. It is an intriguing possibility to obtain the geometric action (14) from a supergravity theory, by expressing the fields of the supergravity multiplet in terms of the fields of the Yang-Mills multiplet.

7. It is rather straightforward to convince oneself that it is not possible to supersymmetrize an  $F^3$ -action. The reason is that all the fermionic terms can be generated by field redefinitions, and one is left with the variation of the  $F^3$ -term which evidently cannot be cancelled.

8. We have considered geometric actions of the form  $(g)^{1/2}$ , and not of the form  $R(F)$ .

With the vielbein given in (17), such an action would yield terms of the form  $(\alpha^2 F \square F + \alpha^4 F^2 (\partial F)^2 + \dots)$ . Note that the first term can be obtained from a redefinition of the Yang-Mills vector field in the usual Yang-Mills action.

9. In constructing  $(F^2 + \alpha^2 F^4 + \dots + \alpha^{2n} F^{2n+2})$  type actions for  $n \geq 2$  which are supersymmetric upto order  $\alpha^{2n}$ , it appears that the group theoretical factors must be generically of the form  $(\text{tr} T^a T^b)^{n+1}$ . Whether the low energy limit of an open superstring action has this property is an open question.

10. An important generalization of our work would be to couple the higher derivative super-Yang-Mills actions to supergravity.

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