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DOUBLE INFLATION: A POSSIBLE RESOLUTION OF THE LARGE-SCALE STRUCTURE PROBLEM

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ABSTRACT

A model is presented for the large-scale structure of the universe in which two successive inflationary phases resulted in large small-scale and small large-scale density fluctuations. This bimodal density fluctuation spectrum in an $\Omega = 1$ universe dominated by hot dark matter leads to large-scale structure of the galaxy distribution that is consistent with recent observational results. In particular, large, nearly empty voids and significant large-scale peculiar velocity fields are produced over scales of ~ 100 Mpc, while the small-scale structure over $\lesssim 10$ Mpc resembles that in a low density universe, as observed. Detailed analytical calculations and numerical simulations are given of the spatial and velocity correlations.

I. INTRODUCTION

Astronomers tell us, convincingly, that the universe is open, with density parameter $\Omega \approx 0.1$. Particle physicists argue plausibly and strongly that it should be marginally closed with $\Omega = 1$. Attempts to reconcile these diverse opinions have not hitherto carried much conviction. These include seemingly ad hoc appeals to biasing of density fluctuations (Davis *et al.* 1985; Bardeen *et al.* 1986) and to decaying particles (Dicus *et al.* 1977; Turner *et al.* 1984; Gelmini *et al.* 1984; Olive *et al.* 1985). Such schemes are designed to leave the dark matter distribution more uniform than the luminous matter, thereby resulting in an apparent Ω that is small. However not only are these schemes somewhat contrived, but they conspicuously fail to resolve other problems of large–scale structure, most notable among these being the clustering of galaxy clusters.

The problem, and the beauty, of inflation is that it provides highly specific initial data for the very early universe. It is precisely this specificity that has raised serious questions about the validity of inflation, and provoked the biasing and decaying particle schemes as rescue operations. However the difficulty may well arise through an oversimplified treatment of inflation itself. Two of us have recently described a novel approach to inflation that promises to resolve the Ω and large–scale structure problems (Silk and Turner 1986). Here we present some quantitative estimates of large–scale structure probes utilizing this new idea. We first review our scheme of double inflation for an astrophysical audience (§II). We next discuss an analytical model and its predictions of large–scale structure (§III), and present numerical simulations of the non-linear structure in §IV. A final section discusses the implications of our results.

II. DOUBLE INFLATION

Guth (1981) proposed inflation to resolve the horizon, flatness, and monopole problems. However, his original model was fatally flawed in that the Universe never exited the inflationary phase back to a radiation–dominated phase. All inflationary scenarios are now based upon the variant, dubbed 'new inflation', proposed independently by Linde (1982) and Albrecht and Steinhardt (1932). A generic prediction of all models of new inflation is adiabatic density perturbations with a scale–invariant spectrum which arises from quantum fluctuations during the inflationary process (Guth and Pi 1982; Starobinskii 1982; Hawking 1982; Bardeen *et al.* 1983). Below we will briefly desribe the aspects of new inflation relevant for double inflation. We refer the reader interested in more details to the recent review by Turner (1985).

According to our present understanding, inflation occurs whenever a weakly–coupled scalar field (denoted by φ) is displaced from the zero–energy minimum of its potential (denoted by $\sigma; V'(\sigma) = V(\sigma) = 0$). While the scalar field φ is displaced from the minimum of its potential, the associated potential energy density $V(\varphi)$, often referred to as 'vacuum energy', drives an exponential expansion of the Universe. Of course, a scalar field displaced from the minimum of its potential will evolve toward that minimum — just like a ball rolling down a hill. However, for a very weakly–coupled scalar field, the time required for φ to 'roll' to $\varphi = \sigma$ is a substantial number of Hubble times, during which time all the scales in the presently observable universe grow from a size smaller than the Hubble radius (during inflation) to a size much greater than the Hubble radius. To be more specific, if the evolution time for φ is greater than about 60 Hubble times, a small, smooth patch of the Universe (before inflation) will grow to a size which encompasses all that we see today.

Quantum mechanical fluctuations in the scalar field φ, which arise as it is evolving to $\varphi = \sigma$, ultimately result in adiabatic density perturbations with constant (albeit model–dependent) amplitude at the time they cross inside the horizon after inflation; this is the so–called Zel'dovich–Harrison spectrum of density perturbations. In particular, as a given scale L crosses outside the horizon during inflation, say at $t = t_1$, quantum fluctuations in the scalar field lead to adiabatic density perturbations on that scale, with amplitude which depends upon the evolution of φ at that instant

$$(\delta\rho/\rho)_{HOR} \simeq (H^2/\dot{\varphi})|_{t_1}. \tag{1}$$

The simplest way to specify when the scale λ crossed outside the horizon is by the number of e–folds of inflation, N, from horizon crossing until the end of inflation; it is straightforward to show that

$$N(L) \simeq 46 + ln(L/Mpc) + \text{'ln term'} \tag{2}$$

where the 'ln term' depends logarithmically upon the vacuum energy during inflation ($V(\varphi)$) and the reheating temperature after inflation. For the simplest scalar potential, $V(\varphi) = \lambda\varphi^4$, the amplitude of the density perturbations on astrophysically interesting scales ($N \simeq 30-50$) at horizon crossing is:

$$(\delta\rho/\rho)_{HOR} \simeq (32/3)^{\frac{1}{2}}\ \lambda^{\frac{1}{2}} N^{\frac{3}{2}}, \\ \simeq 10^3\ \lambda^{\frac{1}{2}}, \tag{3}$$

From Eqn(3) it is clear that to achieve an acceptably small amplitude, $\sim 0(10^{-4})$, the coupling constant, λ, must be very small

$$\lambda \simeq 10^{-14}, \tag{4}$$

i.e., φ must be very weakly–coupled. This result is generic to all models of inflation.

At present there is no truly compelling particle physics model for inflation; however, there are many candidate models. They include models where the scalar field φ is associated with spontaneous symmetry breaking, supersymmetry/supergravity, superstrings, compactification of extra dimensions, exotic theories of gravity, or is just a 'random scalar field'. The plethora of scenarios suggests that the Universe may well have undergone several episodes of inflation. Scales which left the horizon during the same inflationary epoch will have the same amplitude; however, different episodes will necessarily result in different perturbation amplitudes. Silk and Turner (1986) have advocated the simplest version of multiple inflation, 'double inflation', as a means of 'decoupling' perturbations on large and small scales, and have constructed several specific particle physics models where double inflation occurs. We refer the reader interested in further details to their paper.

In this paper we will consider a specific model of double inflation, one where the two episodes of inflation separately produce a Zel'dovich–Harrison spectrum of density fluctuations, of small amplitude ($O(10^{-5})$) on large scales due to the first inflationary phase, and of large amplitude ($O(10^{-1})$) on small scales due to the second phase. In the next sections we discuss in detail the consequences of such a scenario for large–scale structure in the Universe.

III. GALAXY CORRELATIONS AND PECULIAR VELOCITIES

We consider a universe containing hot dark matter. Owing to the large–amplitude density perturbations on small scales, seeds will condense early and eventually develop into galaxies. Here we will not be concerned with the details of this process and will treat this on a purely phenomenological basis. We develop a simple phenomenological model here; in the following section, we shall present numerical simulations of the galaxy distribution.

Double inflation produces a gaussian fluctuation spectrum

$$P(k) = P_S(k) + P_L(k). \tag{5}$$

This is identical to the standard scale–invariant power spectrum $P_L(k)$ on large–scales, with a cut–off below comoving scale $2\pi/k_D = L_D = 13Mpc/\Omega h^2$ due to the free–streaming of the hot dark matter with one species of massive neutrinos, where as usual $h \equiv H_o/100\text{kms}^{-1}\text{Mpc}^{-1}$. We shall subsequently set $h = 0.5$. We write (Bond and Szalay 1983) $P_L(k) = ALk10^{-(k/k_D)^{1.5}}$. In addition, there is a non–linear small–scale seed contribution that we describe phenomenologically by writing

$$P_S(k) = A_s k^4 [1 + (\beta k)^5 \exp \alpha k]^{-1}. \tag{6}$$

The physical origin of the seeds is due to the fact that our double inflation model yields large fluctuations on small scales. These grow by gravitational instability to become non–linear seeds on galactic or subgalactic scales while the large–scale power produces only small amplitude density fluctuations. The seeds could be primordial black holes if they collapsed in the very early universe, or supermassive black holes such as are believed to lurk inside active galactic nuclei if collapse occurred after recombination. Or they could be a sub–population of massive galaxies that formed very early, at $z >> 10$. Such seeds play a crucial role in the explosive amplification theory of galaxy formation (Ikeuchi 1981; Ostriker and Cowie 1981; Bertschinger 1983; Carr and Ikeuchi 1985), and also arise naturally via accretion onto string loops in the cosmic string theory of galaxy formation (Zel'dovich 1980; Vilenkin 1985). We will not discuss the detailed mechanism of how these seeds transform themselves into galaxies, since our emphasis here is on large–scale structure.

The seed contribution to the fluctuation spectrum is designed to match the observed non–linear galaxy correlations over scales $\lesssim 5h^{-1}$ Mpc, and to mimic the "minimal" k^4 tail in the longwave part of the spectrum ($k\beta << 1$), that would inevitably arise due to mode coupling (Zel'dovich 1965). We adjust the parameters $\alpha = 1h^{-1}$ Mpc to yield the separation between small galaxies and $\beta = 10h^{-1}$ Mpc to correspond to the separation between galaxy clusters. On scales larger than or of the order of the mean cluster separation, $P_S(k)$ makes little or no contribution compared to $P_L(k)$. The effective correlation function is

$$\xi(r) = \xi_S(r) + \xi_L(r), \tag{7}$$

and is the Fourier transform of $P_S(k) + P_L(k)$. The choice (6) yields

$$\xi_S(r) = C\frac{(0.8)\sin(0.4\pi)}{2\pi^2}r^{-1.8} \tag{8}$$

for $\alpha << r << \beta$. Normalizing $\xi_S(r)$ to be equal to $(r/r_o)^{-1.8}$ with

$$r_o = 5h^{-1}\mathrm{Mpc} \tag{9}$$

yields

$$C = 324(\mathrm{Mpc})^{1.8}(r_o/5h^{-1}\mathrm{Mpc})^{1.8}. \tag{10}$$

Over larger scales, the galaxy correlations contribute only

$$\xi_S(r) \propto r^{-7}(r \gg \beta). \tag{11}$$

The correlations yield the gravitational potential energy associated with galaxies as a function of increasing scale, and application of the virial theorem (Peebles 1980) should, at least on small scales, provide a fair estimate of the rms velocities of galaxies. Use of the two-point correlation function gives the rms relative velocity of galaxy pairs, and we obtain for the observed correlations

$$v(r) = 505\left(\frac{\Omega}{0.1}\right)^{0.5}\left(\frac{r}{r_o}\right)^{0.1}\left(\frac{r_o}{5h^{-1}\mathrm{Mpc}}\right)\mathrm{km\ s^{-1}}. \tag{12}$$

Now we may take the effective Ω on these scales to be $\Omega_S = 0.1$. This is the contribution from the fluctuations driven by the primordial seed component that are responsible for forming galaxies. The hot dark matter contribution dominates, but only contributes to fluctuations on large scales. Thus we write $\Omega = \Omega_S + \Omega_L = 1$, $\Omega_L = 0.9$, and infer that $v \approx 500$ km s^{-1} for

$r \approx r_o = 5h^{-1}$ Mpc. This is consistent with observations: indeed the contribution of $\xi_L(r)$ to $\xi(r)$ on scales of $(5–10)h^{-1}$ Mpc may result in an observable flattening of the correlation function (Davis and Peebles 1983). This flattening also enhances the pairwise velocity field, and our model produces a slight increase of $v(r)$ with increasing scale (cf. equation (12)) that may match the pairwise velocity distribution as a function of scale better than any cold dark matter simulations (Davis *et al.* 1985).

In Figure 1, we present the results of analytic calculations of $\xi_L(r)$ and the peculiar velocity field $v_p(r)$. Normalization on large scales is accomplished by setting the J_3 integral equal to 600 h^{-3} Mpc^{-3}, where the J_3 integral is defined by

$$J_3(r) = \frac{1}{2\pi^2}\int_0^\infty dlnk P_L(k)[sin(kr) - kr\ cos(kr)]. \tag{13}$$

We have fixed A_s in order to have $\xi(5h^{-1}\text{Mpc}) = 1$. We then have

$$\begin{aligned} \xi(r) &= \frac{1}{2\pi^2}\int_0^\infty k^2\ dk\ P(k)\frac{sin\ kr}{kr} \\ v_p(r) &= \frac{1}{2\pi^2}\int_0^\infty dk P(k) e^{-k^2 r^2}. \end{aligned} \tag{14}$$

Note that $\xi(r)$ remains positive out to about 45 Mpc. This may be sufficient when filtered above a suitable threshold (Kaiser 1984) to account for much of the power in the cluster–cluster correlations. We cannot be very precise on this matter, since an adequate theory for the threshold amplification mechanism is lacking and because the systematic statistical errors in the cluster–cluster correlations are not well understood. However, we have applied the formula given by Politzer and Wise (1985) to compute the cluster correlations ξ_{cc} by setting a biasing threshold of $\nu = 4$. The uncertainties in the data are taken from a recent analysis of the cluster correlations by Ling *et al.* (1986). The agreement between predicted and observed ξ_{cc} is reasonable, especially considering that ξ_{cc} is only the expectation value of the correlation function at a given log r.

The predicted large–scale peculiar velocity field is much harder to reconcile with observation. In particular, we compare it on scales $\sim 5h^{-1}$ Mpc where there is the dipole motion of $\sim$ 600 kms^{-1} (Lubin and Villela 1986), and at $\sim 60\ h^{-1}$ Mpc where there is the recent cosmic drift measurement of $\sim$ 600 kms $^{-1}$ (Dressler *et al.* 1986). Since a randomly placed observer will, five percent of the time, measure a dipole velocity that is below 1/3 of the plotted value and a large scale peculiar velocity that exceeds the plotted values by 1.6, our model only reconciles the large–scale peculiar velocity measurements provided that A is increased by about a factor of 4. This is marginally consistent with the cluster correlations and allows a significant peculiar motion on large scales, thereby avoiding the difficulties arising in biased cold or hot (conventionally normalized : $\langle\delta\rho/\rho\rangle_{\rm rms} = 0.6$ at $z = 3$) dark matter scenarios (Vittorio and Turner 1987). The normalization to cresting today is equivalent to setting $\langle\delta\rho/\rho\rangle_{\rm rms} = 0.6$ at $z = 1$.

Perhaps the most serious potential objection to our model concerns the amplitude of the galaxy peculiar velocities on small scales ($\lesssim 5h^{-1}$Mpc) where the galaxy correlations are unity or larger: can we reconcile the relatively low observed values with an inflationary $\Omega = 1$ cosmology? We address this in the following section.

IV. NUMERICAL SIMULATIONS

No self–respecting theory of large–scale structure is complete today without verification by means of numerical simulations. To help substantiate the phenomenological model given in the previous section, we have studied simple numerical models that we see as first approximations to the problem at hand.

We would like to generate an initial mass distribution with the correct initial power spectrum. However, the number of particles and/or dynamical range in length scales in presently available N–body codes is not sufficient to specify in detail the initial large and small–scale structure present after recombination. For the initial exploration of our phenomenological theory, a simple model will suffice. We approximate the small scale structure by an initial Poisson distribution, which is represented by a flat power spectrum. The large–scale structure is approximated by a single wave with a wavelength the size of the computational box, or a delta function in k-space. We use a 3-D particle mesh code developed by JVV (Villumsen and Davis 1986). We have 32768 particles and the initial conditions are generated using the Zel'dovich approximation (Efstathiou, Davis, Frenk and White 1985; hereafter EDFW).

We have run four models past the point where the wave crests and one control model. In these models the only variable is the strength of the wave relative to the white-noise level. The Poisson noise is taken to be identical in the various runs. We vary the strength of the wave by setting the expansion factor a_{cr} at which the wave crests. We have tried a_{cr}= 4.05(A), 6.075(B), 8.1(C), 16.2(3), ∞(E), with the main difference being the time that the small scale structure has to amplify before cresting. Model C is the model we favor.

Figure 2 shows snapshots of model C(a_{cr}=8.1) seen in two projections. The pancake plane is the x-y plane. The strong clustering is evident even before cresting. The correlation function in Figure 3 shows nearly self-similar evolution over a large range. Points inside log $r \sim -1.6$ with r measured in grid units are strongly affected by the grid softening and are not reliable. The logarithmic slope $\gamma \equiv -dln\xi/dlnr$ is approximately 2.0. This compares favorably with the observed slope of 1.8. EDFW found that the initial power spectrum $|\delta^2(k)|$ had to be steeper than $|\delta^2(k)| \propto k^{-n}$ with $n > 1$. In the present models we vary the relative strengths of the large and small scale structure, which effectively varies the slope of the initial power spectrum. In model A we find that the correlation function is too shallow; $\gamma \sim 1.5$. In model D the slope is slightly steeper than 2, while in model B, $\gamma \sim 1.7$. Model E was run as a test model for the code, and it develops a very steep correlation function $\gamma \sim 3$ at $a = 11$ in agreement with previous simulations (Davis *et al.* 1985). There is an obvious steepening of the correlation function with decreasing strength of the wave. This is equivalent to saying that the correlation function steepens with decreasing slope of the initial power spectrum.

Outside the correlation length, the correlation function becomes shallower due to the effect of the pancake. On scales larger than the thickness of the pancake the density distribution is effectively two-dimensional. This large-scale density contrast, which constitutes the pancake, shows up as a plateau in the correlation function. After the wave crests, the plateau is at a fairly constant amplitude in time because the thickness hardly changes. The strength of the small scale structure, however, grows so the plateau moves to larger and larger scales.

The pairwise velocity dispersion is calculated in units of the Hubble velocity across the box, excluding components in the z–direction, that is normal to the pancake plane. We can translate the velocities into physical units by setting the correlation length to $5h^{-1}$ Mpc. Figure 4 shows the velocity dispersion at various epochs for our favored model (model C). The velocity scaling has been applied at a = 13. We see that the dispersion profile is quite flat at all epochs. At the end of the simulation the velocity dispersion is in the range 400–500 km sec^{-1}. This is the one–dimensional velocity dispersion averaged over the two components in the pancake plane. If an observer were placed inside the pancake after the wave crested and then determined the pairwise velocity dispersion by measuring only radial velocities, he would not measure the true velocity dispersion. Most of his objects would be in the pancake plane and his velocity determinations would not be sensitive to the velocities perpendicular to the plane. He would thus underestimate the true velocity dispersion. If we apply the same scaling to physical units at the various epochs independently, we see that the amplitude of the velocity dispersion changes little in time. Models A and B have velocity dispersion profiles very similar to model C, while in model D the amplitude is over 1000 km $\sec^{-1}$. Due to the grid softening, these profiles are not meaningful inside log r ~-1.6. The magnitude of the coherent part of the velocity field is best seen in phase space; in Figure 5, we display plots of velocity versus height above the symmetry plane. The random component is between five and ten percent of the Hubble flow across the box, while the magnitude of the coherent flow amounts, after cresting, to about 60 percent.

In Figure 6, we show the density profile as a function of height above the symmetry plane. After cresting, the density contrast averaged over a region containing 50 percent of the mass is about 10 to 1 between pancake and the void evacuated by the assymmetric collapse. This compares well with observations: for example, in the Bootes void, a 3σ upper limit on the density of luminous galaxies is 25 percent of the mean background value (Oemler 1986).

V. DISCUSSION

Relaxing the conventional inflationary prediction of a single amplitude Zel'dovich–Harrison spectrum of scale invariant density fluctuations has allowed us to revive hot dark matter as a viable cosmological scenario for the evolution of large-scale structure. Our preferred scheme, referred to as double inflation, yields large amplitude small-scale density fluctuations that form pregalactic objects, capable of clustering and producing the galaxy–galaxy correlations, together with small amplitude, large-scale fluctuations that dominate on scales longward of $\sim 10-20h^{-1}$ Mpc. A wide variety of weakly non-linear initial conditions on small scales are capable of producing the k^{-1} power spectrum that characterizes galaxy clustering over 0.1 – 10 h^{-1} Mpc, and we have developed a phenomenological model that is insensitive to the detailed fine–scale structure of the early universe. It is the large–scale power that produces large–scale voids and sheets of galaxies.

How close are our models to accounting for the observed large–scale structure? Our phenomenological model is designed to reproduce the galaxy–galaxy correlations. Large voids arise naturally: the typical dimension is the coherence length of the hot dark matter component, between 40–100 Mpc. The density contrast between pancake and void is about 10 to 1, and is consistent with upper limits on the absence of luminous galaxies at the voids. The quasi-spherical topology observed for the voids is likely to be a natural outcome of any model in which voids are generated by gravity. Our models have $\Omega = 1$, but the effective Ω as measured by galaxy peculiar velocities is ~ 0.1. Thus no recourse to biasing schemes is necessary for galaxies. The low effective Ω is due to the fact that we take a snapshot of the pancake shortly after cresting occurs, before sufficient time has elapsed to randomize the coherent velocity field. Since we are not constrained by having to form galaxies out of the hot dark matter component, our model is not subject to the usual objections to a neutrino–dominated universe (see, e.g., Frenk *et al.* 1983). Our calculated peculiar velocities ($\sim 400 - 500$ kms^{-1}) are in reasonable agreement with galaxy–galaxy peculiar velocities on small scales ($\lesssim$ 10 h^{-1} Mpc), while on large scales v_p is still consistent with the large peculiar velocities of $\sim$ 600 kms^{-1} recently reported for galaxies clusters over scales up to $\sim 60h^{-1}$ Mpc. Our adopted large–scale normalization must be increased to $J_3 \approx 2000$ Mpc^{-3} in order to match these large–scale streaming motions and simultaneously allow a large enhancement in the cluster correlations. The gaussian nature of the fluctuations means that at least 5 percent of randomly placed observers would expect to measure a value in excess of a factor 1.6 larger than that shown in Figure 1 for the large–scale drift out to $\sim 60h^{-1}$ Mpc around Virgo. In addition, it should be noted that our adopted normalization, using the galaxy correlation length on small scales and the J_3 integral on large scales, is uncertain by at least a factor 2.

The two observations that we have not explicitly evaluated by means of numerical simulations are the cluster–cluster correlation function and the cluster peculiar velocities. Our simulation is of a single supercluster and lacks the dynamical range to examine larger scale structure. However it is clear that hot dark matter simulations must come much closer to producing sufficient clustering of galaxy clusters than do the biased cold dark matter simulations. Now these latter models probably produce sufficient numbers of rich clusters and do not fall far short of accounting for the cluster correlations when account is taken of the large statistical uncertainties in the data (White 1986); hence we suspect that our model should be capable of meeting this challenge. We have not unequivocally demonstrated this, of course, but it may be noted that the galaxy correlations do remain positive on large scales so that Kaiser's (1984) amplification mechanism may play an important role in enhancing the cluster correlations (Figure 1).

We note finally that there are only two models presently capable of meeting the challenge of large–scale structure posed by the triple challenge of the observations of 'Hubble bubbles', cosmic drift and clustering of clusters. Cosmic strings coupled with hot dark matter can possibly explain the latter two phenomena (Brandenberger *et al.* 1986), but it is difficult to see how galaxy correlations can be enhanced relative to the cluster correlations without breaking the self–similarity and how galaxy formation around string loops is suppressed in the voids: moreover, the non–gaussian nature of the fluctuations in this model has hitherto prevented any computation of the probability of our being associated with any specific observed large–scale flows. Double inflation with hot dark matter and decoupled small–scale structure appears to be capable of explaining all three of these phenomena, and the random phase nature of the fluctuations allows us to make specific predictions of probabilities for experimental tests.

Microwave background anisotropy may eventually provide a discriminant between these two competing scenarios, and we note that our renormalized hot dark matter model (non–linear at $z \sim 1$) predicts $\delta T/T \sim 3 \times 10^{-6}$ on large and intermediate angular scales (Silk and Vittorio 1987). Moreover any fine–scale anisotropy is likely to be suppressed due to reionization of the intergalactic medium in models with non–linear seeds developing at early epochs ($z \gtrsim 30$).

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FIGURE CAPTIONS

Fig. 1. Large-scale peculiar velocity field v_p and cluster–cluster correlation function ξ_{cc} versus scale r for either one, two, or three species of equal mass neutrinos with $\Omega = 1$, normalized as described in the text. The cluster correlations are computed by biasing the galaxy correlation function predicted from linear theory with a threshold $\nu = 4$ according to the prescription given by Politzer and Wise (1985). Error bars on the observed cluster–cluster correlations (Bahcall and Soneira 1983) are shown due to sampling uncertainties (Ling, Frenk and Barrow 1986). Data on the peculiar velocity field refers to measurements of the dipole anisotropy (Lubin and Villela 1986), effectively the Local Group motion at $5h^{-1}$ Mpc (Aaronson *et al.* 1986), Collins *et al.* (1986) (at $50h^{-1}$ Mpc), and Dressler *et al.* (1986) (at 60 h^{-1} Mpc).

Fig. 2. Snapshots of the evolution of the particle distribution looking perpendicular (x–z) to the pancake plane and looking down on the pancake plane (x–y). For a = 1,5,7,9,11,13(Model C).

Fig. 3. The correlation function at various epochs for Model C (a=5,7,9,11,13). The abscissa is the separation measured in grid units. Cresting is at a=8.1

Fig. 4. The velocity correlation function for Model C in km sec^{-1} ($V_z = 0$, with the scale normalised to 5 h^{-1} Mpc at $a = 13$). The other curves are rescaled by a factor a/13. The abscissa is the separation measured in grid units. Cresting is at a=8.1

Fig. 5. The reduced phase-space plot (z-V_z) for Model C. The abscissa is height above the pancake plane in grid units. The ordinate is the peculiar velocity in units of the Hubble velocity across the box. The plots are labelled by the expansion factor. Note the absence of small-scale structure near the pancake plane. A zero temperature wave would crest at a=8.1

Fig. 6. Evolution of the mean density $\rho(z)$ as a function of height above the pancake plane for Model C. The abscissa is height above the plane in grid units. The ordinate is the log of the density in units of the mean overall density. The plots are labelled by the expansion factor. 50% of the mass is contained within the vertical dotted lines. A zero-temperature wave would crest at a=8.1

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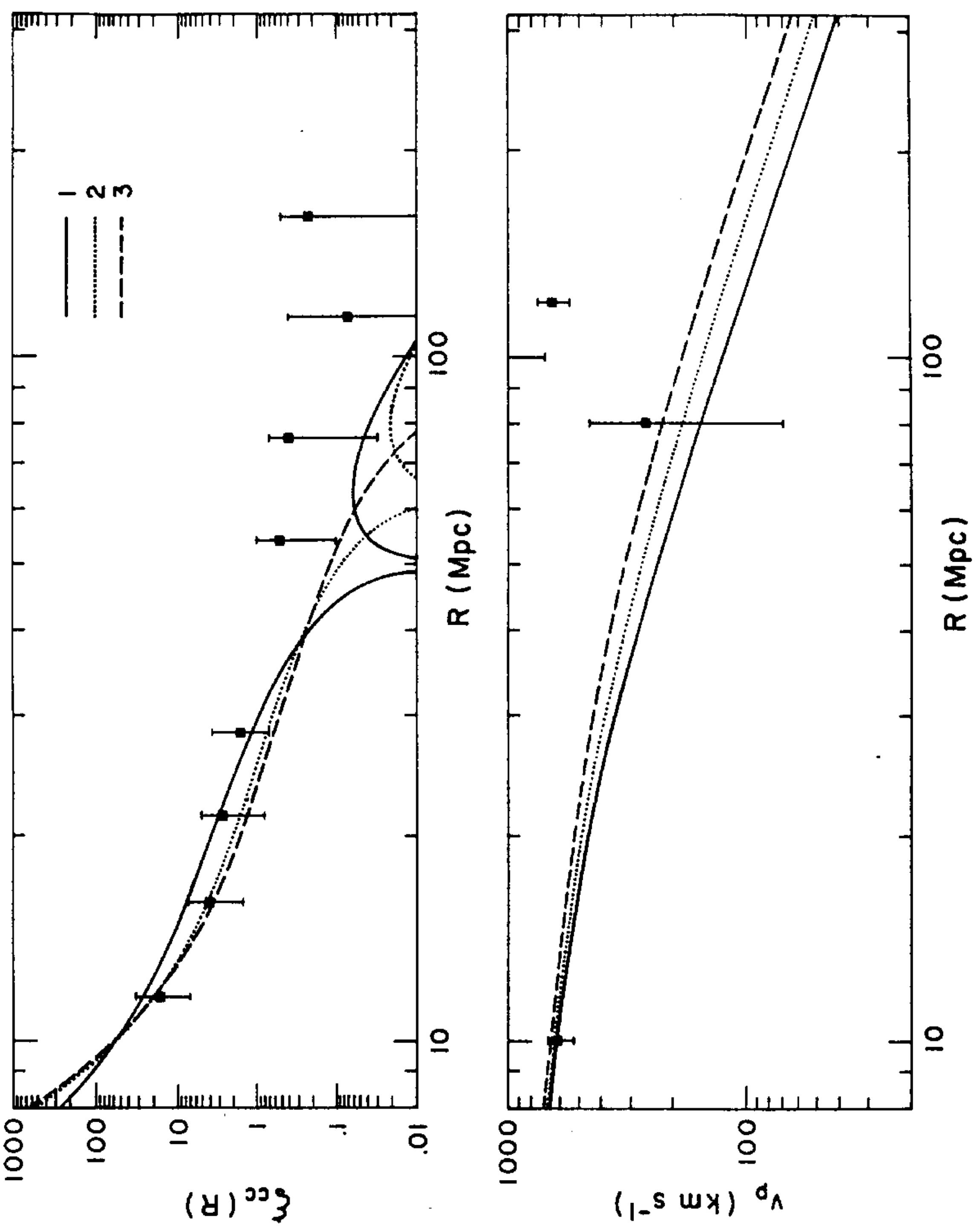
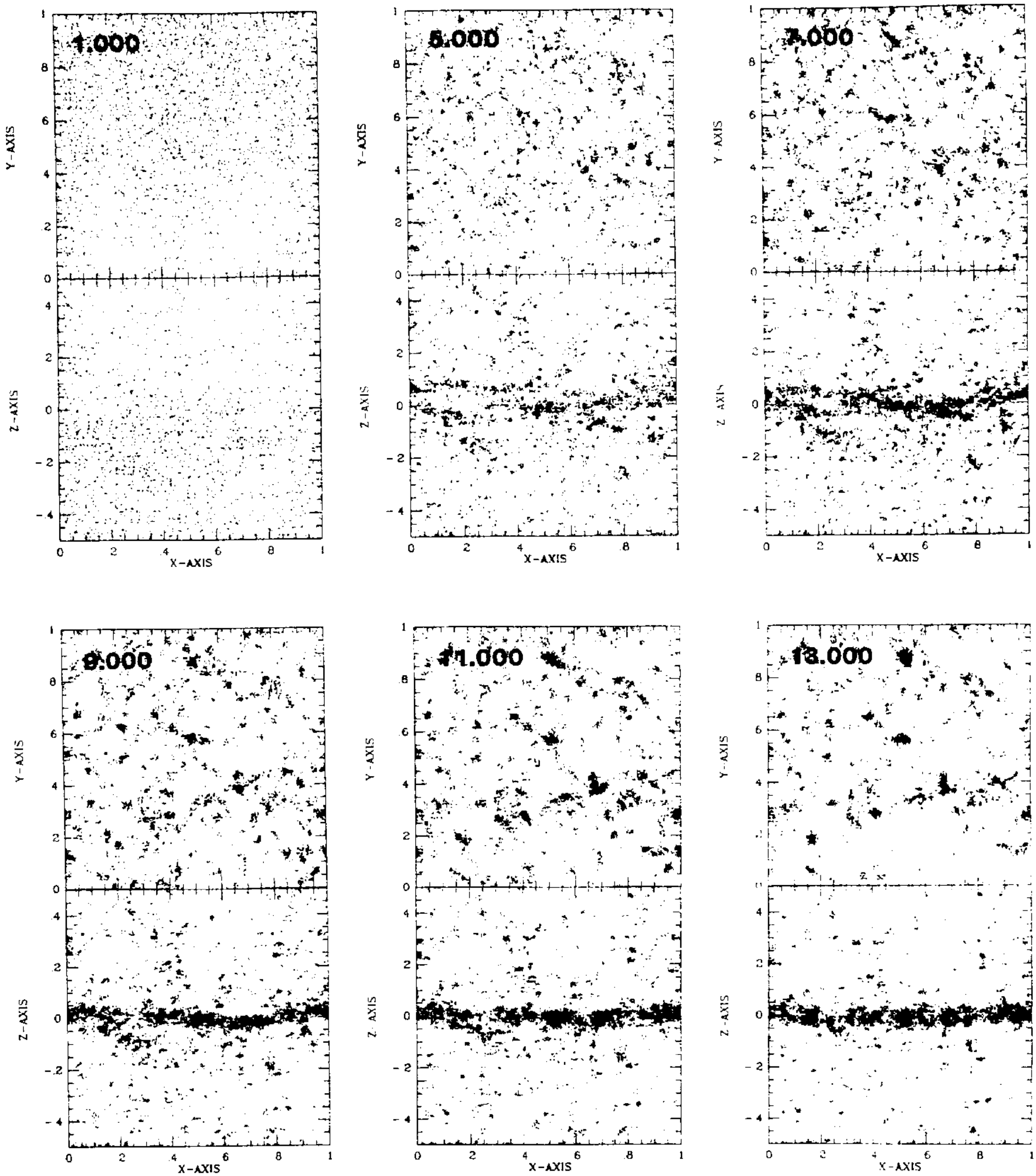
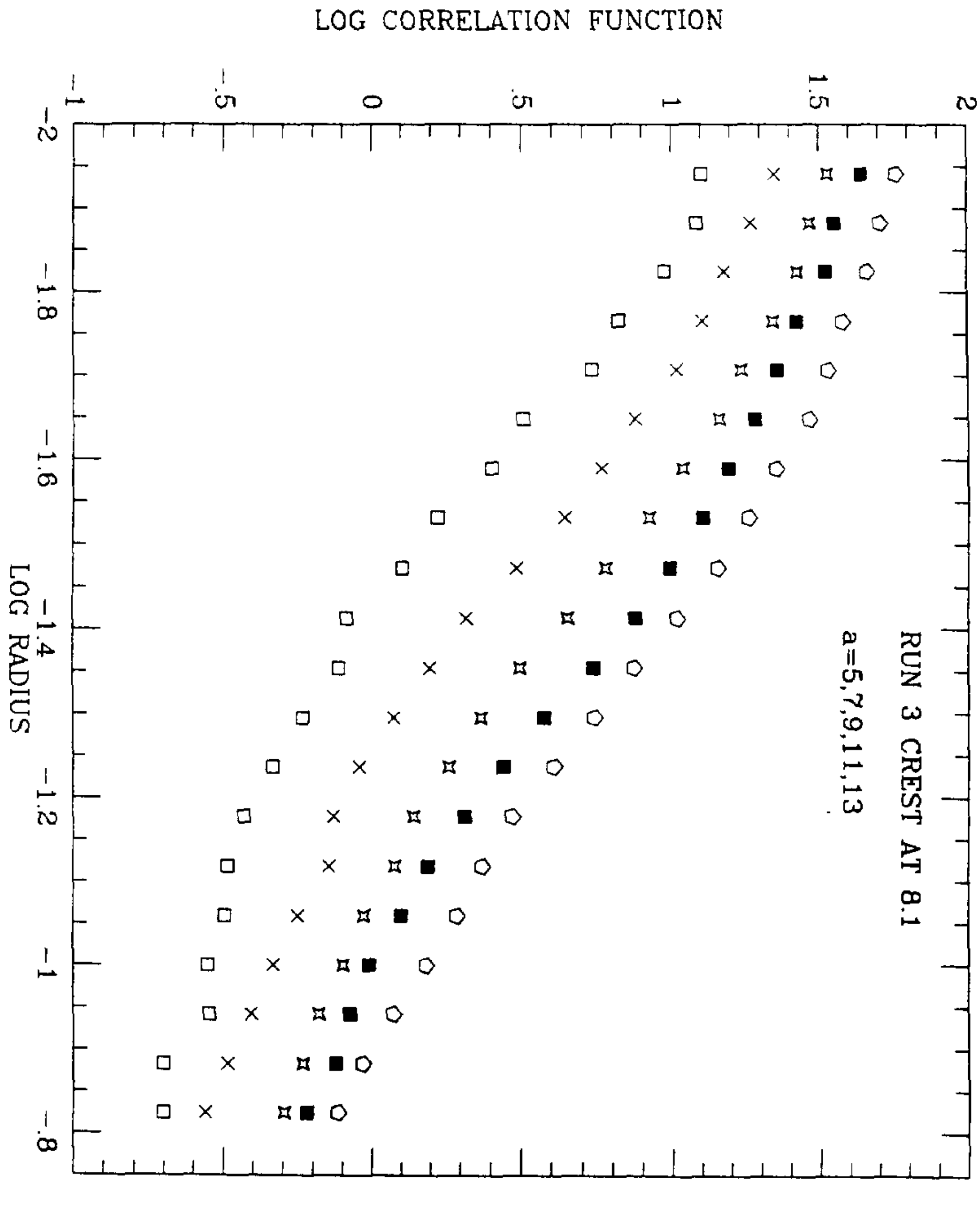


ξcc (R)
1000
100
10
1
.1
.01
1
2
3
10
100
R (Mpc)
vp (km s-1)
1000
100
10
100
R (Mpc)

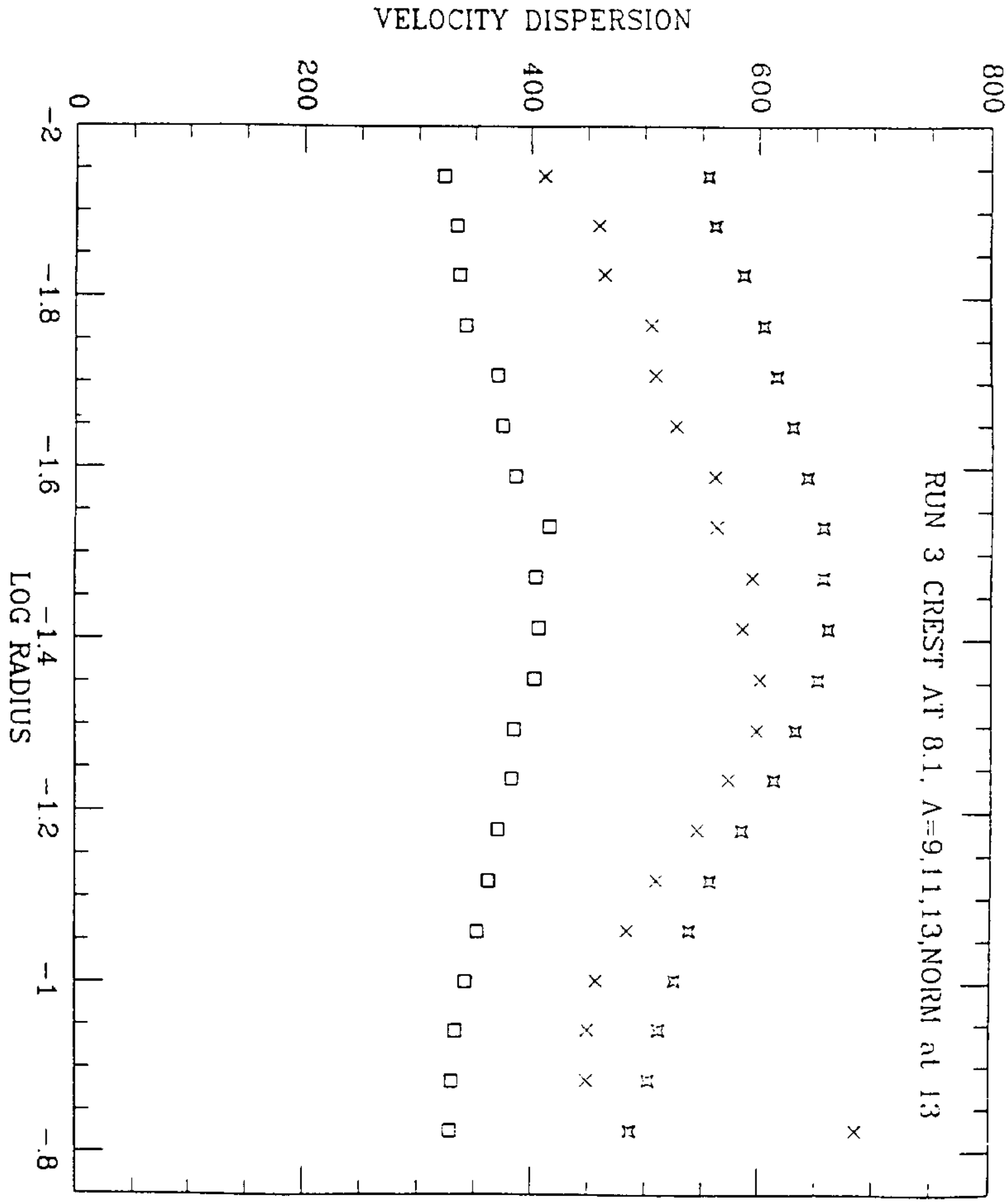
- Figure 1 -



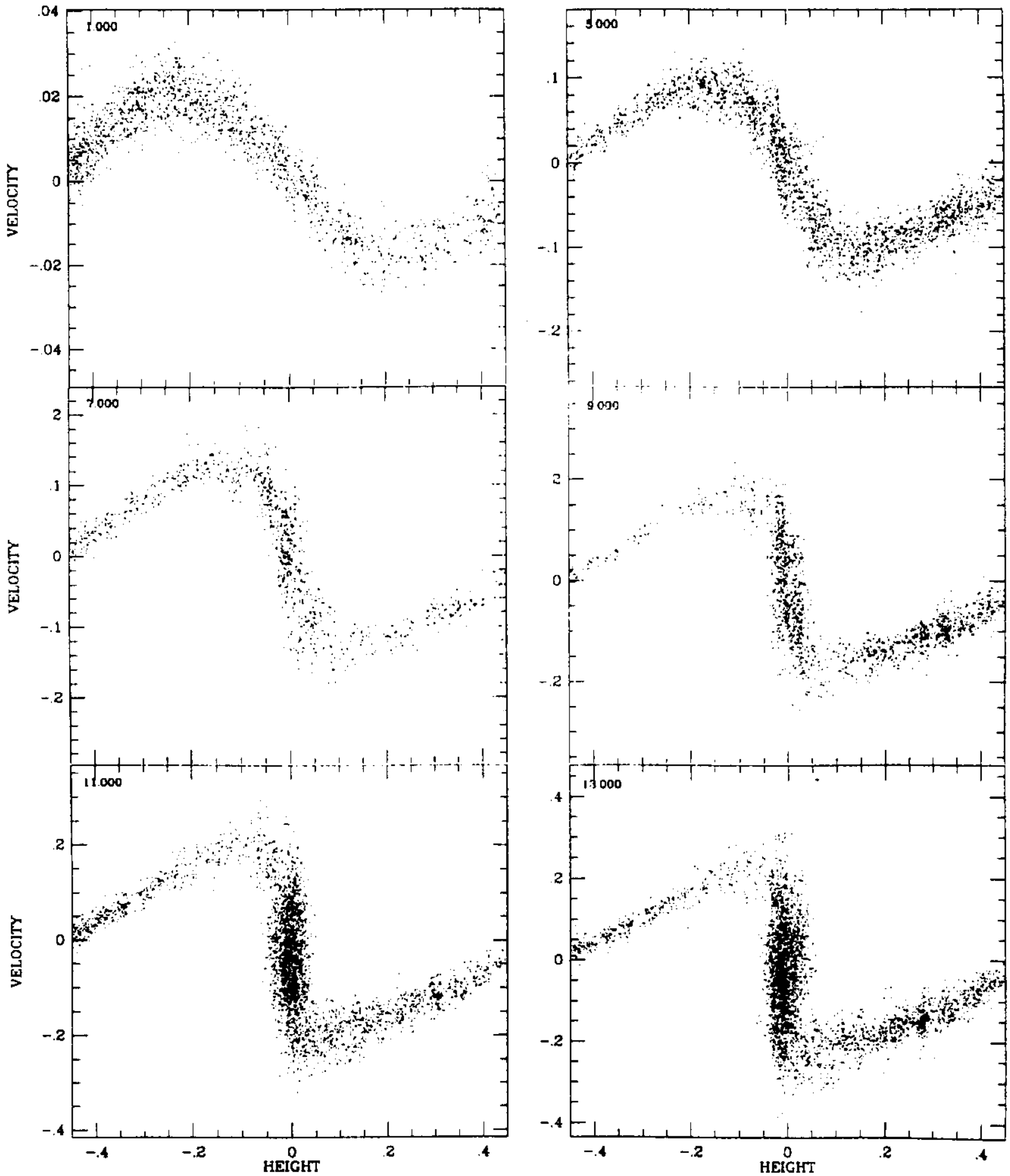
- Figure 2 -


LOG CORRELATION FUNCTION
LOG RADIUS
RUN 3 CREST AT 8.1
a=5,7,9,11,13

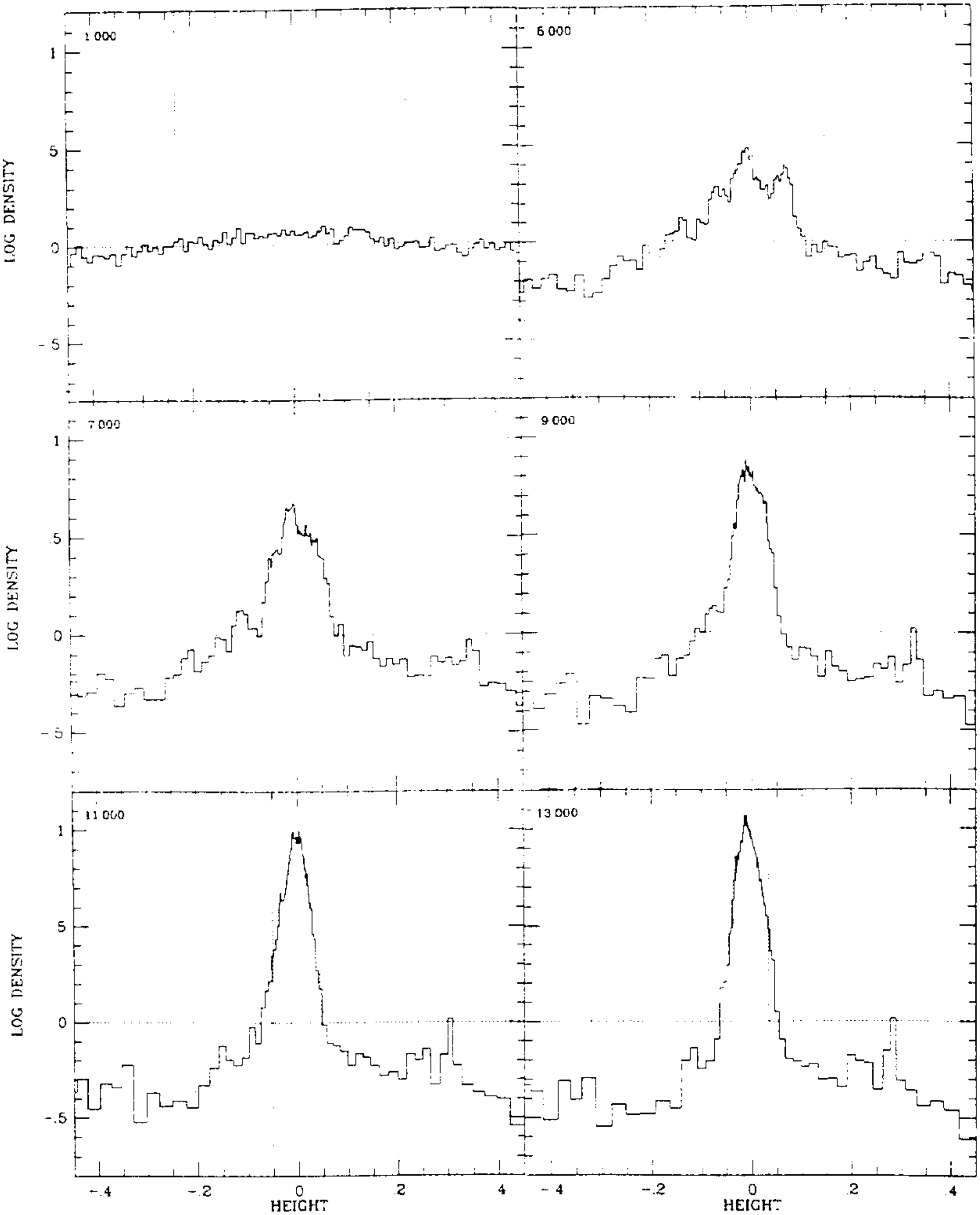
- Figure 3 -


VELOCITY DISPERSION
0
200
400
600
800
−2
−1.8
−1.6
−1.4
−1.2
−1
−.8
LOG RADIUS
RUN 3 CREST AT 8.1, A=9,11,13,NORM at 13

- Figure 4 -


1.000
5.000
7.000
9.000
11.000
13.000
VELOCITY
HEIGHT
.04
.02
0
-.02
-.04
.1
-.1
-.2
.2
.4
-.4

— Figure 5 —


1 000
6 000
7 000
9 000
11 000
13 000
LOG DENSITY
LOG DENSITY
LOG DENSITY
HEIGHT
HEIGHT

- Figure 6 -