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A MODEL FOR DYNAMICAL CHIRAL SYMMETRY BREAKING AND QUARK CONDENSATE

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Abstract

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In the framework of the model, proposed earlier[1,2] to describe non-perturbative QCD, the singularity of the type $1/k^4$ in the gluon propagator is shown to result in dynamical chiral symmetry breaking and appearance of quark condensate. The value, obtained for quark condensate, is close to the phenomenological one.

Аннотация

Некрасов М.Л., Рочев В.Е. Модель динамического нарушения киральной симметрии и кварковый конденсат: Препринт ИФВЭ 86-50. – Серпухов, 1986. – 9 с., библиогр.: 12 назв.

В рамках модели, предложенной ранее[1,2] для описания КХД в непертурбативной области, показано, что наличие сингулярности типа $1/k^4$ в глюонном пропагаторе приводит к динамическому нарушению киральной симметрии и возникновению кваркового конденсата. Полученное значение кваркового конденсата близко к феноменологическому.

INTRODUCTION

Constructing effective nonperturbative approximations of quantum chromodynamics (QCD) is one of the most urgent problems in modern field theory. In ref. [1] the solutions for the quark propagator equation were found in a model, proposed before [2] for the description of the QCD nonperturbative region. The underlying idea (repeatedly stated before [3-5]) is that the gluon self-action in the nonperturbative region causes a singular behavior of a gluon propagator of the type k^{-4}. The existence of such an infrared asymptotics of the gluon propagator corresponding to a linearly increasing potential, is confirmed by a number of the studies of the Schwinger-Dyson equations in the infrared region [4,6-8].

In accordance with this idea the Lagrangians of the Yang-Mills field are approximated in the nonperturbative region by an effective quadratic Lagrangian whose kernel is a gluon propagator with singular infrared asymptotics k^{-4}. Further it becomes possible to apply the N^{-1} expansion methods, which allow one to write down closed equations for the Green's functions in the leading order in $1/N_c$ where N_c is the color group rank.

In ref. [1], the exact solutions have been obtained for the quark propagator equation in the model with the propagator k^{-4} in some special gauge. One of these is chiral-invariant (for massless quarks), the other one is essentially nonperturbative and corresponds to the dynamical chiral symmetry breaking. Both these solutions, however, violate quite a number of general physical requirements (correspond to non-stable vacuum, etc.) and are, therefore, not suitable for constructing a realistic model of elementary particles. As shown in this paper (Section II), the equation for the quark propagator admits a whole class of solutions which are a combination of those obtained in ref. [1]

The auxiliary condition (realness of the vacuum expectations $\langle\bar{\psi}\psi\rangle$ and $\langle\bar{\psi}\, i\hat{\partial}\psi\rangle$) allows one to find a unique solution. This solution satisfies all physical requirements. Further in Section II an exact solution for the quark propagator (in nonperturbative region) has been obtained in the framework of the model, where account is made not only of the nonperturbative contributions to the infrared region but of ultraviolet one (upto logarithm), ie , of the model with a gluon propagator of the type $D(k)\sim\sim k^{-4}+k^{-2}$. The solution obtained satisfies the condition of realness and corresponds to the dynamical breaking of the chiral invariance. Section III is devoted to numerical calculations of the quark condensate based on this solution. The obtained value for $\langle\bar{\psi}\psi\rangle$ is close to the phenomenological one.

1. QUARK PROPAGATOR

The QCD effective model considered here is described by the Lagrangian

$$L=\frac{1}{2}A^{ab}_{\mu}D^{-1}_{\mu\nu}A^{ba}_{\nu}+\psi^{a}(i\hat{\partial}\delta^{ab}+\frac{g}{\sqrt{N_c}}A^{ab})\psi^{b} \quad (1)$$

Here $a,b=1,\ldots,N_c$ are color indices, g is a coupling constant, normalized in a way standard for the N_c^{-1}-expansion; $D_{\mu\nu}$ is a gluon propagator. The quark propagator S in the main approximation of the N_c^{-1} expansion is a solution of the equation[1,2]

$$S^{-1}=Z(-i\hat{\partial})-ig^{2}\gamma_{\mu}S\gamma_{\nu}D_{\nu\mu}. \quad (2)$$

In formula (2) Z is a renormalization constant, which compensates possible ultraviolet divergences. In accordance with what was said above, the gluon propagator is approximated by the expression:

$$D_{\mu\nu}=D^{IR}_{\mu\nu}+D^{UV}_{\mu\nu} \quad (3)$$

where $D^{IR}\sim 1/k^{4}$ is the leading term in the nonperturbative expansion in the infrared region, and $D^{UV}\sim 1/k^{2}$ is a "Coulomb" term, which coincides with the ultraviolet asymptotics of the gluon propagator upto a logarithm.

A choice of the gauge is quite an essential circumstance in finding a solution for equation (2). The gauge $d_l=4$, where there are no infrared divergences[1,7], is quite convenient for this purpose. In this gauge the gluon propagator has the form (4)

$$\frac{g^2}{4\pi}D_{\mu\nu}(k)=\left[\frac{2\pi\varkappa^2}{(k^2)^2}-\frac{a}{k^2}\right]\left(g_{\mu\nu}-4\frac{k_\mu k_\nu}{k^2}\right)=-\frac{1}{2}(2\pi\varkappa^2-ak^2)\partial_\mu\partial_\nu\frac{1}{k^2}$$

In formula (4) $\varkappa$ is a constant of the mass dimension which can be related to the slope parameter of a linearly increasing effective potential. The quantity a is weakly (logarithmically) dependent on k^2. We will neglect this fact, taking a to be a constant.

Going over to the Euclidean variables and fulfilling integration over the angles, one can present equation (2) with kernel (4) as a system of equations for the invariant structures A and B of the quark propagator $S(p)=A\hat{p}+B$:

$$\frac{A}{tA^2+B^2}=Z-\varkappa^2A-\frac{3}{4\pi}a\left\{\frac{1}{t^2}\int_0^t dt'\,t'^2A(t')+\int_t^\infty dt'A(t')\right\}$$

$$\frac{B}{tA^2+B^2}=\varkappa^2B. \qquad (5)$$

Here $t=-p^2$ (in the Euclidean region $t=p_e^2>0$).

From the second equation of system (5) it is seen that at $\varkappa=0$, ie , when nonperturbative contributions of the infrared region are not taken into consideration, only chiral-symmetric solutions (B=0) are permitted for the quark propagator. At $\varkappa\neq0$ system (5) has also anomalous solutions, breaking the chiral symmetry (in this case $B^2=\varkappa^{-2}-tA^2$).

First let us consider a simplified problem, assuming that $a=0$ in system (5). Corresponding solutions can easily be obtained and they have the following form/1,9/ (at Z=1)

$$S_1=\frac{1}{2\varkappa^2}\left(\hat{p}+\sqrt{4\varkappa^2-t}\right)$$

$$S_2=\frac{1}{2\varkappa^2}\left(1-\sqrt{1-4\varkappa^2/t}\right)\hat{p}. \qquad (6)$$

The first solution S_1 breaks the chiral symmetry and is essentially nonperturbative. The second solution S_2 corresponds to the iteration solution of equation (2) and is chiral-invariant. Besides these two solutions system (5) admits also two more analytical solutions, different from (6) in the signs at the roots, moreover it admits any combination of the solutions mentioned, made up with the help of θ-function, of the form

$$S=S_1\theta(t_o-t)+S_2\theta(t-t_o) \qquad (7)$$

with arbitrary t_o.

Among solutions enumerated, however, there is only one solution which possesses the very properties, peculiar for "true" physical solutions: a) realness of the vacuum expectation values $\langle\bar{\psi}\psi\rangle$ and $\langle\bar{\psi}\hat{p}\psi\rangle$ (this condition guarantees the realness of the momentum energy tensor trace[10]); b) correct (asymptotically free) ultraviolet behaviour of the quark propagator at $t\to\infty$ and, finally, c) a proper sign of the quark condensate*) (see below). Solution (7) is the very solution at $t_o=4\varkappa^2$. One can easily get convinced that propagator (7) at $t_o=4\varkappa^2$ is a continuous function of t. Consequently propagator (7) is the only physically acceptable solution of the effective model considered (at $\alpha=0$). Here the parameter t_o separates the nonperturbative ($t<t_o$) and perturbative ($t>t_o$) region of the model in the momentum scale.

Let us proceed now to the solution of a more realistic model, where account is simultaneously made of the ultraviolat region ($\alpha\neq0$) of the theory and of nonperturbative contributions from the infrared region. The solution of system (5) will be looked for in the form

$$A=A_1\,\theta(t_o-t)+A_2\,\theta(t-t_o),$$
$$B=B_1\,\theta(t_o-t). \qquad (8)$$

The point t_o is unambiguosly determined from physical conditions of realness of the VEVs $\langle\bar{\psi}\psi\rangle$ and $\langle\bar{\psi}\hat{p}\psi\rangle$. It can be shown that the quantity t_o determined in this way, is renormalization invariant, which provides the correctness of representation (8) from the renormalization point of view. (See Appendix).

When substituting (8) into system (5) the first equation falls into two, one is in the region of $t>t_o$ and the other in the region of $t<t_o$. The solution of the first equation we define as an iteration expansion (ie , we choose a perturbative solution). The second equation in the nonperturbative region of $t<t_o$ has the following form

$$Z-\frac{3}{4\pi}\alpha\int_{t_o}^{\infty}dt'A_2(t')=2\varkappa^2A_1(t)+\frac{3}{4\pi}\alpha\Big\{\frac{1}{t^2}\int_{o}^{t}dt't'^2A_1(t')+\int_{t}^{t_o}dt'A_1(t')\Big\}. \qquad (9)$$

*) The latter condition is automatically fulfilled as a consequence of the first two if the solutions are studied within the chiral limit $m\to+0$, where m is quark bare mass, added to the operator $-i\hat{\partial}$ in equation (2). Relevant solutions for massive quarks are given in ref.[1].

The renormalization constant Z is, by definition, chosen such that the quantity in the LHS of equation (9), is finite. Let us fix the finite arbitrariness, after removing ultra violet divergences, so that

$$Z-\frac{3}{4\pi}\alpha\int_{t_o}^{\infty}dt'A_2(t')=1. \qquad (10)$$

Normalization condition (10) is equivalent to the subtraction scheme at a fixed point $t=t_o$. Note here one important circumstance, that normalization condition (10) provides closeness of equation (9) with respect to the function $A_1(t)$, ie , the analysis of the solution in the region of $t<t_o$ may be realized independently of the properties of the solution in the region of $t>t_o$.

Equation (9) can easily be reduced to a linear differential equation

$$t\ddot{A}_1(t)+3\dot{A}_1(t)-\frac{3}{4\pi}\frac{\alpha}{\varkappa^2}A_1(t)=0. \qquad (11)$$

The solution of equation (11), finite at t=0 is expressed through the Bessel function of an imaginary argument. The integration constant of this solution is determined from the boundary condition at $t=t_o$, which follows from integral equation (9) together with normalization (10). The condition of realness, which may be presented in the form $B_1^2(t_o)\equiv\varkappa^{-2}-t_oA_1^2(t_o)=0$, yields an equation at t_o. This condition provides simultaneously continuity of the propagator over the momentum variable t.

Finally we obtain the following solution in the non-perturbative region $t<t_o$

$$A_1(t)=\frac{\sqrt{t_o}}{\varkappa t}\,\frac{I_2\left(\sqrt{\frac{3}{\pi}\frac{\alpha}{\varkappa^2}t}\right)}{I_2\left(\sqrt{\frac{3}{\pi}\frac{\alpha}{\varkappa^2}t_o}\right)},$$

$$B_1(t)=\sqrt{\varkappa^{-2}-tA_1^2(t)}, \qquad (12)$$

where I_2 is a modified Bessel function of the first kind and t_o satisfies the equation

$$\sqrt{\frac{3}{4\pi}\alpha}\,I_1\left(\sqrt{\frac{3}{\pi}\frac{\alpha}{\varkappa^2}t_o}\right)=I_2\left(\sqrt{\frac{3}{\pi}\frac{\alpha}{\varkappa^2}t_o}\right) \qquad (13)$$

The quantity t_o is a rising function of α, in this $t_o \to 4æ^2$ at $\alpha \to 0$. Solution (12) is essentially nonperturbative (non-analytical over $æ^2$ in zero. At $\alpha = 0$ it goes into S_1 (see formula (6)).

QUARK CONDENSATE

The solution obtained for the quark propagator in the nonperturbative region (see previous Section) allows us to calculate an important phenomenological parameter - quark condensate $\langle\bar{\psi}\psi\rangle$ which plays a fundamental role in mesonic physics and which is a measure for dynamical breaking of the chiral symmetry. Its value according to formula (8) is

$$\langle\bar{\psi}\psi\rangle = i\int\frac{dp}{(2\pi)^4}\mathrm{tr}S(p) = -\frac{N_c}{(2\pi)^2}\int_0^{t_o} dt' t' B_1(t'), \qquad (14)$$

where the function $B_1(t)$, corresponding to normalization (10) is determined with formula (12).

Thus the quark condensate is expressed through two parameters of our model $æ$ and α. For numerical estimates we are to stick to concrete values of these parameters.

The value for the parameter $æ$ (see formula (4)) is quite naturally to be related to the slope parameter of the linearly increasing effective quark potential $V(r) = Kr$, $r \to \infty$. A corresponding connection has the form $\pi æ^2 = K$. From the phenomenology of heavy quarkonium the value for K is well known and is equal to $K = (420\ \mathrm{MeV})^2$, and, consequently, $æ = 237$ MeV. The parameter α is estimated to be a running coupling constant of QCD $\alpha_S(t)$ at the point separating perturbative and nonperturbative regions in our models, ie , at the point $t = t_o$.*) In other words, for α we take the solution of the equation

$$\alpha = \frac{N_c}{2}\,\frac{4\pi}{b_o \ln(t_o(æ, \alpha)/\Lambda^2_{QCD})}, \qquad (15)$$

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*) There is no strict estimate in the effective potential for the coefficient at the "Coulomb" correction. Usually it is chosen (with an accuracy up to group factor) as a running coupling constant $\alpha_s(m_q)$, where m_q is a heavy quark mass.

where $b_0=11\ N_c/3$ (fermionic loops in the leading $1/N_c$ approximation are not taken into consideration), $t_0(\varkappa, a)$ is a solution for equation (13). The coefficient $N_c/2$ in formula (15) corresponds to the coupling constant normalization adopted here (see formula (1)). At $\varkappa=237$ MeV, $\Lambda_{QCD}=300$ MeV the solution for equation (13) and (15) is $a=1.2$; $\sqrt{t_0}=600$ MeV*).

The quark condensate (14) here is equal to (at $N_c=3$)

$$\langle\bar{\psi}\psi\rangle_{t_0} = -(240\ \text{MeV})^3, \tag{16}$$

which is in agreement with the phenomenological value

$$\langle\bar{\psi}\psi\rangle \approx -(240\text{-}250\ \text{MeV})^3.$$

Note an important fact that a considerable contribution to the quark condensate value is made by the Coulomb-like term k^{-2}. A qualitative explanation for it is as follows; the quark condensate is formed in the region of "average" momenta ($\sim t_0$) where the Coulomb-like contributions and infrared ones ($\sim k^{-4}$) are compatible in value.

The estimates for a, determining the intensity of the "Coulomb" term $1/k^2$, obtained here, are to be compared with the results of other independent approaches. For instance, in ref./8/ the analysis of the Schwinger-Dyson equation in the infrared region, as well as the investigation of the long-wave fluctuations of the string and QCD calculations on the lattice, see ref./12/, make it clear that there do exist Coulomb-like corrections to the leading terms, moreover their coefficient has the value comparable with our estimate (the coefficient at such a correction calculated in/12/, is equal to $\pi/3$, which in fact, coincides with the value for $a=1.2$ obtained by us).

As the main conclusion of our paper we should mark out the fact that the presence of nonperturbative infrared contributions $\sim 1/k^4$ in the gluon propagator results in dynamic breaking of the chiral invariance in QCD, as well as to dynamical appearance of characteristic sizes of the nonperturbative region (in our model it is determined, unambigously and is equal to $\sqrt{t_0}=600$ MeV). The numerical results obtained make it clear that the model considered is applicable not only for qualitative, but for quantitative description of hadrons.

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*) It is interesting to note that this value of t_0 is close to the size of the nonperturbative region obtained in/11/ at calculations of the gluon vacuum expectation values.

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Appendix

The renormalization-invariant property of the quantity t_o greatly depends on the way of fixing the boundary condition in formulae (8). This condition is given, proceeding from the physical principle of realness, which can be presented in the form $B_1(t_o)=0$ or, equivalently, in the form

$$\varkappa^{-2}-t_oA_1^2(t_o)=0. \quad (A.1)$$

Then we demand the invariance of this condition respect to renormalizations

$$\varkappa^2 = z^2 \varkappa'^2, \qquad A_1 = z^{-1} A_1' \tag{A.2}$$

ie , in the renormalized theory the quantity t_o' is determined from the equation

$$\varkappa'^{-2} - t_o' A_1'^2 (t_o') = 0. \tag{A.3}$$

Coming back in formula (A.3) to unprimed quantities one can easily obtain with the help of (A.2) the following relation

$$\varkappa^{-2} - t_o' A_1^2 (t_o') = 0$$

from which and from (A.1) it follows that $t_o' = t_o$, Q.E.D.

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