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RESTRICTIONS ON SU(5) AS A GRAND UNIFIED THEORY*†

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ABSTRACT. We examine some restrictions imposed upon Grand Unified Theories by dynamical symmetry breakdown. We observe, in particular, that theories with SU(5) as symmetry group, with 3 or more fermion families undergo dynamical symmetry breakdown, and some of the fermions will acquire mass at the Grand Unified scale. On the other hand, the SO(10) group, with 3 families is free from this problem.

RESUMO. Examinamos, neste trabalho, restrições impostas a Teorias Grande Unificadas pela quebra dinâmica de simetria. Observamos, em particular, que teorias com SU(5) como grupo de simetria, com 3 ou mais famílias de férmions sofrem quebra dinâmica de simetria, e consequentemente, alguns dos férmions adquirem massa na escala de Grande Unificação. Por outro lado, teorias com o grupo SO(10) estão livres deste problema.

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Quantum Chromodynamics and the Electroweak Interactions seem to be fragments os some Grand Unified Interaction. This possibility is pointed by the similar mathematical structure of both theories, by the quantization of the electric charge of quarks and leptons and by the experimental value of sin θ_W. Some of the consequences of a Grand Unified Theory (GUT) can already by tested experimentally, e.g., the measurement of the lifetime of the proton. These tests can be helpful on setting restrictions on the possible theoretical models for GUT's.

In this note we will examine some theoretical restrictions which can be cast on GUT's, which arises from the dynamical breakdown of the gauge symmetry. Dynamical symmetry breaking is a very attractive mechanism for the generation of gauge boson masses, from the theoretical point of view. It is an elegant and economical mechanism, depending only on structures already present in the theory and avoiding the ugly quadratic divergences of the Higgs mechanism, as well as the proliferation of fundamental scalar fields.

There are at least two different ways of realizing the dynamical symmetry breakdown: a) intrinsic dynamical symmetry breaking[(1-3)] and b) technicolor mechanism[4]. The technicolor mechanism occurs whenever there is a spontaneous breakdown of some continuous global symmetry in the theory, generating consequently *Goldstone* bosons, and such that they are coupled to currents from some other sector of the gauge theory. The gauge bosons coupled to these same currents will acquire mass. So in order to generate the gauge bosons spectrum of the Electroweak Theory, via the Technicolor mechanism, one has to extend the theory incorporating the new interactions.

The intrinsic dynamical symmetry breakdown was used by Cornwall and Norton and Jackiw and Johnson[1] to exhibit the relativistic version of the *Nambu-Jona Lasinio* phenomenon[5] applied to gauge theories. The crucial point here is the existence of anomalous solutions to the Schwinger Dyson equations, *for the fermion propagators, which are* stable[3,6]. Violation of the gauge symmetry will occur whenever the fermion effective mass matrix do not commute with some of the gauge group generators, i.e.,

$$[M,T^a] \neq 0 \qquad (1)$$

for some a, where M is the renormalized fermion mass and T^a the group generators. It can be shown[3,7] that the linearized version of the Schwinger Dyson equation for the fermionic mass term (fig. 1) has an anomalous solution with the asymptotic form:

$$M\,\Sigma_\psi(p^2) \xrightarrow[p^2 \to \infty]{} M[1 + bg^2(\mu^2)\ln(-p^2/\mu^2)]^{-c/2b}$$

$$= M[g^2(p^2)/g^2(\mu^2)]^{-c/2b} \tag{2}$$

where b is the β functions coefficient

$$b = \frac{1}{16\pi^2}\left(\frac{11}{3} S_1(G) - \frac{2}{3}\sum_i S_1(\psi_i)\right) \tag{3}$$

and i the flavours of the handed fermions. The coefficient c is associated with the commutators of the mass matrix M and the generators, and is given by

$$c = \frac{3}{16\pi^2}\left(S_2(\chi) + S_2(\psi) - S_2(M)\right) \tag{4}$$

with $\sum_a (T^a T^a)_{ij} = S_2(\psi)\delta_{ij}$.

The matrix M can be split into irreducible components contained in the product of the fermion representations, $\chi \otimes \psi$, and $S_2(M)$ is a shorthand for the sum of the Casimir operators, over these representations. The matrix M plays a role analogous to the vacuum expectation value of the scalar field in the Higgs mechanism.

The presence of the anomalous term in the fermion propagator, will induce a mass term in the gauge boson propagator. The relationship between the fermionic mass matrix and the gauge boson mass term can be derived from the Schwinger

Dyson equation obeyed by the polarization tensor (see fig.2), and is given by

$$m_{ab} = - \frac{1}{8\pi^2(c-b)} \; Tr\{2T^a M L^b M^+ - T^a M M^+ T^b - M L^a L^b M^+\} \qquad (5)$$

Here T^a and L^a are the group generators in the representation of the fermion χ and ψ respectively. The trace of the braket term in (5) is negative, consequently the denominator (c-b) must be positive. Actually, the necessary and sufficient condition[3] in order to have dynamical symmetry breaking requires the theory to be asymptotically free ($b>0$) and

$$c - b > 0 \qquad (6)$$

In this contribution, we are going to examine these conditions for theories with SU(5) and SO(10) as group symmetries, as a function of the number of fermion families.

We start by studying theories with SU(5) as group symmetry and *f families of left handed fermions*, each of them with the representation content $10 \oplus 5^*$. The coefficient

$$b = \frac{1}{48\pi^2}(55-4f) \; , \qquad (7)$$

thus the theory is *asymptotically free for* $f < 14$. The fermion mass matrix will be in one of the representations contained in the decompositions

$$5^* \otimes 5^* = 15^* \oplus 10 \qquad (8a)$$

$$10 \otimes 10 = 5^* \oplus 45 \oplus 50 \qquad (8b)$$

$$5^* \otimes 10 = 45^* \oplus 5 \qquad (8c)$$

c will be negative if the mass matrix M is within one of the representations 15*, 45, 45* or 50, hence, one has to analyze only the cases where the fermion condensation go through one of channels 5, 5* or 10*. The factor c-b in each case will be:

$$c(5) - b = \frac{1}{16\pi^2} \left(\frac{20f - 113}{15}\right) \qquad (9a)$$

$$c(5^*) - b = \frac{1}{16\pi^2} \left(\frac{20f - 59}{15}\right) \qquad (9b)$$

$$c(10^*) - b = \frac{1}{16\pi^2} (4f - 52) \qquad (9c)$$

The channel 5 and 5*, have a different value for c-b, because the first is associated with the condensation of the representations $5^* \otimes 10$, while the other with $10 \otimes 10$.

The factor c(5)-b is positive for $f \geq 6$, while c(5*)-b will be so for $f \geq 3$. c(10*)-b is positive when $f \geq 14$, but then the theory is not asymptotically free anymore This result imply that a Grand Unified Theory with SU(5) as symmetry group and having 3 or more fermion families will necessarilly under go an intrinsic dynamical symmetry breakdown. A GUT with SU(5) has 3 fermion families and the condensation will go through the channel 5*, so that the symmetry will be spontaneously broken to SU(4) ⊗ U(1), leaving 8 gauge bosons with mass, as well as a combinations of fermions, which will acquire mass at the same scale. So SU(5) with 3 fermion families cannot describe the fermion spectrum of quarks and leptons as we know it.

We will now study the pattern of SO(10) with *f* left handed fermion families belonging to the representations 16. The fermion mass matrix will be in the decomposition of

$$16 \otimes 16 = 10 \oplus 120 \oplus 126 \qquad (10)$$

The β function coefficient $b = (22-f)/270\pi^2$ is positive provided $f<22$. On the other hand

$$c(R) = \frac{3}{32\pi^2}\,(1 - 2.S_2(R)) \qquad (11)$$

where R indicates one of the representations contained in the decomposition (10). This factor is less than zero if R is the 120 or the 126, but

$$c(10)-b = \frac{1}{16\pi^2}\,\left(\frac{16f - 109}{270}\right) \qquad (12)$$

and becomes negative for $f \geq 7$. Thus, a GUT with symmetry SO(10) and 3 fermions families, do not undergo intrinsic dynamical symmetry breakdown.

In conclusion, we have shown that the pattern of the intrinsic dynamical symmetry breaking of theories with the group SU(5) has a spectrum of light fermion families, which is inconsistent with the one observed for quarks and leptons, while theories with the group SU(10) suffers no restriction of this kind. On the other hand, the new experimental limits on the proton lifetime, are inconsistent with a Grand Unified Theory with symmetry SU(5). The restriction imposed by the intrinsic symmetry breaking could provide an explanation on why GUT's cannot attain its simplest form.

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FIGURE CAPTIONS

Figure 1 - Linearized version of the Schwinger Dyson equation for the fermion mass term which violates the gauge symmetry (ψ and χ can belong to different representations).

Figure 2 - Schwinger Dyson equation for the gauge boson mass term.