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ELASTIC pp–SCATTERING IN THE COULOMB–NUCLEAR INTERFERENCE REGION AND LOW ENERGY BEHAVIOUR OF $\bar{p}p$–SCATTERING PARTIAL AMPLITUDES

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Abstract

The experimental data on the low energy elastic $\bar{p}p$ scattering in the Coulomb-nuclear interference region /1/ and on the shift and width of the 1s level of $\bar{p}p$-atom /2,3/ are analysed. The partial wave amplitudes for l=0,1 are extracted. The p-wave amplitude is in fair agreement with the atomic data for the 2p state /2,4/ and exhibits some energy structure. It is shown that the real-to-imaginary ratio of the $\bar{p}p$ forward elastic-scattering amplitude becomes negative in an energy interval just near $\bar{p}p$-threshold.

Recently at the LEAR facility of CERN the differential $\bar{p}p$ cross section was measured at seven beam momenta between 180 and 590 MeV/c /1,5/. The data in the Coulomb-nuclear interference region were used in Ref. /1/ to extract the real-to-imaginary ratio of the elastic-scattering amplitude $f_N(k,\theta)$ at $\theta = 0°$

$$\rho = Re f_N(k,0°) / Im f_N(k,0°). \qquad (1)$$

The results of paper /1/ show that function $\rho(p)$ depends nontrivially on the antiproton lab momentum p in the near-threshold region, see Fig. 1. Namely, ρ is positive for $p \approx 590$ MeV/c, becomes negative for momenta below $p \approx 500$ MeV/c, reaches its minimum at $p \approx 260$ MeV/c and changes sign again at $p \approx 230$ MeV/c.

In this article we investigate the low energy behaviour of the $\bar{p}p$ scattering amplitudes using the results of Ref. /1/ and the $\bar{p}p$-atomic data /2,3/. As we have no experimental information on spin dependence of the $\bar{p}p$ amplitude in this energy region, we use the $\bar{p}p$-amplitude averaged over spin variables. So the amplitude $f_N(k,\theta)$ should be considered as a scalar function of the momentum k and the scattering angle θ in the c.m. frame.

1. On the model-independent analysis of the data in the Coulomb-nuclear interference region

In paper /1/ the differential cross-section in the Coulomb-nuclear interference region was fitted fairly well with nuclear scattering amplitude $f_N(k,\theta)$ of the following form

$$f_N(k,\theta) = \frac{k\sigma_{tot}}{4\pi}(i+\rho)e^{-bt/2}, \tag{2}$$

where parameters σ_{tot}, ρ, and b depend on k. The differential cross-section was fitted with the following formulae:

$$\frac{d\sigma}{d\Omega} = \left|\frac{\alpha m_p}{t}F^2(t)e^{-i\delta(t)} + f_N(k,\theta)\right|^2, \tag{3}$$

$$F(t) = (1+t/0.71(GeV/c)^2)^{-2}, \tag{4}$$

$$\delta(t) = -[\ln(R^2t)+C]\frac{\alpha}{v}. \tag{5}$$

Here $\alpha = 1/137$, v is $\bar{p}p$ relative velocity, F(t) is the proton charge formfactor and (−t) is the 4-momentum transfer squared,

$$t = 2k^2(1-\cos\theta). \tag{6}$$

$R^2 = 9.5\ (GeV/c)^{-2} = (0.6\ fm)^2$, C = 0.5772 is the Euler's constant. Formulae (2)–(5) were obtained /6/ for the problem of charged particles scattering at high energy. For the low energy case the status of Eqs. (2)–(5) is rather questionable. The correct formulae for the cross-section are given below((9)–(13)). These formulae lead to values of σ_{tot} and ρ differ slightly from those obtained in Ref. /1/. Nevertheless the experimental errors in σ_{tot} and ρ are now larger than the discrepancies between values of σ_{tot} and ρ obtained within different approaches (2)-(5) and (9)-(13).

We begin our consideration with the analysis of the nuclear amplitude of the form (2) taken from Ref. /1/. This amplitude surely doesn't work well in the full angular range, but it may be used to estimate contributions to the total cross-section from different partial waves. Making partial expansion of (2) we get the following expression for partial amplitudes $f_\ell(k)$:

$$f_\ell(k) = \frac{k\sigma_{tot}}{4\pi}(i+\rho)e^{-bk^2}\sqrt{\frac{\pi}{2bk^2}}\,I_{\ell+1/2}(bk^2), \qquad (7)$$

where $I_{\ell+1/2}(z)$ is the modified Bessel function. The partial cross-section is determined with the formula

$$\sigma_\ell = \frac{4\pi(2\ell+1)}{k}\,\mathrm{Im}\, f_\ell(k). \qquad (8)$$

Numerical results for σ_ℓ with parameters σ_{tot}, ρ, and b from Ref. /1/ together with those obtained in Ref. /5/ are given in Fig. 2. Notice that in Ref. /5/ the other than in /1/ set of experimental data was used. Namely they used as an input the differential $\bar{p}p$ cross-section $d\sigma_{el}/d\Omega$ at $30° < \theta_{c.m.} < 180°$. In Ref. /5/ the other formula for $f_N(k,\theta)$ was also used.

Nevertheless we have obtained the results for σ_ℓ similar to those in /5/, see Fig. 2. So we conclude that for $p \lesssim 300$ MeV/c contribution to σ_{tot} from s- and p-wave dominates, the d-wave contribution being less than 10%. Notice that different theoretical models of $\bar{N}N$-interaction /7-9/ confirm also the dominance of s- and p-waves in this energy region.

Now discuss, whether the procedure of extracting parameters σ_{tot} and ρ using Eqs. (2)-(5) is correct. It is well known /10/ that for the interaction being the sum of the Coulomb and the short-range potentials the scattering amplitude has the following form:

$$f(k,\theta) = f_c(k,\theta) + \sum_\ell e^{2i\delta_\ell^c} f_\ell^{(NC)}(k), \tag{9}$$

$$f_c(k,\theta) = \frac{m\alpha}{t} \exp\left[2i\left(\delta_0^c + \frac{\alpha}{v} \ln \sin\theta/2\right)\right]. \tag{10}$$

Here $f_c(k,\theta)$ is the Coulomb scattering amplitude, $f_\ell^{(NC)}$ are the partial Coulomb-nuclear amplitudes and δ_ℓ^c are the Coulomb scattering phases,

$$\delta_\ell^c(k) = \arg \Gamma(1+\ell-i\alpha/v). \tag{11}$$

Taking into account that among all $f_\ell^{(NC)}$ only $f_0^{(NC)}$ and $f_1^{(NC)}$ dominate, we obtain *)

$$\frac{d\sigma}{d\Omega} = \left| \frac{m\alpha}{t} \exp\left(2i\frac{\alpha}{v} \ln \sin\theta/2\right) + f_{NC}(k,\theta)\right|^2, \tag{12}$$

where

$$f_{NC}(k,\theta) = f_0^{(NC)}(k) + 3 f_1^{(NC)}(k)\, e^{2i(\delta_1^c - \delta_0^c)} \cos\theta. \tag{13}$$

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*) The charge formfactor effect may be included in $f_{NC}(k,\theta)$.

Comparison of (2) and (13) shows some differences between these formulae. First of all, they differ by phase ϕ :

$$f_{NC}(k,\theta) = e^{-i\phi} f_N \ , \ \phi = \frac{\alpha}{v}\left[2\ln(2kR)+C\right]. \qquad (14)$$

Secondly, partial wave expansion of f_{NC} (13) contains additional phase factors different for each partial wave. At last the amplitude f_N defined by Eq. (2) is determined for the given k by three parameters only (σ_{tot} , b , ρ). But the exact amplitude (13) depends on four parameters - the real and the imaginary parts of $f_0^{(NC)}$ and $f_1^{(NC)}$. One can estimate role of mentioned factors on the extracted values of σ_{tot} and ρ . In the momentum region under consideration $p \lesssim 300$ MeV/c

$$|\phi| \lesssim 2 \cdot 10^{-2}$$

It is less than experimental errors for $\rho(p)$, so the phase difference between f_{NC} and f_N is not essential. The correction of the same order of magnitude to σ_{tot} and ρ comes from the factor $\exp\left[2i(\delta_1^c - \delta_0^c)\right)$ in Eq. (13).

Notice that amplitude $f_{NC}(k,\theta)$ (13) at small angles $1 - \cos\theta \ll 1$ may be transformed to the form

$$f_{NC}(k,\theta) = \frac{k\,\tilde{\sigma}_{tot}}{4\pi}\left(i+\tilde{\rho}\right)e^{-\tilde{b}t/2}. \qquad (15)$$

Here parameter $\tilde{b}$ is complex, in contrast to (2), and its imaginary part produces the angular dependence of the real-to--imaginary ratio of the amplitude $f_{NC}(\theta,k)$. To estimate the importance of this dependence in the Coulomb-nuclear interference region we suppose $\mathcal{J}m\,\tilde{b} \sim Re\,\tilde{b} \sim b$ and obtain that for angles $\theta = 0° \div 25°$ the phase of the total amplitude changes only by $|\delta\varphi| < 2\cdot 10^{-2}$. Notice that the results of Ref. /1/ for σ_{tot}, b, and ρ are to be considered as averaged over the angular range of essential Coulomb-nuclear interference. In principle it is possible to introduce corrections to σ_{tot} and ρ, obtained in /1/, taking into account the true angular - dependent phase of strong amplitude $f_N(k,\theta)$ from Ref. /5/. But it is evident that these corrections are smaller than the existing experimental errors and do not change the values σ_{tot} and ρ seriously.

Nevertheless it would be desirable to use exact formulae (12), (13) for the analysis of all the experimental data /1,5/ and to extract parameters σ_{tot} and ρ in the model-independent way.

2. $p \to 0$ limit of ρ

In the limit $p \to 0$ the s wave dominates the scattering, so we have

$$\rho(0) = Re\,\bar{a} / \mathcal{J}m\,\bar{a} \tag{16}$$

where $\bar{a} = a^s/4 + 3a^t/4$ is the spin-averaged $\bar{p}p$ scattering length.

One can determine both singlet a^s and triplet a^t scattering lengths using the data on nuclear shift ΔE and width Γ of 1S-level for orto-(1^1S_0) and para (1^3S_1)-protonium states with the formula /11/

$$\Delta E - i\,\Gamma/2 = \frac{4\pi a}{m_p}\,|\Psi_{1S}(0)|^2 = a\alpha^3 m_p^2/2. \tag{17}$$

From (16, 17) it follows *)

$$\rho(0) = -2\,\frac{3\,\Delta E(1^3S_1) + \Delta E(1^1S_0)}{3\,\Gamma(1^3S_1) + \Gamma(1^1S_0)}\,. \tag{18}$$

The measurements of the shifts and widths of the 1 levels of $\bar{p}p$-atoms give the following results **)

$$\begin{aligned} &\Delta E\ (1S) = 0.5 \pm 0.3\ \mathrm{KeV}, && \Gamma \simeq 1\ \mathrm{KeV}\ [2] \\ &\Delta E\ (1S) = 0.73 \pm 0.15\ \mathrm{KeV}, && \Gamma = 0.85 \pm 0.39\ \mathrm{KeV}\ [3] \end{aligned} \tag{19}$$

The large experimental errors in (19) do not allow one to determine $\rho(0)$ with high accuracy:

$$\rho(0) \approx -2 \div -1\ . \tag{20}$$

Nevertheless the negative sign of $\rho(0)$ follows in unique way from the fact that the nuclear interaction repulses the S-levels of $\bar{p}p$-atom. The repulsion of the S-level was predicted in papers /12,13/. Different theoretical models of $\bar{N}N$-interaction /14-16/ give the following predictions for $\rho(0)$:

$a^s = (0.81 - i0.91)$ fm, $a^t = (0.91 - i0.76)$ fm, $\rho(0) = -1.1$ /14/

$a^s = (0.54 - i0.51)$ fm, $a^t = (0.76 - i0.45)$ fm, $\rho(0) = -1.5$ /15/

$a^s = (0.50 - i0.68)$ fm, $a^t = (0.76 - i0.50)$ fm, $\rho(0) = -1.3$ /16/

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*) The possibility to determine the sign of Re a from the shift of the 1S level of $\bar{p}p$-atom was pointed out in paper /12/.

**) The splitting of singlet and triplet states in protonium was not found up to now.

After analysing the atomic data /2,3/ we conclude that $\rho(0)<0$. Thus, if the results of Ref. /1/ are correct, $\rho(p)$ should change its sign once more when momentum p decreases from 180 MeV/c to zero. Qualitatively the behaviour of $\rho(p)$ in the nearthreshold region is shown in Fig. 1.

Another method to determine the imaginary part of scattering length a is the extrapolation of -wave annihilation cross-section to the $\bar{p}p$ threshold

$$\mathrm{Im}\,\bar{a} = -\lim_{k\to 0} \frac{k\,\sigma_{tot}}{4\pi} = -\lim_{k\to 0} \frac{k\,\sigma_s^{ann}}{4\pi} \qquad (21)$$

Using σ_s^{ann} from Ref. /5/ we find that $(v\sigma)_0 = 25$ mb, $\mathrm{Im}\,\bar{a} = -0.5$ fm, and $\Gamma_{1S} = 0.8$ keV, in quantitative agreement with the result of Ref. /3/.

3. On the p-wave $\bar{p}p$-amplitude

Noticing that at $p \lesssim 300$ MeV/c the contributions to the nuclear scattering from the higher partial waves ($\ell \geqslant 2$) are negligibly small one can extract the p-wave scattering amplitude using $f_N(k,\theta)$ from Ref. /1/ and determining the s-wave amplitude somehow. Namely, one may suppose that in the near-threshold region the zero effective range expansion works well. Hencefore the amplitude $f_0(k)$ is

$$f_0(k) = \left(-a^{-1} - ik\right)^{-1}. \qquad (22)$$

In Figs. 3a,b we give the Argand diagrams for $f_1(k)$ obtained for two values of scattering lengths $a = (0.6 - i0.6)$ fm and

a = (0.8 - 10.5) fm. Comparing Fig. 3a and 3b we conclude that the result for p-wave amplitude is not very sensitive to the exact value of $\rho(0)$.

The obtained energy dependence of the p-wave amplitude evidences the possible existence of some structure in the near-threshold region. Unfortunately, the existing experimental uncertaities do not allow us to identify this structure in a unique way neither with the p-wave baryonium state /17,18/ nor with the multichannel nearthreshold singularity (CC-pole) /19/. Extrapolating the obtained p-wave amplitude to the $\bar{p}p$-threshold we get the scattering volume

$$A_p = \lim_{k\to 0} k^{-2} f_1(k). \qquad (23)$$

The imaginary part of A_p is equal to $\mathrm{Im}\, A_p \approx 80\ \mathrm{GeV}^{-3}$ and we can obtain from this value the width of the $2p$-state of $\bar{p}p$-atom:

$$\Gamma_{2p} = \frac{3}{2^7} m_p^4 \alpha^5\, \mathrm{Im}\, A_p \approx 3\cdot 10^{-2}\, eV. \qquad (24)$$

The width of $2p$-state was obtained independently in atomic experiments:

$\Gamma_{2p} \simeq 10^{-1}$ eV, $\quad \mathrm{Im}\, A_p \simeq 2\cdot 10^{2}\ \mathrm{GeV}^{-3}$ /3/ ,

$\Gamma_{2p} \simeq 4\cdot 10^{-2}$ eV, $\quad \mathrm{Im}\, A_p \simeq 10^{2}\ \mathrm{GeV}^{-3}$ /4/ .

These results agree qualitatively with result (24) of our extrapolation.

4. Conclusion

Making combined analyses of the low-energy scattering data at small angles /1/ and data on $\bar{p}p$-atom /2,3/ we get the following principal results.

First of all, it is the forecast of $\rho(p)$ behaviour at very small energy, see Fig. 1. Secondly, we obtained the p-wave $\bar{p}p$-amplitude and it shows evidence for some structure. Our solution for p-wave is in agreement with solution A of the partial wave analyses performed in Ref. /5/, where data at angles $30° < \theta_{cm} < 180°$ were used in contrast to /1/.

Finally, emphasize that it would be desirable to make careful analysis of the low energy behaviour of the partial-wave amplitudes using both the data on differential cross sections /1,5/ and the atomic data /2-4/ with the adequate formulae of (12)-(13) type.

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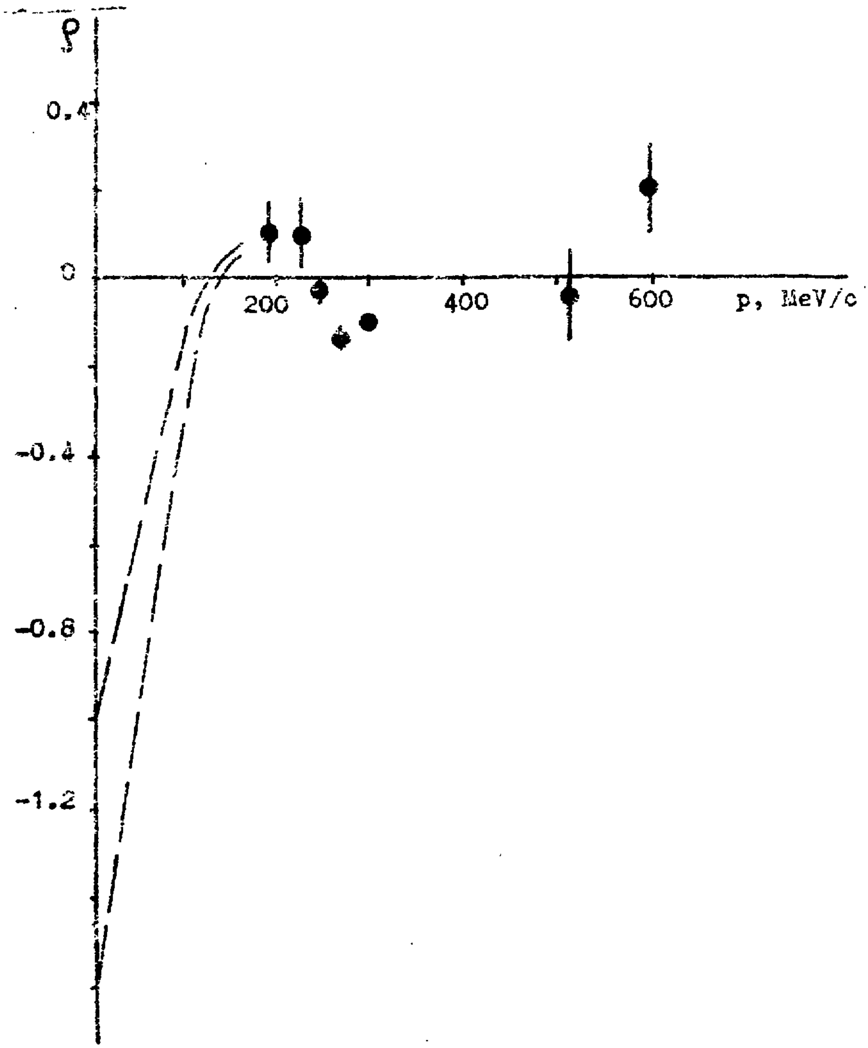

Fig. 1. The real-to-imaginary ratio of the pp forward elastic scattering amplitude vs. p̄ momentum p in the lab.frame /1/. Qualitative behaviour of ρ at very small momentum is shown by dashed line.

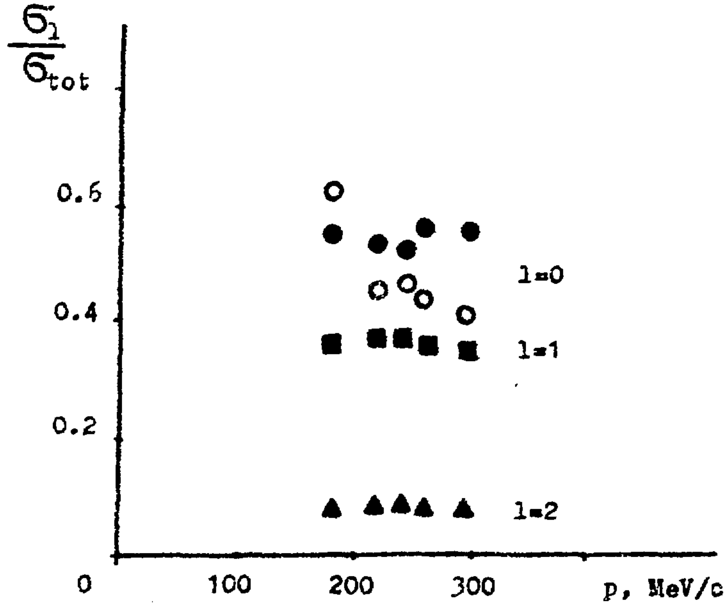

Fig. 2. The contributions from $l = 0, 1, 2$ partial waves to the total $\bar{p}p$ cross section as a function of $\bar{p}$ momenta. The black circles, squares, and triangles are the present results, the open circles are from Ref. /5/.

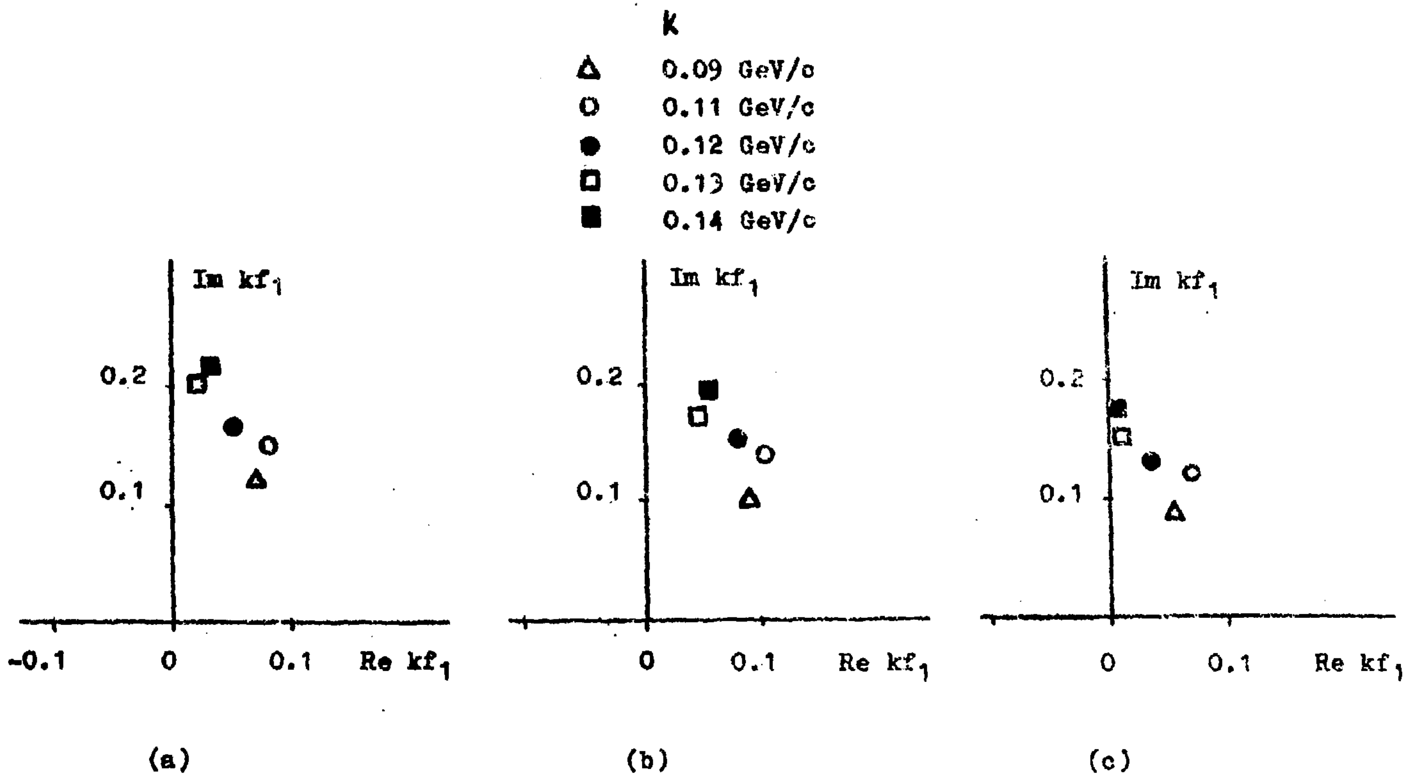

Fig.3. The Argand diagrams for the p-wave scattering amplitude; (a) and (b) correspond to the S-wave scattering length a = (0.6 - i0.6) fm and a = (0.8 - i0.5) fm, respectively, (c) is from Ref./5/ (solution A).

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