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The Editors
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MAGNETIC RECONNECTION IN SPACE AND LABORATORY PLASMAS

S. V. Bulanov, A. G. Frank
General Physics Institute, Academy of Sciences of the USSR
117924 Moscow, Vavilov st., 38, USSR

1. INTRODUCTION

Now a series of phenomena are known under space and laboratory conditions when plasma in a magnetic field can be stable enough for a long period and then suddenly and very fast it turns into an unstable state. The process is often accompanied by burst-type manifestations: in a short time plasma is heated up to high temperature, intensive plasma flows are ejected, the beams of accelerated particles, electrons and ions, are generated as well as radiation in a wide range of electromagnetic spectrum. The best known and most impressive example of the phenomenon of this kind is the solar flare, one of the most powerful manifestation of solar activity. Enormous quantities of energy, up to \(3 \times 10^{32}\) ergs, are released in solar atmosphere during a time interval of a few minutes. Similar processes are also observed on other stars, in magnetospheres of the Earth and other planets, in some well-known laboratory devices for plasma confinement, for example tokamaks, stellarators, compact tori, plasma focus devices and so on. In all the processes mentioned above there is observed directly or it is necessary to assume an extraordinary rapid dissipation of magnetic energy and its transformation into other forms.

The rapid magnetic energy dissipation is accompanied by a change in the magnetic field structure referred to as magnetic field line reconnection [1-7]. The process occurs if the well-known condition that the magnetic field is frozen into high-conductivity plasma[8] becomes no more valid, that corresponds to the appearance of the resistive diffusion of magnetic field lines through the matter.

According to freezing-in condition the magnetic flux through any closed surface which moves with high-conductivity plasma have to be conserved. Then the magnetic field lines can be distinguished and their topology structure have to be maintained. The freezing in condition is known to be correct, if magnetic Reynolds number is large enough:

\[
\Re_m = \frac{\tau_R}{\tau_a} = \frac{L_0 v_a}{\nu_m} \gg 1
\]

(1)

i.e. for the typical distance scale \(L_0\) the resistive dissipation time

\[
\tau_R = \frac{L_0^2}{\nu_m}
\]

(2)

is much more, than the typical magnetohydrodynamical time

\[
\tau_a = \frac{L_0}{v_a}
\]

(3)
where $\nu_m = c^2/4\pi\sigma^\prime$ (4) is the magnetic viscosity, $\sigma^\prime$ - plasma conductivity and

$$v_a = |B_0|/\sqrt{4\pi N_1 M_1}$$

(5) is the typical Alfvén velocity. For space plasmas the condition (1) seems to be always fulfilled: for active regions of solar corona with typical size of $L_0 \sim 10^3$ cm, $R_{em} \sim 10^{13}$, for flares in the atmospheres of red dwarf stars $R_{em} \sim 10^{14}$. But if the distance scale becomes small enough, then the resistive dissipation becomes essential and the magnetic field line reconnection occurs even at rather high conductivity. The decrease of the distance scale results in the electric current density $j$ increase as well as the the dissipation rate $\sim j^2/\sigma^\prime$. Such a situation is realized, for example, in the course of shock wave formation: nonlinear processes cause the shock front steepening, the typical scale decrease and the corresponding increase of the dissipation.

2. CURRENT-SHEET FORMATION

Magnetic reconnection is also quite important in the vicinity of singular magnetic lines, where the current sheet formation is known to be possible [9]. Here the converging plasma flows arise, so that two firstly quite different magnetic lines of force draw together. As a result magnetic reconnection occurs due to finite plasma conductivity and two other magnetic lines are formed, which may be topology quite different from the previous ones. The simplest example of singular line is the $X$-type neutral line of a two-dimensional (2D) magnetic field configuration, fig. 1. The most detailed investigations of the magnetic reconnection have been done just for this case by theoretical methods, including numerical simulation, as well as by means of special laboratory experiments.

The plane 2D magnetic field is usually described by one-component vector-potential

$$\vec{A} = A(x,y,t)\hat{z}$$

(6)

$$\vec{B} = \text{rot}(\vec{A}) = \left\{ \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} ; 0 \right\}$$

(7)

In this case the magnetic force lines are the lines of equal values of the vector-potential

$$A(x,y,t) = \text{const}$$

(8)

In the absence of electric currents the magnetic field in the vicinity of
X-type neutral line can be described in the following way:

\[ \vec{E}_0 = -h \{ y; x; 0 \} \]  
\[ A_0(x, y) = \frac{h}{2} (x^2 - y^2) \]  
\[ |\vec{E}_0'| = h \cdot |r'| ; \quad |r'| = \sqrt{x^2 + y^2} \]

Even the behaviour of small amplitude magnetohydrodynamical (MHD) perturbations of initial equilibrium state allows to conclude, that plasma flows lead to magnetic energy accumulation and electric current density increase in the vicinity of the neutral line [6,10,11]. Let the initial state be characterized by homogeneous density and pressure

\[ \rho_0 = \text{const}; \quad p_0 = \text{const}; \quad \vec{v}_0 = 0 \]  
and the magnetic field of (9)-type. If some perturbation of initial equilibrium state appears, for example, if an electric field is suddenly applied along the neutral line, then the 2D plasma flows in (x,y)-plane arise, fig. 1. In the approximation of strong magnetic field, when

\[ \beta = \frac{8\pi p_0}{B^2} \ll 1 \quad \text{and} \quad v_s \ll v_a \]
everywhere except very small region around the neutral line, the perturbation of the initial state cause the appearance of MHD-waves of magnetoacoustic and Alfvén types, their phase velocity being equal to local Alfvén velocity:

\[ v_{ph} = v_a = \Omega r \]  
\[ \Omega = \frac{h}{(4\pi N_1 N_2)^{1/2}} = \tau_a^{-1} \]

\( t_a \) is the typical time for the wave propagation in the field with the gradient h. In the course of the wave converging towards the neutral line \( (r \to 0) \) the phase velocity decreases, but the corresponding amplitudes of the perturbations of the magnetic field, the plasma velocity, the plasma density and the electric current density increase, namely:

\[ \delta A = \text{const}; \quad |\delta B| \sim r^{-1}; \quad |\vec{v}| \sim r^{-1}; \quad \delta \rho \sim r^{-1}; \quad j \sim r^{-2} \]
i.e. the wave steepens, so that nonlinear and dissipative effects become more and more important.

The peculiarities of converging magnetoacoustic wave were also observed experimentally [12,13] in the laboratory installation shown on fig. 2. The device represents the impulsive electric discharge in the 2D magnetic field (9) - (11), the electric current \( J_x \) parallel to the neutral line, which is arranged on the vacuum chamber axis. The magnetic field does not depend on z-coordinate along the plasma column length, \( \ell = 40 \text{ cm} \).

The process of magnetoacoustic wave propagation from plasma boundaries to the neutral line is shown on fig. 3, where the spatial distributions of magnetic field at successive times are plotted. The electric current excitation at the plasma boundaries, far away from the neutral line, causes the perturbations of the initial magnetic field, which are relatively small. Magnetoacoustic wave, radially converging to the neutral line, carries the
The experimental device for study current sheet formation and the following evolution [14]. (a) - the cross-section: (1) Current carrying conductors for creating the initial quasistationary two-dimensional magnetic field with gradient \( h = (0.5 - 3) \times 10^3 \) G/cm and the neutral line at the chamber axis \( \delta r \). (2) A cylindrical glass vacuum chamber, 10 cm in diameter, 40 cm length, filled with a gas, helium or argon, at the pressure \( 10^{-1} - 10^{-2} \) Torr. The initial plasma with the density \( N_{e0} = 10^{15} \) cm\(^{-3}\) and electron temperature \( T_{e0} = 1 - 3 \) eV is created by an auxiliary discharge. (b) - the side-view of the device and the electric circuit for plasma current excitation and the formation of the current sheet \(-3\), the initial electric field \( E_z = (100 - 400)\) V/cm. A series of magnetic probes arranged at the sheet surface is shown on the fig. (a) by short arrows.

![Fig. 2 a, b](image)

The \( y \)-distributions of \( B_y \)-component of the magnetic field for several moments of time after the start of plasma current \( (t = 0) \); the typical for the experiment conditions MHD-time \( t_a = 2 \times 10^{-7} \) µsec. The data from [13].

![Fig. 3](image)

![Fig. 4](image)

Ion plasma velocities obtained from the broadening of Ar II 4806A spectral line by plasma observation at different directions; the data from [16].

Perturbations, their amplitudes increase and the wave is slowing down (fig. 3, \( t = 0.2; 0.3 \) µsec). The nonlinear stage is completed by the current sheet formation, which changes essentially the magnetic field topology \( (t = 0.4; 0.5 \) µsec).

Plasma dynamics demonstrates also the cumulative nature of flows under consideration: plasma velocities in \((x,y)\)-plasma increase with time, exceeding the ion thermal velocity (fig. 4) The comparison of inflow velocity \( v_y \) with outflow velocity \( v_x \) [15-17] reveals the 2D character of plasma flows

\[
 v_x > v_y > v_{Ti} 
\]  

This results in fast plasma compression into a plane sheet with sharply out-
lined boundaries (fig. 5) and the sheet density which exceeds both the initial and surrounding densities more than 10 times. [18-21].

3. METASTABLE PHASE OF CURRENT SHEET EXISTENCE

In the self-similar solutions [22,23] for a compressible fluid flow in the nonlinear region near the neutral line the singularity was obtained for a finite time interval. In the solution the kind of plasma collapse is realized, when the current density, the plasma density, one component of magnetic field and one component of the velocity become infinite. The conclusion was made that the collapse results in the formation of the pinch current sheet, the magnetic field structure around the sheet was obtained in [24]. The collapse-type behaviour of plasma flow near the neutral line was obtained experimentally [12-21]. The electric current distribution after the formation stage takes the shape, similar to one shown on fig. 6, i.e. the electric current is concentrated in the limits of the plane sheet with the width $2\Delta x \gtrsim 7$ cm and the thickness $2\Delta y \lesssim 1$ cm. The increase of the total sheet current $j_z$ leads to the increase of the sheet width $2\Delta x$, according to the expression [24]:

$$\Delta x = \sqrt{0.4 j_z / h}$$

(17)

which was proved experimentally [13, 18].

Plasma density has also the sheet shape, fig. 5, $t > 0.4 \mu$sec.

The two-dimensional plasma density distributions at successive times during the sheet formation. At $t = 0.20 \mu$sec, $N < 10^{15}$ cm$^{-3}$. The data [21] obtained by five-frames cine-holographic technique [19-21].

The current sheet formation leads to the considerable change in the magnetic field structure, fig. 7: $B_x$ component parallel to the sheet surface is increased essentially by the simultaneous decrease of $B_y$ component normal to the sheet surface, so that $B_y < B_x$ (fig. 7, $t = 0.5; 0.9 \mu$sec) in contrast with the initial field, $t = 0$. The process corresponds to the mag-
The magnetic field structure inside the current sheet at successive times after the start of $j_x$-current. Only one quadrant of $(x,y)$-plane is shown, the picture is symmetrical relatively the planes $(x=0)$ and $(y=0)$. The interval between the next magnetic force lines is $\delta A = 0.25 \text{ kG cm}$. For the two time moments, $t=0.9$ and $t=1.1$ $\mu$sec, the through magnetic energy accumulation in the vicinity of the current sheet.

The sheet-shape distributions of current density and plasma density, similar to the plotted on fig. 6, 5, don't change practically during rather long time interval, which usually exceeds several times the typical MHD-time ($t_{\text{MHD}}$). The situation corresponds to metastable sheet existence. The most important point is that the current sheet is not the static object at metastable phase; it represents principally dynamic configuration: plasma is flowing into the sheet through wide boundaries and is throwing away with higher velocities through narrow side edges [16,27]. Thus the dynamic equilibrium is settled, and plasma density distribution seems to be practically unvariable.

At this stage the magnetic reconnection process also takes place continuously because of the non-infinite value of plasma conductivity: $5 \cdot 10^{13} \leq \sigma \leq 2 \cdot 10^{14}$ $\text{sec}^{-1}$. The time dependences of the voltage on the plasma column, the discharge electric current and its time derivative, fig. 8, are the foundation for the calculation of the time dependences of the plasma resistance $R(t)$ and the conductivity $\sigma(t)$ averaged over the electric current region. The equation for the whole electric circuit, including plasma, is used for the calculations. The local value of plasma conductivity around the neutral line is also obtained [28] using the equation for the vector-potential change:

$$\frac{\partial A}{\partial t} + (\nabla \cdot \mathbf{v}) A = \nabla \times \nabla \cdot A$$

and the data about plasma velocities [27]. It is possible to attribute then to every magnetic force line its own value $A$ and to follow the line's motion in time, i.e. to observe the magnetic reconnection process [29], fig. 9.

The magnetic field structure is rather complicated even at the metastable stage (fig. 9, $t \leq 1.2 \mu$sec) and can include several neutral lines both of X-type and O-type. The reconnection rate varies in time and along the sheet surface because of plasma conductivity is not homogeneous. This is the cause of current density redistribution and magnetic topology change. At $t=0.9\mu$sec the current sheet contains the neutral line of X-type. At the next time-moments, $t = 1.0; 1.05 \mu$sec, the reconnection rate increases in the regions $1 \text{ cm} \leq x \leq 1.5 \text{ cm}$ and as a result the current density increases in the middle of the sheet, where the O-type neutral line appears with the closed mag-
The y-distributions of helium plasma luminosity in spectral lines of helium atom He I 5876 Å and helium ion He II 4686 Å.

Fig. 8
The typical time-dependences of the discharge electrotechnical parameters: $U_c$ - the voltage at the plasma interval, $j_z$ - the total plasma electric current, $j_x$ - its time derivative, $R(t)$ - the calculated value of plasma resistance. The initial conditions: $h=2 \times 10^3$ G/cm, Ar-gas, $p_0=10^{-2}$ Torr, $E_0=190$ V/cm.

Analogous to fig. 7, except the initial conditions: $h=600$ G/cm, He-gas, $p_0=5 \times 10^{-2}$ Torr; $E_0=315$ V/cm. The through numbering of the magnetic force lines is used for all the plotted time-moments. The dashed lines correspond to the initial magnetic field, without plasma current.

Fig. 9

The spectral line Ar II 4880 Å profiles observed in x-direction, i.e. along the sheet surface. At $t=0.6 \mu$sec the profile corresponds to the ion temperature $T_i=20\pm5$ eV. The distortion of the line shape at $t=1.1 \mu$sec reveals the increase of plasma flow velocities along the sheet, $v_x=2.5 \times 10^3$ cm/sec. $v_x \approx 140$ eV. Then the reconnection rate increases in the middle of the sheet, the current density decreases here by the simultaneous increase in the neighbour regions, and the O-type neutral line is gradually transformed into X-type ($t = 1.13$; $1.21 \mu$sec).

Fig. 12
The two-dimensional distributions of the electric current density $j$ and plasma density $\varrho$ in the vicinity of the current sheet, according to the numerical simulation [30].
The current sheet with the electric current density up to 10kA/cm$^2$ is the region of enhanced release of the energy, which is effectively transformed here into thermal and kinetic plasma energy. The direct evidence, that the electron temperature inside the sheet exceeds the temperature of the immediate sheet environment follows from the fig.10, $t = 1.25 \mu$sec [16]. The space distribution of the radiation of helium ion spectral line He II $4686\,\AA$ is very sharp and corresponds to the sheet region only, $2\Delta y \leq 1$ cm, while the radiation of helium atom spectral line He I $5876\,\AA$ is spread practically over whole vacuum chamber. The difference is caused by the difference in the excitation energy for the both spectral lines: 51 eV and 23 eV. The estimated electron temperature inside the sheet is about $T_e \approx 10$ eV.

The experimental data show that the current sheet is the distinguished region of high plasma density and high electron temperature, i.e. the region of the increased plasma pressure, which is balanced by the magnetic field topology of the current sheet in the "UN-Penix"-device at successive time moments; $t=0$ - the start of plasma electric current. $B_z = 400$ G; $N_e = 10^{12}$ cm$^{-3}$; $B_0 = 1300$ G at $t=0.6\mu$sec. Dot lines show the position of zero lines ($B_z=0$).
netic pressure outside:

$$Nk(T_e + T_\pm) \approx \frac{B_x^2}{8\pi}$$  \hspace{1cm} (19)$$

The measurements of magnetic field near the sheet surface $B_x$ and plasma density $N_\infty$ inside the sheet give the value of thermal plasma energy $(T_e + T_\pm) \approx 20-50$ eV at the metastable phase of sheet existence [17,18,21]. The inequality

$$T_\pm \geq T_e$$  \hspace{1cm} (20)$$
is usually fulfilled for the sheet region [16]. We believe that the rise of ion temperature is caused by the relaxation of inflow plasma energy inside the sheet.

The intensive hydrodynamic flows with time increasing velocities, fig. 11, are observed during the metastable phase. The outflow velocities are not uniform along the sheet surface, increasing from the middle to the edges [27]. This is caused by the action of the additional electrodynamic force:

$$f_x = \frac{1}{c} \cdot j_z \cdot B_y$$  \hspace{1cm} (21)$$

For the real experimental conditions the calculated force density $f_x \approx 10^6$ dyne/cm$^3$ [28] can provide plasma acceleration up to velocities $v_x \approx 3 \cdot 10^6$ cm/sec near the sheet edges in agreement with the results of [27].

Numerical simulations reveals also the cumulative nature of plasma flows in the vicinity of neutral line [30]. The statement of the problem was similar to the discussed above. The plasma is taken out of its initial equilibrium state by the action of an electric field applied at the plasma boundary and parallel to the neutral line. During a short time interval a thin current sheet is formed where the electric current and plasma density are compressed, fig. 12. After the formation stage the magnetic field configuration and current structure do not display any radical changes. There are clearly seen the joined shock wave front at the sheet edge in a contrast with the results of laboratory experiments [13]. The difference may be caused by the unhomogeneous character of dissipative processes in real conditions.

4. STABILITY OF CURRENT SHEETS

Thus the current sheet that is formed near the initial neutral line represents a flat sheet of current-carrying plasma which separates the oppositely directed magnetic fields. In such a static configuration the well-known instability of tearing-mode [31] can develop, which leads to the fast magnetic field line reconnection and to the formation of magnetic islands. The instability arises, if the inequality

$$k_x \cdot \Delta y < 1$$  \hspace{1cm} (22)$$
takes place, where $2 \Delta y$ is the current sheet thickness, $k_x = 2\pi/\lambda$ is the component of the wave number directed along the sheet width. The maximum instability growth rate in the high conductivity limit ($\lambda_a/\lambda_c \ll 1$) is
\[
\gamma_{\text{max}} \approx \left( \frac{\gamma_{\sigma}}{\gamma_{\sigma}'} \right)^{1/2}
\]

where

\[
\gamma_{\sigma} = \Delta y / v_a = \Delta y (4 \pi N_{\text{M}1}^\text{max} \cdot M_1)^{-1/2} \quad \gamma_{\sigma}' = (\nu y \nu_m)
\]

In the low conductivity limit \((\gamma_{\sigma} / \gamma_{\sigma}'>1)\) the instability growth rate, according to [31,32] is of the order of magnitude:

\[
\gamma \sim \gamma_{\sigma}^{-1}
\]

For the above discussed experiment [21,28] \(\gamma_{\sigma} / \gamma_{\sigma}'>1\), and the typical instability growth rate is about \(\gamma_{\sigma}^{-1}\). From (22)-(25) expressions it is clear, that current sheets with the elongation \(\Delta x / \Delta y > 2\gamma\) should be unstable practically all the time. But the results of laboratory experiments both reported above and [33] show, that the wide current sheets can exist without significant change of their structure for a long period which exceeds \(\gamma_{\sigma}\) about ten times. It is caused probably by some mechanism of stabilization: the influence of the external plasma, surrounding the sheet [34,35], plasma flow inside the sheet [36,37] and so on. If the typical time for the instability development, \(\gamma_{\sigma}^{-1}\), exceeded the time \(\Delta x / v_x\) which is necessary for initial perturbations to be carried out of the sheet with plasma flow, then the instability would be suppressed. The numerical simulation of the current sheet stability [38] confirms this conclusion. Under the experimental conditions of [21,28] the both times are of the same order of magnitude. Under the boundary conditions and initial conditions, which are quite different from the previous ones, [39,40] the formation and the following evolution of magnetic islands is observed, fig. 13, as a result of tearing-mode instability.

5. EXPLOSIVE DISRUPTION OF THE CURRENT SHEET

Under specific initial conditions a spontaneous explosive disruption of the current sheet was observed experimentally [41] after rather long period of metastable sheet existence. The sheet disruption occurs dramatically: there is a significant change in the magnetic field topology due to electric current redistribution [29,28], plasma is heated, superthermal flows are generated ejecting plasma from the sheet [16,17,27], the electric fields of inductive nature appear, accelerating the changed particles effectively [42,26]. The phenomena of that kind were observed previously in somewhat different experiments: in the annular teta-pinch device [43] and in the double inverse pinch device [44]. In the case of plane current sheet [26,45] the process starts always by the increase of the magnetic reconnection rate in the central region: the magnetic field tangential component \(B_x\) decreases here by the simultaneous and sharp increase of the normal component \(B_y\) (fig. 7, \(t = 0.9-1.1 \mu\text{sec}\); fig. 9, \(t = 1.21-1.29 \mu\text{sec}\)). The process corresponds to the appearance of the local minimum of electric current in the middle of the sheet, fig. 14, \(t > 1.2 \mu\text{sec}, x = 0-0.5 \text{ cm}\), by the simul-
taneous electric current increase in the neighbour regions, $x > 1$ cm. The magnetic flux through the sheet surface increases several times: at $t = 1.29 \mu$sec (fig. 9) the magnetic field contains an X-type neutral line with the gradient which is a factor of three greater than the initial gradient. The start of the fast impulsive phase of magnetic reconnection is accompanied by strong nonlinear wave excitation. The wave propagates along the sheet surface (Ox-axis) from the middle of the sheet to its edges with a super-Alfvén velocity and causes the fast expansion of the intensified magnetic reconnection region and electric current redistribution, fig. 14: the current is thrown away to the sheet periphery without any substantial violation of the sheet symmetry [29].

The increase of magnetic reconnection rate is accompanied by plasma heating and redistribution, fig. 15. The first frame ($t = 1.40 \mu$sec) corresponds to the final stage of metastable sheet existence with the uniform distribution of electron density along sheet width. Then the fast plasma expansion is observed in the middle of the sheet ($t = 1.46 \mu$sec), that cause the plasma density decrease and the increase of the sheet thickness. It is very likely, that the fast plasma heating takes place, which is connected directly with the local energy release. The following frames ($t = 1.52-1.64 \mu$sec) show the expansion of low plasma density region and the decrease of total particle quantity.

The appearance of plasma flows ejected from the sheet in the course of its explosive disruption is displayed in the change of ion spectral line shapes registered at different directions [16,27], in particular in the halfwidth increase, fig. 16. In the time interval 0.7-1.2 \mu sec the both halfwidthes are close to each other and increase smoothly, that corresponds to ion temperature rise from 100 to 200 eV. Under the conditions, when the explosive sheet disruption occurs (the curves(2)) the sharp increase in $A_\perp$ half-width is observed at $t = 1.25 \mu$sec, that is the evidence of local plasma heating and expansion. At the following time interval, $t = 1.4-1.6\mu$sec, is increasing impetuously from 2.8 a to 4 \AA, so that $A_\perp > A_\perp$. Assuming that
the halfwidth $\Delta A \approx 2.5$ Å is connected at this stage with the ion temperature, $T \lesssim 190$ eV, we obtain the plasma flow velocity directed along the sheet surface $v_x \approx 10^7$ cm/sec and the ion energy $W_x \approx 250$ eV. The superthermal plasma flows are observed also by the splitting of the \( \text{Ar II} 4866 \) Å spectral lines [27]. The velocities are $3 \cdot 10^6 \lesssim v_x \lesssim 4.4 \cdot 10^6$ cm/sec and the corresponding energy of argon ions are $200 < W_x < 400$ eV.

At the explosive stage of the sheet existence some peculiarities are observed sometimes in the current density distribution, i.e. the appearance of the reverse currents near the sheet edges [25,28], fig.17, $t > 2 \mu$sec, $x > 3$ cm. It is the evidence of high velocity plasma motion across strong transversal magnetic field resulting in the exitation of the reverse electric field, $E_x \approx \frac{1}{c} v_x B_y$. It is clear from the data above, that both $B_y$-component of magnetic field and plasma velocity $v_x$ increase towards sheet edges, so that reverse electric currents have to be of a maximum value here, that is in a good agreement with the experimental data, fig. 17. The value of reverse electric field near sheet edge was obtained from the direct measurements of magnetic field and plasma velocities[28]. The sharp increase in the calculated value of the reverse electric field and the appearance of the reverse electric current are time correlated with the sheet disruption.

The decrease of the electron density inside the sheet at the explosive stage, fig. 15, causes a sharp increase of the electron current velocity, fig. 18, first of all in the middle of the sheet, $x = 0$, and in a short time interval away from it, $x = 2$ cm [17,21]. Before the start of fast magnetic reconnection ($\gamma \lesssim 0$) and right up to $\gamma = 0.3 - 0.5 \mu$sec the current velocity is $v_c = (2 - 3) \cdot 10^6$ cm/sec, and then increases sharply, exceeding in some cases $10^7$ cm/sec. The sharp increase in the current velocity indicates, that some parts of the sheet could be transformed into turbulent state with anomalous resistance. In fact estimations of magnetic reconnection rate [29,26,28] allow to conclude, that at certain stage of the sheet disruption plasma conductivity falls more than ten times in $0.1 \mu$sec, so that $\sigma_0 = 7 \cdot 10^{12}$ cm$^{-1}$. An abrupt transition of the current region into turbulent state was demonstrated [46] in the double inverse pinch device (DIPD).

Fast changes of the magnetic field in the course of sheet disruption should produce strong enough electric fields of inductive nature $E \approx 300$V/cm in the middle of the sheet [41,29]. The sharp increase of magnetic field in the course of sheet disruption should produce strong enough electric fields of inductive nature $E \approx 300$V/cm in the middle of the sheet [41,29]. Thus the acceleration of changed particles is possible. The electrons with the energies $E_e > 5 - 10$ keV have been observed by X-ray-measurements just at the moment of sheet disruption [42,26]. The spectrum of energetic electrons may be approximated either by the

Fig. 16

Halfwidths of He II 4686 Å spectral line vs time by current sheet observation at different directions: $\Delta A_z$ - along z-axis; $\Delta A_x$ - along x-axis; $\Delta A_y$ - along y-axis. Experimental conditions: $h = 600$ G/cm; helium, $p = 5 \cdot 10^{-2}$Torr; (1) $E_x = 250$ V/cm, (2) $E_z = 406$ V/cm. The data from [16].
The electric current density vs time at different regions along the sheet width [28]: I - x=0; II - x=0.8 cm; III - x=1.7 cm; IV - x=2.6 cm; V - x=3.5 cm.

Initial conditions: h=2.3kG/cm; Ar-gas, p=10^{-2} Torr; E_z=250V/cm.

steep power function with the index \( y = -(6-7) \) or by the exponential function in agreement with [47]. The power energy spectrum with the index \( y = -4 \) was obtained for energetic electrons in DIPD [48].

Now we shall consider the nonlinear model of a current sheet decay following to [35,49]. If the stabilization condition breaks down, for example, due to the excitation of a plasma turbulence and the onset of anomalous resistivity, then the increment of the tearing mode instability increases. In this case the nonlinear stage of the instability starts very fast, that results in macroscopic disruption of the current sheet, the plasma density becomes approximately zero in the region of a finite width during a finite time, the sheet breaks into separate pieces, and the fast reconnection of magnetic field lines occurs [49].

Let us consider the motion of the coming off pieces of the sheet. The distance between them (the length of discontinuity) is equal to \( L \). The magnitude of the magnetic field flux through the discontinuity can be evaluated as \( \Phi \approx B_0 L^2 \). The magnetic field lines tension, acting on the discontinuity edges, is of the order of magnitude \( F(L) \approx B_0^2 L^2 \). The edge moves with the velocity \( \dot{d}L/dt \) under the influence of this force. The mass of the plasma participating in the motion can be evaluated as \( M(L) = n_L L^2 m_p + n_p L^3 \), where \( n_a \) and \( n_p \) is the plasma concentration in the sheet and beyond the sheet, respectively. The equation of the motion of the ends of the discontinuity, written in the "snow plough" approximation, commonly used to describe Z-pinch dynamics, is as follows

\[
\frac{d}{dt} M(L) \frac{dL}{dt} = F(L) \tag{26}
\]

One can see, that for \( n_p < n_a \) the motion of the sheet ends is uniformly accelerated

\[
\frac{dL}{dt} = v_a (L/a)^{1/2}, \text{ or } L(t) = v_a^2 t^2/2a \tag{27}
\]

The presence of the plasma with the concentration \( n_p \) beyond the sheet results in a velocity \( \dot{d}L/dt \) limited by the value \( v_a^* = B_o/(4\pi n_p m_i)^{1/2} \), which is the Alfvén velocity beyond the sheet.
6. THE CURRENT SHEET FORMATION IN THREE-DIMENSIONAL MAGNETIC FIELD STRUCTURES

At present the main features of the processes of the magnetic field lines reconnection in the planar two-dimensional configurations near the null lines of a magnetic field have been investigated theoretically and experimentally. A study with spatially nonuniform configurations leads to essentially new structures in which a formation of pinch current sheets is also possible [50,51]. In distinguish to the planar two-dimensional structures the magnetic field separatrix surfaces play the fundamental role in this case. Besides the rotation of a plasma, which arises in a selfconsistent manner, can prevent from pinch effect and hence from the formation of current sheet. The situation dependson the initial direction of the electric current relatively to the magnetic field separatrix surface.

Now we shall discuss the dynamics of a plasma and a magnetic field near null points of a magnetic field. Our aim is the elucidation of the conditions under which the origination of current sheets is possible. The pinch current sheets are the objects for which similarly to the shock waves the dissipation and nonlinearity are essential and comparable. Hence it is useful to find the dimensionless parameters describing the dissipation and nonlinearity, the large values of which correspond to the condition of formation of current sheets.

The magnetic field strength is the linear function of the distance from the origin of the coordinates near the null point. It can be approximated by the expression

\[ B_i = A_{ij} x_j \]  

(28)

where \( x_i \) is the displacement, and \( A_{ij} = \frac{\partial B_j}{\partial x_j} |_{x=0} \). The structure of magnetic field lines is determined by the eigenvalues \( \lambda \) of \( A_{ij} \). If all eigenvalues are vanish, we have the neighbourhood of a null surface. If one eigenvalue is equal to zero, then we have a null line, and for real nonvanishing eigenvalues \( \lambda \) one have a zero point. The magnetic field lines enter the vicinity of the zero point along one axis and leave along the separatrix plane, fig. 20. The null line of a magnetic field is the place of a crosssection of two separatrix surfaces.

From (28) it follows that

\[ \begin{vmatrix} B \end{vmatrix} = h \begin{vmatrix} x \end{vmatrix} \]  

(29)

where \( h \) is the typical quantity of the elements of the matrix \( A_{ij} \). The time
of propagation of MHD waves towards the null point is equal to infinity
\[ t = \int \frac{d\omega}{v_a} = 2, \int \frac{v}{\gamma} \rightarrow \infty \]
if one does not take into account the effects of the dissipation and nonlinearity.

Hence, the dimensionless parameters must depend on the value \( \omega \), where \( \omega \) is the typical time of the variation of the boundary conditions. Dissipative effects in a magnetized plasma are described by magnetic Reynolds number
\[ \text{Re}_m = \frac{v_a \cdot J}{\nu_m} = \frac{\Omega}{v_m} \]

The analysis of the propagation of waves in the vicinity of a null line of a magnetic field \([10,11]\) demonstrates that the nonlinear effects become important at the distance from the null line
\[ r_a = \frac{b_0}{h} \]
for Alfvén waves, and at the distance
\[ r_m = (b_0 R/h)^{1/2} \]
for magnetoacoustic waves. Here \( b_0 \) is the amplitude of the wave at the distance \( R \), for example, at the boundary. The combination of the dimensionless numbers \( \Omega \), \( R_m(r) \), and \( r_a \) or \( r_m \) gives the parameters \([52]\):
\[ I_a = \frac{\Omega \cdot J}{\nu_m} \]
\[ I_m = \frac{b_0 R \cdot \Omega \cdot \gamma}{h} \]

which do not depend upon the distance from zero point, see also \([9]\).

The computer simulation \([30]\) and the laboratory experiments \([53]\) demonstrate that the parameter \( I_m \) is quite important for the problem of the formation of the current sheet along a null line of a magnetic field (the condition is \( I_m > 1 \)), and \( I_a \) is the main parameter for the problem of the formation of the current sheet along a separatrix surface of a magnetic field (the condition is \( I_a > 1 \)\([52]\)).

The essentially nonlinear stage of the plasma motion near zero point of a magnetic field is described by exact selfsimilar solutions of the equations of magnetohydrodynamics \([2,51]\):
\[ v_i = \omega_i^{(t)} \cdot x_i = dM_{ik}/dt \cdot M_{kj}^{-1} x_j \]

\[ B_i = A_{ij}^{(t)} \cdot x_j, \quad A_{kk} = 0 \]

\[ f = f(t) \]

\[ P = \frac{1}{2} x_i P_{ij}(t) x_j, \]

where the plasma density has only a time dependence, \( v_i \) is the velocity vector, \( B_i \) is the magnetic field, \( P_{ij} \) is the plasma pressure. One can write the MHD equations as an equations for the matrix \( M_{ij} \)

\[ \frac{d^2 M_{ij}}{dt^2} = - \frac{M_{ij} P_{ij}(0)}{f(0) D(2-t)} + \frac{M_{ik} A_{k\ell}(0) A_{\ell j}(0) - (M_{ik} A_{k\ell}(0) A_{\ell m} M_{mj}) M_{kj} A_{ij}(0)}{4\pi f(0) D(t)} \]

where \( D(t) = \det (M_{ij}(t)) \), the initial values of the matrices \( M_{ij} \), \( A_{ij} \) and \( P_{ij} \) are \( \delta_{ij} \); \( \omega_j^{(0)} \); \( A_{ij}(0) \); \( P_{ij}(0) \), respectively.

The time evolution of \( A_{ij}(t) \), \( P_{ij}(t) \) and \( f(t) \) is determined by that of the matrix \( M_{ij}(t) \):

\[ A_{ij}(t) = \frac{M_{ik}(t) A_{k\ell}(0) M_{kj}(t)}{D(t)} ; \quad P_{ij}(t) = \frac{M_{ik}(t) P_{k\ell}(0) M_{kj}(t)}{[D(t)]^{1/3}} ; \quad f(t) = \frac{f(0)}{D(t)^{2/3}}. \]

The results of the numerical solution of the equation (36) for \( P_{ij}(0) = 0 \) are shown at the fig. 21a for the case, when the electric current is perpendicular to the separatrix plane in the vicinity of a zero point of a magnetic field. There is not any singularity in the solution. The rotation \( \omega \neq 0 \) leads to the expansion of a plasma \( d = \text{div} \vec{v} > 0 \), and to the gradual decrease of the values \( |J|, |W| \) and \(|d|\). But if the electric current has the component along the separatrix plane, then the solution is quite different, fig. 21b: the singularity is attained in a finite time interval. The fast compression of a plasma takes place \( d < 0 \) in spite of the presence of the rotation \( \omega \neq 0 \), and the electric current density tends to the infinity. The solution describes the origination of a current sheet along the separatrix surface of a magnetic field.
field in the shearing flows of a plasma [52].

The pinch current sheet formation near the separatrices of a magnetic field have been studied in [52] on the basis of numerical MHD simulation for two-dimensional nonplanar configurations. The initial and boundary conditions corresponded to current excitation orthogonal to the null lines of a magnetic field. The flows of the plasma near the separatrix plane passing through two zero lines and in the vicinity of the line of crossing of two separatrices have been investigated, fig. 22 a. It was shown that on quasi-stationary stage the electric current distributions have form of the sheets directed along the separatrices, fig. 22 b,c.

7. THE ACCELERATION OF CHARGED PARTICLES

The rapid change in a magnetic field structure in the course of decay of the current sheet gives rise to an induced electric field $E = v_B/cB$. This electric field can be nonvanishing in the vicinities of the magnetic null lines and points, and the acceleration of charged particles can take place [9,47,54]. In these fields nonadiabatic regions exist near zero points. Their dimension is equal to [9,47,54]:

$$R_{N.A.} = \max \left\{ \left( \frac{E}{m_e c} \right)^{1/3}, \left( \frac{e P_0}{m_e h} \right)^{1/2}, \frac{E}{h} \right\}$$

where $e$, $m$, $P_0$ are charge, mass and initial momentum of a particle. Particles are not magnetized, and the straight acceleration by the electric field takes place in this region. Some time later particles get into the drift region, where the adiabatic invariant $J_\perp = P^2/|B|$ is conserved. If the inequality $2eE m J_\perp / |h|$ is valid then the trajectory of a particle passes to the infinity and acceleration occurs. The maximum value of the energy of a particle is determined by the electric field strength, the charge of a particle, and the typical scale of the nonuniformity of a magnetic field ($E_{max} \approx eE_0$). If the inequality $2eE m < J_\perp |h|$ takes place, then the particle is trapped in a finite volume and no acceleration occurs. In one- and two-dimensional cases the motion of particles is always unlimited.

Typical trajectories of positively charged particles in neighbourhoods of magnetic zeroes are shown in figures 23, 24 for the electric field parallel to the $z$-axis. Hence the energy gain of a particle is $\Delta E = eE_0 z$. At the figure 23 on can see the straight acceleration in the nonadiabatic region near null line of a magnetic field ($|x| < 4, |y| < 2$) and the change of the energy of the particle in the drift region due to conservation of the adiabatic invariant $J_\perp = |P_\perp|^2 / |B|$. The motion and acceleration of untrapped particle in the vicinity of a zero point of a magnetic field is shown at the figure 24.

The distribution function of fast particles can be determined with the help of the conservation of particle flux in phase space. Depending on the structure of a magnetic field near zero point the energy spectrum can be of
Fig. 23
Trajectory of charged particle near \( \Pi \) type zero line in uniform electric field parallel to the \( z \)-axis\(^{[54]}\).

Fig. 24
Typical trajectory of untrapped charged particle in the vicinity of zero point in uniform electric field parallel to the \( z \)-axis\(^{[54]}\).

Typical trajectory of untrapped charged particle in the vicinity of zero point in uniform electric field parallel to the \( z \)-axis\(^{[54]}\).

Note, that the main feature of energy spectrum and the value of typical energy of fast electrons detected during the decay of a current sheet in experiments \(^{[42]}\) are in agreement with the estimations based on the discussed theory of acceleration of particles.

8. LABORATORY EXPERIMENTS AND SOLAR FLARES

The main purpose of the above reported experiments consisted in an investigation of a magnetic reconnection process and related phenomena. Another aspect is a simulation of solar flares and some astrophysical phenomena of flare-type. Typical linear dimensions of the flare region differ from that of laboratory devices by a factor of \( 10^8 - 10^9 \), so the exact laboratory simulation of the flare should require one to obtain a superdense plasma in a superstrong magnetic field \(^{[55]}\), that seems unrealizable. Nevertheless the simulation of the flare is possible in the framework of restricted-model principle \(^{[56]}\), comparing values of characteristic dimensionless parameters. If some parameter is greater than unity in the astrophysical context, it should also be greater than unity in laboratory, even though the actual values may differ by many orders of magnitude. Then the
Table 1

<table>
<thead>
<tr>
<th>Dimensionless parameters</th>
<th>Laboratory experiments</th>
</tr>
</thead>
<tbody>
<tr>
<td>DIPD, UC, Riverside Phys. Inst., USA</td>
<td>CS-3, General Phys.Inst., Moscow, USSR</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>( 3 \times 10^{-4} )</td>
</tr>
<tr>
<td>( \varepsilon )</td>
<td>0.13</td>
</tr>
<tr>
<td>( R_{\text{em}} )</td>
<td>50</td>
</tr>
<tr>
<td>( \Omega C' )</td>
<td>30</td>
</tr>
<tr>
<td>( L )</td>
<td>6.5</td>
</tr>
<tr>
<td>( S )</td>
<td>10^{3}</td>
</tr>
</tbody>
</table>

experiment should be capable of reproducing the basic features of the processes occurring in space.

To illustrate we have assembled in Table 1 [57] the dimensionless parameters, which are typical for active regions of solar corona [58-60], for flare-star atmospheres [61,62] and for several laboratory experiments designed to study the phenomena in the vicinity of magnetic neutral lines: the double inverse pinch device (DIPD) [44,63], the current-sheet experiment (CS) [14,53] and the device with two parallel plates for excitation of the current (PPD) [33]. Four of the dimensionless parameters are generally accepted:

\[
\beta = 8 \pi p B^2; \quad \varepsilon = \nu / \nu_a = \delta B / B \approx cE / Bv_a; \quad R_{\text{em}}; \quad \Omega C' = t / t_a \quad (39)
\]

i.e. the ratio of gas-kinetic and magnetic pressures; the nonlinearity parameter; the magnetic Reynolds number; the ratio of the duration of the process and Alfvén time. For plasma flows in essentially nonuniform magnetic fields with singular lines Syrovatskii's numbers

\[
L = \varepsilon R_{\text{em}}; \quad S = 2 \varepsilon / \beta \quad (40)
\]

are of the most importance [64].

One can see from the Table 1, that in the frames of restricted simulation only some of the above mentioned experiments can reproduce satisfactorily the preflare state, in particular the possibility of magnetic energy accumulation \((R_{\text{em}} \gg 1; L \gg 1)\). Therefore these experiments could be adequate to physical processes which take place during solar flares.

A comparison of typical plasma frequencies and microscopic scales, which characterize the dense high-temperature plasma regions of a solar flare and of the laboratory produced current sheet, Table 2, shows, that in the both cases the following inequalities are valid for electrons:
Table 2

<table>
<thead>
<tr>
<th>Current sheet, General Phys. Inst., Moscow, USSR</th>
<th>Solar flares</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \omega_{de}, s^{-1} )</td>
<td>( (6-60) \times 10^{11} )</td>
</tr>
<tr>
<td>( \omega_{bh}, s^{-1} )</td>
<td>( (3.5-9) \times 10^{10} )</td>
</tr>
<tr>
<td>( v_{e}(v_{eff}), s^{-1} )</td>
<td>( (1-2.5) \times 10^{10} )</td>
</tr>
<tr>
<td>( \zeta_{el}, s )</td>
<td>( 7 \times 10^{-6} )</td>
</tr>
<tr>
<td>( r_{de}, \text{cm} )</td>
<td>( 5 \times 10^{-5} )</td>
</tr>
<tr>
<td>( f_{e}, \text{cm} )</td>
<td>( 3 \times 10^{-3} )</td>
</tr>
<tr>
<td>( c/\omega_{oe}, \text{cm} )</td>
<td>( (5-0.5) \times 10^{-2} )</td>
</tr>
<tr>
<td>( \lambda_{e}, \text{cm} )</td>
<td>( 2 \times 10^{-2} )</td>
</tr>
<tr>
<td>( \Omega_{oi}, s^{-1} )</td>
<td>( 3 \times 10^{9} )</td>
</tr>
<tr>
<td>( \Omega_{bl}, s^{-1} )</td>
<td>( 1.2 \times 10^{6} )</td>
</tr>
<tr>
<td>( \nu_{i}, s^{-1} )</td>
<td>( 8 \times 10^{5} )</td>
</tr>
<tr>
<td>( p_{i}, \text{cm} )</td>
<td>( 1 )</td>
</tr>
<tr>
<td>( \lambda_{i}, \text{cm} )</td>
<td>( 2 )</td>
</tr>
</tbody>
</table>

\[ \omega_{de} > \omega_{bh} > \nu_{e} > \frac{1}{\zeta_{el}} \]
\[ \lambda_{e} > \frac{c}{\omega_{oe}} > f_{e} > \nu_{e} \]

Thus the current sheet experiments seem to be a rather good model for the impulsive phase of solar flares.

REFERENCES

An important and widely used approach to determine macroscopic properties of plasmas, especially transport and rate coefficients of electrons, is provided by the Boltzmann equation whose solution yields the electron velocity distribution function \( F(r, v, t) \) resulting from the electric field action and the binary collision processes suffered by the electrons in the gas. In fact, by appropriate velocity-space averaging over the distribution \( F \) all the mentioned macroscopic quantities can be obtained.

Several methods have been developed to solve Boltzmann's equation for electron swarms as well as for electrons of stationary and spatially homogeneous, weakly ionized plasmas, in a (d.c.) electric field \( E \). The most convenient approximation of the velocity distribution is the well-known Lorentz approximation which represents \( F \) by the first two terms of an expansion in spherical harmonics or, due to the special symmetry of the problem, in Legendre polynomials. Recent analyses have shown that the conventional two-term approximation (TTA) of the velocity distribution is inadequate in several cases of large practical interest. Since the connection between cross sections for elastic and inelastic collisions and macroscopic properties is provided by solutions of Boltzmann's equation, for the inference of accurate cross sections from measured swarm data (or for the determination of accurate transport and rate coefficients from given cross sections) a precise technique of solution of Boltzmann's equation is needed. In the last years, several efforts have been done to improve the accuracy of the conventional solution by using a velocity moment method /1/, a path integral method /2/, a finite difference approach /3/ and, very recently, a finite element approach /4/. The most comprehensive studies of the mentioned question became possible by two further methods /5, 6/ which include higher order terms than the second of the expansion in Legendre polynomials and are strict generalizations of the two-term Lorentz approximation. The solution of the resulting singular differential-difference equation system for the expansion coefficients is based upon the Galerkin method with cubic B-spline approximation /5/ or upon a direct numerical integration supported by an analysis of the mathematical structure of the general solution /6/. The results given in this paper have been obtained by this last technique.

1. Homogeneous or space-averaged case

Let us consider first the higher order approximation to the electron velocity distribution for stationary and spatially homogeneous plasmas, or to
the space-averaged velocity distribution of electron swarms in the hydrodynamic stage. In both cases the velocity distribution \( F^{(o)}(v) \), defined by the relation \( \frac{dn_e}{m_e} F^{(o)}(v) = \int F^{(o)}(v) d\xi \), satisfies the Boltzmann equation

\[
\frac{d}{d\tau} C (F^{(o)}) = \frac{dC}{d\tau} (F^{(o)}) + \sum_{k} C_{k}^{(o)} (F^{(o)}) + \lambda \frac{d}{d\tau} (F^{(o)}) = 0 \tag{1}
\]

with the collision integrals \( C_{k}^{el} \) and \( C_{k}^{in} \) for elastic and inelastic electron-neutral collisions all preserving the electron number. With an electric field \( \vec{E} = E \hat{z} \) (i.e., parallel to the z direction) the distribution function has the structure \( F^{(o)}(v, v_{z}/v) \) and thus can be given the Legendre polynomial expansion

\[
F^{(o)}(v, v_{z}/v) = \sum_{n=0}^{2m-1} \frac{2m-1}{n!} \tilde{p}_{n}(v) \tilde{P}_{n}(v_{z}/v) \tag{2}
\]

which here we write in an approximate form involving only an even number \( 2m \) of terms. Substitution of (2) into (1) and truncation of a second expansion of the collision integrals with respect to the mass ratio \( m/M \) after the leading terms, yield the differential-difference equation system

\[
\frac{1}{3} \frac{d}{d\tau} \frac{d}{dU} f_0 + \frac{1}{3} f_1 + \delta U^{2} \frac{d}{dU} f_0 + [\delta (U + U_{e})^{2} - U_{q}] f_0 + \sum_{k} (U + U_{e})^{2} q_{k} f_{k}^{(o)} (U + U_{e})^{2} = 0,
\]

\[
\frac{n}{n} \left( \frac{d}{dU} f_{n-4} - \frac{n-4}{2} f_{n-4} \right) + \frac{n+1}{n+3} \left( \frac{d}{dU} f_{n+1} + \frac{n+2}{2} f_{n+1} \right) - U (p+q) f_{o} = 0, \quad 1 \leq n \leq 2l+1, \quad l \geq 1, \quad f_{2l+1} = 0 \tag{3}
\]

where it is assumed that the normalization condition

\[
\int_{0}^{\infty} U^{n} f_{o} dU = 1 \tag{4}
\]

is satisfied. Here, \( U = mv^{2}/2, f_{0}^{(U)}(U) = \frac{2m(2/m)^{3/2} \gamma_{0}^{(U)}}{(2U/m)^{1/2}} \) and \( p = q_{k}^{el}(U)/(e_{e}/e_{o}), q_{k}^{in} = q_{k}^{in}(U)/(e_{e}/e_{o}), \theta = \sum_{k} q_{k}, \delta = 2m/M, (q_{k}^{el}, q_{k}^{in} \text{ are total cross sections}) \). Moreover, isotropic scattering in all of the collision processes is assumed. The ratio of the electron number in the volume element \( dU \cdot d(v_{z}/v) \) to the number of electrons in \( dU \) is given by

\[
(\frac{dn_e}{m_e} \psi_{v_{z}/v})_{U} = \psi(U, v_{z}/v) d(v_{z}/v), \psi = \sum_{n=0}^{2m-1} \frac{2m-1}{n!} \tilde{p}_{n}(U) \tilde{P}_{n}(v_{z}/v) / f_{o}(U) \tag{5}
\]

where \( \psi \) is the (normalized) angular distribution of the electrons with energy \( U \). A pronounced dependence of \( \psi \) on \( v_{z}/v \) reflects a pronounced anisotropy of \( F^{(o)}(v) \) in the velocity space whilst in the limit of small anisotropy (i.e., for \( f_{0}(U) \approx f_{0}(U) \), \( n \geq 1 \) the angular distribution approaches the value \( 0.5 \) independent of \( v_{z}/v \).

Concerning the macroscopic quantities, mean energy, drift velocity, mean power input from the electric field and average collision frequencies are obtained with (2) as

\[
\overline{U} = \int_{0}^{\infty} U^{1/2} f_{o} dU, \quad \overline{W} = -\frac{1}{3} \int_{0}^{\infty} \overline{U}^{2} f_{o} dU, \quad \overline{N} = \frac{W}{E}, \quad \overline{V_{z}/v} = (\frac{2}{3})^{1/4} \int_{0}^{\infty} U^{1/2} f_{o} dU \tag{6}
\]

and thus are only determined by \( f_{0} \) and \( f_{1} \) independent of the chosen approximation order \( 2m \).

The solution procedure of system (3) recently developed in /G/ is based on
an analysis of the weak singularity for \( U \to 0 \) and the strong singularity for \( U \to \infty \). It can be shown that the general solution of (3) at low as well as high energies contains 1 singular and 1 non-singular fundamental solutions when using the 21 term approximation (2) of \( F(0) \). The isolation of the non-singular part of the general solution in the two energy regions is found to be numerically possible. This permits the construction of the physically relevant solution by a continuous connection of both the non-singular parts and a final normalization according to (4).

In the following, solutions of (1) are reported which are obtained by solving (3) when increasing the order 2l of approximation beyond the TTA, up to the converged approximation (CA). This makes an estimation of the errors of the usual TTA also possible. Quite different situations have been studied in model /6, 7, 9/ as well as real gases /8/ and some general conclusions seem possible as concern the conditions where the CA deviates significantly from the TTA. A large anisotropy of the velocity distribution and thus the need of higher order approximations, generally arises when the frequency of inelastic collisions is large and becomes comparable with that of elastic impacts. This can be well characterized by a large sum \( \sum_k Q_k^{\text{in}} \) of the total cross sections and by similarly large values of \( \sum_k Q_k^{\text{el}} \) as of \( Q^{\text{el}} \). These conditions have the most important impact. Often, in real gases, the occurrence of these conditions is markedly dependent on the electron energy and is only verified over certain energy regions, e.g. at large \( U \) in inert gases with intense (high threshold) inelastic processes, or at energies of some eV in molecular gases because of intense vibrational excitation processes or even at small energies in molecular gases presenting a significant Ramsauer effect or a pronounced electron attachment.

Since in a given gas the energy regions of interest can be quite different depending on the value of \( E/N \) the need of higher order approximations will depend on \( E/N \) too. Thus, it must be expected that the corrections to the different macroscopic quantities provided by higher order approximations will not be the same as the electrons of different energy regions contribute to their values. This will be illustrated in the following for model as well as real gases.

1.1 Model gas - isotropic scattering

In order to give some of the main features of the corrections produced by higher order approximations a simple model plasma /7/ with one inelastic collision process having a threshold at \( U^{\text{in}} = 1 \) eV is considered first

\[ Q^{\text{el}} = 6 \cdot 10^{-16} \text{ cm}^2, \quad Q^{\text{el}}_1(1 < U < 1.2 \text{ eV}) = Q^4 \cdot 5(U-1), \quad Q^{\text{el}}_1(U > 1.2) = Q^4, \quad M = 4 \text{ atomic mass units}. \]

By this model with increase of \( Q^4 \) an increasing intensity of the inelastic collision process can be modelled and its impact on the correction of the TTA due to higher order approximations can be studied.

Fig. 1 reports a comparison between the isotropic distribution \( f_0 \) obtained by TTA and CA (i.e. for 2l=8) at \( E/N = 50 \text{ Td} \) for several values of \( Q^4 \).
An increasing difference between TTA and CA when increasing the electron energy above the threshold can be observed for each $Q^*$. The higher values of the CA show a monotone growth when increasing $Q^*$, which even reaches one order of magnitude and which is completely confirmed by the results (points) of accurate MC simulations relevant to the same model. In correspondence of the enlargement of $f_0$ the corresponding angular distribution $\Psi$ reported in Fig. 2 reveals a monotone increase of the anisotropy of $P^{(0)}$ when increasing $Q^*$. Both results clearly show how the TTA leads to increasing errors at larger electron energies when $Q^*$ is increased.

In order to illustrate the impact of the electric field on the correction to the velocity distribution, Fig. 3 reports comparisons of $f_0$ in TTA and CA at three $E/N$ values for $Q^* = 6 \times 10^{-16}$ cm$^2$. It can be seen that the correction to $f_0$ becomes higher at medium $E/N$ values. Furthermore, the converged isotropic distributions always agree with corresponding accurate Monte Carlo (MC) simulations.

1.2 Real gases - isotropic scattering

We start by considering first at large $E/N$'s the inert gas Ar /8/, with inelastic processes of high thresholds. The energy dependence of the corresponding collision cross sections $Q_{el}^*$ and $Q_{in}^{in}$ is given in the inset of Fig. 4. The figure reports $f_0$ and $f_1$ at 350 Td as obtained in TTA (dashed lines) and CA (full lines). There is a pronounced increase of the two distributions in the CA when compared with those in the TTA only at large energies (above 20 eV) where the lumped cross section $Q_{k}^{in}$ becomes large and comparable with $Q_{el}^*$. Note how the increase is again in perfect agreement with that of corresponding MC results. In addition, the representation of $f_0$, ..., $f_3$ reported in Fig. 5 demonstrates that the TTA is a fairly good approximation for small energies but fails for energies above 20 eV. According to these results, a study of the macroscopic properties in the $E/N$ region from 70 to 400 Td has shown that the TTA results for $U$, $W$ and $P/N$ are only in error by less than 0.5% and those relevant to the collision frequencies $\nu_k^{in}/N$ by less than 2%. In fact, all these macroscopic quantities are accumulated by electrons at relatively small energies when considering the regions where the corrections to $f_0$ and $f_1$ of Fig. 4 become significant.

When going from an inert to a molecular gas, as mentioned above, a distinct impact of higher order terms in the region of small energies (i.e. of intensive vibrational excitations) can be expected. This is confirmed by the behaviours of $f_0$ and $f_1$ as obtained in the CA and the TTA for the two gases CO$_2$ and He at medium field strengths 35 and 40 Td /8/. Some results are presented in Figs. 6 and 7 which show how pronounced corrections to the TTA are already required at energies of some eV's. Indeed, from the energy-dependence of the cross sections for CO$_2$ presented in Fig. 8, a large intensity of inelastic processes at a few eV's can be seen.

The larger corrections to $f_0$ in He than in CO$_2$ at nearly the same $E/N$, is certainly a consequence of the known remarkably larger intensity of vibra-
tional excitations in $N_2$ than in $CO_2$. In correspondence of the inset of the correction to $f_0$ at energies between 2 and 3 eV (cf. Fig. 7), the angular distributions for $N_2$ given in Fig. 9 at 40 and 100 Td show a particularly large anisotropy in the energy region of intense vibrational excitation. In order to give an idea of the impact of higher order approximation on transport and rate coefficients, in Table 1 the ratio between macroscopic quantities in the CA and the TTA is given for $\bar{U}$, $W$, $P/N$ and for some selected frequencies $\nu_{\text{in}}/N$ of vibrational and electronic excitation (ve and ee) as well as ionization (i) in $CO_2$ and $N_2$ (corresponding energy losses or term symbols are given in parentheses) at several values of $E/N$. As one can see the correction to the TTA values are markedly dependent on $E/N$. For $\bar{U}$, $W$ and $P/N$, which are quantities accumulated in the body of the distributions, the corrections amount to less than 10% whilst for the inelastic collision frequencies, which are accumulated in the tail of the distributions, corrections which amount to 100% and more are found.

### CO$_2$

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<th>$W$</th>
<th>$P/N$</th>
<th>ve(0.083)</th>
<th>ve(0.339)</th>
<th>ee(7.)</th>
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<td>1.01</td>
<td>1.01</td>
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<td>-</td>
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### $N_2$

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<th>$W$</th>
<th>$P/N$</th>
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<th>ve(v=5)</th>
<th>ee(A$^3\Sigma$) ve(C$^3\Pi$)</th>
<th>i(15.6)</th>
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</thead>
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<tr>
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<td>0.94</td>
<td>0.93</td>
<td>1.19</td>
<td>1.21</td>
</tr>
</tbody>
</table>

Table 1

As a final observation it will be noticed that the results found for model and real gases allow to conclude that even under the condition of large anisotropy of $P^{(o)}$, the CA for $f_0$ and $f_1$ (up to about 3 significant figures) and thus for the macroscopic quantities is obtained with up to 21=8 terms. On the contrary, the CA of $P^{(o)}(v)$ and thus of $\gamma(U,v,N)/v_0$ needs a somewhat higher approximation, that is up to 21=14 terms. The first conclusion is underlined by the behaviour with increasing approximation order 21 of some macroscopic quantities, i.e. of $\bar{U}$ (in eV), $W$ (in $10^6$ cm/s), $P/N$ (in $10^{-9}$ eV cm$^3$/s) and of the mean collision frequency $\gamma/N$ (in $10^{-12}$ cm$^3$/s) for electronic excitation with the energy loss of 7 eV in $CO_2$ and for vibrational excitation from the ground state to the level v=5 in $N_2$, respectively, which is given in Table 2 at 35 Td in $CO_2$ and at 40 Td in $N_2$. A comparison is also reported with accurate MC results in this table.
Table 2

1.3 Non-isotropie scattering

In the case of non-isotropic scattering the differential cross sections $\sigma_k$ and $\sigma_k^{\text{in}}$ are dependent on the scattering angle. This leads to a generalization /9/ of system (3) and needs also a generalization of the solution technique of the isotropic case, used in /6/. To study the impact of non-isotropic scattering, we consider the simple differential cross section $\delta(U,x)=\alpha(U)R(x)/2\pi$, $x=\cos \theta$ ($Q$ is total cross section) with the normalization $\int_{-\pi}^{\pi} R(x)dx=1$. The scattering angle distribution $R$ is given the form $R(x)=\exp\left(-\left(x-x_0^s\right)^2/s^2\right)$, i.e. a Gaussian profile with a maximum scattering probability at $x_0^s$ and a width $s$.

For the same model of sect. 1.1 (with $Q^*=6\cdot10^{-16}$ cm$^2$), as an example of the non-isotropic scattering effect in elastic collisions, Fig. 10 shows the pronounced variation of the isotropic distribution in the CA with the width $s$ for forward scattering ($x_0^s=1$) at 50 Td /9/. For the same situation, Fig. 11 gives the ratio between the isotropic distribution in the CA (i.e. for $21=8$) and that in the TTA. As one can see, there is a comparably large correction to $f_0$ when passing from the TTA to the CA as found in the isotropic case. Corresponding corrections to the drift speed when we pass from the TTA to the CA are presented for forward, transversal ($x_0^s=0$) and backward ($x_0^s=-1$) scattering in Fig. 12. The results show widely differing behaviours of $\mathcal{W}$ with the width $s$, however the corrections are nearly of the same magnitude. For the same values of $E/N$ and $Q^*$, corrections to $\mathcal{W}$ are reported in Fig. 13 for different situations of isotropic and non-isotropic ($1s$ and $\text{in}$) forward scattering in elastic and inelastic (el or in) collisions, which further show a particularly large dependence of $\mathcal{W}$ on $s$, when the scattering is non-isotropic in elastic as well as in inelastic collisions.

2. Inhomogeneous case

To this point we have considered only higher order approximations to the solution of the homogeneous (i.e. space-independent) Boltzmann's equation (1). Recently, the solution of the kinetic equations which determine the anisotropic diffusion of electron swarms in the hydrodynamic stage (and which are derived in the frame of the modern transport theory from the general
Boltzmann equation) has also been made possible to higher orders of approximation /5/, /10/. Substitution of the following expansion

$$F(t, v_i, t) = F^{(e)}(v_i) n_e + F^{(e)}(v_i) (-\nabla n_e) n_e + \cdots$$  \((7)\)

for the velocity distribution (i.e. with respect to the gradient of the electron density \(n_e(x,t)\)) into the general Boltzmann equation with \(E = E_{le}\) yields, in addition to (1), the two further kinetic equations

$$-\frac{e}{m} E \frac{\partial}{\partial v_i} F^{(e)} = C (F^{(e)}_i + (v_i - \bar{v}_e) F^{(e)}),$$

$$-\frac{e}{m} E \frac{\partial}{\partial v_i} F^{(e)} = C (F^{(e)}_i + v_i F^{(e)}), \quad i = x, y, z$$  \((8)\)

for the components of \(F^{(1)}\) with the additional normalization \(\int F^{(1)}(v) dv = 0\), \(j=x,y,z\). Note how the inhomogeneities are determined by \(F^{(1)}\), i.e. by the solution of (1). As shown in /10/, the particular velocity dependences following from eqs. (8) are: \(F^{(1)}_i(\mathbf{v}, v_i/v)\) and \(F^{(1)}_i(v_i/v_i) F^{(1)}(v, v_i/v)\), \(i = x, y\) with \(v_{\perp} = (v_x^2 + v_y^2)^{1/2}\) and with \(F_{\perp}\) as a common function for \(F^{(1)}_i\).

Substitution of the two even order expansions

$$F^{(e)}(v_i, v, v_i) = \sum_{n=0}^{2l-1} \frac{1}{N} \tilde{P}_n(v) \tilde{P}_n(v_i/v), \quad F^{(e)}(v_i, v, v_i) = \sum_{n=1}^{2l} \frac{1}{N} \tilde{P}_n(v) \tilde{P}_n(v_i/v)$$  \((9)\)

in Legendre and associated Legendre polynomials into (8) leads, in analogy to (3), to two additional inhomogeneous differential-difference equation systems for the expansion coefficients \(\tilde{g}_n^e\) and \(\tilde{h}_n^e\) and to the following expressions

$$N\Delta = \frac{e^2}{3} \int_0^{\infty} v^3 \tilde{g}_n^e dv, \quad N\Delta_a = \frac{e^2}{3} \int_0^{\infty} v^3 \tilde{h}_n^e dv$$  \((10)\)

for the longitudinal and transverse diffusion coefficients. Again /10/, both the resulting equation systems are, as system (3), weakly singular for \(U \to 0\) and strongly singular for \(U \to \infty\) and can be solved by properly generalizing and modifying the technique used in /6/, /7/ and /8/ for the homogeneous system (3).

For isotropic scattering, solutions of the kinetic equations (8) could be obtained by this technique for models as well as real gases, by solving the corresponding systems of equations for \(\tilde{g}_n^b\) and \(\tilde{h}_n^b\) with an increasing (even) order \(2l+1\) of approximation, up to the CA. In order to illustrate with some examples the convergence properties of the diffusion coefficients, Table 3 gives values of \(N\Delta_{\|}\) and \(N\Delta_{\perp}\) (in \(10^{-22} \text{s}^{-1} \text{cm}^{-1}\)) as found for \(\text{CH}_4\), \(\text{Ar}\), \(\text{CO}_2\) and \(\text{N}_2\) when increasing the order \(2l+1\). A comparison is also reported with accurate LC results. Similar convergence properties as found for \(\tilde{g}_n^b\), \(\tilde{h}_n^b\), \(P/N\) and \(\tilde{g}_{\perp/n}^b\) can also be seen for the diffusion coefficients.

Note how the LC results are always in very good agreement (i.e. within the expected errors influencing the LC simulations) with the diffusion coefficients in the CA. Concerning the Legendre expansion coefficients, Fig. 14 reports for \(\text{N}_2\) at 40 Td the converged functions \(\tilde{g}_1^b\) and \(\tilde{h}_1^b\) (related to \(\tilde{g}_n^b\) and \(\tilde{h}_n^b\)), which determine the diffusion coefficients.
Table 3

A comparison with the isotropic part \( f_0 \) of \( F^{(o)} \) is also shown. Fig. 15 illustrates, for the same physical situation, the corrections to \( g_1 \) and \( h_1 \) introduced when going from the TTA to the CA (i.e. \( 21 = 8 \)). As one can see, the corrections are very similar to those found for \( f_0 \) and \( f_1 \) and given in Fig. 7 for the same conditions.

3. Future activities

An aspect of the higher order approximation which deserves further study concerns the hydrodynamic stage with inclusion of non-conservative phenomena such as attachment and ionization, i.e. processes producing alterations of the electron number in collisions. First investigations in this direction have been related to stationary homogeneous plasmas or space-averaged electron distributions. In the latter case in particular, eq. (1) must be replaced by

\[
-\frac{1}{\mathbf{m}} \mathbf{E} \cdot \nabla \Phi \ F^{(o)} = C(F^{(o)}) + C^T(F^{(o)}) - \nabla^T F^{(o)} \tag{11}
\]

where \( \nabla^T = \int \mathbf{c}^T(F^{(o)}) \, \mathrm{d}v \) is the total mean collision frequency relevant to processes which change the electron number, e.g. \( \nabla^T = \nabla^a - \nabla^i \) (i.e. the difference between ionization and attachment frequencies) and \( C^T \) the corresponding collision integrals. First results for pure \( N_2 \) and for \( N_2 \) with admixtures of \( SF_6 \) based on the Galerkin method are given in \cite{11}. Here we will present an example relevant to the model gas considered in 1.1 (with \( \Phi^* = 4 \times 10^{-16} \, \text{cm}^2 \)) we have studied with our new approach \cite{6-10}. We shall include one attachment process with cross section \( q^a(0.5 \leq U \leq 1) = 2(1-U) \times 10^{-15} \) and one ionization process with an energy loss of 10 eV and with the total cross section \( q^i(10 \leq U \leq 60 \, \text{eV}) = (U-10) \times 10^{-17} \, \text{cm}^2, q^i(U \geq 60) = 5 \times 10^{-16} \). For the ionization it was assumed that both electrons after each ionization event have the same energy, nume-
ly a half the remaining kinetic energy. Solutions can be obtained by an appropriate extension of our solution technique /6/, which permits to solve the eigenvalue problem resulting from (11) using expansion (2). To illustrate the corrections produced by higher orders of approximation, Fig. 16 presents the expansion coefficients $f_0$ and $f_1$ as obtained in the TTA and the CA (2l=8) for 20 Td. The results clearly show how larger corrections are needed in the energy region of intense attachment (i.e. below 1 eV). Furthermore, the corresponding behaviour of some macroscopic quantities with increasing approximation order 2l, i.e. of $U$ (in eV), $P/N$ (in $10^{-9}$ eV cm$^2$/s) and of the collision frequencies $\nu^{a}/N$ and $\nu^{i}/N$ (in $10^{-9}$ cm$^3$/s) for attachment and ionization, respectively, is shown in Table 4 for 20 and 300 Td. It can be seen that the corrections of $\nu^{a}/N$ and $\nu^{i}/N$, when increasing the approximation order beyond the TTA, are not larger than 10% in these cases.

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<tr>
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</table>

Table 4

The extension of investigations on this subject, is one of the problems of present interest. But also the study of the impact of non-isotropic scattering in collisions, especially on the diffusion coefficients, is of particular importance and interest. In fact, this should also be a subject of further activities.

References


/6/ R. Winkler, J. Wilhelm, A. Heß, Beitr. Plasmaphys. 23 (1983) 483


Fig. 12

Fig. 13

Fig. 14

Fig. 15
model with attachment and ionization
$E/N=20 \text{Td}$

$2\ell=2$
$2\ell=8$

Figure 16
Grad's 13-moment approximation for the derivation of balance equations from the Boltzmann equation is generalized to multitemperature plasmas. The coefficients of the collision moments and the transport coefficients are expressed by suitably defined transport collision frequencies.

1. Kinetic equations and balance equations

For the description of transport phenomena (e.g. electrical and heat conduction, diffusion) relations between fluxes and forces are needed. Examples for fluxes are mass flow, heat flow, momentum flow; examples for forces are gradients of pressure, temperature, velocity as well as outer forces like electromagnetic and gravity fields. Besides of the outer forces all other quantities mentioned are average values of microscopic quantities attached to single particles. For the averaging process a distribution function \( f(\xi, \zeta, t) \) for the particle positions \( \xi \) and velocities \( \zeta \) is needed. It obeys a kinetic equation of the form

\[
\frac{\partial f}{\partial t} + \frac{\partial}{\partial \xi} (\xi \cdot f) + \frac{\partial}{\partial \zeta} (\zeta \cdot f) = \frac{\delta f}{\delta t}.
\]

If the acceleration \( \zeta \) describes only the part caused by outer forces, then \( \delta f/\delta t \) must express the change of \( f \) (in unit time) due to the interaction of the particles. If these are described by binary collisions and the corresponding Boltzmann collision integral is written for \( \delta f/\delta t \) then the kinetic equation (1.1) is named Boltzmann equation. If, however, the plasma is fully ionized the Coulomb interaction among the particles is often described by a microscopic electric field incorporated in the acceleration \( \zeta \) and consequently \( \delta f/\delta t \) is put equal to zero. Then the kinetic equation (1.1) is named Vlasov equation. In the following Boltzmann's form is always used.

Instead of solving (approximately) Boltzmann's equation (1.1) for the distribution function \( f(\xi, \zeta, t) \) and then using \( f \) for the averaging process to describe fluxes like the mass flow

\[
m \cdot n(\zeta) := m \int d^3 \zeta f \zeta
\]
(with $n$ as number density and $m$ as particles mass), Maxwell derived a balance equation for an averaged velocity quantity $\phi(\xi)$ from Boltzmann's equation (1.1) as

$$\frac{\partial}{\partial t} n\langle \phi \rangle + \frac{\partial}{\partial \xi} n\langle \xi \phi \rangle - n\langle \xi \cdot \frac{\partial}{\partial \xi} \phi \rangle = C(\phi). \tag{1.3}$$

Putting for the velocity quantity $\phi(\xi)$ dyadic powers of $\xi$, viz. $1$, $\xi$, $\xi^2$ etc., Maxwell obtained a coupled set of balance equations for mass, momentum, energy etc., where fluxes (moments)

$$n\langle \phi \rangle := \int d^3 \xi \ f(\phi) \tag{1.4}$$

and forces appear explicitly. With a corresponding expansion of the distribution function $f(x, \xi, t)$, starting with the Maxwell distribution

$$f^M(x, \xi, t) = n(x, t) (m/2\pi \kappa_B T)^{3/2} \exp\left(-m \xi^2 / 2 \kappa_B T\right) \tag{1.5}$$

and adding products of $f^M$ with the dyadic powers of $\xi$ mentioned above, Maxwell expressed the collision moments $C(\phi)$ in his balance equations (1.3) by the fluxes (1.4). Therewith the desired relations between fluxes and forces were established.

Maxwell represented the coefficients of the fluxes in the expansions of the fluxes in the expansions of the collision integrals $C(\phi)$ by suitable averages over collision cross sections $Q(\xi)(\xi)$ (2.3a). The results are most conveniently expressed as linear combinations of temperature-dependent transport collision frequencies $\nu(\xi)(\tau)$ (2.10b), which in turn enter the expressions for the transport coefficients (e.g. electrical and heat conductivity, viscosity etc.).

Grad (1949) refined Maxwell's expansion of the distribution function $f(\xi)$ using an orthogonal system of three-dimensional Hermite polynomials

$$1 \quad \xi \quad \xi^2 \quad \frac{\kappa_B T}{m} \mathbb{1} \quad \text{etc.} \tag{1.6}$$

(with $\mathbb{1}$ as unit tensor) instead of the dyadic powers $1$, $\xi$, $\xi^2$ etc. Other three-dimensional orthogonal polynomials can also be used, see a review by Weinert (1982).

Since plasmas are always mixtures of different particle species $i$ with often different temperatures $T_i$, a generalization of the Maxwell-Grad procedure to multi-species and multi-temperature gases is necessary (Suchy 1961). With

$$\nu := \sum_i m_i n_i \langle \xi_i \rangle / \sum_i m_i n_i \tag{1.7}$$

as mean mass velocity of the plasma and

$$\nu_i := \xi_i - \mu \tag{1.8}$$
as intrinsic (peculiar) velocity of a particle of species \( j \) the Maxwell distributions (1.5) for different particle species in partial local thermodynamic equilibrium become

\[
f_j^M(x, u_j, t) = n_j(x, t) \left[ m_j / 2 \pi k_b T_j(x, t) \right]^{3/2} \exp \left[-m_j u_j^2 / 2 k_b T_j(x, t) \right]. \tag{1.9}\]

Therefore the particle velocities \( u_j \) in the orthogonal polynomials (1.6) are to be replaced by the intrinsic velocities \( \xi_j := u_j - \mathbf{v} \) (1.8) and Maxwell's balance equation (1.3) is transformed into

\[
\frac{\partial}{\partial t} n_j \langle \phi_j \rangle + \frac{\partial}{\partial x} \cdot n_j \left( \mathbf{v} + \langle u_j \rangle \phi_j \right) + n_j \langle u_j \phi_j \rangle \frac{\partial}{\partial x} \mathbf{v} = C(\phi_j) \tag{1.10a}
\]

\[
+ \left( \frac{D\mathbf{v}}{Dt} - \langle \xi_j \rangle \cdot \frac{\partial}{\partial u_j^2} \phi_j \right) = C(\phi_j).
\]

with

\[
\frac{D}{Dt} := \frac{\partial}{\partial t} + \mathbf{v} \cdot \frac{\partial}{\partial x}
\]

as barycentric derivative.

The fluxes are:

mass flow vector:

\[
\mathbf{j}_j := n_j m_j \langle \xi_j \rangle = n_j m_j \langle u_j \rangle \tag{1.11a}
\]

momentum flow (pressure) tensor of second rank

\[
\mathbf{p}_j := n_j m_j \langle u_j u_j \rangle = p_j I + \mathbf{\Pi}_j \tag{1.11b}
\]

with the hydrostatic scalar partial pressure

\[
p_j := n_j m_j u_j^2 / 3 = n_j k_b T_j \tag{1.11c}
\]

and the (trace-free) stress tensor

\[
\mathbf{\Pi}_j := n_j m_j \langle u_j u_j \rangle - \frac{k_b T_j}{m_j} I \tag{1.11d}
\]

heat flow vector

\[
Q_j := n_j m_j \left( \frac{u_j^2}{2} - \frac{5}{4} \langle u_j \rangle \right) = q_j - \frac{5 k_b T_j}{2 m_j} \xi_j \tag{1.11e}
\]

As buter forces \( m_j \xi_j \) only electromagnetic forces

\[
m_j \xi_j = q_j \left( \mathbf{E} + \xi_j \times \mathbf{B} \right) = q_j \left( \mathbf{E} + \mathbf{v} \times \mathbf{B} \right) + q_j u_j \times \mathbf{B} \tag{1.12}
\]

are taken into account.
The comparison of the definitions (1.11a, d, e) for the mass flow \( \dot{m}_j \), the stress tensor \( \tau_{ij} \), and the heat flow \( Q_j \) with the orthogonal Hermite polynomials (1.6) show that

\[
\begin{align*}
1 & \left( \frac{\beta \theta}{m_j} \right)^{\frac{1}{2}} \dot{m}_j & \frac{\tau_{ij}}{p_j} & \left( \frac{\beta \theta}{m_j} \right)^{-\frac{1}{2}} Q_j
\end{align*}
\]

are proportional to the coefficients of the orthogonal Hermite expansion of the distribution function \( f_j(x, u_j, t) \) (Suchy 1961, eq.23). For weak deviations from partial local thermodynamic equilibrium, expressed by the Maxwell distribution (1.9), all coefficients (1.13) besides the initial one (= unity) must be small against unity. That means

\[
\dot{m}_j \ll m_j \left( \frac{\beta \theta}{m_j} \right)^{\frac{1}{2}} \quad \tau_{ij} \ll p_j \quad Q_j \ll p_j \left( \frac{\beta \theta}{m_j} \right)^{\frac{1}{2}}
\]

Before writing down the (coupled) set of balance equations (1.3) for orthogonal polynomials (1.6) as velocity quantities \( \phi_j \), we have to define the transport collision frequencies \( \nu^{(\phi)}(T) \) (2.10b) for the expressions of the collision moments \( C(\phi) \) in (1.3).

2. Collision cross sections and collision frequencies

Collisions between electrons and gas particles (neutrals and ions) play an important role in plasmas. Due to the small mass of the electrons a quantum mechanical approach is necessary to describe these collisions correctly. It connects the scattering amplitude \( f(\Omega, E) \) obtained from the Schrödinger equation, with the differential cross section \( \sigma(\Omega, E) \) as

\[
\sigma(\Omega, E) = \left| f(\Omega, E) \right|^2
\]

for distinguishable particles (2.1).

Here \( d^2\Omega = d\Omega d(\cos \chi) \) is the element of the solid scattering angle and

\[
E_{\text{coll}} := \frac{1}{2} \mu \nu^{(\phi)} \frac{c_i^2}{c_j^2}
\]

the (kinetic) collision energy with \( \mu \) as reduced mass and \( \nu^{(\phi)} \) as relative speed of the colliding particles of species \( j \) and \( k \).

For the averaging over the deflection angle \( \chi \) Maxwell introduced transfer cross sections (for spherically symmetrical particles)

\[
Q_{jk}(E, \chi) := \int d^2\Omega \sigma(\chi, E_{\text{coll}}) \left[ 1 - P_{\nu^{(\phi)}}(\cos \chi) \right]
\]

using Legendre polynomials \( P_{\nu^{(\phi)}}(\cos \chi) \), while Chapman and Cowling (1970) used powers of \( \cos \chi \) and define

\[
\phi^{(\nu^{(\phi)})}(E) := \int d^2\Omega \sigma(\chi, E) \left[ 1 - \cos \nu^{(\phi)} \chi \right]
\]
with the connections

\[ Q^{(1)} = \phi^{(1)} \]
\[ Q^{(2)} = \frac{3}{2} \phi^{(2)}. \]  

(2.3v)

For the collisions between gas particles classical mechanics can be employed in general. In the classical limit \( \hbar \to \infty \) there holds

\[ \lim_{\hbar \to \infty} \sigma(\chi, E) d(\cos \chi) = \mathcal{G}(\chi, E) d\mathcal{G} \]

(2.4)

with \( \mathcal{G} \) as impact parameter. The connection between \( \mathcal{G} \) and the deflection angle \( \chi \) can then be obtained with the classical integral for the particle trajectory if the interaction potential \( \varphi(r) \) is known. This will be done in the next section 3.

Before averaging over the collision energy \( E \) (2.2) transfer collision frequencies

\[ \nu_{j \mathcal{G}}^{(\xi)} (E) := \epsilon_{j \mathcal{G}} Q_{j \mathcal{G}}^{(\xi)} (E) n_\mathcal{G} = \sqrt{\frac{2E}{\mu}} Q_{j \mathcal{G}}^{(\xi)} (E) n_\mathcal{G} \]

are defined. While the transfer cross sections (2.3) are symmetric in the subscripts \( j, \mathcal{G} \) for the particle species the transfer collision frequencies (2.5) are not.

But it holds

\[ \nu_{j \mathcal{G}}^{(\xi)} = n_j \nu_{j \mathcal{G}}^{(\xi)} \]  

(2.6)

Another useful definition is the combined temperature

\[ \frac{T_{j \mathcal{G}}}{\epsilon_{j \mathcal{G}}} := \mu_{j \mathcal{G}} \left( \frac{T_j}{m_j} + \frac{T_{\mathcal{G}}}{m_{\mathcal{G}}} \right). \]

(2.7)

For equal masses \( m_j \) and \( m_{\mathcal{G}} \) this becomes the mean temperature

\[ \frac{T_{j \mathcal{G}}}{\epsilon_{j \mathcal{G}}} = \frac{1}{2} \left( T_j + T_{\mathcal{G}} \right) \]

(2.8)

and for collisions between electrons and gas particles we obtain

\[ \frac{T_{e \mathcal{G}}}{\epsilon_{e \mathcal{G}}} \approx \frac{T_e}{\epsilon_{e \mathcal{G}}} \text{ for } T_j \ll \left( \frac{m_j}{m_e} \right) T_e. \]

(2.9)

We now expand the transfer collision frequencies \( \nu^{(\xi)} \) (2.5) with the set of orthogonal Laguerre (Sonine) polynomials as

\[ \nu^{(\xi)} (E) = \sum_{s=0}^{\infty} (-1)^s \nu^{(\xi s)} \left( \frac{\tilde{T}}{2} \right) L_{s}^{(\ell + \frac{1}{2})} \left( \frac{E}{\hbar \tilde{T}} \right) \]

(2.10a)

with the transport collision frequencies (Suchy 1969)

\[ \nu^{(\xi s)} (\tilde{T}) \]

\[ = \frac{(-1)^s s!}{(\ell + s + \frac{1}{2})!} \int_{0}^{\infty} d \left( \frac{E}{\hbar \tilde{T}} \right) e^{- \frac{E}{\hbar \tilde{T}}} \left( \frac{E}{\hbar \tilde{T}} \right)^{\ell + \frac{1}{2}} L_{s}^{(\ell + \frac{1}{2})} \left( \frac{E}{\hbar \tilde{T}} \right) \nu^{(\xi)} (E) \]

(2.10b)
as expansion coefficients. Their absolute values decrease with increasing $s$.
This is important for later estimates (5.5b and sect. 6).

Instead of Sonine polynomials $L_s^{(e)} \left( E / \hbar \hat{T} \right)$ (Abramowitz and Stegun 1964) powers of $E / \hbar \hat{T}$ are often used, leading together with the transfer cross sections $\phi^{(e)}(E)$ (2.3b) to the omega integrals

$$\Omega^{(e,s)}(\hat{T}) := \left[ \frac{\hbar}{\mu \hat{T}} \right] \int_0^\infty \left( \frac{E}{\hbar \hat{T}} \right) e^{-E / \hbar \hat{T}} \left( \frac{E}{\hbar \hat{T}} \right)^{s+1} \phi^{(e)}(E). \quad (2.10c)$$

Connection formulas between the transport collision frequencies $\nu^{(e)}$ (2.10b) and the omega integrals $\Omega^{(e,s)}$ (2.10b) are given by Weinert, Kratzsch and Oberhage (1978). It should be stressed that for inverse power interaction potentials the omega integrals increase with increasing $s$ in contrast to the transport collision frequencies $\nu^{(e)}$ (3.5). Therefore the omega integrals cannot be used for estimates.

3. Collisions between charged particles and neutrals

If a particle with charge $q$ encounters a neutral particle with polarizability $\alpha$ its electric field induces a dipole and therefore gives rise to a Maxwell interaction potential

$$\varphi(r) = \frac{C_4}{r^4} \quad \text{with} \quad C_4 := -\frac{1}{2} \frac{q^2}{4\pi \varepsilon_0} \frac{\alpha}{4\pi \varepsilon_0}. \quad (3.1)$$

This is a special case of a class of inverse power potentials

$$\varphi(r) = \frac{C_n}{r^n} \quad (3.2)$$

often used to approximate more complicated interaction potentials. The classical transfer cross sections $Q^{(e)}$ (2.3a) are (Suchy 1984, eq. 6.19)

$$Q^{(e)}(E) = \pi \left( \frac{|e|}{E} \right)^{\frac{n}{2}} Q^{(e)*}(n, \text{sgn } C_n) \quad (3.3)$$

with dimensionless factors $Q^{(e)*}$ depending only on the power $n$ and the sign of the potential (3.2). Their tabulated values (Suchy 1984, sect. 6, table 1) show

$$Q^{(e)*}(4, -) = 2.2092 \quad Q^{(e)*}(4, -) = 2.3076. \quad (3.4)$$

The transport collision frequencies (2.10b) for inverse power interaction potentials (3.2) are (Suchy 1984, eq. 19.8b)

$$\nu^{(e,s)}(\hat{T}) = \nu^{(e)}(\hbar \hat{T}) \frac{(\ell + 1 - \frac{2}{n})!(\ell + 1 - \frac{2}{n})!}{(\ell + 1 + s)! \left( \frac{1}{2} - \frac{2}{n} - s \right)!} \quad (3.5)$$

with

$$\nu^{(e)}(\hbar \hat{T}) = \left[ \frac{2 \hbar \hat{T}}{\mu \ell} \right] Q^{(e)}(\hbar \hat{T}) n_k. \quad [2.5]$$
For Maxwell interaction \( n = 4 \) they become

\[
V^{(c)} = V (e) \delta_{50} \quad \text{for} \quad n = 4
\]  

(3.6a)

with

\[
V^{(c)} = \frac{2 |e|^2}{\mu} Q^{(c)*} (4, \pm) n_n.
\]

(3.6b)

This is the fastest possible convergence with increasing \( n \). For \( n > 4 \) the convergence is rather rapid in the range \( 2 \leq n \leq \infty \) (Suchy 1984, sect. 19, Fig. 1).

The classical transfer cross sections (3.3) with \( n = 4 \) and the corresponding transport collision frequencies (3.6) hold for collisions between ions and neutrals with the exception of light ions with light atoms (see below).

For collisions of ions in their parent gas the momentum transfer cross section via resonant charge exchange

\[
2Q^{ex}(E) = \pi r_{A}^2 \left( \ln \left| \frac{E_{A}}{E} \right| \right)^2 = 2 \left( A - B \log_{10} \frac{E}{eV} \right)^2
\]

(3.7)

has to be added to the classical momentum transfer cross section \( Q^{(e)} \) (3.3), but not to \( Q^{(e)} \). Values of the parameters \( r_{A}, E_{A}, A \) and \( B \) are tabulated for:

\[
\begin{array}{cccccc}
H & H_{z} & He & O & N_{2} & O_{2} \\
\end{array}
\]

(Suchy 1984, sect. 15, table 9). The corresponding transport collision frequencies \( \nu^{(e)ex}(E) \) (2.10b) can be calculated exactly. The somewhat lengthy results can be obtained from Suchy (1984, sect. 22 b).

During a collision between a light ion and a light atom an intermediate molecule is formed. The potential curves for its different quantum mechanical states are approximated mostly with Morse potentials

\[
\varphi(r) = \varphi_{m} e^{\alpha \left( 1 - \frac{r}{r_{m}} \right)} - 2 e^{\alpha \left( 1 - \frac{r}{r_{m}} \right)}
\]

(3.8)

and with Born-Mayer potentials

\[
\varphi(r) = \varphi_{R} e^{\alpha \left( 1 - \frac{r}{r_{R}} \right)} \quad \varphi_{R} > 0
\]

(3.9b)

In the first case curves for transfer cross sections \( Q^{(e)}(E) \) (2.3a) (normalized with \( \pi r_{m}^2 n_{n} \)) and transport collision frequencies \( \nu^{(e)ex}(E) \) (normalized with \( \frac{1}{2} \nu_{m}^{n} \mu_{*} \) \( \pi r_{m}^2 n_{A} \)) are given for various values of the parameter \( \alpha \) by Suchy (1984, sect. 7, Fig. 2 and sect. 20, Fig. 25). The transfer cross sections of the Born-Mayer potential (3.9) can be approximated as (Suchy 1984, eq. 8.3)

\[
Q^{(e)}(E) = \pi r_{m}^2 \left( \ln \left| \frac{E_{n}}{E} \right| \right)^2 = \left( A^{(e)} + B^{(e)} \log_{10} \frac{E}{\varphi_{R}} \right)^2
\]

(3.10a)
The transport collision frequencies \( V^{(c)}(T) \) (2.10b) for the Born-Mayer potential (3.9) are obtained from the transfer cross sections \( Q^{(c)}(E) \) (3.10) either in a similar manner as those for the charge exchange cross sections (3.7) or from tabulated values (Monchick 1959; Suchy 1984, sect. 20, Fig. 30).

The transfer cross sections and transport collision frequencies corresponding to different quantum mechanical states of the intermediate molecule must be combined with a weighted mean. The weights are proportional to the degeneracies (Mason, Vanderslice, and Yos 1959; McDaniel and Mason 1973).

The quantum mechanical calculations of the transfer cross sections \( Q^{(c)}(E) \) (2.3a) for the collisions between electrons and neutrals are rather involved for low collision energies between 0.01 eV and 10 eV. Therefore results from measurements are usually taken. Curves of the momentum transfer cross sections \( Q^{(c)}(E) \) are listed by Suchy (1984, sect. 11, Figs. 11 to 17) for collisions of electrons with

\[
\begin{align*}
\text{H} & \quad \text{H}_2 & \quad \text{He} & \quad \text{O} & \quad \text{N}_2 & \quad \text{O}_2 & \quad \text{CO} \\
\text{NO} & \quad \text{O}_2 & \quad \text{Ar} & \quad \text{CO}_2 & \quad \text{N}_2\text{O} & \quad \text{Kr} & \quad \text{Xe} & \quad \text{Cs} & \quad \text{Hg} \\
\end{align*}
\]

In the first case the corresponding momentum transport collision frequencies \( V^{(c)}(T_e) \) were computed numerically and are plotted by Suchy (1984, sect. 21, Figs. 32 to 38).

4. Collisions among charged particles and among neutrals

For a Coulomb potential the Rutherford differential cross section \( \sigma(\chi, E) \) leads to divergent expressions (2.3a) for the transfer cross sections \( Q^{(c)}(E) \). Therefore the shielding of the Coulomb potential by a cloud of neighbouring charged particles is modeled with a screened Coulomb potential

\[
\sigma(r) = \frac{q_i q_h}{4\pi\varepsilon_0 r} \exp\left(-\frac{r}{\lambda_D}\right) = \varphi(R) \frac{\lambda_D}{r} \exp\left(1 - \frac{r}{\lambda_D}\right),
\]

(4.1)

The screening length \( \lambda_D \) is the Debye-Hückel length defined by

\[
\lambda_D^2 = \sum_j \frac{q_j^2 n_j}{\varepsilon_0 \varepsilon B T_j}.
\]

(4.2)

The classical transfer cross sections \( Q^{(c)}(E) \) (2.3a) must in general be computed numerically but can be approximated for high energies as (Liboff 1959)

\[
Q^{(c)}_{\chi k}(E) \approx \frac{\mathcal{C}(\mathcal{C}+1)}{2} \pi \left(\frac{q_i q_h}{4\pi\varepsilon_0 E}\right)^2 \left[ \ln \left(\frac{4\lambda_D E 4\pi\varepsilon_0}{e^\varphi |q_i q_h|}\right) - \frac{\mathcal{C}}{2} \right]
\]

(4.3)
for $E \gg |q_j q_R| / 4\pi \varepsilon_0 \lambda_D$ and $s = 1, 2$

with $\nu := 0.5272$ as Euler's constant. The results can be represented with $\pi \lambda_D^2$ and $|q_R| = |q_j q_R| / 4\pi \varepsilon_0 \lambda_D e^4$ as normalizing cross section and normalizing energy, resp. (Suchy 1984, sect. 6, Fig. 9)

The transport collision frequencies $v^{(\varepsilon)} (\hat{T})$ (2.10b) must also be computed numerically in general and can only be approximated for rarefied hot (ideal, weakly coupled) plasmas as (Suchy 1984, eq. 20.4)

$$
\frac{v^{(\varepsilon)} (\hat{T})}{v_n} \approx \frac{\varepsilon (\varepsilon + 1)}{2} \left( \left| \frac{E_n}{\varepsilon T} \right|^{3 \varepsilon} \left( \varepsilon - \frac{3}{2} \right) \right) \left\{ \ln \left[ \frac{4}{e^2 \varepsilon} \left( \frac{\varepsilon T}{|E_n|} \right)^{3 \varepsilon} \right] + O(1) \right\}
$$

for $\varepsilon T >> |E_n|$. The previously used normalizing quantities $\pi \lambda_D^2$ and $|q_R| \sim 1/\lambda_D$ can not be used to normalize the temperature since the Debye-Hückel length $\lambda_D$ (4.2) depends on the temperature itself. Instead we introduced a normalization length $r_n (j \hat{\mathbf{e}})$ as a mean distance of charged particles defined by

$$
\frac{1}{r_n^3 (j \hat{\mathbf{e}})} := 4\pi \sum_{j'} \frac{q_{j'}^2}{|q_j q_{j'}|} \frac{T_{j \hat{\mathbf{e}}}}{T_{j'}},
$$

depending merely on ratios of temperatures. The normalization energy $E_n$ is defined as the Coulomb energy at the normalization length:

$$
E_n (j \hat{\mathbf{e}}) := q_j q_{j \hat{\mathbf{e}}} / 4\pi \varepsilon_0 r_n (j \hat{\mathbf{e}}).
$$

The normalization frequency $v_n (j \hat{\mathbf{e}})$ is consequently

$$
v_n (j \hat{\mathbf{e}}) := \sqrt{2 |E_n| / \mu} r_n^2 n_{j \hat{\mathbf{e}}},
$$

and the power $-3/2$ of the normalized temperature $\varepsilon T / |E_n|$ is the plasma parameter:

$$
\left( \frac{E_n}{\varepsilon T} \right)^{3 \varepsilon} \left( \frac{r_n}{\lambda_D} \right)^3 = \left( 4\pi \lambda_D^3 \sum_{j'} \frac{q_{j'}^2}{|q_j q_{j'}|} \frac{T_{j \hat{\mathbf{e}}}}{T_{j'}}, n_{j'} \right)^{-1}.
$$

It has been shown (Suchy 1984, sect. 20, fig. 31) that for rarefied hot plasmas the different transport collision frequencies $v^{(\varepsilon)} (\hat{T})$ (4.4) are of the same order of magnitude, hence the convergence with increasing $s$ is very slow there. For dense cold (non-ideal, strongly coupled) plasmas, however, the convergence is much better.

For collisions between neutrals the interaction potential can often be modeled by a Buckingham ($exp \beta, n$) potential (Suchy 1984, sects. 7\beta, 17, 36 \&):
\[
\frac{\varphi(r)}{-\varphi_m} = \frac{n}{\beta - n} \exp \left[ \beta \left( \frac{r - r_m}{r_m} \right) \right] - \frac{\beta}{\beta - n} \left( \frac{r_m}{r} \right)^n. \quad (4.9)
\]

Tables for transfer cross sections \( Q^{(c)}(E) \) (2.3a) and omega integrals \( \Omega^{(s)}(\hat{\tau}) \) (2.10c) are given by Mason (19.54) for \( n = 6 \) and \( \beta = 12, 13, 14, 15 \). Curves of \( Q^{(c)}(E) \) and transport collision frequencies \( v^{(c)}(\hat{\tau}) \) (2.10b) are plotted by Suchy (1984, sect. 7, Fig. 3 and sect. 20, Fig. 26).

If different quantum mechanical states must be taken into account a weighted mean of cross sections (and collision frequencies) has to be used analogously to collisions between light ions and light atoms (sect. 3 above).

5. Balance equations for mass, momentum, and energy

After the definition and discussion of the transport collision frequencies \( v^{(c)}(\hat{\tau}) \) (2.10b) we turn now to the derivation of balance equations for fluxes (1.11) and forces (e.g. 1.12) with the methods described in sect. 1, starting from Maxwell's general balance equation (1.10) and inserting for the velocity quantity \( \varphi(x) \) the sequence of three-dimensional Hermite polynomials (1.6) with the intrinsic velocities \( \xi_j = \xi_j - \nu \) (1.8) as argument.

For \( \varphi_j = m_j \), we obtain the mass balances (or equations of continuity)

\[
\frac{\partial n_j m_j}{\partial t} + \frac{\partial}{\partial x} \left( n_j m_j \nu + \frac{j_j}{2} \right) = C(m_j). \quad (5.1)
\]

The collision moments \( C(m_j) \) describe gains and losses of masses for different species \( j \), due e.g. to ionization, recombination or chemical reactions.

At the left-hand side the mass density \( n_j m_j \) (flux of zeroth order) is coupled to the mass flow \( \frac{j_j}{2} \) (1.11a), the flux of next higher (first) order.

For the summation over all species of particles \( j \) we use

\[
\sum_j \frac{j_j}{2} = 0 \quad (5.2)
\]

obtained with the definitions of the mean mass velocity \( \nu \) (1.7) and of the mass fluxes \( j_j \) (1.11a). Due to the conservation of masses all collisional gain and loss terms must cancel:

\[
\sum_j C(m_j) = 0. \quad (5.3)
\]
Therefore the summation of the mass balances (5.1) yields the total mass balance:

$$\frac{\partial}{\partial t} \sum_j n_j m_j + \frac{\partial}{\partial x^i} \cdot \nu \sum_j n_j m_j = 0.$$  \hspace{1cm} (5.4)

With \( \phi_j = m_j u_j \), we obtain the momentum balances (or equations of motion):

$$\frac{\partial}{\partial t} J_j + \frac{\partial}{\partial x^i} \left( (\nu J_j + p_j) + J_j \cdot \frac{\partial}{\partial x^i} \nu + n_j m_j \frac{D\nu}{Dt} \right) = -n_j q_j \left( E + \nu \times B \right) - \frac{q_j}{m_j} J_j \times B = C \left( n_j u_j \right)$$  \hspace{1cm} (5.5a)

with the elastic collision moments (Suchy 1963, eq. 860)

$$C_{el} \left( n_j u_j \right) = n_j \sum_k \left\{ \nu_j \left( \frac{\delta_k}{n_k m_k} - \frac{\delta_j}{n_j m_j} \right) \right\}$$  \hspace{1cm} (5.5b)

Again at the left-hand side the first-order mass flow \( J_j \) (1.1a) is coupled with the second order momentum flow \( \frac{D\nu}{Dt} \). But in the collision moment \( C_{el} \left( n_j u_j \right) \) the first-order flow \( J_j \) is linearly coupled with all fluxes of odd order and nonlinearly with even order fluxes. But the coupling coefficients \( \nu^{(4)} \) decrease with increasing deviation from \( J_j \). Terms quadratic in the fluxes \( J_j, Q_j, \) can be neglected for weak deviations from partial local thermodynamic equilibrium (1.14).

With (5.2) the summation over all particle species yields

$$\frac{D\nu}{Dt} \sum_j n_j m_j + \frac{\partial}{\partial x^i} \cdot \sum_j p_j = (E + \nu \times B) \sum_j n_j q_j + B \times J_j = 0.$$  \hspace{1cm} (5.6a)

with the total electric current density

$$J_j := \sum_j \frac{q_j}{m_j} J_j.$$  \hspace{1cm} (5.6b)

The right-hand side vanishes because of the conservation of the total momentum:

$$\sum_j C_{el} \left( n_j u_j \right) = 0.$$  \hspace{1cm} (5.6c)

The summation of the partial pressures \( p_j \) (1.1b) gives the total pressure due to the use of the intrinsic velocities \( \nu_j = \nu_j - \nu_j \) in the definition (1.1b) for the partial pressures (Chapman and Cowling 1970, sect. 2.3' and 2.5). The sums over \( n_j, m_j \), and \( n_j, q_j \) are the total mass density and charge density, respectively.

To obtain the balances for the energy density \( 3p_j/2 = n_j \frac{3k_B}{m_j} T_j/2 \) and the stress
tensor $\tau_{ij}$ is convenient to start with a pressure balance for $P_{ij}$ with $\phi_i = m_i u_i^2$ (Suchy 1963, eqs. 61, 62). Taking then half of the trace yields the energy balance

$$\frac{\partial}{\partial t} \frac{3p_i}{2} + \frac{\partial}{\partial x^i} \left( \nu \frac{3p_i}{2} + q_i \right) + p_{ij} : \frac{\partial}{\partial x^i} \nu$$

$$\left[ \frac{D\nu}{Dt} - \frac{q_i}{m_j} \left( \mathcal{E} + \nu \times \mathbf{B} \right) \right] \cdot \frac{j}{j} = C \left( \frac{m_i}{2} u_i^2 \right)$$

with the elastic collision part

$$C_{ij} \left( \frac{m_i}{2} u_i^2 \right) = 3 n_i \sum_k \mu_{ik} \nu^{(10)} \frac{T_k - T_i}{m_i + m_k} + O(\nu^{11}). \quad (5.7b)$$

The sum over all particle species is

$$\frac{\partial}{\partial t} \frac{3}{2} \sum_j p_j + \frac{\partial}{\partial x^i} \left( \nu \frac{3}{2} \sum_j p_j + \sum_j q_j \right) + \frac{\partial}{\partial x^i} \nu \left( \mathcal{E} + \nu \times \mathbf{B} \right) \cdot \frac{j}{j} = 0 \quad (5.8a)$$

because of the conservation of energy:

$$\sum_j C \left( \frac{m_j}{2} u_j^2 \right) = 0. \quad (5.8b)$$


The stress balance is the trace-free part of the pressure balance.

With the neglects (1.14) for weak deviations from partial local thermodynamic equilibrium (1.9) and with $D/Dt \ll \nu^{(10)} \gg q_j B/m_j$ the stress balance is approximately

$$p_{ij} \left[ \frac{\partial}{\partial x^i} \nu + \frac{\partial}{\partial x^j} \nu \right] - \frac{2}{3} \left( \frac{\partial}{\partial x^j} \mathcal{E} \right) \approx C \left( m_i u_i u_{ij} - \frac{m_i}{3} u_i^2 \right). \quad (6.1)$$

The elastic part of the collision moment is

$$C_{el} \left( m_j u_j u_{ij} - \frac{m_j}{3} u_j^2 \right) = - n_j \sum_k \alpha_{jk} \approx \mathbf{A} \quad (6.2a)$$

with the diagonal terms

$$n_j \alpha_{jj} = \frac{1}{2} \nu^{(10)} + \sum_{k \neq j} \left( \frac{\mu_{jk}^2}{m_i} \nu_{jk}^{(10)} + \frac{2 \mu_{jk}}{m_j + m_k} \nu_{jk}^{(10)} \right) \quad (6.2b)$$

and the off-diagonal terms

$$\alpha_{ji} = \frac{\mu_{ji}}{m_i + m_k} \frac{\nu_{ji}^{(10)} - 2 \nu_{ij}^{(10)}}{m_j} = \alpha_{ji} \quad (6.2c)$$

for $j \neq k$
Inversion of the matrix \( \left[ a_{jk} \right] \) yields
\[
\tilde{\omega}_{ij} = -\eta_j \left[ \frac{\partial}{\partial x_j} \nu + \left( \frac{\partial}{\partial x} \nu \right)^T - \frac{2}{3} \left( \frac{\partial}{\partial x} \nu \right) \right] (6.3a)
\]
with the partial viscosities
\[
\eta_j = \sum_k \left( a^{-1}\right)_{jk} h_k T_H. \tag{6.3b}
\]
The result for the summation over all particle species \( j \) is straightforward.

With the same neglects as for the approximation of the stress balance (6.1) the momentum balances (5.5a) become
\[
\frac{\partial p_i}{\partial x_j} - n_j q_i \left( E + \nu \times B \right) \approx \varepsilon \left( m_j \nu_i \right). \tag{6.4}
\]
The leading order \( O(\nu^{10}) \) of the elastic part of the collision moment (5.5b) is
\[
\varepsilon_{el} \left( m_j \nu_i \right) \approx -n_j \sum_k \varepsilon_{jk} \frac{\eta_i}{\eta_k}. \tag{6.5a}
\]
with the diagonal elements
\[
\eta_i \varepsilon_{ij} = \sum_k \frac{\mu_{ik}}{m_j} \nu_{ik} \tag{6.5b}
\]
and the off-diagonal elements
\[
\varepsilon_{jk} = -\frac{\mu_{ik}}{m_i} \frac{\nu_{ik}}{\eta_k} = \frac{m_i}{m_j} \varepsilon_{jk} \text{ for } i \neq j. \tag{6.5c}
\]
Inversion of the matrix \( \left[ \varepsilon_{jk} \right] \) leads to
\[
\frac{\partial p_i}{\partial x_j} = \varepsilon_{jk} \left[ n_j q_i \left( E + \nu \times B \right) - \frac{\partial p_i}{\partial x_j} \right] \tag{6.6a}
\]
with the mobilities
\[
\varepsilon_{ij} = \sum_k \left( \varepsilon^{-1}\right)_{jk} \frac{1}{\eta_k}. \tag{6.6b}
\]
The leading order of the electrical conductivity is obtained by multiplication of (6.6a) with \( q_j / m_j \) and summation over all particle species \( j \) as
\[
\sigma = \sum_j \frac{n_j q_j^2}{m_j} \varepsilon_{ij} = \sum_j \frac{q_j^2 n_j}{m_j} \sum_k \left( \varepsilon^{-1}\right)_{jk} \frac{1}{\eta_k} \tag{6.7}
\]
The results for the balances of the heat flow vectors \( Q_j \) (1.11e) are given by Suchy (1973). The approximate results with neglects similar to those made in the derivation of the stress balance (6.1.2) are
The collision moment \(6.8a\) has expressions proportional to the mass flows \(J_{i\alpha}\) as in \(C_{i\alpha}(m_{i\alpha},v_{i\alpha})\) for the momentum balance (5.5). But their coefficients are of the order \(O(\nu)\) in contrast to \(O(\nu^{(10)})\) in the momentum balance (5.5b), where the coefficients of the heat flow vectors \(Q_{i\alpha}\) are of order \(O(\nu^{(10)})\). Therefore the collision moments of the momentum balance (5.5b) and of the heat flow balance (6.8a) are coupled by terms of order \(O(\nu)\). These coupling terms give rise to the thermal diffusion (or Soret) effect and to the diffusion thermo (or Peltier) effect. The dominant terms of order \(O(\nu)\) are the leading terms in the expressions (6.7) for the mobilities (6.6b) and the electrical conductivity (6.7) as well as for the partial heat conductivities \(\kappa_{j}\) obtained by the inversion of the matrix \([c_{j\alpha}^{-1}]\) in the heat flow balances (6.3):

\[
\frac{5}{2} \frac{p_{\alpha} k_{B}}{m_{\alpha}} \frac{\partial T_{i}}{\partial x_{i}} \approx - \frac{5}{2} \sum_{\alpha} \frac{\mu_{i\alpha}^{2}}{m_{i}^{2}} \frac{\partial}{\partial x_{i}} \left( \frac{1}{n_{i} m_{\alpha}} \frac{\nu_{i\alpha}^{(1)}}{n_{i} m_{\alpha}} \right) \left( \frac{1}{n_{i} m_{\alpha}} - \frac{1}{n_{i} m_{j}} \right) - \sum_{\alpha} c_{j\alpha} Q_{i\alpha} + O(\nu^{(10)})
\]

\(6.8a\)

with the diagonal terms

\[c_{j\alpha} = \frac{1}{3} \nu_{i\alpha}^{(10)} \]

\(6.8b\)

and the off-diagonal terms

\[c_{j\alpha} = \frac{\mu_{i\alpha}^{3}}{m_{j}^{2} m_{\alpha}} \left[ \frac{4}{3} \nu_{i\alpha}^{(10)} + \frac{m_{\alpha}}{m_{j}} \left( \nu_{i\alpha}^{(10)} + \frac{7}{2} \nu_{i\alpha}^{(12)} \right) + 3 \frac{m_{j}}{m_{\alpha}} \nu_{i\alpha}^{(12)} \right] \]

\(6.8c\)

\[\text{for } j \neq \alpha.\]

For a plasma with uniform temperature \(T = T_{i} = T_{\alpha}\) the summation over all particle species \(j\) yields the total heat conductivity \(\kappa = \sum_{j} \kappa_{j}\).

\[
Q_{j} = - \kappa_{j} \frac{\partial T_{j}}{\partial x_{j}} \quad \text{with} \quad \kappa_{j} = \sum_{\alpha} (c^{-1})_{j\alpha} \frac{5}{2} \frac{p_{\alpha} k_{B}}{m_{\alpha}}.
\]

\(6.9\)

Correction terms to the leading order expressions (6.6b) (6.7) (6.9) are of the order \(O(\left[\nu^{(12)}/\nu^{(10)}\right]^{2})\). Detailed calculations are given by Suchy (1984, chapter C).
7. Conclusions

Two expansions with orthogonal polynomials were used to compare terms in the balance equations and thence to introduce approximations and truncations. The first was Grad's expansion of the velocity distribution function with three-dimensional Hermite polynomials. It yielded estimates for the terms at the left-hand sides of the balance equations. The second was the expansion of the (energy dependent) transfer collision frequencies with Sonine polynomials. Its (temperature-dependent) coefficients, the transport collision frequencies \( \nu^{(T)} (T) \) (2.10b) allowed estimates of the terms in the expressions for the collision moments at the right-hand sides of the balance equations. Expressions to leading order were given for electrical and heat conductivities and for viscosities.

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The breakdown mechanism of SF$_6$ and air is modelled by stochastic processes for the primary electron generation and the development of avalanches. For the avalanche development a birth-and-death process and for the electron generation an inhomogenous Poisson process is used. The results of this model are in agreement with classic theories and with the experience. They enable the pre-calculation of breakdown voltage and breakdown time distribution functions of small, well conditioned test objects. But there seems to be no chance for a sufficient theoretical pre-calculation for technical gas insulations because of field disturbances. For this application the intensities of the Poisson process must be determined experimentally.

1. Introduction

In high-voltage technology the application of insulations with slightly non-uniform electric fields led to the better utilization of materials and space. The success of this trend is especially well-known for SF$_6$-insulations, in which the principle of slightly non-uniform field is supported by the high electric strength of the compressed SF$_6$ (Fig. 1). But also for air insulations slightly non-uniform fields can be used successfully if the disturbance of the field by environmental influences (dust, rain, ice, snow, etc.) can be limited. Examples for this are air insulations of medium-voltage switchgear or high-voltage testing equipment (Fig. 2).

Problems associated with the application of slightly non-uniform fields in SF$_6$ and air are thus of considerable importance and have therefore been subject to extensive

Fig. 1: Compressed gas capacitor for 200 kV - slightly non-uniform SF$_6$ insulation

Fig. 2: AC voltage testing equipment for 1800 kV - slightly non-uniform air insulation
studies. Such investigations concern the measurement of the breakdown voltage or the breakdown time characteristics because the inception of a self-sustaining discharge means the breakdown of the gas in a slightly non-uniform field. But the measured breakdown characteristics are normally subject to statistical variation (Fig. 3), and this is of remarkable technical consequences. The statistical variation arises from different stochastic influences, especially
- the stochastic generation of primary electrons,
- the stochastic development of electron avalanches, and
- the really acting micro-field, which is determined by electrode roughness, particles or micro space charges.

The investigation of the problem directed to technical applications demands on the one hand large scale experiments (as e.g. described in detail in /1/), on the other hand a model for the understanding of the processes. Such a model seems to be necessary for the generalization of the experimental results, also, if there will not be a chance for a purely theoretical pre-calculation of breakdown voltages or times for technical purposes.

It is the intention of this lecture to describe the breakdown process of gases (especially SF$_6$) by means of stochastic processes /2//3/ and to show the application of such models to the semi-empirical design of slightly non-uniform SF$_6$ and air insulations. Necessarily the model must correspond to the classical theory of avalanches (Townsend /4/), to the streamer theory of breakdown (Raether /5/) and its modification for SF$_6$ by the fundamental investigations of Pedersen /6//7//8/.

Further on our investigations were stimulated by those of Legler /9/ and by the concept of critical volume which was assumed for long air gaps by the Les Renardiers Group /10/ and more detailed formulated for SF$_6$ by Boeck /11/. The publication of Boeck has inspired very extensive experimental and theoretical studies of the impulse voltage-time characteristics of SF$_6$ insulations (e.g. /12//13//14//15/). This lecture summarizes the investigations at the Dresden Technical University carried out since 1970 and published in several theses (e.g. /16/ /17//18/), a detailed paper /19/ and some chapters of two books /1/ /20/.

![Fig. 3: Examples for the dispersion of the breakdown process (distribution functions)](image)
2. The electron avalanche formation as a birth-and-death process

2.1. Critical avalanche probability

The development of an avalanche is supposed as in Townsend's model /4/: At any starting point \( x_0 \) (preferably at the cathode) \( n_0 \) electrons are injected into a field (Fig. 4) where the field strength is partly high enough for the development of an avalanche \( (E > E_c) \). The number of electrons in the avalanche is a random variable \( N(x) \). If it is supposed that the number of electrons in a point \( x \) is \( N(x) = i \) and in the near point \( x + \Delta x \) yields \( N(x + \Delta x) = j \), the probability for the alteration of the state of the avalanche (this means of its number of electrons) can be written as

\[
P_{ij} = P[N(x + \Delta x) = j | N(x) = i] = \frac{j}{N(x)}.
\] (1)

The new state \( j = i + 1 \) means the "birth" of an additional electron by ionization, the new state \( j = i - 1 \) describes the "death" of a free electron by attachment and the state \( j = i \) is the result of elastic collisions. In statistics such processes are classified as "birth-and-death processes" /2/, they belong to the group of linear Markow processes. The probabilities \( p_{ij} \) of the birth-and-death process can be calculated by means of its intensities /2/. It was shown /1//21//22/, that these intensities can be understood as the well-known coefficients for ionisation (\( \alpha \)) and attachment (\( \eta \)), therefore it yields

for electron generation:

\[
p_{i+1} = \alpha(x) \cdot i \cdot \Delta x,
\] (2)

for electron loss:

\[
p_{i-1} = \eta(x) \cdot i \cdot \Delta x,
\] (3)

and for elastic collisions:

\[
p_{ii} = 1 - (\alpha(x) + \eta(x)) \cdot i \cdot \Delta x.
\] (4)

For \( N(x) = 0 \) the avalanche dies out and the process stops completely ("absorbing state"). For \( N(x) = n_0 \) in any point \( 0 < x < d \) a critical number of electrons sufficient to form a streamer (/5/ to /9/, /1/, /19/), is reached, and the mechanism of the process is changed. Also this state is modelled as an absorbing state with the "critical avalanche probability":

\[
p_c = P \left[ \max_{0 \leq x \leq d} N(x) = n_0 / N(x_0) = n_0 \right].
\] (5)

Fig. 4: Definitions for the stochastic model

Fig. 5: Simulation of six electron avalanches in uniform field
2.2. Uniform field

On the basis of the equations (2) to (4) the formation of single avalanches was investigated by computer simulations (Fig. 5) /1/9/ /22/. The results show that most avalanches die out after a short distance (Fig. 5: cases 1, 4, 5) and their maximum electron number is very small. Only in rare cases (Fig. 5: case 3) the electron number \( n(x) \) shows the well known exponential growth. But if \( N(x) > 30 \) it is very likely that the avalanche will reach the critical number \( n_0 \) necessary for the formation of a streamer. All this confirms the assumption of Pedersen /7/ /8/ that the growth of an avalanche initiated at \( x = x_0 = 0 \) will not be exponential until a certain value \( x_1 \) is reached.

Also the expected value \( EN(x) \) and the standard deviation \( DN(x) \) of the number of electrons can be calculated by the birth-and-death process (Fig. 6). The expected value is identical with the well-known formula for the number of electrons (/4/ to /6/)

\[
EN(x) = n_0 \exp \left[ (\alpha-\gamma) x \right],
\]

but the tremendous variation (Fig. 6: \( DN(x) > EN(x) \)) explains that a simple breakdown criterion on the basis of the expected value (e. g. \( EN(x) = n_0 \)) will not be very realistic. Better adapted seems a further investigation of the critical avalanche probability, which was derived from eq. (5) for \( n_0 = 1 \) and an uniform field with the gap distance \( d /1/9//22/ \):

\[
P_c = \left( \frac{(1-\exp \left[ - (\alpha-\gamma) d \right])^{n_0-1}}{(1-\alpha_0 \exp \left[ - (\alpha-\gamma) d \right])^{n_0}} \right.
\]

(7)

For \( d \to \infty (\alpha>\gamma \) and \( n_0 \) large enough) this relation is identical with those of Legler /9/:

---

Fig. 6: Expected value and dispersion of the number of electrons (uniform field)

Fig. 7: Critical avalanche probability for a uniform field \( (p_c \to \infty) \)

Fig. 8: Expected value and dispersion of the number of electrons (non-uniform field)
The critical avalanche probability can be considered as a measure of the breakdown probability. Therefore its relation to the field strength (Fig. 7) gives interesting information about the distribution function of the breakdown field strength (or voltage) of an uniform field. Only for a very limited number of starting electrons this means for a very short stressing time (e.g. lightning impulse voltage), a remarkable variation of the breakdown voltage caused by the formation process of avalanches can be expected. For all voltages of longer duration the avalanche process does not contribute to the stochastic character of the breakdown voltage.

2.3. Non-uniform field

The modelling of the avalanche process in a non-uniform field also delivers an expected value

\[ \text{EN}(x) = n_0 \exp \left[ \int_0^x (\alpha(y) - \eta(y)) \, dy \right], \]

which is identical with the well-known relation for the number of electrons (/4/ to /6/). The decreasing field strength causes a maximum of both the expected value and the standard deviation of the number of electrons (Fig. 8). This maximum is closely related to the critical avalanche probability, but eq. (5) cannot be solved for non-uniform fields directly. An approximated solution was found by replacing the non-uniform field by \( m \) uniform intervals (Fig. 9), \( n_0 = 1 \) and \( m \to \infty \)

\[ \frac{1 - \frac{v}{v_c}}{1 - \left( \frac{v}{v_c} \right)^{m-1}} \approx \alpha - \rho. \]  

(8)

![Fig. 9: Discretisation of a non-uniform field by five uniform sections](image)

![Fig. 10: Critical avalanche probability for negative coaxial cylinders](image)

where \( x_0 \) is the starting point and \( x_n \) is the polarity depending final point of the avalanche development (negative: \( x_n = x_0 = x \left( E(x) = E_c \right) \); positive: \( x_n = 0 \) (position of cathode)). For given non-uniform fields eq. (10) must be solved numerically. The results for negative coaxial cylinders are shown in Fig. 10 (\( x_0 = 0 \); \( n_0 = 1 \)). The critical avalanche probability increases only if the maximum field strength \( E_{\text{max}} \) is in the region of its critical
value $E_{\text{max}} c$ which can be calculated by the streamer criterion /1/6/.

For $E_{\text{max}} > E_{\text{max}} c$ the critical avalanche probability $p_c$ is practically identical with the value for an uniform field. As an approximation

$$p_c = \begin{cases} 0 & \text{for } E_{\text{max}} < E_{\text{max}} c \\ \frac{\alpha - E}{\alpha} & \text{for } E_{\text{max}} \geq E_{\text{max}} c \end{cases}$$

(11)

can be used /19//14/.

If the starting point $x_o$ is not at the cathode ($x_o = 0 \leq r_j$), $p_c$ decreases with increasing $x_o$ and reaches $p_c = 0$ if a minimum distance $x_{\text{lim}}$ to the critical point $x_c$ ($E = E_c$) is reached (Fig. 11). Under the given circumstances at least $x_{\text{lim}}$ is necessary for the development of a critical avalanche.

For the positive polarity the direction of the avalanche process is opposite, the model delivers now different values for $p_c$ and $x_{\text{lim}}$ (Fig. 11). Therefore the polarity effect is well described by the model.

Similar to eq. (11) the critical avalanche probability depending on starting point can be described for both polarities by the following approximation:

$$p_c = \begin{cases} 0 & \text{for } /x_o - x_c/ < x_{\text{lim}} \\ \frac{\alpha - E}{\alpha} & \text{for } /x_o - x_c/ \geq x_{\text{lim}} \end{cases}$$

(12)

The application of the equations (11) and (12) instead of eq. (10) simplifies further calculations.

3. Primary electron generation and breakdown as a Poisson process

The breakdown process is not only influenced by the avalanche formation, but also by the generation process of primary electrons. The following mechanisms for primary electron generation must be taken into consideration:

- **External ionization** caused by cosmic radiation or natural radioactivity is independent of field strength and takes place in each volume element ($dv = dx dy dz$) of the gas. Therefore a constant rate of primary electrons can be assumed

$$n_{oi} = f (E(v, t)),$$

$$n_{oi} = \text{const.}$$

(13)

Re-calculation from measured breakdown times deliver $n_{oi}$ in the range between 0.25 and 2 cm$^{-3}$m$^{-1}$ /14//17//19/.

- **Electron detachment** from negative ions is a very probable, but field strength dependent generation process in the whole volume. A linear dependence of the rate from field strength was assumed

$$n_{od} = k_d (\Delta(v, t) - \Delta c),$$

(14)
such a linearly depending function was also found experimentally /15/.

Electron emission and following processes (attachment and detachment /15/) can also be described by a linearly depending primary electron rate /15//19:/

\[
\begin{align*}
  n_{oe} &= \begin{cases} 
    k_e (E(v, t) - E_0) & \text{on surface of cathode} \\
    0 & \text{in volume}
  \end{cases}
\end{align*}
\]

(15)

The whole breakdown process is the connection between the primary electron generation and the avalanche process which can be modelled by an inhomogenous Poisson process as described in /19/. The intensity (or hazard rate) of this process is on the one hand

\[
\lambda(V, t) = \int \left( \rho_e(V, t) / (n_{ai} + n_{bd} + n_{oe}) \right) dv
\]

(16)
on the other hand the hazard rate is generally related to the corresponding distribution function

\[
\lambda(V, t) = \frac{dP(V, t)}{dt}/(1 - F(V, t)) \quad \text{(17)}
\]

\[
F(V, t_b) = 1 - \exp[-\int_0^{t_b} \lambda(V, t) dt] \quad \text{(18)}
\]

and for the considered temporal problem the breakdown probability is completely given by

\[
F(V, t_b) = 1 - \exp[-\int_0^{t_b} \lambda(V, t) dt]
\]

(19)

This equation describes the variation of breakdown voltage or time caused by statistic effects before the avalanche has reaches its critical size. In the well known manner (/11/ to /19/) it can be used for the calculation of the distribution function of the statistical time lag \(F(t_b)\), of the breakdown time lag \(F(t_g)\), or of the breakdown voltage \(F(u_b)\). The comparison of calculated and measured breakdown time distribution (Fig. 12) shows for model insulations a more or less good agreement. But there seems to be no chance for a sufficient pre-calculation for technical gas insulations, because of their microfield determined by particles, roughness or micro space charges. The described conception is useful for a better understanding of the breakdown phenomena and for semi-empirical extrapolations as described in the following.

\[\text{Fig. 12: Measured and calculated breakdown time distribution functions (coaxial cylinders)}\]
4. Poisson process and enlargement problems

The extrapolation of breakdown characteristics measured at small test objects to large insulations can be done for special cases on the basis the multiplication rule for statistically independent events /20/. The application of a Poisson process offers the possibility to introduce an empirically determined hazard rate \( \lambda(u_d(v, t)) \) for a small element (volume \( V_0 \)) and a well defined stressing time \( T_0 \) and to find the general solution for such enlargement problems (insulations with volume \( V_n \) and stressing time \( T_n \)) by modifying of eq. (18):

\[
F_n(u_d) = 1 - \exp \left[ -\frac{1}{A_n} \int \lambda(u_d(v(t)) \, dv \, dt \right]
\]

(20)

If only an area problem is considered (reference area \( A_0 \), insulation area \( A_n \)) the general equation for the area effect is

\[
\lambda(u_d, A_0) = \exp \left[ \frac{u_{d63}(A_0) - u_{d63}(A_n)}{\mu'(A_n)} \right]
\]

(22)

For \( SF_6 \) insulations the Gumbel distribution was found to be useful for the approximation of the breakdown distribution. Its hazard rate is given by

\[
\lambda(u_d, A_0) = \exp \left[ \frac{u_{d63}(A_0) - u_{d63}(A_n)}{\mu'(A_n)} \right]
\]

(22)

where \( u_{d63}(A_0) \) and \( \mu'(A_n) \) are empirically determined for \( A_0 = 10 \, \text{cm}^2 \). On the basis of eq. (21) and (22) the distribution function of the breakdown voltage of a compressed gas capacitor (Fig. 1) was calculated /23/ /18/. Its comparison with measurements (Fig. 13) shows an satisfactory agreement.

For the design of large shielding electrodes of h. v. testing equipment (Fig. 2) at alternating and switching impulse voltages a combined area-time effect of the streamer inception field strength

Fig. 13: Measured and calculated breakdown voltage distribution of a compressed gas capacitor (\( SF_6 \))

Fig. 14: Measured and calculated streamer inception field strength for a toroid (3.5 m/1.1 m) in air
\[ F_n(E_0) = 1 - \exp \left[ -\frac{1}{A_0 T_0} \int_{T_0(A_n)} \int \lambda(\xi_0, a, t) \, da \, dt \right] \]  

(23)

was investigated \((A_0 = 100 \text{ cm}^2, T_0 = 100 \mu\text{s})\). From a measured Gaussian distribution the intensity

\[ \lambda(\xi_0) = \Delta_n (1 - F_0 (\xi_0, a, t))] \]  

(24)

was determined. Eq. (23) can be simplified for small probabilities \(/16/\):

\[ F_n(\xi_0) = \frac{1}{A_0 T_0} \int_{T_0(A_n)} \int \xi_0 (\xi_0, a, t) \, da \, dt \]  

(25)

With the same reference function \(F_0\) \((F_g)\) the distributions for alternating and switching impulse voltage were calculated and confirmed by measurement (Fig. 14).

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THE EFFECT OF HUMIDITY ON POSITIVE CORONA DISCHARGES IN AIR

N. L. Allen

The University of Leeds
Leeds LS2 9JT
U.K.

1. INTRODUCTION

In most environments each cubic centimetre of atmospheric air contains a few hundred ions, of both polarities. The equilibrium concentrations of these ions are determined by the rates of creation, resulting from natural radioactivity, cosmic rays, combustion processes, etc., and the rates of loss by combination to oppositely charged ions or by attachment to dust particles or other neutral nuclei. In the case of the negative ions, the additional process of dissociation into electrons and neutrals can take place, though with very low probability at normal temperatures.

Since the initiation of a corona of positive polarity, which is the subject of this article, does not involve a cathode mechanism, the detachment of an electron from one of these negative ions is regarded as the essential first step. The efficiency of this process becomes critically important where the voltage producing the corona is time-dependent. The presence of water vapour in air determines the composition of the negative ions formed; it is therefore necessary to examine first the effects of water vapour and the consequences for corona formation.

2. NEGATIVE IONS IN AIR

Electrons, created by natural or other processes, very rapidly attach to the oxygen molecules in the three-body reaction.

\[ O_2 + e + M \xrightarrow{k_o} O_2^- + M + Q_o \]  

(1)

where \( M \) is a third body (a nitrogen or another oxygen molecule), \( Q_o \) is the energy released and \( k_o, k_o' \) are the rate constants for the reversible reaction(1). If water vapour is present, hydration of the ion occurs:

\[ O_2^- + H_2O + M \xrightarrow{k_1} O_2(H_2O) + M + Q_1 \]  

(2)

Other stages of the hydration process occur

\[ O_2(H_2O)^p_{-1} + H_2O + M \xrightarrow{k_p} O_2^-(H_2O)^p + M + Q_p \]  

(3)

where, for practical purposes, the maximum value of \( p \) that need be considered is \( p = 5 \).

Consideration of the equations of the form of (3) and use of the Law of Mass Action, with the approximation that the concentration of neutral \( O_2 \) is negligibly affected by the formation of ions, yields the following equation for the concen-
tation of the $p^\text{th}$ ion and of free electrons for the equilibrium condition:

$$\frac{[O_2^-(H_2O)_p]}{[e]} = \frac{k_p}{k_p'} \cdots \frac{k_0}{k_0} [H_2O]^p 0_2 \quad (4)$$

The ratio of the rate constants, $k_p/k_p'$, is the equilibrium constant $K_p$ for the $p$th reaction and is calculated from the equation:

$$K_p = \frac{1}{m} \exp \left( \frac{Q_p}{RT} \right) \quad (5)$$

where $m$ is the concentration of the species $M$. Values of $Q_p$ have been given in the literature (see, for example, BASTIEN et al., 1975) and are summarized in Table I. Using equation (4) together with the fact that

$$[O_2^- (H_2O)_p] + [e] = \text{constant} \quad (6)$$

the relative concentrations of the ions $[O_2^-]$ and $[O_2^-(H_2O)_p]$ can be calculated as a function of the water vapour content. These are shown in Figure 1 for a temperature of 300 K.

It is clear that over the humidity range covered, the most populous ions are $O_2^-(H_2O)_2$ and $O_2^-(H_2O)_3$. The ions $O_2^-$ and $O_2^-(H_2O)$ are reduced in relative concentration as humidity increases, due to the increasing probability of formation of the ions with $p = 2, 3$, while ions with $p > 3$ show a reduced probability of formation and consequent lower concentration.

It follows from equation (5) that an increased temperature $T$ reduces the value of $K_p$ so that the rate constant of dissociation $k_p'$ to a simpler ion is increased. Figure 2 shows the variation in relative ion concentration with temperature at a fixed humidity. It is seen that the concentration of free electrons, by the dissociative reaction of equation (1) is increased, though remaining very small, by the temperature increase.

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Energy of reaction</th>
</tr>
</thead>
<tbody>
<tr>
<td>$O_2/O_2^-$</td>
<td>0.22 eV</td>
</tr>
<tr>
<td>$O_2^-/O_2^-(H_2O)$</td>
<td>0.46</td>
</tr>
<tr>
<td>$O_2^-(H_2O)/O_2^-(H_2O)_2$</td>
<td>0.28</td>
</tr>
<tr>
<td>$O_2^-(H_2O)_2/O_2^-(H_2O)_3$</td>
<td>0.18</td>
</tr>
<tr>
<td>$O_2^-(H_2O)_3/O_2^-(H_2O)_4$</td>
<td>0.106</td>
</tr>
<tr>
<td>$O_2^-(H_2O)_4/O_2^-(H_2O)_5$</td>
<td>0.056</td>
</tr>
</tbody>
</table>
3. ELECTRON DETACHMENT IN AN ELECTRIC FIELD

Where an electric field is applied to air containing negative ions, for example, in the vicinity of an electrode, the ions gain energy, with an effective increase of temperature, so increasing the probability that free electrons will be liberated. It is assumed that the increase in temperature is directly proportional to the electric field (GALLIMBERTI, 1977; BERGER, 1980) so that equation (5) becomes:

\[ k_p = \frac{1}{m} \exp \left( \frac{Q_p}{\kappa(T+aE)} \right) \]  

where "a" is a constant for a given pressure and ambient temperature.

The experimental justification for this modification of equation (5) is slight and there is no significant data available for detachment probability as a function of electric field. However, consideration of the work of VARNEY (1959) with \( N_2^+ \) ions in \( N_2 \) has led BERGER (loc. cit.) to suggest a value in the region of \( 10^{-4} \text{KmV}^{-1} \) for the constant a.

Thus, in a field of the order \( 3 \times 10^6 \text{Vm}^{-1} \), which is characteristic of the value at which ionization occurs in air, there is an effective increase of temperature of the order 300K, giving \( T + aE = 600 \text{K} \), with a very greatly increased probability of electron detachment (Figure 2).

Use of equation (7) with the Law of Mass Action has enabled GALLIMBERTI (1977) to give an expression for the overall equivalent rate of detachment of electrons from negative ions up to \( p = 3 \) of the form:

\[ \frac{n_n}{\tau} = \frac{n_n}{\hbar \tau_0} \exp \left( \frac{p=3}{p=1} \frac{Q_p}{k(T+aE)} \right) \]  

where \( n_n \) is the total concentration of negative ions, \( h \) is the fractional concentration of water molecules, \( \tau \) is the mean detachment ion lifetime and \( \tau_0 \) is the detachment lifetime of \( O_2^- \) ions. Detachment rates are presented, but depend upon an assumed value of \( n_n \) which, in this case, depended in turn upon experimental conditions and may have been in error. The lifetime of the \( O_2^- \) ions is given by an expression of FROMMOLD (1964):

\[ \tau_0 = A - B \ln E/p \]  

Figure 2.

Relative concentrations of negative ions in air as a function of temperature at a constant humidity of 13,700 ppm (11 gm m\(^{-3}\) moisture content).

\[ \text{HUMIDITY, 13,700 ppm (11 gm m}^{-2}\text{)} \]
Taking the example quoted, of a field of $3 \times 10^6 \text{V/m}$ applied to a volume of air containing 100 ions/cc, use of equation (7) and Figure 2 indicates that the density of free electrons would rise to the order of 1 per cc for the commonly encountered humidity of $8 \text{ gm m}^{-3}$ (10,000 ppm fractional concentration). This indicates that the process of detachment is plausible as the source of free electrons in electric fields which are of a value around the minimum required to ensure avalanche propagation.

4. PROBABILITY OF AVALANCHE FORMATION

When a steady electric field is applied to an electrode system, the probability of electron detachment is of no consequence, since a relatively long time is available in which detachment can occur. Where the field is applied at a finite rate, however, the probability of detachment is a factor determining the time-lag to avalanche initiation.

Von LAUE (1925) showed that for a step-function voltage application, the time lag to avalanche initiation was determined by the law of chance. Thus, if $P$ is the probability of detachment within a time $t$, then

$$P = 1 - \exp\left(-\frac{t}{\tau}\right)$$

where $\tau$ is the mean statistical time lag, a quantity which depends on the initial condition of negative ion density, electric field and atmospheric conditions of temperature and pressure. A plot of $\ln P$ against $t$ is a straight line of slope $-1/\tau$.

In practice, the impulse voltage, rising to crest in a specified time, is technologically important. At a fixed point in space, relative to an electrode, the probability of detachment then changes with time, and may not become significant until the value of the electric field has increased above a critical value. An avalanche may then develop, but the probability of development leading to streamer formation is subject to limitations defined by the geometry of the system. For example, in the region very close to the
electrode, there may be insufficient distance to permit an avalanche to develop into the critical size needed for streamer formation (see §5 below). Alternatively, at an appreciable distance from the electrode, the electric field may be too weak to permit nett ionization growth (see §5). These two conditions define boundaries, around the electrode, outside which critical avalanche development cannot be achieved; the closed volume so defined is known as the critical volume. An example is shown in Figure 3 for the case of a cylindrical, hemispherically-ended rod of 1 cm. radius; the critical volumes are defined for several voltages (ALLEN et al 1981).

The size of the critical volume is affected by humidity variations. As will be shown the inner boundary is extended outwards by humidity increase, at a constant electrode voltage, since in the constant field configuration around the electrode, a greater avalanche length is required to achieve the critical condition for streamer formation. The outer boundary contracts, since a larger value of electric field is required for nett ionization growth to occur. Thus, the critical volume becomes smaller.

It is clear that the probability of formation of a critical avalanche near an electrode depends upon the negative ion density, the ion lifetime, the critical volume and the rate at which the voltage increases. These conditions have been investigated by SOMERVILLE and TEDFORD (1982), and BERGER and HAHN (1980).

Where an impulse voltage is applied, for example, to a rod/plane gap, the theoretical probability of critical avalanche formation (leading to corona) can be compared with the experimentally determined probability by measurement of the distribution of time-lags to corona inception. The results obtained, showing agreement with theoretical predictions in each case, depend upon the manner in which the experiments are carried out.

Figure 4

Histograms of inception times of first coronas at two humidities, using 700 impulses in each case, with a hemispherically-ended gold rod electrode. Impulse voltage profile is shown for comparison.

Shaded: Humidity = 12.6 gm m$^{-3}$
Not shaded: Humidity = 7.7 gm m$^{-3}$

In the case of an impulse rising to crest in 16μsec, ALLEN et al (1981) found distributions of time lags as shown in Figure 4, at two
humidities. When integrated, each distribution showed the form of the VON LAUE plot of Figure 5 (modified for the finite rate of rise of voltage). Comparison of the practical and theoretical curves permitted an estimate of the negative ion density present immediately prior to each successive impulse. The results showed an increase of negative ion density with increasing humidity, a fact which has been confirmed by consideration of combination rates between impulses as a function of humidity (ALLEN et al. 1982). However, these results were obtained in a closed laboratory with no ventilation. A similar sequence of tests has been carried out by POLI (1985) where humidity was varied and the air circulated between impulses. Thus, the initial negative ion density remained substantially constant and the trend of time lags showed the opposite tendency to that of ALLEN et al, namely, towards later inception at higher humidity. This result was in accord with simple expectations, since the smaller critical volume and greater lifetime of negative ions at higher humidity must yield longer time lags where the ion density is constant. The importance of careful control of experimental conditions is thus emphasized; the comparison between the results indicates, for example, that results obtained in laboratory and outdoor testing are likely to differ as a consequence of the differences in atmospheric conditions.

![Figure 5](image)

Von Laue plot for rod-plane gap compared with experimental results (shown as crosses) for the impulse voltage of Figure 4; crest voltage 266 kV, atmosphere negative ion density 2 cc⁻¹ and humidity 10 gm m⁻³.

5. CRITICAL AVALANCHE FOR STREAMER FORMATION

There is no analytic expression describing electron multiplication in an electric field. However, HARTMANN (1980) has described an empirical formula for the nett ionization coefficient which holds over a wide range of E/p:

\[
\frac{\lambda}{P} = \frac{a - n + \delta}{N(E/p)^3} \exp \frac{-BP}{E} + Q
\]

(11)
where \( A = 1.75 \times 10^3 \), \( B = 4 \times 10^4 \), 
\( C = 1.15 \times 10^{12} \), 
\( N = 1 + 3.2 \times 10^{-2} H \), \( M = 1 + 10^{-2} H \), 
\( Q = 1 + 1.15 \times 10^{-1} H^{0.1} \), and 
\( \nu = 0.9/[1.49 + \exp (-p/587)] \). \( H \) is the humidity in \( \text{gm} \ \text{m}^{-3} \) and all other values are in MKSA units. Values of \( \lambda_1/p \) as a fraction of \( \varepsilon/p \) and humidity are given in Figure 6. \( \eta \) and \( \delta \) are the attachment and detachment coefficients, respectively, of electrons.

![Figure 6](image)

Figure 6.

Values of \( \lambda/p \) (ion pairs \( \text{m}^{-1} \ \text{torr}^{-1} \)) as a function of humidity \( H \ \text{gm} \ \text{m}^{-3} \) for values of \( \varepsilon/p \ \text{V cm}^{-1} \ \text{torr}^{-1} \) as follows: A 100, B 80, C 70, D 60, E 50, F 44, G 40.

In determining the size of avalanche required for streamer initiation, taking for example, the Meek condition as an appropriate space charge condition, the crucial quantities are the number of positive ions created and the volume which they occupy. It is often assumed that the value of the space charge is determined by the free radial diffusion of electrons, but this assumption cannot be sustained when the positive ion space charge becomes appreciable. In this case a change from free to ambipolar diffusion occurs. The transition is governed by the relation (BROWN, 1959)

\[
e \exp \left( \frac{a-n+\delta}{4\varepsilon e} \right) = \left[ \frac{\varepsilon}{Q\mu} \frac{1}{(B)} \right]^{1/2}
\]

(12)

where \( x \) is the distance from the initiating electron to the transition, \( D/\mu \) is the random energy of the electrons and \( E \) is the applied electric field. The values of \( D/\mu \) increase with humidity for values of \( \varepsilon/p < 100 \) as shown in Figure 7 (MALLER and NAIDU 1975). Using

![Figure 7](image)

Figure 7

Values of \( D/\mu \) for electrons as a function of \( \varepsilon/p \) in dry air and air of humidity 9.6 \( \text{gm} \ \text{m}^{-3} \).

values of \( a-n+\delta \) derived from Equation (11), and substituting in Equation (12), it is found that the avalanche length \( x \) to the free/ambipolar transition increases
with humidity. Further calculation shows that the avalanche length $x_m$ to the Meek condition also increases with humidity:

$$\frac{(a-n+\delta) \exp (a-n+\delta)x_m}{3\pi \varepsilon_A \varepsilon_0} = E \quad (13)$$

These results are shown in Figure 8 (ALLEN 1985)

Physically, the increases with humidity in $x$ and $x_m$ are due partly to the decrease in the value of $(a-n+\delta)$ at constant electric field and partly to the higher random electron energy, $D/\mu$ resulting in a larger space charge volume. It is as a result of the increase in $x_m$ that the distance of the inner boundary of the critical volume from the electrode surface is increased with increasing humidity.

The calculations show that to maintain the critical condition for streamer initiation for a value of $E/p \sim 40 \text{ Vcm}^{-1}\text{torr}^{-1}$ an increase of 0.35 per cent in electric field per $(\text{gm m}^{-3})$ is required. The careful measurements of BLAIR et al (1965) in a uniform field gap gave a rate of change of $E/p$ of 0.44 per cent per $(\text{gm m}^{-3})$ to maintain a critical condition. ALLIBONE and DRING (1972) found a rate of change of 0.3 per cent per $(\text{gm m}^{-3})$ for sphere-gap breakdown with impulse voltages. Agreement between theory and experiment is therefore quite good.

6. STREAMER PROPAGATION AND HUMIDITY

When the critical avalanche size has been achieved, the streamer begins to propagate towards the cathode due to avalanche formation in the space charge field. On the anode side, the nett electric field is reduced to a small value, reducing the nett ionization to zero (HARTMANN, 1980) but sufficient to permit the electrons to progress towards the anode. The length of time for which these electrons remain free depends upon the moisture content in the air and upon the temperature in the streamer; these factors in turn influence the progress of the streamer.

The dynamics of streamer propagation have been investigated by GALLIMBERTI (1972) who considered the energy balance between that lost in ionization processes in the gas and that gained from the electric field. Computations of the rate of
extension and of the rate of current rise of the streamer, and of the spatial development of the corona, agree well with experiment. NIKOLOPOULOS (1983) has developed this work to include humidity effects and shows the following rates of change of various parameters with moisture content in the range $7 < H < 15 \text{ gm m}^{-3}$.

Peak current and total charge:  
2.5 per cent per (gm m$^{-3}$)  
Maximum length of streamer:  
0.17 per cent per (gm m$^{-3}$)  
Initial streamer velocity:  
0.21 per cent per (gm m$^{-3}$)

The calculated rate of variation of total charge can be compared with the experimental results of ALLEN and DRING (1985), where a value of 9.5 per cent per (gm m$^{-3}$) was obtained, by measurement of the charge drifting to the plane of a rod-plane arrangement. These results are, however, open to the interpretation that it is the modification of the (predominantly) positive charge, by an unknown variation of a possible negative ion component that is being measured. Recent work (ALLEN et al 1985) shows that the estimation of the negative ion component is made difficult by combination effects. The experimental result is at present inconclusive but could equally be due to a large real variation with humidity of the positive component.

The rate of change of streamer length can also be inferred from the work of PHELPS and GRIFFITHS (1976)

and of ALLEN and DRING (loc.cit.) which presented a rate of change of 1.6 per cent per (gm m$^{-3}$) in the ambient electric field required, as a function of humidity, for maintenance of a constant streamer length. This result suggested a similar rate of change of streamer length with humidity at a constant electric field. Again, the value is considerably higher than that predicted by theory.

The significance of the moisture content is seen by consideration of the simple model of PHELPS and GRIFFITHS (loc.cit.) in which the energy gained per unit distance advanced by the nett charge $q$ (assumed constant) in the head of the streamer is assumed to be $qE_0$, where $E_0$ is the ambient field. This is balanced, as in GALLIMBERTI's model, by the energy dissipated in re-creating the tip by ionization, as it advances each unit of distance. This quantity is $r^2n_+eV_1^i$ where $r$ is the radius of the tip, $n_+$ the positive ion density and $V_1^i$ is the average energy needed to creating each charge. Equating these two quantities:

$$E_0 = \frac{r^2n_+eV_1^i}{q} \quad (14)$$

Since the nett charge $q$ must be expected to decrease with an increase of moisture content, due to more rapid negative ion formation, as discussed by Phelps and Griffiths, and the value of $V_1^i$ increases due to the changes in the ionization coefficient, equation (14) indicates that the value of $E_0$ is expected to increase with increase of humidity.
The value of the rate of change of $E_0$ depends also, however, upon the gas temperature in the tip, since the balance of negative ion species present, the density of free electrons and, therefore, the conductivity, depend upon temperature. The results of SPYROU et al (1983) using a steady corona discharge, indicated a temperature near the tip of about 400K, but the theoretical work of GALLIMBERTI (1977) suggests a value in the order of 900K. Figure 2 indicates that at the higher temperatures, the proportion of free electrons present becomes very significant and would greatly influence the conductivity of the streamer channel in the region of the tip. In this case the condition (14) would be significantly affected so influencing the variation of the electric field needed for propagation as the humidity is varied. Further investigation is needed to relate more precisely the humidity to the field needed for propagation of the streamer.

7. CONCLUSION

The study of humidity effects on corona in air gives a useful new insight into the general investigation of corona initiation and propagation. Under normal atmospheric conditions, the picture of the dissociation of hydrated negative ions gives a plausible mechanism for the origin of a corona in a non-uniform electric field, while the processes of combination can also be related to moisture content and thereby verify the hypothesis that negative ion densities are reduced to low levels during impulse corona investigations. Work on corona initiation times provides confirmation also of the concept of the critical volume which varies with humidity while studies of the corona itself yield additional information on the mode of propagation of streamers. The use of humidity as a variable, in experimental and theoretical work thus provides an "indicator" which is valuable in the elucidation of corona properties.

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RESONANT ABSORPTION AND MODE CONVERSION IN A NONUNIFORM PLASMA

L.P.J. Kamp

University of Technology, Physics Department
P.O.B. 513, NL-5600 MB Eindhoven, Netherlands

Introduction

Resonant absorption of an electromagnetic wave propagating into a nonuniform plasma has been known for quite some time since Budden’s early paper in 1955 [1]. The absorption appears due to a singular behavior of the wave equation for a cold and nonuniform plasma at the point where the frequency of the incident wave matches with that of some local plasma resonance. Extensive studies of this absorption revealed that it is a manifestation of a mode conversion of the incident long-wavelength mode into a short-wavelength kinetic mode that propagates away from the resonance region.

The absorption and conversion of electromagnetic waves in an inhomogeneous plasma is among other things important to the understanding of problems concerned with the heating of both magnetically and inertially confined thermonuclear plasmas. The relevance of the so-called mode conversion process stems in this case from the fact, that it provides a possible mechanism for the conversion of the energy of the incident electromagnetic radiation into short-wavelength plasma waves that can resonantly transfer energy to the particles e.g. by Landau damping or cyclotron damping. In the lossless cold-plasma limit these plasma waves cannot propagate and thus do not carry off the incident radiation energy that is accumulated in the resonance region where in the warm-plasma case the wave transformation would take place. This accumulation or resonant absorption of energy in the resonance region of a cold plasma (even if the plasma is lossless) continues in the linear stationary situation the electromagnetic field will become singular in the resonance. This paradoxical situation of this resonant energy absorption by a lossless plasma is known as the Herlofson paradox [2]. It were Pilya [3] for the unmagnetized plasma and Stix [4] for the magnetized plasma who showed for the first time that the cold-plasma singularity is removed by adding warm plasma corrections to the relevant differential equations that describe the various modes, and, consequently, that a warm electron or ion mode is generated that couples to the original cold-plasma mode.

The theory of resonant energy absorption and mode conversion is also relevant for the behavior of antennas or probes in plasmas. It is well-known that an antenna that is immersed in a plasma will absorb some of the electrons from its neighborhood. As a result of this an inhomogeneous positive space charge is formed. This so-called ion-sheath acts like a coating of the antenna. In an unmagnetized plasma an interesting situation arises when somewhere in the ion-sheath a resonance is present where the local plasma frequency matches the driving frequency of the antenna. In the cold-plasma limit this resonance acts like an energy sink where most of the radiation from the antenna is absorbed or more correctly accumulated. Such a resonance in the ion-sheath will strongly influence the radiation properties of the antenna (radiation pattern, antenna impedance, antenna efficiency etc.) [5]. Knowledge of these radiation properties is important if the antenna is intended as a wave injection facility or as a diagnostic tool.

Finally the theory of resonant energy absorption and mode conversion may also be
useful in astrophysical research e.g. for the explanation of saturnian myriametric radiation [6].

Classical theory of linear wave transformation and resonant energy absorption

As is well-known a plasma can usually support two or more modes at the same frequency. For most choices of the plasma parameters (temperature, density, magnetic field strength etc.) the wavenumbers of these waves are widely differing. But it may also happen that at some setting of these parameters the wavenumbers will coalesce. One way of constructing such a coalescence of wavenumbers is by considering a wave propagating through an inhomogeneous plasma and approaching a region where the local plasma parameters correspond to that coalescence. One may ask now what happens with the wave; is it reflected or absorbed or converted into another mode or a combination of all these? It is this question to which this paper is addressed.

In this context it is important to identify a serious shortcoming in the literature on this subject. By introducing the above-mentioned concept of local wavenumbers one assumes that the WKB-approach is valid, especially in the region of coalescence of wavenumbers. However, it turns out that in most cases where conversion and coupling of one mode to another takes place the WKB-approach and therefore the concept of a local dispersion relation is invalidated. This means that the investigation of a local dispersion relation can at most give some indication where absorption and/or conversion of the various modes could take place but one has then to construct full-wave solutions in the critical layer in the plasma where the local plasma parameters would correspond to a coalescence in order to settle the above-mentioned question. Also the calculation of the absorption and/or conversion by integrating the imaginary part of the wavenumber of the incident radiation across the resonance region as is sometimes seen in the literature needs further justification.

Another limitation that goes back to Stix's theory [4] of mode conversion is also still often encountered in the literature. It was Stix's idea of associating a differential equation with the local dispersion relation [4], usually one of fourth-order, whose asymptotic solutions can be found by suitable integral-transform methods or phase-integral methods. These methods thus provide a technique by which one can study power flow, mode conversion, absorption etc. The limitation of the application of these methods is related to the foundation of the above-described local method in that the form of a differential equation corresponding to a local dispersion relation can be ambiguous since no a priori prescription is available for the commutation properties between functions and the differential operators multiplying them. Moreover it turned out that these methods are rather laborious and often do not give explicit solutions in the mode conversion region.

Overview of resonances

In a plasma there can exist lots of resonances (in the above-given sense) where absorption and/or conversion of a mode that can propagate in the plasma can take place. During the last thirty years a great many investigations on all kinds of resonances has been performed. For an unmagnetized, inhomogeneous plasma resonant energy absorption or mode conversion can take place in the region where the local electron-plasma frequency (the ions are assumed to be immobile at the high frequencies under consideration) matches the frequency of a propagating electromagnetic wave. In a cold plasma the energy of the electromagnetic wave, which is assumed to have an electric field component parallel to
the density gradient, is partially absorbed in the resonance. In this case the electric field components become, at least in the lossless linear theory, singular at the position of the resonance [7]. In a warm plasma the electric field strengths remain finite but may, for a low-temperature plasma, become large in a narrow region around the resonance. In this case the electromagnetic wave is partially converted into a quasi-longitudinal Langmuir wave which travels towards the lower-density side of the plasma [8]. This example of resonant energy absorption and linear wave transformation has been investigated by many people either analytically [8] or numerically [9].

If an external magnetic field is added to the plasma the number of resonances is considerably enlarged. In the first place there are the hybrid resonances that are determined by $S = 0$ where $S$ is given by

$$S = 1 - \sum \frac{\omega_p^2}{s \omega - \omega_c^2},$$

$$\omega_p^2 = \frac{n_s q_s^2}{\omega_c^2}, \quad \omega_c = \frac{q_s B}{m_s}$$

where $s$ designates particle species and $q_s$ is the particle charge including its algebraic sign. The other symbols have their usual meaning. It can be shown that $S = 0$ is satisfied by

$$\omega^2 = \omega_{UH}^2 = \omega_p^2 + \omega_c^2 \text{ i.e. the upper-hybrid resonance},$$

$$\frac{1}{\omega_{LH}^2} = \frac{1}{\omega_p^2} + \frac{1}{\omega_c^2} \text{ i.e. the lower-hybrid resonance}$$

and in the high-density limit by

$$\omega_{1j}^2 = \omega_c^2 + x_j \omega_c^2, \quad x_j = \frac{n_j}{n_1}$$

Now just as for the unmagnetized plasma, finite temperature modifies the hybrid resonances in that it eliminates the singular behavior of the electric field strength in the resonance. It can be shown that for a warm plasma the paradoxical resonant absorption of electromagnetic radiation can be understood in terms of linear wave transformation of the long-wavelength electromagnetic wave into short-wavelength so-called Bernstein modes. This process can also be reversed so that energy of a short-wavelength electrostatic Bernstein mode is converted into a long-wavelength electromagnetic mode, and thus account for the observability of the so-called Landauer radiation [10].

A similar singular behavior of the wave components that can be attributed to a mode conversion into a short-wavelength kinetic wave can be found in the magnetohydrodynamic frequency range. In this case the singularity appears at the location where the externally applied frequency $\omega$ matches with the local Alfvén frequency, $k \parallel V_A$ where $k$ is the component of the local wavenumber parallel to the external magnetic field and $V_A$ is the local Alfvén speed. The introduction of additional physical effects e.g. finite ion Larmor radius affects this singularity at the Alfvén resonance layer, but also raises the relevant differential equation from second to fourth or higher order, thus introducing new solutions which represent short-wavelength modes. Analysis by mode conversion theory shows that the wave energy which according to the MHD-model would have been absorbed at the Alfvén resonance is, in fact, converted into these new short-wavelength modes [11].

A third group of resonances that is relevant for heating of fusion plasmas are the
electron and ion cyclotron harmonics. Analogous to the preceding resonances, the singular behavior and the resonant energy absorption near these cyclotron harmonics can, at least for some polarizations of the electromagnetic mode, be understood in terms of mode conversion into a kinetic Bernstein mode.

As has been argued in the preceding overview for rather low temperatures the various field components can become very large in the resonance regions. Therefore it is not astonishing that for a low temperature plasma various nonlinear phenomena can take place near the resonance e.g. profile modification by ponderomotive forces, generation of quasi-static magnetic fields, parametric decay instabilities, harmonic generation, wavebreaking, modulational instability and generation of solitons and cavitons. A detailed review of these nonlinear mode-conversion effects is, however, beyond the scope of the present paper.

General theory of a class of mode conversion problems

In order to overcome the previously mentioned shortcomings a unified theory of a number of mode conversion problems will be developed in what follows. Moreover the following approach of the mode conversion problem has some additional advantages over the tackling of it by other investigators [12] as will turn out.

The treatment is based on boundary-layer theory [13] for the resonance region and Langer’s approach [14] for the regions far away from the resonance.

It turns out that most of the linear mode-conversion problems can be modelled by the following equation

\[ s^2 \frac{d^4 \phi}{dx^4} + f(x) \frac{d^2 \phi}{dx^2} + \gamma f'(x) \frac{d \phi}{dx} - [sf(x)(1-\alpha f(x))+\beta] \phi = 0 \]

with \( f(0) = 0, f'(0) < 0 \) (\( f(x) \) is monotonically decreasing) and \( \beta \geq 0 \). To arrive at equation (1) a harmonic time dependence according to \( \exp(\pm i \omega t) \) has been assumed. The one-dimensional non-uniformity in the plasma is modelled by the function \( f(x) \). Henceforth the plasma is assumed to be only weakly non-uniform so that \( f = f(x/L) \) with \( L \gg 1 \). \( \delta \) is proportional to the thermal speed of the electrons or ions. The plasma is assumed to be weakly relativistic so \( \delta \ll 1 \). In physical applications two situations are relevant. These situations can be described by different values for \( s \) and \( \alpha \). For \( s = +1 \) and \( \alpha \geq 0 \) the above-given equation will be called the Reflecting Equation and for \( s = -1 \) and \( \alpha = 0 \) the resulting equation will be called the Tunneling Equation. Both types of equations do not only describe a resonance phenomenon at \( x = 0 \) but also introduce one or two cutoffs approximately there where \( \beta = sf(x)(af(x) - 1) \).

The exact values of the parameters \( \gamma \) and \( \beta \) depend upon the kind of resonance that is considered. If the plasma is unmagnetized then \( \beta = 0 \). In what follows \( \gamma \) is assumed to be an integer because this is physically the most interesting situation. In Fig.1 and 2 the variation of the local wavenumber \( k_x \), that could be associated with the Reflecting Equation and the Tunneling Equation respectively, is indicated in a qualitative way. Note that these sketches should be looked at with a good deal of suspicion because actually the local wavenumber \( k_x \) has no physical meaning anymore in the neighborhood of the resonance and the cutoff. From Fig.1 and 2 the name-giving of both equations becomes clear.

Reflecting Equation \( (s=+1, \alpha \geq 0) \):

The fourth-order differential equation (1) for \( \phi \) can be solved approximately by means of boundary-layer theory [13] and Langer’s method [14]. The singular perturbation method is based on the small parameter \( \delta \). This implies that there has to be made a distinction between two so-called outer regions and in the case
at issue one inner region. The outer regions are characterized by the absence of rapid variation in $\phi$. In these regions $\phi$, $\phi'$, $\phi''$, and $\phi'''$ remain finite if $\delta + 0$. As a consequence of this $\delta$ may be taken zero in these outer regions as a first approximation. The resulting second-order differential equation is then solved for a weakly inhomogeneous plasma by means of Langer's method [14]. The gist of this method is that approximately identical differential equations have approximately identical solutions. As will be shown in the neighborhood of the resonance $\phi$ and /or its derivatives do not remain finite anymore if $\delta + 0$. Around the resonance a narrow region with rapid variations in $\phi$ will develop for $\delta + 0$.

Hence, the inner region is some neighborhood of the resonance at $x = 0$ and the outer regions extend to the left and to the right of this resonance region and are sufficiently far away from the resonance proper.

For the solutions in the outer regions the following expansion is proposed:

$$
\phi_{\text{outer}} = \phi_0^0 + \delta \phi_1^0 + \delta^2 \phi_2^0 + \ldots
$$

The equation for $\phi_0^0$ is thus given by

$$
\frac{d^2 \phi_0^0}{dx^2} + \frac{f'}{f} \frac{d\phi_0^0}{dx} - \left[1 - \alpha f + \frac{B}{f}\right] \phi_0^0 = 0.
$$

Hence in lowest order the outer regions behave like a cold plasma. Equation (3) is rewritten as follows:

$$
\frac{d^2 \psi}{dz^2} + \left[L^2 q(z) + r(z)\right] \psi = 0
$$

where $q(z) = af - 1 - \frac{\delta}{f}$,

$$
r(z) = \frac{\gamma}{2} \left(1 - \frac{\gamma}{2} \frac{f'}{f}\right)^2 - \frac{\gamma}{2} \frac{f''}{f},
$$

$$
z = \frac{x}{L}, \quad \phi_0^0 = f - \frac{\gamma}{2} \psi.
$$

Fig. 2

The application of Langer's method [14] to equation (4) with (6) results in the first term of a 1/L-expansion of $\phi_0^0$ around the resonance at $x = 0$

$$
\phi_0^0 = \frac{q}{f^2} \frac{1}{\psi} \frac{1}{\phi_0^0} H^{(2)}(\phi)
$$

where

$$
\phi = \int_0^x \sqrt{q(x')} dx', \quad \psi = \int_0^x \sqrt{af(x')} - 1 - \frac{\beta}{f(x')} dx',
$$

$$
\nu = \frac{1 + 1}{p}, \quad p = 1 \text{ for } \beta \neq 0 \text{ and } p = 2 \text{ for } \beta = 0.
$$
$H^{(2)}_\nu(x)$ is the Hankel function of the second kind and of order $\nu$ [15]. The other solution has been discarded in order to satisfy the boundary condition at infinity. If $\beta \neq 0$ equation (4) has two cutoffs approximately there where $q = 0$. If $\beta$ becomes zero (e.g. when the plasma is unmagnetized) then one of these cutoffs merges into the resonance while the other retains its identity. In order to demonstrate the technique and without loss of generality the most simple situation is considered that is $\beta = 0$ (one cutoff and one resonance). The generalization to $\beta \neq 0$ is straightforward.

In the neighborhood of the turning point $x_\ell^*(x^I) where $f(x_\ell^*) = 1/\alpha$, the functions $q$ and $r$ behave as follows:

$q(z) = \alpha f'(x_\ell^*) (z - z_\ell^*) \left[ 1 + O(z - z_\ell^*) \right]$, \hspace{1cm} (9)

$r(z) = \left[ \frac{2}{\alpha} (1 - \frac{2}{\alpha}) (\alpha f'(x_\ell^*))^2 - \frac{2}{\alpha} \alpha f''(z_\ell^*) \right] \cdot \left[ 1 + O(z - z_\ell^*) \right]$

where $z_\ell^* = x_\ell^*/L < 0$. The application of Langer's method [14] to equation (4) with (9) now gives the following first term in a $1/L$-expansion of $\phi_0$ around the cutoff $x = x_\ell^*$:

$\phi_0 = \frac{2}{\alpha} q(1 - \frac{2}{\alpha})^2 \left[ C_2 H^{(1)}_{1/3}(\Psi) + C_3 H^{(2)}_{1/3}(\Psi) \right] \hspace{1cm} (10)$

where $\Psi = \int_{x_\ell^*}^{x} \sqrt{q(x')} \ dx'$ = \int_{x_\ell^*}^{x} \sqrt{\alpha f(x')} - 1 \ dx'$, \hspace{1cm} (11)

$H^{(1)}_{1/3}(x)$ and $H^{(2)}_{1/3}(x)$ are the Hankel functions of order $1/3$ [15]. Similar solutions could be found around the other cutoff that would be present if $\beta \neq 0$.

If it is assumed now that the resonance ($x = 0$) and the cutoff ($x = x_\ell^*$) are well separated than it is possible to match both solutions (7) and (10) in their common region of validity by identifying their asymptotic behavior in this region. By making use of the analytical continuation of the Hankel functions via the complex upper half-plane and by taking into account the Stokes phenomenon it is found that

$c_2 = -2 \cos (\nu \pi) \exp \left\{ \left( \frac{1}{2} \nu + \frac{1}{6} \right) \pi - \Delta \right\} c_1$, \hspace{1cm} (12)

c_3 = \exp \left\{ \left( \frac{1}{2} \nu - \frac{1}{6} \right) \pi + \Delta \right\} c_1$

where $0 < x_\ell^* \sqrt{1 - \alpha f(x')} \ dx' >> 1$. \hspace{1cm} (13)

If $x = 0$ and $x = x_\ell^*$ are too close to each other the above-described technique fails and the resonance and the cutoff have to be treated in some way as one point (for $\gamma = -1$ this has been done in [16]).

The absorption coefficient that can be found from the asymptotic evaluation of (10) for $x << x_\ell^*$ turns out to be given by

$A = -4 \cos (\nu \pi) e^{-2 \Delta}$. \hspace{1cm} (14)

Since $\nu = |\gamma - 1|/2$, this is, in the present case, physically only acceptable for $2 + 4n \leq \gamma \leq 4 + 4n, n = 0, 1, 2, \ldots$. Note that for $\gamma$ even the absorption becomes zero. Previously it has been mentioned that $\gamma$ is assumed to be an integer. Furthermore it has been shown that for $\gamma$ even there is no absorption and no conversion and that for physically acceptable models $2 + 4n \leq \gamma \leq 4+4n$.

So obviously the most interesting values for $\gamma$ are $\gamma = 3 + 4n$. So $\nu = |\gamma - 1|/2 = 2m + 1, m = 0, 1, 2, \ldots$.

If $\delta \rightarrow 0$, the solutions of the warm plasma equation (1) develop a narrow region around $x = 0$ of rapid variations. Since the thickness of this region approaches zero if $\delta \rightarrow 0$, it is called a boundary layer [13]. In this internal boundary layer around $x = 0$, it is no longer allowed to neglect the fourth derivative in (1); the complete fourth order differential equation has to be used. In order to determine the region of nonuniformity of the outer expansion, a stretching of the $x$-coordinate is applied according to $\zeta = \delta^{-\mu} x$ with $\mu > 0$. If $\delta \rightarrow 0$ the resulting equation depends on the
value of \( \mu \). By applying the method of dominant balance [13] it is found that the only acceptable limit is the so-called distinguished limit. With this method of dominant balance it is found that \( \mu = 2/3 \). Only this limit gives a nontrivial boundary-layer structure around the resonance \( x = 0 \) which is asymptotically matchable to the outer solutions.

For the inner solutions the following inner expansions are used:

\[
\phi_{\text{inner}} = \phi_{00} + \phi_{10} t + \phi_{11} t^2 + \phi_{20} t^3 + \phi_{21} t^4 + \ldots
\]

For \( \gamma = 3 + 4\ m \) and
\[
\phi_{\text{inner}} = \phi_{00} + \phi_{10} t^{1/2} + \phi_{11} t + \phi_{20} t^{3/2} + \phi_{21} t^{2} + \phi_{22} t^{3} + \ldots
\]  

for \( \gamma = -1 - 4\ m \). Near the resonance at \( x = 0 \) the function \( f(x) \) is Taylor expanded as follows:

\[
t(x) = f'(0) \delta x + \frac{1}{2} f''(0) \delta x^2 + \ldots
\]

For \( \phi_{\text{inner}} \) the following reduced fourth-order differential equation is now found:

\[
\frac{d^4 \phi_{\text{inner}}}{dt^4} - A^3 \left( \frac{d^2 \phi_{\text{inner}}}{dt^2} + \gamma \frac{d \phi_{\text{inner}}}{dt} \right) = 0
\]  

where \( \gamma = 3 + 4\ n \) and \( A^3 = |f'(0)| > 0 \). This equation can be solved exactly, for \( \gamma = 3 + 4\ m \) and
\[
\phi_{00} = a_0 + a_1 t^{2+4m} + \int \ldots \int \left\{ a_2 A(A) + a_3 B(A) \right\} dt
\]

for \( \gamma = -1 - 4\ m \). \( A(x) \) and \( B(x) \) are the Airy functions [15] and \( G(x) \) can be expressed in terms of integrals of the Airy functions [15]. The Airy functions in (18b) have to be integrated \((3 + 4\ m)\)-times from \( t = 0 \) to \( t = \zeta \). Since the terms with \( B(\zeta) \) grow exponentially for \( \zeta \to +\infty \), \( a_3 = 0 \).

For \( \zeta \to \infty \) it becomes possible to distinguish between the short-wavelength plasma mode and the long-wavelength electromagnetic mode in \( \phi_{\text{inner}} \). The matching of \( \phi_{\text{inner}} \) with \( \phi_{\text{outer}} \) is based on the identification of the long-wavelength part of the inner solution with the long-wavelength outer solutions near the resonance

\[
\lim_{\zeta \to -\infty} \phi_{\text{inner}}(\zeta) = \lim_{t \to 0} \phi_{\text{outer}}(t)
\]

By making use of the asymptotic behavior of the Airy functions it is thus found that

\[
a_0 = 0, \quad a_1 = a_2 = \frac{-1}{3} \frac{2^{2m+1}}{(2m)!} \left| f'(0) \right| \quad C_1
\]

for \( \gamma = 3 + 4\ m \) and
\[
a_0 = \frac{1}{\pi} \frac{2^{2m+1}}{(2m)!} \left| f'(0) \right| \quad C_1
\]

for \( \gamma = -1 - 4\ m \).

From (12) and (20) it is clear that if \( C_1 \) is known, (e.g. by fixing the amplitude of an incident electromagnetic wave) all solutions are completely determined. A single uniform approximate solution of the fourth-order differential equation that is valid for all
x can be constructed now by combining inner and outer solutions in the following way:
\[ \phi_{\text{uniform}} = \phi_{\text{outer}} + \lim_{x \to 0} \phi_{\text{inner}} \]  
(21)

It turns out that in the case at issue for \( x < 0 \) in addition to the long-wavelength electromagnetic wave a short-wavelength plasma wave propagates away from the resonance. This plasma mode is the result of the partial conversion of the incident electromagnetic mode and it carries off the energy that in the cold-plasma limit is accumulated in the resonance thus leading to the singular behavior. By taking into account thermal corrections in the outer regions, this plasma mode can be continued analytically into these outer regions. As a result it is found that
\[ \phi = \frac{(-1)^m}{2} \int_0^x \sqrt{f(x')} \, dx' \]  
(22)

for \( \gamma = 3 + 4m \) and \( \phi = 0 \) for \( \gamma = -1 - 4m \) where the superscript \( p \) indicates the short-wavelength plasma wave part of \( \phi \). The fact that \( \phi = 0 \) for \( \gamma = -1 - 4m \) does not necessarily mean that there is no mode conversion altogether for these \( \gamma \)'s. There only is no mode conversion up to the considered order; higher order contributions according to (15b) could reveal, however, the generation of a plasma mode.

It is of course also possible to reverse the above-described process and to consider an incident plasma mode (instead of an incident electromagnetic mode) that is partially converted into an electromagnetic mode near the resonance and partially reflected.

**Tunneling Equation \((s = -1, \, \alpha = 0)\):**

The mathematical treatment of this case is identical with that of the Reflecting Equation. With the same methods it is found that the approximate solutions near the resonance at \( x = 0 \) and near the turning point at \( x = x_t \), where in this case \( f(x_t) = \beta \), are identical with (7) and (10) respectively, but, the function \( q \) and the order \( \nu \) of the Hankel function in (7) are now given by
\[ q = 1 - \frac{\beta}{\epsilon}, \quad \nu = |\gamma - 1| \]  
(23)

Due to this different \( q \), the solution \( \phi_0^0 \) according to (7) is now not evanescent for \( x > 0 \) but, since \( q = 0 \) for \( x > 0 \), \( \phi_0^0 \) is now an outwards propagating electromagnetic mode. This electromagnetic mode stems from the tunneling of wave energy through the region between the cutoff and the resonance where the solution \( \phi_0 \) is evanescent. Also result (12) remains unaltered but \( \Lambda \) now becomes
\[ \Lambda = \int_{x_t}^0 \sqrt{1 - \frac{\beta}{f(x')}} \, dx' \gg 1. \]  
(24)

If the absorption coefficient is calculated, the above-mentioned outwards propagating mode has to be taken into account. As a result for the absorption coefficient different from that for the Reflecting Equation it is found that
\[ A = - \left( 1 + 4 \cos(\nu n) \right) e^{-2\Lambda}. \]  
(25)

Since \( \gamma \) and thus \( \nu \) are assumed to be integers, this result is physically only acceptable for \( \gamma \) even. So \( \gamma = 2n \).

The inner expansions are again given by (15a) for \( \gamma = 2m+2 \) and by (15b) for \( \gamma = -2m \). The reduced fourth-order differential equation for \( \phi_0^1 \) is now found to be given by
\[ \frac{d^4 \phi_0^1}{d\zeta^4} + \Lambda \left[ \frac{d^2 \phi_0^1}{d\zeta^2} + \frac{\gamma}{2} \frac{d \phi_0^1}{d\zeta} \right] = 0 \]  
(26)

where \( \gamma = 2n \). The general solutions of this equation are
\[ \phi_0^1 = a_0 + a_1 \frac{d^2 \phi_0^1}{d\zeta^2} + \frac{d \phi_0^1}{d\zeta} \begin{cases} a_1 \left( \phi_0^0 + a_2 \phi_0^2 + a_3 \phi_0^3 \right) \\
2m + 1, \end{cases} \]  
(27a)

for \( \gamma = 2m+2 \) and
\[ \phi_0^1 = a_0 + a_1 \frac{d^2 \phi_0^1}{d\zeta^2} + \int_0^r \left( a_2 \phi_0^2 + a_3 \phi_0^3 \right) \, dt \]  
(27b)
for $\gamma = -2m$. The matching of inner and outer solutions finally yields

$$J_0 = 0,$$

$$a_1 a_2 = \frac{\gamma}{2} \left( \frac{\beta - \frac{1}{2}}{\beta} \right)^{2m} \sum_{n=0}^{\infty} \frac{1}{n!} \left( \frac{1}{\beta} - 2m \frac{1}{\beta} \right)^n C_n \tag{28a}$$

$$a_2 = 0$$

for $\gamma = 2m+2$ and

$$a_0 = i^{m} \left( \frac{\beta}{2} \right)^{2m} \sum_{n=0}^{\infty} \frac{1}{n!} \left( \frac{1}{\beta} - 2m \frac{1}{\beta} \right)^n C_n \tag{28b}$$

for $\gamma = -2m$. It turns out that for the Tunneling Equation the short-wavelength mode that is generated in the resonance region propagates for $x > 0$ and away from the resonance. Once again by taking into account thermal corrections in the outer regions, this plasma mode can be continued analytically into these outer regions. Thus it is found that:

$$\phi^P = (-1)^{2m+1} \left( \frac{2}{\beta} \right)^{1/2} \left( 2m \beta - \frac{1}{2} \right) \sum_{n=0}^{\infty} \frac{1}{n!} \left( \frac{1}{\beta} - 2m \frac{1}{\beta} \right)^n C_n \tag{29}$$

for $\gamma = 2m+2$ and $\phi^P = 0$ for $\gamma = -2m$. As before $\phi^P = 0$ for $\gamma = -2m$ does not necessarily mean that there is no mode conversion for these $\gamma$'s.

CONCLUSIONS

In the preceding part of this paper a method has been presented by which it is possible to tackle rather general mode conversion problems. The method is based on boundary-layer theory [13] and Langer's method [14] and is applicable to lukewarm (weakly relativistic) and weakly nonuniform plasmas. The nonuniformity is arbitrary provided that it is monotonous and weak. Although demonstrated for a plasma, the applicability of the technique is by no means restricted to wave propagation and wave conversion in plasmas but may also be used in other areas e.g. in studying the propagation of internal gravity waves near the so-called Brunt-Väisälä frequency [17].

The advantages of the present approach to that of other investigators [12] are:

- Instead of implicit solutions in integral form that have been obtained in a rather laborious way, boundary-layer theory gives explicit solutions in terms of special functions that are valid in the whole resonance region.

- It is not necessary to linearize the function $f(x)$ on forehand; Langer's method is applicable for an arbitrary $f(x)$ provided that it depends only weakly upon $x$. Also the incorporation of higher order derivatives in the boundary layer is easy and straightforward.

- Higher order corrections according to (15) can be determined straightforwardly. For a Reflecting Equation with $\gamma = -1$ and $\beta = 0$ this has been done in [16].

- If the fourth-order differential equation (1) is determined in a correct manner, then the technique does not suffer from the previously mentioned disadvantages of a local-dispersion relation approach.

Although the present model with its calculations is rather general, the problem can even be generalized further by letting the quantity $s$ that multiplies the fourth-order derivative of (1) and the other parameter $s$ in (1) become $+1$ or $-1$ independently from each other.

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1 Introduction

Though the potential of wave coupling and resonant absorption as a means of plasma heating has been recognized for a long time, in recent years the interest in this field has increased considerably, in particular due to large scale application in fusion work. The wide range of frequencies available allows for a lot of different approaches. The following studies concentrate on the regime of the lower hybrid frequency, which represents a fundamental frequency of magneto-plasmas due to the electrostatic coupling of the positive and negative plasma components. In view of the very basic nature of this coupling, in this regime a variety of possibilities and problems are to be expected. The following experiments are performed at the University of Bochum against the background of fusion applications as well as basic understanding.

At a fundamental resonance such as the lower hybrid resonance large values of the refractive index are to be encountered; even for idealized conditions considerable wave absorption may be expected due to collisions. Features of realistic plasma experiments, however, lead to modifications implying not only complications, but also additional channels of absorption. Four experiments, designed and selected to address some important features of this type, will be discussed in the following. Finally a spectroscopic method of verifying enhanced absorption will be presented.

The finite geometry, length, of plasma and coupling structure precludes wave propagation strictly perpendicular to the direction of the magnetic field: thus $k_y$ is finite. Aside from implying the presence of Landau-damping, this leads to two wave modes, more or less sharing properties opportune for wave heating: the "slow" wave, more susceptible to capacitive, and the "fast" wave, more susceptible to inductive coupling. Capacitive coupling is favoured in the first and fourth experiment below, whereas inductive coupling is addressed in the second and third case.

The added complication of radial density gradient opens up a much favoured channel of ion heating (pointed out by Stix [1] many years ago): mode conversion. Experimental evidence for this process will be presented as well as observations on other aspects: coupling with low frequency fluctuations not only modifies the wave propagation due to scattering, but may lead to additional absorption channels due to enhanced $k$-spectra and decorrelation; channels of electron heating may result such as:
enhanced Landau-damping and - more important - turbulent heating. Experiments show enhanced ion heating, too. Increased wave amplitude may lead to various effects, one of which will be considered here: parametric decay processes, of considerable interest in recent years.

2. Linear arrangement - mode conversion

The first investigation is mainly concerned with mode conversion at the lower hybrid resonance. In Fig. 1 the horizontal axis depicts growing plasma density. Towards the plasma interior the refractive index (vertical axis) is essentially real and grows, as quite well predicted by cold plasma theory, provided $k_g$ is large enough. The arrows visualize the wave energy entering the plasma and turning over into a second branch of refractive index (existing due to the presence of a warm ion component) which cannot easily be supplied with wave energy directly from the outside. The turnover occurs in a region where the wave frequency is close to the lower hybrid frequency. The excitation of the - well damped - ion mode is aimed at in large scale fusion experiments but could not be clearly demonstrated so far.

Fig. 1

Fig. 2
Fig. 2 shows (schematically) the linear set-up used at Bochum [2]. A slow-wave-antenna, thus the necessary \( k_\parallel \) and predominantly capacitive coupling to the desired mode are realized by rings of changing polarity. The wave enters the hydrogen plasma on paths of "resonance cones" depicted towards the conversion regime. By a method of detection eliminating the perturbing influence of density fluctuations it has been possible to pick up the phase informations due to the converted ion wave. Fig. 3 shows the wave length thus obtained in good agreement with the curves calculated from known plasma data. A large fraction of the incoming energy reaches the conversion regime, as can be deduced from the measured damping of the incoming wave. In Fig. 4 (damping versus wave frequency) at the right hand side the damping is somewhat larger than expected from collisions, whereas for low frequency electron Landau-damping takes over. So far the wave energies were small in accordance with linear perturbation theory. For increased wave energies changes of plasma density can be detected (Fig. 5; density versus radius). Below the frequency is chosen, so that no lower hybrid regime is present. The upper picture demonstrates that - aside from additional ionization outside - the central density inside the hybrid layer is increased. Taking into account the observed electron temperature increase also observed - particularly in the hybrid regime - the steepening of the density towards the center can be attributed to a changed diffusion equilibrium; ponderomotive forces appear to play a minor role [3]. Increases of ion temperature could be detected, too (Fig. 6), some of which are attributable to the converted wave.
This experiment demonstrates the functioning of the mode conversion process, but also reveals that its effectiveness is limited by a competing process, by coupling of the incoming wave with low density fluctuations.

3. Mirror system - geometrical resonances

The influence of low frequency fluctuations on wave absorption is also found in a related investigation at a magnetic mirror arrangement with a hydrogen target plasma generated by microwaves [4]. Again the experiments are performed under linear conditions, i.e. with low rf power; inductive coupling near the lower hybrid frequency is used, thus the fast wave is predominantly excited [4]. Fig. 7 depicts radial geometrical resonances of this mode: absorbed power versus frequency. These resonances are due to large values of the refractive index and appear when half a radial wavelength fits the radius about once, twice etc.

The third axis towards the back demonstrates that the resonances are lowered and broadened with increasing level of fluctuations. The observed damping is always considerably above the one expected due to collisions. The absorption channel via fluctuations will be followed up in more detail in a torus experiment yet to be discussed. But prior to that the role of parametric effects shall be commented on.
4. Linear arrangement - parametric decay

With increasing wave amplitude parametric decay processes are expected opening up additional channels of wave absorption. For some time these channels had been held in high esteem, particularly in view of ion heating. In a linear arrangement with heated cathodes and target plasmas selected for very low levels of fluctuations again the fast mode has been excited [5]. Again inductive coupling to the fast mode is used and considerable pain has been taken to generate reproducible quiet neon plasmas. Fig. 8 shows - versus radius - the amplitude of the primary (pump) wave and of the two decay modes, i.e. of a neighbouring lower hybrid mode and an ion cyclotron wave at lower frequency. The typical feature of decay processes "pump depletion" is most pronounced in the central region. Whereas electron density and temperature remain essentially unchanged, the ion temperature is noticeably enhanced, as shown in Fig. 8.

Thus ion heating via parametric decay is observable, even at relatively low rf amplitudes. The detailed studies with correlation analysis show that the thresholds for parametric decay are relatively low; they may even be lowered to zero by the presence of low frequency drift wave modes coupled to ion cyclotron waves and the rf may influence their coupling at the expense of the drift modes [6]. On the other hand the studies show again and again that the saturation
level of decay processes is very low, too. This heating appears to be easily drowned out by other channels of absorption. So far the observations at various experiments suggest: parametric decay is easily observable, but its effectiveness as heating channel is subjected to strong competition by other effects.

5. Stellarator system - low frequency scattering

This finding concerning parametric heating (of ions, but also of electrons) was corroborated by investigations at a stellarator configuration in particular. For the fourth experiment considered here, again coupling to the slow wave is used. The hydrogen target plasmas are generated by electron cyclotron resonance and heated by 150 MHz pulses with powers of the order of 1 kW. The arrangement is shown schematically in Fig. 9. The process of coupling to low frequency fluctuations - twice mentioned before as a competitive channel - certainly dominates the heating in this experiment, whereas parametric activity as well as presence of mode conversion is not noticeably correlated to effective heating. Fig. 10 depicts the frequency spectrum of the rf heating with sidebands corresponding to the low frequency fluctuations (at the left), caused essentially by drift wave turbulence; the positions of parametric signals are indicated schematically. The level of fluctuations is relatively strong at the device considered [7]; this situation is used for correlation analysis the results of which are demonstrated in Fig. 11.

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Fig. 9

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The distribution (the spectral energy density) of $k_r$ in the sidebands of the rf pump wave is shown for the radial component, the dotted curve indicating the original spectrum launched by the coupler. A substantial broadening is to be seen. Further measurements reveal similar spectra for $k_\theta$ (though it vanishes originally) and severe decorrelation of the pump wave, obviously caused by scattering due to the low frequency fluctuations.

The absorption can be analysed by means of an energy balance for the electrons using measured values (in particular containment times), as indicated by the scheme of Fig. 12. Collisional absorption is not sufficient as confirmed by many experiments. The dissipation of the oscillatory energy parallel to the magnetic field is enlarged by enhanced $k_u$-spectra via Landau damping, moreover by enhanced (transient) high energy tails of the electrons. However for the cases considered in the following major contributions from dissipation of the perpendicular energy are required to fit the observations. This can be accounted for by an effective damping frequency connected to a turbulent diffusion coefficient as indicated. Here values are tested which are derived from a simple (random walk) model of a test particle diffusion coefficient based on the observed properties of the scattering low frequency fluctuations (Fig. 13). As indicated schematically the observed spectral broadening $\Delta \omega_k$ of these fluctuations and the displacement $\Delta k$ proportional to the Fourier component of the density fluctuation $\hat{n}(k)$ are used. Fig. 14 depicts the results (for wave frequencies of 50 and 100 MHz) plotted versus the values obtained from the electron energy balance: the circles include collisional terms only, the squares the turbulent terms just outlined; there are of course considerable error bars, but the latter values are close to the matching 1:1 line. Similar results are obtained when the values of specific shots are compared as a function of radius. This turbulent electron heating may account for the observed strong wave damping and the fast rise of electron temperature within a few $\mu$s.
\[
\frac{1}{2} N_e \frac{\partial k_{Te}}{\partial t} = P_e - \text{const.}
\]

measured loss term

wave power density absorbed

dissipation of "parallel" oscillatory energy by collisions
Landau damping \( k_a \) enhanced / high energy tails

\[ P_e = \frac{\omega_{pe}^2}{\omega_{ce}^2} \sum_k E_k^2(k) v_0^k \]

\[ v_0^k = k^2 D_1^k \]

\( D_1^k \) diffusion coeff. for turbulent phase perturbations due to low frequ. fluct.)

Fig. 12

\[
\frac{\gamma}{\text{d}r} \text{d}w dr
\]

fluctuation energy

Fig. 13

Fig. 14

Fig. 15
Though a similarly fast rise of the ion temperature is observed, the specific channel of absorption is not completely pinned down by the experiments so far. For the stellarator experiments considered mode conversion and parametric decay may contribute, but can certainly be ruled out as dominant mechanisms. Fig. 15 demonstrates that the rise of ion temperature with power is closely correlated to that of the energy of the low frequency fluctuations, suggesting turbulent heating either in a direct process or via electron component by fast transfer.

6. Spectroscopic diagnostics of enhanced absorption

In the previous four experiments some mechanisms of enhanced wave damping have been considered, and in particular the potential of turbulent heating has been stressed, for instance in the presence of strong low frequency turbulence. A relatively simple method shall be presented to corroborate enhanced damping by observing enhanced emission of atomic lines. The range of applicability is essentially that of a corona model for the excitation of an atomic emission line as indicated in Fig. 16. The energy balance of Fig. 16 contains the various excitation rates and the ionization rate all which may be assumed to be affected by the wave fields in quite similar manners. Elastic losses are usually negligible and diffusion losses on a different time scale (or excluded by a modulation technique [8]). Thus the square of the electric wave field times the effective damping frequency are proportional to the excitation rate and consequently to the observed line intensity. This method has been tested in various cases of strong rf fields by comparing the results obtained from the enhancement of the line intensities with those from yet another independent spectroscopic method: line broadening and shifts due to rf Stark effect. Fig. 17 depicts the measured enhanced line intensities versus $E^2$ determined by Stark effect; the solid line indicates the expected agreement (hydrogen plasmas generated and heated by microwaves). Thus the method of enhanced line intensity may be used in many cases (even for relatively low field strengths) to determine the actual damping. For Fig. 18 strong inductive rf coupling is investigated at an arrangement somewhat similar to that used for low amplitude studies in part 3. Fig. 18 depicts the absorbed power density as a function of radius (from end on measurements in helium). The crosses are determined by this method.

**Corona Model:**

\[
0 = \sum_{n=1}^{\infty} N_n N_m \langle \sigma_n \nu \rangle - \sum_{m=1}^{\infty} N_m A_{mn}
\]

\[
\sim I_{\text{line}} \sim N_r A_{\text{line}} \sim N_e \nu_r
\]

**Energy Balance:**

\[
N_e \nu_r X_1 + N_e \sum_j V_j X_j + \frac{3}{2} M \nu_{\text{elast}} k T_e + \frac{3}{2} N_e k T_e \frac{B^2}{\lambda^2}
\]

\[
- E_{\text{wave}}^2 \nu_{\text{eff}}
\]

\[
- N_e \nu_r
\]
The smooth curve is obtained from calculation when the effective damping frequency is 200 times the collision frequency. It should be stressed that changing this value by a factor of 2 already results in an appreciable mismatch. Moreover it should be stressed that enhancements of the damping frequency of this order of magnitude are found again and again by experiment.

7. Summary

Among the various possible channels of enhanced wave absorption a few important ones have been investigated: mode conversion, parametric decay, but obviously most important turbulent damping. Particularly in case of strong fluctuations low frequency turbulence appears to be a good candidate, supported by observations presented here. Finally among remaining candidates, turbulence originating from process of higher frequencies (such as two-stream instabilities) should be stressed; particularly in large amplitude wave experiments it deserves further investigations.

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References

An exact knowledge of the phenomena in the electrode regions of high-pressure discharges is of major interest for welding arcs, circuit breakers and arc lamps. Especially in low-power lamps the hot spots in front of the electrodes contribute considerably to the electrical and radiative properties of the arc. For such arcs a method has been developed to describe the properties of the arc plasma from the centre to the electrodes in a closed model.

With this model it is possible to determine the integral and local data of the discharge and to study their dependence on the properties of the electrodes (geometry, heat resistance, emissivity) and of the filling gas (transport properties, operating pressure). The model gives a quantitative explanation of the transition from diffuse to constricted electrode spots. The numerical results are compared with measured data from high-pressure mercury lamps.

1. Introduction

Most of the scientific work that has been done in the field of arc discharges is concerned either with the arc column, treating the influence of the electrode regions as negligible or as a small disturbance, or with surface layers at the electrodes, using the plasma state in the column only as a boundary condition at infinity. There are, however, some arc types of wide practical interest in which such a separation of arc column and electrodes is not adequate for an accurate model. In many circuit breakers, welding arcs and lamp plasmas the hot spot region in front of the electrodes is the most significant part of the arc, determining the performance of the device.

Thus a satisfactory theoretical description of such arcs should cover the whole arc volume from the centre to the electrode surfaces including the properties of the gas fill as well as those of the electrodes.

In this paper a model is developed which allows an accurate description of the plasma state in high pressure arcs including the processes in the electrode fall regions. With a suitable choice of the coordinate system the basic equations are solved for a typical arc geometry for low-wattage high-pressure (HP) discharge lamps.

2. Properties of HP discharge lamps

Typical high-pressure discharge lamps consist of a quartz vessel with tungsten electrodes sealed in at the ends of the vessel, which in high power lamps mostly consists of a cylindrical part and spherical end constructions, while for low wattage lamps elliptical forms are preferred. Typical dimensions of a 30 W lamp are of the order of 5 mm. Fig. 1 shows such a quartz vessel designed for 30 to 35 W arc power. The filling consists mainly of mercury at an operation pressure of up to 40 bar with the addi-
tion of argon as a starting gas and in some cases of various metal halide additives to improve the radiation output. Such lamp plasmas are very complex with respect to their chemical composition, but on the other hand due to the high mercury pressure the transport processes are comparatively simple. All diffusion processes are dominated by collisions with mercury atoms. Energy transport is mediated preferably by transfer between mercury atoms at lower temperatures and by optically thick resonant radiation in mercury lines at high temperatures. Due to the large electron-atom collision cross-section in mercury vapour the mobility of the electrons is also determined by the density of the mercury atoms. The estimated electron mean free path for a mercury pressure of 30 bar is of the order of $10^{-7}$ cm, comparable to the magnitude of the atomic distance.

There are two major consequences of this fact: Even at a field strength of $10^7$ V/cm, which is the maximum to be expected in front of the cathode if pure field emission is responsible for the arc current, the energy gain of the electrons from the electric field between two collisions is so low that a thermal energy distribution can be assumed for the electron gas all over the arc volume. Thus secondary emission processes from the cathode [1, 2] induced by high-energy electrons or ions can be neglected in the high-pressure mercury arc. The second point is that the method commonly used to calculate the space charge distribution in the electrode boundary layer cannot be applied here. Most theoretical investigations assume that, by analogy with low-pressure discharges, there exists a collision-free zone in which the electrons take up energy from the electric field, which is created by the electrons and ions themselves (see e.g. [3]). The space charge distribution in this zone, the thickness of which is roughly equivalent to one electron mean free path, results from the field equations assuming that the field strength is negligibly small outside the zone. The use of such a model is obviously ruled out, however, if the mean free path is comparable to the atomic distance. In this case the space charge distribution must be determined from the diffusion equations.

3. Simplified Arc Model

3.1. Energy balance

As a first step in the evaluation of a quantitative model of low-power arc lamps we will consider a simplified description of the arc plasma that allows the solution of the basic equations for the given geometry of the lamp with acceptable numerical effort. For a plasma in local thermal equilibrium (LTE) the energy balance of a stationary arc has the form
\[ \text{div grad } S + \sigma (\text{grad } \phi)^2 - U = 0, \quad (1) \]

where \( S \) is the heat flux potential, defined as
\[
S = \int_{T_0}^{T} \lambda \, dT
\]
(\( \lambda \) is the thermal conductivity). For the reference temperature \( T_0 \) it is convenient to choose the mean wall temperature so that \( S \) is approximately zero at the boundaries. \( \sigma \) is the electrical conductivity, \( \phi \) the electric potential.

The radiation term is very complicated in most practical lamps as it contains optically thin radiation in spectral lines, molecular bands and continua as well as radiation transport in optically thick resonant lines. A lot of numerical work has been done to describe the radiation processes in detail (see e.g. \([4-6]\)), but for our purpose we can confine ourselves to a very simple approximation. As has been shown by several authors \([4,5]\), optically thick radiation dominates the energy flux in lamp plasmas containing large amounts of mercury vapour, so that radiation transport acts like an additional thermal conduction. So the radiation term can be completely omitted in the energy balance if the thermal conductivity is replaced by the sum of the classical conductivity and this radiative conductivity.

In the energy balance convective transport has been neglected. This is obviously a serious restriction of the model. In practical arc lamps the asymmetric shape of the arc due to convection can always be recognized, but the inclusion of this effect in the model would strongly increase the numerical effort without giving any better insight into the phenomena in the electrode regions.

Thus the model is restricted to the integration of the energy balance in the form of eq. (1), which has to be solved simultaneously with the basic equation for the electric field distribution given by
\[
\text{div } \mathbf{J} = -\text{div } \sigma \cdot \text{grad } \phi = 0.
\]

3.2 Transport properties

When one speaks of an LTE plasma one normally means that the properties of the plasma depend only on the local temperature (if pressure is assumed to be constant). This is not true, however, if the plasma contains a mixture of gases. Especially if there are molecular species together with atomic gases, strong segregation effects may occur \([7-9]\) caused by the simultaneous action of convection, diffusion and electrical forces. Inclusion of these processes in a model would require an exact solution of the equations of motion for all particle species before the local properties of the plasma can be computed. In our model we have not included these segregation effects. Thus the numerical results should be reliable only for pure gases (Hg lamps). Where calculations for gas mixtures are reported, it should be borne in mind that the model consistently assumes a constant stoichiometry of the plasma. The great advantage of this simplification is that the equilibrium transport coefficients depend only on the local temperature and can be determined for the given lamp filling, before the local distribution of any plasma parameters is calculated. To determine the temperature dependence of the plasma composition we used a software package developed by Schnedler \([10]\). This package gives the partial pressures \( p_j \) of all neutral and charged components and, if one species is declared as the dominant one (in our case atomic mercury), the binary
diffusion coefficients $D_i$ in the hard core approximation.

From the partial pressure of the electrons the electrical conductivity can easily be calculated. For a weakly ionized high-pressure mercury plasma the mobility of the electrons is determined by collisions with mercury atoms. Using the momentum transfer cross-sections given by Rockwood [11] the conductivity can be expressed by

$$\delta = \frac{e^2 n D_{eo} p_e}{kT} = 2.97 \times 10^{10} T^{-3/2} \frac{p_e}{p} \text{C} \text{m}^{-1} \text{kg}^{-1}$$

(3)

The thermal conductivity consists of three fractions, one of which is from the transport of kinetic energy, the others are due to radiation transport and transfer of reaction energy. In the low-temperature range the classical process of kinetic energy transfer dominates. Here the conductivity can be approximated by a square root law. The more detailed calculations done by Stormberg [12] for pure mercury can be fitted by

$$\lambda_C/[\text{Wm}^{-1}\text{K}^{-1}] = 7.78 \times 10^{-4} \sqrt{T/K}$$

(4)

in the temperature range up to 3000 K. For higher temperatures radiation transport becomes the dominant transfer mechanism. To determine this fraction of the conductivity we calculated the local net emission in the arc center for a number of test profiles by a method developed by Stormberg [12]. From the resulting net energy loss $Q_0$ in the arc center we then calculated the radiative conductivity from the formula

$$\lambda_R \left( \frac{n^2 T}{dr^2} \right) = Q_0$$

In the pressure range from 5 to 30 bar the function $R(T)$ can be approximated by

$$\lambda_R/[\text{Wm}^{-1}\text{K}^{-1}] = 1.22 \times 10^{-10} \frac{p}{\text{bar}} \cdot \exp \left\{ \frac{\Pi(T)}{T} \right\}$$

(5)

Here $\Pi(T)$ represents the polynomial

$$\Pi = 0.0189 x^4 - 0.40356 x^3 + 2.859 x^2 - 5.303 x$$

with $x = T/1000$ K.

The third fraction of the conductivity is transport of reaction energy. In a pure mercury plasma only transport of ionization energy has to be considered, whereas in lamps with metal halide admixtures dissociation must be taken into account as well. If neutral mercury is the dominant species in the plasma, the flux of reaction energy is given by

$$\mathbf{q}_r = \sum_i n_i \mathbf{J}_i$$

where $n_i$ is the internal energy of some component $i$ relative to the reference states of the elements, and $\mathbf{J}_i$ are the corresponding flux densities, which are coupled to the partial pressure gradients by the diffusion equations:

$$\mathbf{J}_i = - \frac{n_i D_i}{p} \cdot \nabla p_i.$$ 

In an equilibrium plasma the partial pressures depend only on the plasma temperature, so that the energy flux can be written as

$$q_r = - \sum \frac{u_i D_i}{kT} \cdot \nabla p_i$$

$$= - \sum \frac{u_i D_i}{kT} \frac{dr}{dT} \cdot \nabla T$$

$$= - \lambda_p \cdot \nabla T$$

(6)

In the case of charged particles the $D_i$ must of course be replaced by the ambipolar diffusion coefficients.

The three quantities $\lambda_C$, $\lambda_R$, and $\lambda_D$ add up to the total thermal conductivity, from which the heat flux potential can be determined:

$$S = \int_{T_0}^{T} \left( \lambda_C + \lambda_R + \lambda_D \right) dT$$

(7)
The reference temperature $T_0$ has been set to 1100 K in the calculations, which is a reasonable value for the mean wall temperature of low-power arc lamps.

3.3 Lamp geometry

In most of the high-pressure arc lamps that are produced now or are being developed the ends of the arc vessel have a spherical or elliptical form. In small lamps the shape of the whole lamp is elliptical. Thus it is convenient to restrict the model calculations to such elliptical lamps. This has the advantage that the boundary conditions become very simple if we choose an appropriate system of elliptical coordinates $(\xi, \eta)$ which are connected to cylindrical coordinates by

$$r = \ell \cdot \sqrt{(\xi^2 - 1) \cdot (1 - \eta^2)}$$
$$z = \ell \cdot \xi \cdot \eta$$

Fig. 2 shows a typical grid of such coordinate lines. The boundary conditions for the heat flux potential in these coordinates are

$$S = 0 \quad \text{at} \quad \xi = \xi_0$$
$$\frac{\partial S}{\partial \xi} = 0 \quad \text{at} \quad \xi = 1$$

If in addition one assumes that the electrodes have the shape of hyperboloids given by $\eta = \eta_0$, the boundary conditions for the electric potential are

$$\phi = \pm U_0/2 \quad \text{at} \quad \eta = \eta_0$$

where $U_0$ is the arc voltage. Of course nobody will build lamps with such hyperboloid-shaped electrodes, but for the calculation of the arc properties this shape represents a good approximation as long as the radius of curvature at the electrode tip agrees with that of the real electrode, because the energy flux from the plasma to the electrode and also the electrical current is concentrated at the tip area. The energy balance for the electrode itself must be solved of course for the exact real shape. In the energy balance of the plasma the most important feature of the electrodes - besides that of being the source of the electric current - is to represent an efficient heat sink, as their temperature is very low - low at least compared to the temperatures in the plasma. Thus for a first approximation we can use the additional boundary condition

$$S = 0 \quad \text{at} \quad \eta = \pm \eta_0$$

in the numerical determination of the plasma temperature distribution.
3.4 Numerical solution

To solve the coupled system of differential equations formed by the energy balance (1) and the electric field equation (2) for the elliptical arc problem it is convenient to use eq. (2) in integral form. Integration over an area $\Omega = \text{const}$ yields the total arc current

$$I = \int \nabla \cdot \mathbf{E} = 2\pi \int_0^\infty \sigma (1 - \eta^2) \frac{d\phi}{d\eta} d\xi \quad (8)$$

The electrostatic potential for the geometry discussed here would only depend on the coordinate $\eta$. In a current-carrying plasma there will be a dependence on $\xi$, too, that vanishes exactly only near the electrode surfaces and in the arc centre, but the derivatives with respect to $\xi$ will be small compared to those with respect to $\eta$ as long as there are no abrupt changes of $\sigma$ in the direction of $\xi$ as would be the case at the boundary of a copper wire, for example). Thus in eq. (8) the quantity $\phi/\eta$ can be placed in front of the integral.

Neglecting $\phi/\eta$ in eq. (1) and substituting $S/\eta$ according to eq. (8), the energy balance takes the form

$$\frac{\partial}{\partial \xi} (\xi^2 \frac{dS}{d\xi}) + \frac{\partial}{\partial \eta} (1 - \eta^2) \frac{dS}{d\eta} = -\frac{1}{\pi \xi^2} \frac{1}{1 - \eta^2} \left( \frac{\eta}{\int^\infty_{\eta_0} \xi \, d\xi} \right)^2 \quad (9)$$

This equation can be solved by an iteration process for a given arc current $I$ and an arc geometry described by the values of $I$, $\xi_0$ and $\eta_0$. To reduce the numerical effort it is convenient to use series expansions of the heat flux potential $S$ and of the right-hand side of eq. (9):

$$S = \sum q_v(\eta) \cdot f_v(\xi)$$

$$H(\xi, \eta) = \left( \frac{1}{2\pi \xi} \right)^2 \frac{\eta}{\int^\infty_{\eta_0} \xi \, d\xi} \left( \frac{\eta}{\int^\infty_{\eta_0} \xi \, d\xi} \right)^2$$

where $f_v(\xi)$ are the solutions of the differential equation

$$\frac{d}{d\xi} (\xi^2 \frac{df_v}{d\xi}) + \frac{d}{d\eta} (1 - \eta^2) \frac{df_v}{d\eta} = \lambda_v f_v \quad (11)$$

with the boundary conditions

$$f_v(\xi_0) = 1, \quad f'_v(\xi_0) = 0 \quad \text{at} \quad \xi_0 = 1$$

and

$$f_v(1) = 0 \quad \text{at} \quad \xi = \xi_0$$

These conditions define a set of radial eigenfunctions $f_v$ and the corresponding eigenvalues $\lambda_v$.

With this set of eigenvalues a second set of axial eigenfunctions $\gamma(\eta)$ can be defined as the solutions of the homogeneous equations

$$\lambda_v \gamma + \frac{d}{d\eta} (1 - \eta^2) \frac{d\gamma}{d\eta} = 0 \quad (12)$$

with the boundary conditions

$$\gamma(\eta_0) = 1, \quad \gamma'(\eta_0) = 0 \quad \text{at} \quad \eta = \eta_0$$

The eigenfunctions $f_v$ and $\gamma_v$ depend only on the value of $\xi_0$ and thus on the geometry of the arc. Thus the solutions can be calculated in advance before the iterative procedure for the solution of eq. (9) is started. The nonlinear part of the problem is reduced to a simple integration. For the expansion coefficients $g_v$ we find

$$g_v(\eta) = \gamma_v(\eta) \left( \frac{\gamma_v(\eta_0)}{\gamma_v(\eta_0) \gamma(\eta) \, d\eta} \right)$$

$$- \frac{1}{\gamma(\eta_0)^2} \int_{\eta_0}^\eta \gamma_v(\eta) \, d\eta \, d\gamma$$

The integration constants $g_v(\eta_0)$ are equal to zero in the simplified model ($S(\eta_0) = 0$). For a more detailed determination they must be calculated from the electrode surface temperature. The individual steps of the numerical procedure are shown schematically in fig. 3.

First, for a given geometry (diameter of the arc vessel $d$, electrode separation $a$ and radius of curvature of the electrode tip $r_0$),
Given: arc geometry, \( I \)

- Calculation of radial eigenfunctions \( f_i(\xi) \) and eigenvalues \( \lambda_i \)
- Calculation of axial eigenfunctions \( y_i(\eta) \)
- Transport properties, \( \sigma(\eta), \alpha \)
- Series expansion: \( \sigma = \sum \lambda_i f_i(\xi) \)
- Calculation of the heat flux potential \( S(\xi, \eta) \)
- Calculation of the expansion coefficients \( y_i(\eta) \)
- Given: arc current \( I \)
- Arc voltage \( V \)
- Heat flux to wall \( \text{heat flux} \)
- Temperature distribution \( T(\xi) \)

Fig. 3. Scheme of numerical solution for the LTE model

For a given arc current \( I \), we determine the parameters

\[
\gamma_0 = \sqrt{\frac{\lambda}{4 + \frac{2}{\alpha}}} \quad i = \frac{\alpha}{2 \gamma_0} \quad \xi_0 = \sqrt{\frac{1 + \alpha}{2 \xi}}
\]  

(14)

With this value of \( \xi_0 \), the radial eigenvalues \( \lambda_i \) and the eigenfunctions \( f_i(\xi) \) are calculated and tabulated for a fixed set of grid points \( \xi_0 \leq \xi_i \leq \xi_0 \). Then the corresponding axial functions \( y_i(\eta) \) are tabulated for \( 0 \leq \eta_i \leq \eta_0 \). The spacing of the grid points should be taken very narrow near \( \eta = \eta_0 \), as here the functions become very steep if \( \eta_0 \) is close to 1.

The second preliminary step is the determination of the transport properties of the plasma (see section 3.2) and tabulating the functions \( \sigma(\Sigma) \) and \( T(\Sigma) \). Next, starting values for the heat flux potential \( S(\xi, \eta) \) are estimated, for which then the conductivity profile \( \sigma(\Sigma) \) is determined from the tables. From the \( \sigma \)-field the function \( H(\xi, \eta) \) is given by eq. (10) and then the expansion coefficients \( h_\nu(\eta) \) can be calculated for a given arc current \( I \). As the functions \( f_\nu(\xi) \) are orthogonal in the interval \( \xi = 1 \) to \( \xi = \xi_0 \), the expansion coefficients are defined by

\[
h_\nu(\eta) = \int_{\xi_0}^{\xi} H(\xi, \eta) f_\nu(\xi) d\xi / \int_{\xi_0}^{\xi} f_\nu^2(\xi) d\xi
\]

(15)

With this set of functions new expansion coefficients \( g_\nu(\eta) \) are determined, which then yield an improved profile of the heat flux potential \( S(\xi, \eta) \). The iteration procedure converges to an accuracy of better than 1% within less than ten steps for all numerical examples that have been computed. A typical result of such a calculation is shown in fig. 4. This plot of the S-field shows the typical arc shape observed in real discharge lamps. Fig. 5 shows the temperature profile of such a lamp determined spectroscopically from the intensity of an optically thin mercury line. The arc constriction and the formation of hot plasma spots immediately in front of the electrode surface clearly show up in the model calculations. Thus we can conclude that it is predominantly the cooling of the plasma by the electrodes which causes the arc to form a hot plasma spot near the surface to compensate for the additional energy loss. Variation of the electrode geometry (of the parameter \( \gamma_0 \)) shows that the enhancement of the field strength in front of a narrow electrode tip is favorable for the formation of small spots, but constriction and spot formation also occur with nearly flat electrodes (fig. 6).

The calculated data yield not only the temperature distribution, but also the arc voltage and the heat flux to the wall and to the electrodes. But while the plasma temperature and the heat flux to the walls agree well with experimental data, the heat flux to the electrodes and still more the arc
Voltage show very large discrepancies. This is caused to some extent by the assumption of $S = 0$ on the electrode surface. This assumption means that the electrical conductivity is practically zero at the surface. Thus the electric field had to be infinite to carry the arc current through this non-conductive layer. Though this singularity is only of order 1/2 so that the integral

$$U_0 = 2 \cdot \int_0^\gamma \langle \text{grad} \phi \rangle \; d\gamma$$

remains finite, the resulting arc voltage exceeds the measured values considerably.

Assuming a more realistic temperature on the electrode surface ($T_0 \neq 0$) reduces the discrepancy slightly, but does not bring the results close to the experimental data. This can be reached only by means of a more detailed model that takes into account non-equilibrium effects at the electrode boundaries.

4. Non-equilibrium model

4.1 Enhancement of conductivity

The main reason for the failure of the equilibrium model presented in the last section is that the electrical conductivity values in the boundary layer at the electrodes is too low. The results show, however, that the temperature gradients in this boundary layer are extremely high (up to $10^9$ K/m in the example of fig. 6). Thus we must expect strong diffusion of charged particles from the plasma to the electrodes, which leads to an enhancement of the electrical conduc-
tivity in this zone. To determine the electron density in the boundary layer we must solve the diffusion equations for the electrons and ions under the influence of the electrical forces caused by the particles themselves and by the outer electric field.

For a weakly ionized plasma in which diffusion is dominated by collisions with neutrals (in our case Hg atoms), the diffusion equations have the form [13]

\[
\frac{1}{n^2 D_{i0}} (n_e \vec{j}_e - n_i \vec{j}_i) = \text{grad} \left( \frac{n_e e \vec{E}}{n^2} \right) \tag{17a}
\]

\[
\frac{1}{n^2 D_{e0}} (n_i \vec{j}_i - n_e \vec{j}_e) = \text{grad} \left( \frac{n_i e \vec{E}}{n^2} \right) \tag{17b}
\]

These equations relate the particle densities to the corresponding fluxes, which are connected to each other by the conservation laws for mass and electric charge

\[\vec{j}_0 = -\vec{j}_i\] \hspace{1cm} (18)

and \(\text{div} \vec{J} = e \cdot \text{div} (\vec{j}_e - \vec{j}_i) = 0\), \hspace{1cm} (19)

where \(\vec{J}\) is the net electric current density.

In the continuity equations for the individual particle species it is necessary of course to consider production and annihilation:

\[\text{div} \vec{j}_i = R^+ - R^- \] \hspace{1cm} (20)

Production of ions and electrons takes place predominantly by collisional ionization with a production rate given by

\[R^+ = q^I \cdot n_e \cdot n_0\] \hspace{1cm} (21)

Loss of charged particles is mainly by radiative recombination with a neutral atom being the third particle:

\[R^- = q^R \cdot n_e n_i + q^S \cdot n_e n_i n_0\] \hspace{1cm} (22)

\(q^I, q^R\) and \(q^S\) are the rate coefficients corresponding to the three processes. For the practical calculations we can make use of the fact that in equilibrium the rates \(R^+\) and \(R^-\) must cancel each other. This leads to the relation between the rate coefficients:

\[q^R q^S n_0 = q^I \cdot \frac{n_i}{n_{i0}}\] \hspace{1cm} (23)

where \(n_{i0}\) is the ion density at equilibrium. Thus we get for the net rate of ion production:

\[\text{div} j_i = q^I n_e n_0 (1 - \frac{n_i}{n_{i0}})\] \hspace{1cm} (24)

The rate coefficient \(q^I\) has been determined by integrating the ionization cross-section given by Brown [14] for a Maxwellian energy distribution with a mean energy corresponding to \(T = 4000\) K. This value has been incorporated in the numerical procedure as a constant. The quantities \(nD_i\), which are proportional to \(\sqrt{n}\), have also been set constant to their values at 4000 K. Considering in addition that \(D_{i0} \gg D_{i0}\) and that the ratio \(n_e/n\) is proportional to the conductivity \(\sigma\), eqs. (17) to (24) can be combined to give the relation

\[\Delta \sigma = -\frac{q^R n_i}{2 D_{i0} \alpha} \left(1 - \alpha \frac{\sigma}{\sigma_0}\right)\] \hspace{1cm} (25)

where \(\sigma_0\) is the equilibrium value of the conductivity as used in the simplified model, and \(\alpha\) is the ratio \(n_i/n_e\).

The space charge distribution can be calculated from the electrical field equations
\[
\text{div} \mathbf{\varepsilon} \text{ grad} \phi = e(n_e - n_i) \quad (26)
\]
and
\[
\text{div} \mathbf{\sigma} \text{ grad} \phi = 0 . \quad (27)
\]

Knowing that space charge plays a role only in the electrode boundary layers, where the plasma parameters vary much less in the direction of the surface than perpendicular to it, we can integrate eqs. (25) and (27).

For the space charge distribution we find:

\[
n_e - n_i = - \frac{e}{e_l} \cdot \frac{4}{\xi - \gamma^2} \cdot \frac{1}{\sigma} \cdot \frac{d \mathbf{b_0}}{d \gamma} \cdot C(\xi) \quad (28)
\]

The function \(C(\xi)\) results from the integration. It can be determined from the fact that integrating the electrical field along any line \(\xi = \text{const}\) must yield the arc voltage \(U_0\):

\[
C(\xi) = \frac{\pm U_0}{\int_0^{\xi_o} \frac{dy}{\sigma(\xi^2 - \gamma^2)(\xi - \gamma^2)}} \quad (29)
\]

4.2 Boundary conditions

In the model developed up to now there is no point where we must distinguish between cathode and anode. This difference comes in only in the boundary conditions of eq. (25). Using the approximate formula (3) for the electrical conductivity and the simplifying assumptions discussed above, the diffusion equation (17a) can be written as

\[
\text{grad} \sigma = \frac{e^2}{2kT} \left( E \cdot \mathbf{E} - \frac{\sigma \mathbf{a_0}}{\alpha \mathbf{D_0}} \cdot \mathbf{j}_i \right). \quad (30)
\]

On the anode surface the ion current is practically zero, so that eq. (30) leads to the boundary condition

\[
\frac{\partial \sigma}{\partial \gamma} = - \frac{e^2}{2kT} \frac{\partial \phi}{\partial \gamma} \text{ at } \gamma = \gamma_o. \quad (31)
\]

On the cathode the ions may contribute considerably to the arc current. Here however the contribution of the electrons is limited by the maximum emissivity of the surface. Thus we get the boundary condition

\[
\frac{\partial \sigma}{\partial \gamma} = - \frac{e}{2kT} \left[ \frac{1}{\alpha \mathbf{D_0}} \cdot \mathbf{j}_{\text{em}} \right]
\]

where \(\partial \phi / \partial \gamma\) is the maximum emission current.

4.3 Electron emission

In the case of a dense mercury plasma the emissivity of the surface can be easily estimated. As has been mentioned above, the mean free path in a high-pressure mercury plasma is so small that even at a field strength of \(10^9 \text{ V/m}\) the energy gain of electrons between two collisions is of the order of 1 eV so that all secondary emission processes from high-energy particles can be neglected. Thus the well-known equations for thermionic emission, field-enhanced thermal emission and possibly field emission can be applied. For completeness these equations are given in the form derived by Dyke and Dolan [15]:

\[
j_{\text{em}} = \frac{4 \pi m_e kT}{h^2} \int_{-\infty}^{\infty} \text{D}(E, \xi, E_a) d\xi \quad (33)
\]

with

\[
\text{D}(E, \xi, E_a) = \exp \left\{ - 6.83 \cdot 10^7 (E_a - E)^{3/2} \cdot f(y)/E \right\}
\]

and

\[
y = 3.79 \cdot 10^{-4} \cdot \sqrt{E} (E_a - E)
\]

where \(E\) is the field strength in V/cm, and \(E_a\) is the work function in eV. The function \(f(y)\) is of the order one. The exact values are tabulated in [15].
4.4 Energy balance of the electrodes

The determination of the emission current requires exact knowledge of the electric field strength at the surface and of the surface temperature. While the electric field results from the solution of the basic set of equations discussed above, the surface temperature must be determined in a separate step by solving the energy balance of the electrode. In practical lamps various types of electrodes are used: rods, hollow coils, coils with a central rod or rods with spheres fused into the top. For the model calculations we have chosen the form shown in fig. 7. The model electrode consists of a cylindrical rod with a sphere on top and a second half sphere of different radius on the top of the first one. This construction has the advantage that the relevant parameters of most real electrodes can be simulated by a suitable choice of the radii of the spheres and of the dimensions of the rod. The radius of the upper sphere determines the electric potential surface. Its curvature is fitted to the hyperbolic shape used in the arc model. The radius of the second sphere determines the radiating surface of the electrode; the heat resistance can be fixed by the dimensions of the rod. The energy input from the plasma is concentrated on the electrode tip, in our model electrode to the surface of the upper half sphere. The heating and cooling mechanisms that must be considered are heat conduction and radiation from the plasma, heating of the anode and cooling of the cathode by the electron current, heating of the cathode by the ion current and the radiative loss of energy from the electrode surface:

\[
q(\vec{v}) = -(\nabla S)_n - c_0 \rho \nabla T + (E_{\text{eff}} \cdot j_e + (E_i - E_{\text{eff}}) \cdot j_i)
\]  

(34)

where \(j_e\) has to be taken negative at the cathode, \(E_{\text{eff}}\) is the effective work function, which can be determined from eq. (33), and \(E_i\) is the ionization energy (or its weighted mean value in a gas mixture). Radiation is the only relevant energy loss mechanism on the surface of the lower sphere and of the rod.

With these boundary conditions and assuming a fixed temperature of \(T_0 = 1400\) K at the electrode base point we solved the heat flux equation for the interior of the electrode numerically, using a one-dimensional approximation for the rod and series expansion into spherical harmonics for the spherical parts.

4.5 Numerical procedure

The numerical procedure for the extended model is basically the same as that described in the previous chapter, but a few addi-
5. Results

The arc model derived in the foregoing chapter allows the description of high pressure discharges, where the plasma is collision-dominated in the electrode boundary layer as well. For such arcs it delivers not only the temperature distribution in the plasma, but also the voltage-current characteristics, the energy flux to the wall, the energy loss to the electrodes and the temperature and electric field distribution on the electrode surfaces. The model contains no adjustable constants, but is based solely on fundamental data of the gas contained in the arc. Thus the results can only be as good as the input data, which are not all known with high accuracy. An exact coincidence of the results with measured data cannot be expected, but on the other hand the relatively good agreement in the data is a strong indication, that at least the most relevant physical effects have been taken into account in the right way.

As an example of the good agreement between model calculations and measured data fig. 9 shows some current-voltage-characteristics of mercury arcs. The various curves in the graph correspond to different curvatures of the electrode tips. The experimental curve belongs to a lamp with electrodes etched from a tungsten bar to a tip diameter of about 350 μm. The operating pressure of 30 bar was not measured but estimated from the lamp filling and from the calculated temperature profile. As the temperature distribution behind the electrodes is not accurately known - it is strongly affected by convection, which has been neglected in the model - an error of ±10% must be allowed in the determination of the pressure. The second set of curves in fig. 9 gives the temperature of the cathode calculated for...
a model electrode according to fig. 7. The results show that although the arc voltage - and also the cathode fall - are higher for smaller tip diameters, the heat loss to the electrodes and thus also the tip temperature are lower. This result shows that conduction and radiation from the plasma are the most relevant heating mechanisms in the high-pressure mercury arc, whereas ion bombardment, which predominates in low pressure discharges, is less important. The measured curve shown in the graph also belongs to the lamp mentioned above. The agreement with the numerical results is quite satisfactory having regard to the fact that for the absolute value of the electrode tip temperature an error margin of at least ± 200 K must be admitted to allow for disturbances of the pyrometric measurements by stray light from the plasma and for uncertainty of the surface emissivity.

The discussion in the last section serves only to illustrate the reliability of the data obtained from the model. Results for special arc parameters will not be discussed in more detail here. Instead another form of presentation of numerical results will be given below that allows somewhat more insight into the phenomena occurring upon variation of the arc parameters. For that purpose the scheme of solution of the basic equations has been changed slightly (see fig. 10). Instead of varying all the quantities in the iteration process, the field strength at the electrode surface has been regarded as an independent parameter in the determination of the electron current and in the solution of the diffusion equations. The field strength value at the surface resulting from the field equation can then be plotted as a function of the input value.

Such a graph is shown in fig. 11. The true solution is then given by the point where the curve crosses the diagonal. Fig. 11 shows some curves for a variation of the cathode tip diameter. All the curves exhibit
a wide range in which the slope is very close to one. Varying the tip diameter leads to a shift of this nearly parallel part from above to below the diagonal. Thus the position of the crossing point may change substantially with small variations of the tip diameter. The field strength at the electrode changes by three orders of magnitude in this transition from values of the order of up to \(10^9\) V/m, which is close to the pure field emission regime, to field strengths of some \(10^5\) V/m, where only thermionic emission and the Schottky effect must be considered. In this burning mode the ion current makes up a reasonable fraction of the total current, while in the high-field mode the electron current is dominant.

Similar abrupt changes in the burning conditions also occur upon variation of other arc parameters. Fig. 12 shows solutions for a diameter variation of the electrode rod, which means variation of the heat resistance of the cathode. In Fig. 13 curves for various arc currents are plotted. Here again the crossing point changes from the high-field mode to the low-field mode with increasing current. Fig. 13 also contains curves for the anode. The curves have a somewhat similar shape except that they do not exhibit the sharp kink in the low-field region that occurs in the cathode curves due to the influence of the ion current. Though in principle strong variations in field strength may also be expected at the anode, in the parameter range covered by our experimental lamps the solutions were always of the high-field mode.

A qualitative demonstration of a transition between the two burning modes of a cathode spot is given in Fig. 14, which shows the voltage of an a.c. operated lamp with a sinusoidal arc current. If the temperature
of the cathode is high enough, the arc voltage will show a smooth behaviour as shown in the upper photograph. The arc burns in the low-field mode all the time. If the arc power is decreased, the tip temperature is lowered. This would correspond to an upward shift of the solution curve similar to the effect of decreasing heat resistance (fig. 12). Thus after current reversal the arc first starts in the high-field mode.

With increasing current the conditions on the electrode change. Higher current and higher tip temperature favour the low-field mode. Within some tenths of a millisecond the arc switches to this mode. The transition shows up in the time dependence of the arc voltage as a sudden drop of the order of 10 V (fig. 14, centre). If it occurs during the reignition peak, it may even be as large as 50 V (fig. 14, bottom). The model presented in this report can of course only give a qualitative explanation of the transition phenomena; a quantitative description of the effect must await an extension of the model to a.c. operated lamps.

The examples discussed up to now were all concerned with the influence of geometric or thermal conditions at the electrodes on the properties of the discharge. The fact that the composition of the arc plasma also influences the shape of the hot plasma spot in front of the electrodes is demonstrated in fig. 15. The upper part shows the field of isotherms in the cathode region of the mercury arc lamp, the lower part shows an arc in otherwise identical conditions, but with the addition of NaI, TlI and InI. In the latter case the cathode spot is much less constricted. This difference, which also results in a lower temperature of the electrode tip, is observed in experiments, too. That the reduction of the tip temperature, which should be of the order of 300 K according to the model calculations both for the cathode and for the anode, is much smaller in real a.c. lamps, can be explained by the fact that during the anode half cycle most of the sodium that is nearly totally
ionized is removed from the anode boundary region by diffusion in the electric field, so that the arc is more like a mercury discharge in this half cycle.

For the practical design of discharge lamps a question of importance is what the heat resistance of an electrode must be to achieve a given tip temperature and what burning voltage and energy losses to the electrodes will occur under these conditions. Fig. 16 shows a graph of the arc voltage versus the tip temperature of the cathode for a mercury arc. As can be read from the curve, any attempt to reduce the tip temperature will result in an increase of the arc voltage. The additional power introduced into the plasma will be conducted to the electrodes. The dependence of the energy loss to the cathode on the tip temperature is shown as a dashed-dotted line in the graph. The plasma counteracts an enhanced cooling by an increase in the amount of energy that is delivered to the electrode surface from the plasma. Cooling to below a certain limit, which is about 2660 K in the example given here (\(E_A = 4.5\) eV), is practically impossible. The attempt to remove more heat from the cathode would only have the effect of increasing the energy transfer from the plasma by the same amount.

Further reduction of the tip temperature below the limit is possible only by reducing the work function of the electrode material, as demonstrated by the second set of curves shown in fig. 16. A reduction from 4.5 eV to 4 eV would lead to a lowering of the limiting temperature by 200 K. The results shown in fig. 16 can also be looked at from another point of view, however. The model calculations show that in small high-pressure discharge lamps (30 W in the example shown here) the electrode losses are about 20%, even if the tip temperature is very high. Thus a further increase of these losses would considerably reduce the efficiency of the lamp, and therefore one should not try too hard to reduce the tip temperature below 2800 K as it would make the lamp less efficient. If however a tip temperature of 2800 K is tolerable, it does not make much sense to look for emitters to reduce the work function, as in this temperature range a change of the work function has only a minor influence on the energy loss of the electrode.

6. Conclusions

Though the model presented in this report is far from giving an exact description of all the processes that may take place in the electrode region of high-pressure arcs, it offers an explanation for the most important phenomena that determine the integral data of arc lamps. The mathematical procedures developed here are tailored to the best possible description of elliptical lamps, but this geometry can be regarded as a good approximation for many other arc types. The main features of the model are the facts
that the electrode is not only regarded as the source of the electric current, but also as a strong heat sink. It is this property that is responsible for many of the phenomena in the boundary region. The fact that within the scope of the model the radial dependence of the local arc properties can be calculated with reasonable numerical effort makes the model superior to any one-dimensional or channel model. A knowledge of the local plasma composition, current and temperature distribution is the indispensable premise for a description of the gas flow in the spot region, which one needs in order to obtain information on the material flow from the electrode to the walls.

From one point of view the high-pressure mercury plasma is much simpler than any other arc plasma: it is characterized by an extremely short mean free path of the electrons. Thus in spite of the steep gradients in the electrode boundaries it is close to LTE, and secondary emission processes at the cathode are unimportant. In rare-gas arc plasmas additional deviations from LTE may occur and emission processes caused by high-energy electrons may lead to an additional complication of the model, but arcs in rare gases or air at atmospheric pressure are also much closer to the conditions described in this report than to those found in low-pressure discharges and which are used by some authors to describe the phenomena in the boundary layer of the electrodes in high-pressure arcs.

References

12. H.P. Stormberg, private communication
1. The discussed problems. Cathode parts of gas discharge include regions between cathode and positive column: cathode fall layer, negative glow and dark Faraday space. Longitudinal structure of glow discharge of a long gap, namely, the distributions of luminity L, electric field E, electron and ion densities \( n_e, n_i \) between the electrodes are presented at the Fig. 1. The fact that current occupies a definite area \( S \) on cathode is the main peculiarity of transverse structure. When there is a free place on cathode, current density there \( j = i/S \) has a definite value of \( j_{\text{max}} \). When current varies, area of current spot on cathode changes proportionally to \( i \). Such glow discharge is called a normal one.

For normal discharge the cathode potential fall \( V_K \) and thickness of fall layer \( d \) \( (V_n, d_n) \) do not depend on current.

In an abnormal discharge in which current occupies the whole cathode area \( S_K, V_K \) and \( j \) grow with current increasing.

We consider three problems which are of principal value for understanding the discharge structure:

a) Effect of non-locality of electron spectrum in cathode fall and negative glow regions. There is no one-valued relation between spectrum and field strength at a given point of space, as in the case of uniform field.

b) The reasons of the arising of weak field, less than in positive column. This weak field results in dark Faraday space.

c) Why does the current occupy definite area of cathode surface? What is the mechanism of normal current spot formation?

Let us note that the first two problems (which are closely related) are rather clear qualitatively but there are some essential questions in the quantitatively theory. As to the third problem it has no satisfactory answer up to now in spite of the correct theoretical determination of normal parameters.

2. One-dimensional theory of cathode fall. Being formulated by von Engle and Steenbeck [1] 50 years ago it allows us to estimate correctly the main parameters. It is accepted that current self-maintenance is provided by electron impact ionization in the cathode layer with the Townsend coefficient

\[
\alpha = Ap \exp \left( -BP/E \right)
\]

and by secondary emission from cathode with the coefficient \( \gamma \). Therefore the Townsend condition

\[
\int d_\alpha |E(x)| \, dx = \ln\left( 1 + 1/\gamma \right), \quad E \equiv |E|/2
\]

is valid. On the base of probe data it is assumed that in the layer

\[
E = E_K (1 - x/d), \quad V_K = E_K d/2
\]

The dependence of \( V_K \) upon \( p_d \) following from (1), (2), (3) is simi-
lar to the Paschen breakdown curve. The latter corresponds to \( E(x) = \text{const} \). In the cathode fall layer \( n_+ \gg n_e \) therefore the electrostatic equation

\[
\frac{dE}{dx} = 4\pi e (n_+ - n_e) \tag{4}
\]
gives \( n_+ \approx E_K/4\pi e d \). As \( j = (l+\gamma)\mu_+ E_K (\mu_+ - \text{ion mobility}) \) current–voltage characteristics of cathode layer \( V_K (j) \) is obtained (Fig. 2). The parameters of minimum point \( V_n, (pd)_n \) are of order of \( V_{\text{min}} \), \((pd)_n \) for Paschen curve and

\[
\frac{j_n}{p^2} = (l+\gamma)/(\mu_+ p) V_n^2 / (\pi (pd)_n)^3 \tag{5}
\]

The right branch of the curve \( V_K (j) \) for \( j > j_n \) describes correctly abnormal discharge where \( j = i/S_K \) but the left branch is not observed. Postulating that normal discharge of arbitrary current corresponds to minimum \( V_K (j) \) one obtains good agreement with experimental data. It seems to be natural that optimal conditions for electron multiplication requiring minimal voltage are automatically chosen. But this fact does not follow from one-dimensional model. The existence of a boundary at cathode which separates regions with and without current is essentially two-dimensional effect.

3. Integration of one-dimensional differential equations. Self-maintained current in a discharge gap can be described by the simplest system including the equations

\[
d_e/dx = \alpha(E) j_e , j_e = e n_e / e E \tag{5}
\]

\[
j_+ = e n_+ / e E , j = j_e + j_+ = \text{const}
\]

the electrostatic equation (4) and the boundary conditions on cathode and anode

\[
j_e K = \gamma j/(1+\gamma) , j_e A = j \tag{6}
\]

Integral relation (2) follows from (5), (6), if we neglect ion current \( j_+ \) at \( x = d \). The equations of the type of (5), (4), (6) were integrated numerically by Ward [2].

The calculated distributions \( E, n_e, n_+ \) (Fig. 3) confirm linear behaviour of \( E(x) \) in the region where \( n_+ \gg n_e \). One can see how the field becomes rather weak in electroneutral plasma. But the equations (5), (4), (6) can not describe a uniform positive column which is usually formed in a long discharge gap. Electron losses due to recombination or ambipolar diffusion to the walls are not taken into account. It is justified only for short gaps and weak currents when the field does not fall very low and charge density is not very high. In these cases the ionization rate even in minimal field exceeds essentially diffusion and recombination losses. In the opposite cases of long gap and (or) high current the loss terms must be included in the first equation of (5), for example

\[
d_e/ox = \alpha(E) j_e - e n_e n_+ \tag{7}
\]

where \( \beta \) – recombination coefficient

Improved system (7), (4), (6) describes the arising of uniform electroneutral column in a long gap, where charge losses are compensated by the birth and \( j_e, E, n_e = n_+ \) are constant. However, it would be mistaken to think that in such a way one can obtain a correct transition from cathode layer to inform positive column through the Faraday space. The appearance of field minimum and sharp charge density maximum before the positive column (Fig. 1) do not follow from the equations considered. They give a direct transition from
cathode fall to quasiuniform column in a long gap, as is shown at Fig. 4. This result is obtained both by qualitative analysis of the equation \( \frac{dE}{dj} = f(E, j) \) (obtained by dividing (4) by (7)) and numerical integration of the equations (7), (4), (6) \([3]\). Such solution behaviour is a result of accepted dependence of ionization coefficient \( \alpha \) on local field.

The electron birth rate falls monotonically with decreasing of \( E \) from cathode until it is compensated by losses. The situation does not change if one takes into account the electron and ion diffusion currents.

4. Electron spectrum nonlocality. It was noted long ago that a small amount of fast electrons - "a beam" - goes out from cathode layer of glow discharge. These are electrons which manage to pass the layer without unelastic collisions. The atom exitation in the region of negative glow after cathode layer (where the field is weak and electrons seem to have very small energy) was explained by beam action. Latter three electron groups were observed in the negative glow region. According to one of the recent experiments \([4]\) there are electrons of mean energy of 2, 22,5 and 150 eV in He (for \( V_{th} \approx 150 V \)). The most numerous group of slow electrons are born at the end of cathode layer where field is weak or they have lost their energy at unelastic collisions. The number of electrons of moderate energy is of two orders less. They were born in a middle part of cathode layer and did not spend their energy.

The presence of energetic electrons in spectrum is related to sharp space non-uniformity of the field in cathode layer. Some length along drift way is required for formation of equilibrium electron spectrum in a field. In the case considered the field decreases markedly along this relaxation length. Therefore electron spectrum in a point with given field is a non-equilibrium one. It consists of more energetic particles, than in the case of uniform field of the same magnitude.

Presently the efforts in cathode layer treating are directed to take into account nonlocal effects. Generally it is necessary to solve a kinetic equation in a non-uniform field. We have to find electron impact ionization rate which is necessary for discharge structure investigation. This is very difficult problem, therefore some approximate approaches are looking for. In particular, model equation for electron mean energy \( \overline{E} \)

\[
\frac{d\overline{E}}{dx} = c \overline{E}(\lambda) - \alpha(\overline{E})(\overline{I} + \overline{E})
\]

is considered \([5]\). Here \( \overline{I} \) is the ionization potential, \( \alpha(\overline{E}) \) - reverse value of the free path for ionization. On the base of kinetic equation analysis it was shown \([6]\) that one can approximately use the ionization coefficient \( \alpha \) in the form (1) but to substitute instead of local field \( E(\lambda) \) the mean field from the foregoing path interval \( \Delta \lambda \) where electron acquires the energy \( \int_{\lambda}^{\lambda + \Delta \lambda} E(\lambda) d\lambda = \overline{I} \).

The most comprehensive and
reliable information is obtained by
the Monte-Carlo method which provides
numerical simulation of stochastic
process for electrons in cathode
layer. In this case it is equivalent
to the kinetic equation solution.
New such results [7] are given at
Fig. 5. The calculations were made
for He at pressure $p = 1$ Torr
and distance between electrodes of
1.5 cm. It were assumed the law (3)
at $0 < x < d_0$ with $d = d_n = 1.3$
cm, $E_n = 230 \text{ V/cm}$, $V_n = 150 \text{ V}$
and $E = 1 \text{ V/cm}$ in the rest
part of the gap.

Mean energy of electron spectrum at the end of cathode layer
($x = d_0$) was $\bar{E} \approx 9 \text{ eV}$. At
this point a calculated ionization coefficient $\alpha \approx 0.6 \text{ cm}^{-1}$ only two
time less that its maximal value in
the middle of the layer. Distribution of ionization probability $\alpha(x)$
is essentially different from equilibrium monotone function
$\alpha[E(x)]$ falling from its maxi-
mum 1.7 cm$^{-1}$ near cathode to almost
zero at $x \approx d_n$. These calculations did not confirm the existence
of three electron groups which were observed experimentally though there
is a small number of electrons with $E = eV_K = 150 \text{ V}$. Perhaps this is
the result of small distance between the end of cathode layer and
anode, otherwise the calculations would require too much computer
time.

5. Longitudinal discharge structure. Three factors are of
importance for structure formation and they probably are responsible
for the Faraday space existence.

a) The presence of ionization source which is not related to local
field magnitude. The birth of the great number of electrons at the
weak field region leads to maximum of electron density $n_e$ in front of
positive column though the field there is stronger.

b) Local charge losses which are not related to charge extraction to
electrodes. These losses provide uniform positive column formation
for the case of long distance between electrodes. The field in the
column does not depend on $x$. The slow charge birth and losses are
compensated in constant drift flows.

c) Longitudinal electron diffusion. In some cases it results in
electric current in a region where electron density falls after maximum
Polarization field, arised due to diffusion cancels partly external
field. The field can even reverse the sign here.

Generally, all three elements were considered in old theories of
negative glow. The electron source being prescribed to a beam.
According to modern data it should be prescribed to electrons of in-
termediate energy - a beam is too small for it. Old theory overesti-
mate the role of electric current carried by a beam. In fact, only a
small part of electrons emitted from cathode manage to cross catho-
de layer without electron losses. Besides, electron current from ca-
thode is a small part ($\gamma/(\gamma+1)$) of the whole current.

As it was already mentioned there is no Faraday space for
$\alpha(x) = \alpha[E(x)]$. But if we take into
account only non-local effects (factor a)) without losses (factor b)) we obtain nothing qualitatively new in comparison with Fig.3 for distributions $E$, $n_e$, $n_+$. One can see that from the calculations [8]. Model calculations [9], when all three factors were taken into account gave qualitatively correct field behaviour $E(x)$.

There is no additional ionization source in a transitional region between the Faraday space and positive column. Past electrons depleted their energy. After $n_e$ maximum a field is small and there is no equilibrium ionization yet. Electron density falls due to losses and electron diffusion current arises. Charge densities are high here and plasma is electroneutral. In this case

$$j_e = e n_e \mu_e E - e D_e \frac{dn_e}{dx} = \left(\frac{\mu_e}{(\mu_e + \mu_+)}\right) j - e D_a \frac{dn_e}{dx}$$

where $D_e$ and $D_a$ are free and ambipolar diffusion coefficients respectively. Substituting the second expression of (9) into (7) and taking into account $d_j/dx = 0$ we obtain an equation to describe $n_e$ in the transitional region. Value of $E$ in $\alpha(E)$ is determined by (9).

$$E = \frac{j}{e n_e \mu_e} - \frac{T_e}{n_e} \frac{dn_e}{dx}$$

As far as we know there is no calculations of longitudinal discharge structure from cathode to anode as in [2,8], but for long discharge gap and with taking into account all the abovementioned factors. We consider this problem as a nearest necessary step in discharge study.

6. Why does a normal discharge occur? Von Engel and Steenbeck [1] pointed out to instability of $j < j_n$ states. It is connected with the falling character of the left branch of current-voltage characteristics Fig.2 (see also [10, 11]). It is less obviously why cathode spot with $j > j_n$ should spread inspite of stability of states corresponding the right growing branch of $V_k(j)$ and generally stability considerations can not answer many questions. What is the mechanism of normal state formation and non-normal destruction, what does stabilize the boundary of current spot on cathode etc.

Twenty years later von Engel [12] leaving apart his previous stability arguments spoke about not understood phenomenon, about possible action of some unknown forces and appealed to the Steenbeck minimal power principle (1932). Really in normal cathode layer the voltage and power $P = i V_k$ are minimal respect to current area $S$ variation while current $i = j S$ is conserved. But this principle is not the physical law and explains nothing [13]. One should treat phenomena in terms of charge motions, ionisation, electrostatics etc.

The process on cathode has much in common with constriction effect.
Really, there are regions in the field, with and without current. The effect can be considered on this base \([14]\). Let us direct axis \(y\) along cathode surface and consider the layer of the variable thickness \(d(y)\) bounded by equipotential surface \(V = V_K\) (Fig. 9). Let \(\overline{n}_+\) be the average (along normal axis \(X\)) density in the layer. It is assumed in \([14]\) that the steady distribution \(\overline{n}_+(y)\) is a result of the processes occurring only in this layer. The existence of nonlinearly growing dependence of ionisation rate on charge density is necessary for constriction \([13]\). In this case it arises due to space charge action. According to (4) \(d \approx (V_K / 4\pi e n_+)^{1/2}\) and \(E \approx (4\pi e n_+ V_K)^{1/2} \sqrt{\overline{n}_+}\). Therefore \(\alpha(E)\) increases with \(\overline{n}_+\). The charge birth compensates drift losses in current spot (at \(y \to -\infty\) on Fig. 6). There is no such compensation in the region where \(\overline{n}_+\) is small (at \(y \to \infty\)). The lack of the birth is compensated there by charge inflow due to diffusion from the region where \(\overline{n}_+\) is higher. Let us note that usually there is no steady constriction without diffusion or heat conduction interaction between regions with and without current. The solution of simplified diffusion problem gives that value \(V_K\), for which there are steady distributions of \(\overline{n}_+(y)\) and \(j(y)\). It appears to be close to normal \(V_n \approx [V_K (d)]_{\text{min}}\). It seemed that in \([14]\) the effect was qualitatively explained (in \([15]\) mechanism of nonlinear charge birth connected with gas density termal variation was used).

But recently two-dimensional problem of unsteady breakdown process and glow discharge formation was solved numerically \([16]\). A spot with current density \(j\) close to normal one was formed at cathode (Fig. 7). When current was increased twice the spot area was also increased approximately twice, but \(V_K\) and \(d\) vary only slightly. All these results were obtained from the equations generalizing the equations (7), (4), (6) in which diffusion was not taken into account at all. The width of transitional zone between the cathode regions with and without current is of order of cathode layer thickness itself, \(d \approx d_n\). It doesn't look like constriction model \([14]\), which looks to be rather attractive.

Transition width \(\Delta\) is determined there by ion diffusion coefficient \(D_+\) and \(\Delta \approx d\).

It is rather difficult to make any final conclusions because the calculations \([16]\), were not led to quite steady situation (otherwise computer time expense would be too high). It is possible that stabilisation would not be reached without taking into account diffusion flows. The other hand, in transitional zone the field component along cathode is comparable with normal one and drift flow is more essential than diffusion flow. The calculated velocity of current spot expansion for \(V > V_n\) based on drift ion motion only \([17]\) gave the estimation
Calculated values of \( v^* \) agree with Steenbeck experimental data \([18]\) and are higher than diffusion velocity which we would obtain if we consider unsteady constriction model for \( V_k \neq V_n \).

7. Approaches to the problem.
The first our problem now is to try to find numerically really steady solutions of two-dimensional discharge problem without diffusion. The equations are

\[
\begin{align*}
-\nabla V = \alpha \cdot \nabla \Phi & \Rightarrow j_e = e \beta n_e n_+ + j = \frac{e n_e}{\mu e} \nabla E, \\
\frac{dV}{dt} = \frac{e n_+ + \mu E}{j} = \frac{j}{\mu} & \Rightarrow \frac{dV}{dt} = \frac{e n_+ + \mu E}{j} \\
\n\end{align*}
\]

The boundary conditions (6) are valid on the electrode surfaces and longitudinal potential gradient \( \partial \Phi / \partial x \) is constant far from current channel where \( n_e, n_+ = 0 \). Discharge voltage \( V \) is related to given supply voltage \( V_S \) and external circuit resistance \( R \) by the equation \( V_S = V + i R \).

Perhaps cathode spot stabilization is a result of two-dimensional space charge and external resistance action only (the latter usually regulates the stable state setting). In this case there would be a sharp current boundary in a gap and on cathode with tangent current discontinuity. In the current region the nonequality

\[
A \int A \left[ E(\xi) \right] d\xi > \ln(1 + 1/\gamma)
\]

must be valid, in the currentless region - the opposite one. Here integration is taken over \( \xi \) along electric force line from cathode to anode.

If we find a steady solution of the diffusionless system (11) and if it confirms normal current density law one should probably consider diffusion mechanism of constriction to be unnecessary. In this case diffusion will only smooth tangent discontinuity. If the system (11) has no steady solution one should add diffusion flow terms into \( J^* \) as in the first expression (9).

We hope that detailed analysis of steady and unsteady solutions of two-dimensional problem with and without diffusion for essentially different currents and cathode spot areas helps us to understand at last the real mechanism of normal current density arising.

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Fig. 1. Longitudinal structure of glow discharge in a long tube.

Fig. 2. Current-voltage characteristics of cathode layer.

Fig. 3. The distributions of $E$, $n_e$, $n_+$; calculated by Ward [2] for argon, $p = 10$ Torr, $j = 5.62 \times 10^{-4}$ A/cm$^2$.

Fig. 4. Distributions of $E$, $n_e$, $n_+$ in a long gap, obtained under the assumption that $\alpha = \alpha(E)$. 
Fig. 5. Electron spectrum at the end of cathode fall (a), distributions of the mean electron energy (b) and the ionization coefficient (c) inside of cathode layer obtained by Monte-Carlo method [7]. (Helium, \( p = 1 \) Torr, \( d = 1.3 \) cm).

Fig. 6. The constriction model current spot on cathode [14]. Equipotential line \( V_K \) and \( n_e \) distribution along cathode surface.

Fig. 7. Results of numerical integration of nonsteady two-dimensional diffusionless equations of type of (11) for planar geometry [16]. Nitrogen, \( p = 5 \) Torr, \( V_s = 700 \) V, \( R = 250 \) k\( \Omega \) gap length (along \( X \)) 1 cm, half width (along \( Y \)) 1 cm and -3 cm.

a) Line of equal \( n_e \) (in \( 10^9 \) cm\(^{-3} \))

b) Line of equal \( n_e \) (in \( 10^9 \) cm\(^{-3} \))

c) Equipotential and current lines. They are marked by the magnitude of volts and total current fractions.

da) Current density on cathode.
The aim of this lecture cannot be to give a complete review on all experimental activities in the field of nonideal plasmas. We would rather like to introduce some few problems thought to be of topical interest in experimental nonideal-plasma research. The state of the art will be discussed in verifying existing theories by experimental results or in explaining measured effects by theory. Of course, the selection of the experiments for demonstrating this was to a certain extent arbitrary.

After an introduction, the generation of nonideal plasmas and the problems related with their diagnostics are discussed. In chapter 4 on results of the experimental research the following topics are dealt with: electrical conductivity, continuum-radiation absorption coefficient, shift and broadening of spectral lines, and plasma phase transition. Finally, some conclusions are given.

1. Introduction

A well-known field of plasma physics is arc physics. An arc plasma is created by an electric discharge at about atmospheric pressure, and it is assumed to be in local thermodynamic equilibrium. The behaviour of arc plasmas can be almost completely understood by well-established theories.

Its thermodynamics is that of an ideal gas with ionisation. At higher densities, in the ionisation equation a lowering of the ionisation energy has to be taken into account in the usual Debye form /1/. Its transport properties can be described by the theories of Chapman and Cowling /2/ and of Spitzer /3/. The radiation of spectral lines is covered by the work of Griem /4/, whereas the continuum radiation theory was worked out by Gibeiman and Norman /5/ and by Schluter /6/.

The theoretical description of arc plasmas and its experimental verification were essentially finished in the sixties. The last twenty years since that time were characterised by applications of arc plasmas as spectroscopic light sources, as light sources generally, as circuit breakers, and as plasma jets for welding, cutting, and for plasma chemistry.

At the beginning of the seventies a new field of plasma physics was opened especially by activities in the Soviet Union. It is characterised by nearly the same temperatures as arc plasmas but by much higher pressures. That means the particle density is also much higher than in usual arc plasmas. The correspondingly small interparticle distance leads to a
strong interaction between the particles what has a non-negligible influence on the physical properties of these dense plasmas.

The interaction causes a reduction of the pressure, an enhancement of the ionisation, a smaller electrical conductivity, and there are also indications for a lowering of the continuum-radiation absorption coefficient and for a reduction of the spectral-line shift and broadening at higher densities. These statements must be understood related to expectations from calculations using arc physics theories.

The research in this more recent field of plasma physics is motivated and can be justified by the following topics: Firstly, those strongly coupled plasmas offer interesting opportunities for basic research. Secondly, those plasmas are of high interest for astrophysics, and thirdly, there is a lot of technological applications and possibilities connected with those dense plasmas as for instance: Light sources, MHD generators, circuit breakers, laser mirrors and shutters, high-temperature gas-phase fission reactors, material surface treatment, and laser fusion.

The technological capabilities of dense plasmas result from the high heat capacity which enables them to transfer or to accept high amounts of heat on a high temperature level and, further, from the short recovery period after operating which makes them an undestroyable working fluid able to fast actions with a high repetition rate.

Those plasmas, introduced now, are called nonideal, strongly coupled, non-Debye, or simply dense plasmas. Contrary to low pressure plasmas, where the kinetic energy $E_{\text{kin}}$ of a particle is always high in comparison to the potential energy $E_{\text{pot}}$ between two neighboured particles, in dense plasmas it is comparable or even lower than the potential energy. The so-called nonideality parameter $\gamma$ characterises this behaviour:

$$\gamma = \frac{E_{\text{pot}}}{E_{\text{kin}}} = \frac{e^2(n_0+n_f)^{1/3}}{kT}.$$  \hspace{1cm} (1)

Here we restrict ourselves to charged-particle interaction but at higher neutral density nonideality due to neutrals is also possible.

There exist different theories for describing the thermodynamics of nonideal plasmas. We prefer a theory given by Ebeling and co-workers /7/ which appears as an extension of the Debye theory. The lowering of the ionisation energy $\Delta E_i$ and the interaction term of the pressure $\Delta \rho$ are given here in the first approximation:

$$\Delta E_i = \frac{e^2}{r_D \Lambda / 8},$$  \hspace{1cm} (2)

$\Lambda$ - electron de Broglie wavelength,
$r_D$ - Debye-radius,
The underlined terms are the additions to the usual Debye expressions which can be found in textbooks (cf. e.g. /8/).

For a better identification of the plasma we are dealing with a diagram of state is given where the free electron density is shown versus the temperature (Fig. 1). As an orientation, the degeneration limit for electrons is given and further two values of the nonideality parameter can be seen which are characteristic of weakly ($\gamma = 0.2$) and strongly nonideal plasmas ($\gamma = 1$).

Weakly nonideal means about one charged particle in the Debye sphere ($N_D = 1$), conditions which we meet in electric pulse discharges and in high-pressure cascade arcs. A lot of work was done in this field during the last fifteen years (cf. /9/ and the references cited therein).

Strongly nonideal means about one tenth or less particles in the Debye sphere. From this condition it becomes once more obvious that the Debye theory here can be no longer valid. Examples for strongly nonideal plasmas are shock wave plasmas as they have been produced by Fortov and co-workers by means of explosives /10/. Further, strongly nonideal conditions can be reached by so-called ballistic compressors. Such devices have been used during the last years in several laboratories for nonideal plasma research /11 - 14/. The dashed region marked by "ballistic compressor" corresponds to plasma states reached in our laboratory in rare gases using two different devices (one of them is called AICA - Adiabatic Impulse Compression Apparatus).

In alkali plasmas, high nonideality can be obtained at lower parameters than, for instance, in rare gases due to their lower ionisation energy. Shock wave experiments in alkali vapours done in the Moscow Institute of Chemical Physics are shown here /15/. Further the parameters of a cesium experiment in an electric pulse discharge can be seen (the cross in the lower left) /16/.

Finally, a region is shown (in the upper left) which is characteristic of shock wave experiments in solids where strongly nonideal plasmas were produced which are at the same time strongly degenerated /17/. We restrict ourselves on non-degenerated plasmas for the first.
2. Generation of nonideal plasmas

The generation of such extreme plasma states as necessary to get non-ideal plasmas is an independent scientific task. Here three different experimental set-ups are given as examples.

In Fig. 2, a discharge tube is shown for generating weakly nonideal plasmas by a quasi-stationary discharge as it was used by Radtke and Guenthör /9/. In the lower part the discharge current and the light output can be seen. The auxiliary electrode is movable and can be used for diagnostic purposes such as field strength measurements.

![Fig. 2: Discharge tube for generating weakly nonideal plasmas](image)

In Fig. 3, a shock tube is shown as it is used by Fortov and co-workers /10/. It is driven by an explosive; the four electrodes shown here are used for measurements of the electrical conductivity. In Fig. 4, a ballistic compressor is shown as we are using it in our laboratory. The piston compresses a test gas which is simultaneously heated because the process is fast and therefore nearly adiabatic.

![Fig. 3: Shock tube explosively driven for nonideal plasma generation](image)
3. Diagnostic problems

There are three reasons which make diagnostics in nonideal plasmas a difficult task: Firstly, many relations between measured quantities and the parameters wanted are in discussion. Examples are: the relation between the electron density and the index of refraction, between the electron density and the broadening as well as the shift of spectral lines, or the relation between the temperature and the intensity of continuum or line radiation. Therefore, these relations should rather be subject of research than diagnostic tools.

Secondly, the thermodynamic relations between the plasma parameters themselves as the equation of state and the ionisation equation are in question to a certain extent. Therefore, it is not possible to determine a complete set of plasma parameters by measuring only two of them as we can do it in arc plasmas.

Thirdly, the high particle density leads to a high optical density. Therefore, radiation often comes from anyone point of a boundary layer and cannot give any information about the plasma core. If one uses extremely thin layers to avoid this effect the plasma necessarily becomes very inhomogeneous.

On the other side, in some devices for nonideal-plasma generation as
in shock tubes and ballistic compressors an additional source of information exists: the dynamics of the compression process itself which is closely related to the plasma parameters. In shock tubes, from the measured shock velocity the plasma state can be derived from the Rankine-Hugoniot equations. In ballistic compressors, the measured compression ratio should give the plasma state. In both cases, however, one has to use an equation of state and an ionisation equation for which the interaction parts are at least in discussion. Further, different loss processes play a role, in the slower ballistic compressors surely more than in shock waves.

Due to these problems in nonideal-plasma diagnostics we would like to underline the necessity for the experimentalist to publish his results at first in its primary form. As basic parameters which are measurable relatively free from assumptions the pressure, the mass density, and - under certain conditions - the temperature should be used. If the electron concentration or any kind of nonideality parameter is used the procedure of getting these quantities should be given in detail.

These are presumptions necessary for an adequate comparison of different experiments and of experimental results with theoretical work.

4. Experimental results

Now some selected results of recent experiments will be given. As already mentioned, the selection is of course to a certain extent arbitrary. It should only serve to show the problems. Firstly, we will be dealt with the electrical conductivity of cesium and rare gas plasmas. Secondly, the absorption coefficient in rare gases and in hydrogen will be discussed. Thirdly, the spectral line behaviour at higher densities will be our subject and, finally, the problem of a predicted phase transition will be touched on.

4.1. Electrical conductivity

Measurements of the electrical conductivity in cesium are shown in Fig. 5. The conductivity is given in dependence upon the temperature with the pressure as parameter. The curves are calculated for constant pressure the value of which is indicated. The curve marked by (1) is from Likalter /22/ whereas the curves marked by (2) are from Gryaznov et al /24/. As can be seen, in wide ranges of temperatures the electrical conductivity of cesium is only weakly pressure-dependent. The agreement with theory is not excellent.

Fig. 5: Electrical conductivity in cesium versus temperature
- note the logarithmic scale - but the qualitative behaviour can be
described. Remarkable is a deep minimum near the saturation line which is
followed by a steep rise to lower temperatures. This is the range of the
so-called anomalously high electrical conductivity in cesium vapours.

What is the reason for this high conductivity at low temperatures?
In the high temperature range, the conductivity is mainly determined by
charged particle interaction. Under these conditions, its pressure depend-
ence indeed is weak (the conductivity increases with increasing pressure).
It shows the well-known behaviour according to Spitzer proportional to
$T^{3/2}$. Starting from about $100 \, \Omega^{-1} \, \text{cm}^{-1}$ characteristic of such plasmas, the
conductivity at first decreases with decreasing temperature according to
this law.

At lower degrees of ionisation, however, the conductivity at constant
pressure changes linearly with the electron density, and a decrease of the
temperature induces an exponential decrease of the conductivity according
to the Saha equation (the conductivity here decreases with increasing pres-
sure).

The minimum around 2 000 K and the following steep rise to lower tem-
peratures cannot be explained on the basis of usual plasma considerations.
This effect is rather due to the formation of clusters consisting each of
two and more atoms. (In the region of the minima the pressure dependnce is
once more reversed: rising pressure means now again rising conductivity.)

In Fig. 6 the effective ionisation energy of these clusters can be
seen versus the number of
atoms per cluster /25/. It is
the lower the more atoms the
cluster consists of. Therefore
the ionisation equilibrium is
shifted to a higher density of
the free electrons. This is
the reason for the strong in-
crease in the electrical con-
ductivity in cesium beyond
this minimum to lower temper-
atures exceeding the standard
plasma estimates by more than
five orders of magnitude.

Electrical conductivi-
ties measured in rare gases
under nonideal conditions are
frequently compared with the
well-known Spitzer formula
valid at medium pressures and
higher degrees of ionisation.
In Fig. 7 the conductivity of argon is shown in dependence upon the pressure at two different temperatures. The Spitzer values do not take into account electron-atom collisions. The experimental points as well as the other theoretical approaches shown contain additionally the interaction with the neutrals.

This representation of conductivity data seems to us suitable for a comparison between experiment and theory because the experimental data have not undergone any more or less non-transparent manipulation.

At low temperatures, say 10,000 K, the nonideality is low \( \gamma = 0.13 \) and simultaneously, the neutral-particle influence on the electrical conductivity dominates (the degree of ionisation \( \alpha = 0.01 \)). In this range, deviations from Spitzer above all are due to the neutral particle influence and only to a less extent due to nonideality.

The electrical conductivity can be described by theoretical methods as given, for example, by Devoto (short-dashed line). Corrections taking into account nonideality are only small (full line). Therefore, at low temperatures nonideality is hidden behind the neutral particle influence, and measurements in this region are not suitable for obtaining informations on nonideality effects.

At higher temperatures, say 20,000 K, the nonideality is higher \( \gamma = 0.7 \) as a maximum), and, at the same time, the neutral-particle influence on the electrical conductivity is negligible. Therefore, deviations from Spitzer here essentially are due to nonideality effects, that is, roughly spoken, due to a reduction of the charge mobility by the strong particle interaction.

The full curves are calculated using a semi-empirical correction of the shielding parameter as it has been successfully used by Radtke and Guenther for weakly nonideal plasmas. It seems to approximate the experimental values quite well also in the range of higher nonideality /9/.

Fig. 7: Electrical conductivity in argon versus pressure
4.2. Continuum absorption coefficient

In the last ten years, some measurements of the continuum absorption coefficient in the visible spectral range have been published under conditions of stronger nonideality. These measurements were done in argon, xenon, and air at only few wavelengths, and they seemed to indicate a decrease of the absorption coefficient with increasing nonideality in comparison with the theory established for arc plasmas. The published results are of different quality, only few authors have given a full set of plasma parameters.

The real physical dependences are often hidden behind complex parameters which are used in the given diagrams. We would like to suppose to present the measured absorption coefficient as a function of the temperature which has the strongest influence on it. If it is to compare with theory, the neutral density must be known, and the stimulated emission has to be taken into account. On the basis of these data, the dependence of the absorption coefficient on the electron density or on the degree of nonideality may be discussed using electron densities obtained by more or less direct measurement or by calculation from other plasma parameters via equation of state and mass action law.

In Fig. 8 the results of two groups are shown from similar shock tube experiments. As can be seen, the measurements exhibit a completely different behaviour when comparing them with theory. (The full lines represent absorption coefficients calculated according to Hofsaess /26/ including the influence of free-free transitions and stimulated emission.)

The measurements in xenon /27/ are in very good agreement with theory. Only at medium electron densities there is a deviation to lower values which is a little bit outside the error bars. In this experiment, the electron density increases with the temperature by one order of magnitude up to $4 \times 10^{19}$ cm$^{-3}$. This good agreement is the more astonishing since the theory is derived for vanishing electron density.
The measurements in argon /28/, however, show a decrease of the absorption coefficient per atom with increasing temperature in contrast to theory. This behaviour indicates a strong influence of nonideality: an influence which cannot be explained satisfactorily till now. In difference to the preceding measurements, the electron density here is about one order of magnitude larger, it increases up to \(1.5 \times 10^{20} \text{ cm}^{-3}\).

There exist simple ideas what happens with the absorption coefficient when high-lying levels vanish due to the plasma interaction. In short, it should be described by a shift of the series limits to longer wavelengths and by a supression of that part of the free-bound radiation which stems from the extinguished levels. Further there are some attempts to calculate the influence of the plasma interaction on the radiation. We mention here the prediction of a transparency window near the series limit of spectral lines by Norman, Kobzov, and Kurilenkov /29/ and the arguments by Hoehne and Zimmermann /30/ against its existence, at least in the case of hydrogen. Further there are attempts by Fortov and his co-workers using the so-called confined-atom model /28/. Of interest in this connection is the contribution of Sevastyanenko and Soloukhin at this conference /31/.

As an experimental contribution to the "transparency-window" discussion in Fig. 9 two hydrogen spectra are shown measured around the Balmer series limit at electron densities of about \(2 \times 10^{17} \text{ cm}^{-3}\) and \(8 \times 10^{17} \text{ cm}^{-3}\) /32/. In agreement with Hoehne and Zimmermann, at least in this range of weak nonideality no transparency window could be found.

A theory, however, which is generally accepted and applicable also for more complex atoms does not exist. Further theoretical work should be done, but the state of the experiments in this field is also insufficient for a comprehensive description of the effects. More careful experimental work is necessary covering the whole accessible spectral range, and more groups must investigate the absorption coefficient under comparable conditions.
4.3. Shift and broadening of spectral lines

More recently, we have begun to study the spectral line shift at high neutral and electron densities by means of a ballistic compressor /54-56/.

The pressure pulse shown in Fig. 10 is generated by a fast mechanical compression of gaseous xenon. The maximum pressure reached in this pulse was 30 MPa and the pulse length was about 1 ms. The maximum temperature was 10,000 K and the corresponding electron and neutral density was $3 \times 10^{18}$ cm$^{-3}$ and $2 \times 10^{20}$ cm$^{-3}$, respectively. The electron density was not directly measured; it was calculated from compression data using an equation of state and a mass-action law according to Ebeling and co-workers /7/.

In Fig. 11, we have selected two of the strongest xenon lines which exhibit also the strongest shift. The time development of these lines was recorded by a rotating mirror spectrograph. Here photometer records are shown; the compression proceeds from bottom to top. That means the temperature and also the electron and neutral density increase in this direction. The numbers on the curves give the time before maximum compression in milliseconds.

Fig. 10: Pressure pulse connected with plasma generation in a ballistic compressor

Fig. 11: Xenon-plasma emission around 468 nm during compression
With increasing electron density, the lines become broader and the shift which is always to the red increases. At the bottom a low-pressure xenon spectrum is given as a reference. A maximum shift of about 3 nm was obtained as can be seen here. The line 473.4 nm is nearly unshifted as expected.

In Fig. 12 the measured shift in dependence upon the electron density can be seen for the line 467.1 nm. Earlier work on the shift of this special line had been carried out up to electron densities of about $10^{17}$ cm\(^{-3}\). The triangles and the circles are from Klein and Meiners /33/ and from Truong-Bach and co-workers /34/, respectively.

We extended the experimental range by more than one order of magnitude till $3 \times 10^{10}$ cm\(^{-3}\). Our measured shifts at the higher densities are below a linear extrapolation of the low-density values which should be correct for quadratic Stark effect. This deviation may stem from the plasma nonideality but till now, no theory of such effects exists. Due to the high neutral density, an influence of the neutrals on the line shift may be expected. Under the conditions near the pressure maximum, however, this influence can probably be neglected because of the neutrals' small shift constant.

Meanwhile, similar results on line shift at higher density have been reported in cesium /35/ and in argon /36/. Measurements on the same lines in xenon have been done in our institute in an electric pulse discharge, but only up to electron densities of $10^{18}$ cm\(^{-3}\) /37/. In these experiments where the electron density has been measured by laser interferometry, no deviation from a linear extrapolation could be found.

### 4.4. Plasma phase transition

Nearly 20 years ago, a plasma phase transition in a dense plasma was predicted by Norman and Starostin /38/ from a weakly ionised to a strongly ionised plasma state. To get such a transition theoretically, it is sufficient to suppose a density-dependent lowering of the ionisation energy. The parameters of this transition, however, are strongly dependent upon the...
plasma-interaction model used for their calculation. In Fig. 13 a p-v diagram of xenon is shown. Besides some isothermes /46-43/ experimental shock Hugoniots /44,45,49,50/ are shown as squares and triangles. A dielectric-to-metal transition seems to occur above 100 GPa /39-41/. The dashed area corresponds to the plasma phase transition calculated by Dienemann and co-workers /42/. These calculations are obviously - at least in part - not correct because the transition region is partly beyond the zero-Kelvin isotherme. New calculations done by Ebeling and co-workers will be published in the near future.

Provided that such a plasma phase transition exists, what can we say about possibilities to attain the critical region by experiment? In Fig. 14 a p-T phase diagram is given. It shows the three ordinary phases with the triple point in the lower left. Further, shock compression experiments can be seen starting from liquid low-temperature xenon /43-45/ and from gaseous xenon at room temperature /49,50/, respectively. The shock curves are given only schematically. At last, piston compression experiments are given and the maximum values are indicated obtained with our ballistic compressors AICA and LAICA.

The critical region for the plasma phase transition should be at about 10,000 K and some Gigapascals according to Ebeling and co-workers /51/ as well as according to Dienemann and co-workers /42/. This region is obviously free from experiments up to now. The shocks going out from gaseous xenon pass the 10,000 K below 1 GPa, the other one from liquid xenon above 10 GPa.

Starting from liquid, lower pressures at this temperature can hardly be reached. Starting from gas, very high initial pressures are necessary to attain the critical region (some 10 Megapascals). Using a ballistic compressor, initial pressures lower by about one order of magnitude are already sufficient to get this goal.

The reason for the different behaviour of a ballistic compressor in comparison to a shock tube lies in the isentropic nature of the piston compression whereas the shock compression is nonisentropic. As a result, the shock pressure is over wide ranges proportional to the temperature whereas in the piston compressor the pressure is proportional to T^{5/2}. This behaviour, of course, is modified by the real-gas properties of the investi-

Fig. 13: State diagram of xenon (p-v plane)
gated species (ionisation, nonideality).

Contrary to rare gases, the plasma phase transition in cesium seems to overlap with the ordinary liquid-gas phase transition, in the vicinity of which clusters take an essential part. Such clusters may be active also in those few cases where experimental evidence for plasma phase transitions was reported.

Till now, however, no experimental evidence for the existence of a plasma phase transition in its pure form has been found.

Fig. 14: State diagram of xenon (p-T plane)
5. Conclusions

The study of nonideal plasmas is of interest from a basic-research point of view as well as from technological aspects. In this lecture, we have restricted ourselves on the basic-research aspects. We have discussed some properties of nonideal plasmas and have tried to show how these plasmas differ from usual arc plasmas.

The thermodynamics of nonideal plasmas can be verified only by a direct measurement of the particle densities beside the temperature and the pressure. The theory is satisfactorily developed; an experimental verification requires a refinement of methods especially for the free electron determination.

Today, the theory of the transport processes has an advantage of the corresponding experiments. To create better suppositions for its verification at higher densities, the measurement error has to be reduced appreciably and more different groups should measure on the same species. The effect of a lowering of the electrical conductivity under nonideal-plasma conditions in comparison with arc plasma extrapolations appears to be established. (Therefore, the reason for the measured increase of the conductivity with increasing nonideality as reported by Vallinga at this conference /52/ is not clear.)

The radiation properties are less well studied than the transport properties. More theoretical work is necessary to get a consistent description of the plasma radiation under highly nonideal conditions and also for more complex atoms. It will be useful to simulate the whole spectrum including line and continuum radiation and compare it with corresponding measurements in an extended wavelength range. The influence of plasma nonideality on its radiation properties seems to us still an open and challenging field for experiments as well for theory.

Concerning the phase transition, its existence and its parameters have to be shown at first by theory to limit the experimental effort. Further, theory has to give us an idea how this transition can unambiguously be identified in experiment. (More recently, a behaviour similar to a plasma phase transition has been observed in a so-called Rydberg gas consisting of highly excited Cs atoms /53/.)

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ENERGY TRANSPORT IN LASER PRODUCED PLASMAS

M H Key
SERC Rutherford Appleton Laboratory
Chilton Oxfordshire England

Introduction

Transport of energy in laser produced plasmas is of considerable pure physics interest but is also of central importance in major applications of laser produced plasmas which include laser fusion [1], intense X-ray source applications [2] and X-ray laser research [3], as illustrated in Fig 1.

Fig 1 Plasma formation in major applications of laser produced plasmas (a) implosion of a spherical shell target in laser fusion, (b) intense source of X-rays on a plane target, (c) plasma at a line focus to X-ray laser research.

Fig 2 shows schematically the steady state spatial distribution of density temperature and flow velocity produced by laser irradiation of a solid surface in vacuum [4], for typical conditions of interest (Irradiance $I = 10^{14} \text{ W cm}^{-2}$, $\lambda = 1 \mu\text{m}$).

Fig 2 Schematic radial variation of plasma temperature $kT$, density $\rho$, flow Mach number $M$ and X-ray emission, for a spherical target.
The laser radiation penetrates the plasma up to the critical density and, in the plasma flow below critical density, inverse Bremsstrahlung absorption heats the thermal electrons, while non linear coupling of the laser EM wave and plasma waves creates large amplitude plasma waves whose damping generates energetic 'hot' electrons. Above the critical density and closer to the target surface a steep temperature gradient is created and heat is carried to the target by thermal electrons. This conduction zone is the main source of thermal radiation which occurs predominantly in the form of soft X-rays ($0.1 < h^* < 1keV$). A high pressure, the ablation pressure, is created in this region and reaches its maximum level at the ablation front when the flow velocity is zero. The plasma flow accelerates through the region, driven toward the vacuum by the ablation pressure, and the density decreases. The Mach number of the flow reaches unity close to the region of laser energy deposition near the critical density. Further into the target beyond the ablation front the solid is compressed behind a shock wave driven by the ablation pressure and there is additional preheating by long range hot electrons and X-rays. The separation of the critical density and ablation front is shown as $r_c - r$ in Fig 2 and sometimes termed the 'standoff' distance.

The relevance of energy transport processes to the applications illustrated in Fig 1 can be summarised as follows. In laser fusion the magnitude of the ablation pressure at the surface of the target is determined by energy transport in the surface plasma by thermal electrons and photons. The density achieved in the implosion depends on the level of preheating of the interior by energetic electrons and photons emanating from the surface plasma. The symmetry of the implosion and hence the compression ratio depend on the uniformity of the ablation pressure which is influenced by the competing processes of thermal conduction smoothing of the effects of non uniform laser intensity and the possible occurrence of thermal transport instabilities creating localised plasma jets which may enhance non uniformity.

The X-ray emission of laser produced plasmas is characterised by its uniquely high spectral brightness and small source dimensions which depend respectively on axial and transverse energy transport.

In X-ray laser research it is necessary to create plasmas of chosen density temperature and ionic composition in long cylindrical geometry with good axial uniformity. The design of experiments requires a good understanding of both axial and transverse energy transport which initiate formation of
the plasma plume. Thermal instabilities which may destroy axial uniformity are also of importance.

This paper will summarise current understanding of the energy transport by:

1. thermal electrons (in large temperature gradients such that the electron range $l_e$ may exceed the scale length of the temperature gradient $L$)

2. photons (typically thermal soft x-rays)

3. Thermal smoothing and thermal transport instabilities (the latter leading to the formation of localised plasma jets)

1) Thermal electrons

A useful model for describing heat transport by thermal electrons assumes that the laser energy is deposited at a plane in the plasma. On one side outward flow of energy advected by the plasma 'W' balances the inward flow of laser energy 'I', and on the other side, inward flow of heat carried by thermal electrons, 'S', balances the outward energy advection by the plasma 'W', the latter being continuous across the plane. Thus $I = S = W$ and we can express $W$ in terms of the flow of enthalpy and kinetic energy using the local velocity of sound $c$, density $\rho$ and Mach number $M$ giving,

$$ I = S = 3c^3\rho (0.8M + 0.2M^3) \quad (1) $$

with the first term in the bracket being enthalpy and the second kinetic energy with constants evaluated for the specific heat ratio of an ideal gas.

Heat flow cannot carry energy more rapidly than the free streaming limit for the electrons.

$$ S_M = (nkT_e) (kT_e/m_e)^{1/2} \quad (2) $$

and therefore from Eqs [1] and [2]

$$ S/S_M = 3 (m_e/2m)^{1/2} (0.8M + 0.2M^3) \quad (3) $$

assuming a fully ionised plasma with 2 amu per electron. It follows that for $M = 1$, we have $S/S_M = 0.05$. 
An important consequence, which is explained below, is that electron heat transport in laser produced plasmas cannot be described by the usual \( S = KVT \) Spitzer Harm model. The model becomes invalid \[5\] in the limit of steep temperature gradients for \( t_e/L > 10^{-2} \) where \( L = VT/T \) is the gradient scale length and \( t_e \) is the electron mean free path. This criterion can also be expressed in the form \( S/S_H > 0.02 \) or from Eq \[3\] as \( M > 0.4 \).

The Mach number at the energy deposition plane (close to critical density) is therefore an important parameter in the problem. The two limiting cases of planar and spherical geometry are useful approximations here.

In planar geometry, ie when the stand off distance \( r_c - r \) in Fig 2 is small compared to the focal spot diameter \( \phi \) or the radius of spherical target \( r \), it has been shown \[6\] that,

\[
 r_c - r = (46/\mu m) \left( I/10^{13} \text{ W cm}^{-2} \right)^{4/3} \left( \lambda/1\mu m \right)^{14/3}
\]

and that \( M = 1 \) at the energy deposition plane.

In spherical geometry (which means \( r_c - r > r \) or \( \phi \)) there is a divergence of the flow between the ablation and energy deposition region causing acceleration and a tendency to supersonic flow. To assess when the flow is best described as spherical a numerical modelling analysis \[7\] with the result,

\[
 r_c - r = 0.14r \left( I/10^{13} \text{ W cm}^{-2} \right)^{0.7} \left( \lambda/1\mu m \right)^{2.7}
\]

is useful. In spherical geometry, which from Eq 5 occurs for high irradiance, long wavelength and small focal spots, it follows that the Mach number is limited by the magnitude of the heat flow which the plasma can sustain.

It is clear that the Spitzer model is usually invalid because \( M > 0.4 \) and new theoretical treatments based on solution of the Fokker Planck equation have given a more accurate description of the heat flow \[8\]. In the most recent work \[9\] a numerical Fokker Planck solution for the heat flow has been coupled with numerical modelling of the ablation hydrodynamics for a typical spherical geometry problem, illustrated in Fig 3. Here a target of
initial radius 60μm is irradiated with laser light of wavelength 1μm at an absorbed irradiance of $1.7 \times 10^{11}$ W cm$^{-2}$. Density temperature and flow Mach number are plotted as a function of radius for steady state ablation. It is interesting to note that macroscopic quantities such as mass ablation rate $\dot{m}$ and stand off distance $(r_c - r)$ are very similar when the Fokker Planck analysis is replaced by the simple Spitzer Harm treatment. There is a significant difference however in the temperature profile which shows a local maximum at the energy deposition zone and much steeper temperature gradients driving heat flow both into the target and outwards to maintain the temperature of the adiabatically cooling diverging plasma flow. The Mach number exhibits a plateau at $M = 1.2$ which correlates through Equation [3] with the maximum heat flow $S/S_M \sim 0.06$ predicted by the Fokker Planck model. This is shown more explicitly in Fig 4 where $S/S_M$ is plotted against the local value of $L/e$ for points at intervals along the temperature profile. Maximum heat flow of about $S/S_M = 0.06$ is seen for values of $L/e/L$ ranging from $10^{-2}$ to $10^{-1}$ corresponding to fractions 1 to $10^{-1}$ respectively of the Spitzer Harm heat flow.
Thus classical energy transport by thermal electrons may give heat flow significantly less than the Spitzer Harm value for steep gradients, with saturation occurring at $S/S_{M} \sim 0.06$. This limits the flow Mach number in the region of maximum heat flow to $M \sim 1.2$. Moreover the heat flow is not a single valued function of the temperature gradient but is influenced by long range electrons from elsewhere on the temperature profile, eg at the foot of the temperature profile long range but electrons from the hot region give heat flow exceeding the Spitzer Harm value as illustrated in Fig (4).

Experiments designed to test understanding of the transport problem have typically measured parameters such as mass ablation rate, density profile, acceleration, shock speed, ablated ion velocity, laser light absorption and X-ray intensity [10].

Measurements have been compared with predictions from numerical hydro codes in which Spitzer Harm conductivity has been assumed with an imposed heat flux limit at 'f' times the free streaming limit. Many experiments have involved irradiation of plane targets with small focal spots and analysis has been complicated by edge effects [10]. Better experiments have used uniformly irradiated spherical targets [11].

The most recent experiment of the latter kind [12] gave results for mass ablation rate (measured by streak time resolved X-ray spectroscopy of the burn through of alternating thin layers of polymer and Al) and density profile (measured by short pulse interferometry) for irradiation at laser
wavelength 1.05μm, with an absorbed irradiance between $5 \times 10^{13}$ and $2 \times 10^{14}$ W cm$^{-2}$, target radius 60μm and pulse duration 1nsec. Simulation with the LASNEX code [12] and experimental data are shown in Figs [5] and [6] and data points from the above Fokker Planck calculation (scaled to the slightly different experimental conditions) are also shown.

There is good agreement between the experiment and the FP calculation (but the latter is steady state and does not include a treatment of energy transport by soft X radiation). There is agreement also with LASNEX if $f = 0.08 \pm 0.2$ and here the modelling includes both a multigroup treatment of energy transport by radiation and the explicit temporal behaviour.

It can be concluded that there is no evidence for 'strong' flux limitation of the heat flow, but rather that the result seems consistent with the classical FP calculation or with numerical modelling with $f \sim 0.08$. It should be noted however that these two models differ in their description of the temperature profile and give results which also differ from the pure Spitzer Harm calculation with no flux limit. Both models however predict $\dot{m}$ and $r_C - r$ values similar to those obtained by assuming Spitzer Harm conductivity and no flux limit. This chance occurrence can be attributed to reduced heat flow near the temperature peak and enhanced heat flow at the
foot of the temperature profile which cancel each other in their net effect on $m$ and $r_c - r$.

There are still some discrepancies between experiments however [12], which may be due to the different experimental conditions or due to experimental inaccuracies.

**Soft X-rays**

Emission of soft X radiation from the thermal conduction region (Fig 2) can carry as much as half the absorbed laser intensity for high Z targets [13] as illustrated in Fig [7]. For example a black body of temperature $kT = 100\text{eV}$ radiates $10^{13}\text{ W cm}^{-2}$ with peak spectral intensity at $h\nu \sim 3kT$. The soft X-ray source is typically optically thick for soft X-rays ($h\nu < 300\text{eV}$) and the spectrum is close to that of a Black Body [13] as illustrated in Fig [8]. The range of the soft X-ray photons in cold carbon is also shown in Fig [8] to show how the harder X-rays ($h\nu > 1\text{keV}$) cause preheating at significant depth within the target, while the soft X-rays carry energy in a diffusive fashion within the thermal conduction region of the plasma.

Energy transport by radiation may cause significant additional ablation of cool material of temperature lower than the effective black body temperature of the radiation. The lack of a description of this process in sophisticated Fokker Planck models of electron thermal conduction may be an important limitation in making comparison with experiments, except perhaps for low Z targets (see Fig 7) as in the case discussed above where radiation is a relatively small effect. Detailed calculation of the radiation effects is a difficult but developing area [14], though fairly good approximate methods are used in some codes, eg LASNEX [12].

![Graph](image)

**Fig 7** Conversion of absorbed laser radiation into X-rays for targets of varying atomic number irradiated at $5 \times 10^{14}\text{ W cm}^{-2}$ and $\lambda < 0.53\mu\text{m}$. Overall conversion efficiencies reach about 50% for angle integrated emission from high Z targets (with permission from Ref 13).
For very high Z elements the soft X-ray intensity may itself generate large ablation pressure, with several times higher pressure being obtained for a given absorbed intensity than with visible/UV laser radiation because of the cooler denser ablation, and this effect is used in some laser fusion target designs [15].

**Thermal Smoothing**

Energy transport by diffusion of thermal electrons causes a reduction in spatial non-uniformity of heat flow. The magnitude of the smoothing effect can be estimated for simple diffusion between the absorption zone near critical density and the ablation front [7] as,

$$\frac{\Delta S_a}{S} = \frac{\Delta S_c}{S} \exp \left[ - \frac{2\pi (r_c - r)}{L_p} \right]$$

(6)

where $L_p$ is the transverse spatial scale of the perturbation and $\Delta S_c/S$ and $\Delta S_a/S$ are the perturbation amplitudes at critical density and the ablation front respectively.

The stand off distance must be comparable to $L_p$ for significant smoothing and it can be estimated from Eqs [4] and [5].
There is a transient phase during build up of the stand off distance when smoothing is less effective \[16\]. Experiments have verified the behaviour predicted by Eq \[6\]. Scaling with wavelength is such that smoothing is severely reduced for short wavelengths Eq \[5\] and Eq \[6\], though experimental data \[17\] are lacking for \(\lambda < 0.35\mu m\).

**Thermal Instabilities**

Experimental observation of plasma jets in the direction of the plasma flow from high Z targets \[18,19\], as illustrated schematically in Fig \[9\], has prompted considerable interest in the stability of the energy transport process and has raised the question whether inhomogeneity in energy flow can develop spontaneously with deleterious effects in laser fusion and X-ray laser research. Possible instability mechanisms include the magnetic field generating thermal instability (occurring where \(Vn//VT\) below critical density) \[20\], the radiation cooling instability (occurring where there is strong radiation cooling at densities above critical) \[21\] and the Weibel instability \[22\] occurring where heat flow causes a directional asymmetry in the electron velocity distribution function, \(f(v_x) \neq f(v_y)\).

![Fig 9 Schematic of plasma jets created by heat flow instability in laser irradiated targets.](image)

These mechanisms have been explored theoretically, in the small amplitude linearised limit, but there has so far been no conclusive correlation of theory and experiment.

Important questions which remain to be answered include whether the jets originate at densities above or below critical and by which mechanism or mechanisms.

The large amplitude behaviour of the instabilities has not been determined either experimentally or theoretically. Does refraction of the laser radiation enhance the jets by causing self focusing? Does Nernst convection \[23\] amplify the magnetic fields around the jets? Do the B fields inhibit electron energy transport or cause pinching effects? Do the instabilities
create non uniform ablation pressure? Can they be avoided in laser fusion and X-ray laser research by choice of target materials of low Z or by other methods?

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in many experiments on REB transportation in gas the ions, accelerated in direction of the electron beam up to energies exceeding the energy of beam electrons, were observed. This effect was obtained with a good repetition in a fixed parameter range. For the first time the accelerated ions were discovered in 1968 [1]. Many experimental and theoretical investigations were devoted to this subject. The excellent review was given in ref. [2]. The main results of the experimental research can be presented as the following statements:

- the accelerated ions are observed in a relatively narrow range of pressures from 0.1 up to 1 torr;
- the acceleration takes place exclusively in the case of the beam current being over the critical value, i.e. during the electron beam injection the virtual cathode is formed by its own space charge causing a self-locking;
- one or two pulses of accelerated ions can be observed during one beam shot;
- the ion velocity in the first pulse is approximately equal or exceeds the front velocity of the REB;
- different type of ions, which take part in the acceleration process receive (in the same shot) the equal energy per nucleon, several times higher (up to 10 times) than the energy of the beam electrons;
- in a typical experiment (electron beam energy 0.5-2 MeV, current 10-30 kA, pulse duration 50 nsec) the total number of accelerated ions is about \(10^{12}-10^{13}\);
- ion acceleration is observed also in non-relativistic beams.

The discovered phenomenon has evoked great interest. Many experimental and theoretical researches were pointed to discover both the acceleration mechanism and the development of a practical acceleration method. Among the proposed models for explanation of the acceleration process the model of N. Rostoker [3] should be mentioned as a popular one. According to this model the ion acceleration occurs at the moving ionization front where the beam forms a virtual cathode. The beam space charge at the front is not compensated by the plasma ions caused by the ionization process at the front. The field of the space charge traps and accelerates the ions. If the front potential well moves with a relatively low velocity \(B\) and at the bottom of the well an electron-ion pair appears as a result of the ionization, the electron leaves the front region and the ion being trap-
ped recieves the velocity \( \sim 2B \).

Since the value of the electric potential in the well is equal to the beam electrons energy, the ion can receive the energy four times higher than the beam electrons one. This process was observed in most experiments. The serious objects to Rostokers model are advanced usually. All experimental efforts to obtain Cherenkov resonance of a potential solitary wave with the accelerated ions by means of gas pressure gradient had no positive results. The model of Rostoker [3] is not able to explain the appearance of the second pulse of accelerated ions.

In an interesting investigation [8], the ion acceleration was obtained by an electron beam injection into the gas with the nonrelativistic beam current being more than the critical one too. The acceleration was obtained with the electron energy about 1 keV and the current less than 1 A. The measurements showed convincingly that the acceleration can be realized by a quasi-stationary beam injection and the mechanism of the ion acceleration in Ref. [8] is not linked with the beam front. Other proposed models like the Putnam's one [4] (which deals with the acceleration by the REB pinching) and the similar model of Khodataev-Tsytovich [5] (which deals with the acceleration by the focusing instability of the beam) explaining the appearance of multy pulses of accelerated ions, but they are not able to explain the totality of the experimental data.

The Rostoker model was given a careful theoretical revision and its results were reviewed in [6]. The final idea is to ensure the Cherenkov resonance with an accelerated ion by control of the front velocity with an external ionization source [7]. According to [7], this method gives a possibility to accelerate ions up to energies of the order of 1 GeV per nucleon. It should be mentioned that the ion acceleration was observed in case of the REB injection into the limited plasma cloud with the current being over the critical value. In this case the ambipolar expansion of the cloud which contains beam relativistic electrons ensures the ion acceleration up to energies being higher than the beam electron energy. This mechanism which has no perspectives to obtain subrelativistic or relativistic ions will be excluded from this published work.

To solve the uncertainty in the identification of the acceleration mechanism by a REB injection into the gas, a research of the process was made with a 2D kinetic nonstationary model. This investigation deals with the Maxwell-Vlasov equation for the distribution functions of plasma and beam electrons and ions. The ionization process was taken into consideration by introducing a source of electron-ion pairs, the influx of which in a unit volume is determined by an ionization crosssection and an electron distribution function.

The previous efforts [9] to solve this problem based on a 2D Maxwell-Vlasov kinetic description
and quasi-stationary assumptions, which is not allowed in case of relativistic beams.

The performed numerical experiments correspond to the typical conditions in real experiments. At \( t=0 \) a monoenergetical beam with a homogeneous density distribution is injected through a thin metal foil into a cylindrical drift tube filled with a neutral gas. All the injected electrons have the same direction along the axis of the tube.

The computation was realized by the particle in cell method according to the program developed by V.F. Diatchenko et al. [10]. The parameters of the problem were the following: \( \gamma_0 m \omega^2 \) - the energy of the injected electrons; \( J \cdot \frac{m c^2}{4e} \) - the beam current; \( m M \) - ion mass; \( \rho \) - the concentration of the neutral gas; \( L \cdot R_T \) and \( L \cdot Z_T \) - the radius and length of the drift tube; \( c \) - is the light velocity, \( m \) - is the electron mass, \( e \) - is the electron charge, \( L \) - is the beam radius in the injection plane, \( \sigma(y) \) - is the ionization cross section. Two typical cases were investigated: the first one with \( J < J_{cr} \approx (\gamma_0^2/3) \) and the second one with \( J > J_{cr} \).

The computation showed that at \( J < J_{cr} \) after time interval higher than \( L \cdot Z_T \left( L + \frac{1}{\gamma} \sqrt{\frac{1}{p_0}} \right) \), where \( p_0 = \sqrt{\frac{e}{\gamma}} \), \( p_0 = p_0/\gamma_0 \), the system reaches a quasi-stationary state, which depends on \( \gamma \) and changes along the beam slowly. The problem was investigated for a typical case with \( J_0 = 1 \), \( \gamma_0 = 7 \), \( M = 100 \), \( \Phi = 0.01 \), \( R_T = 6 \), \( Z_T = 50 \) (the ion mass was chosen under the real one to reduce the technical difficulties in the numerical modeling).

It appears that the beam forms a self-focusing structure. In these conditions the ions are created continuously in the volume occupied by the beam and then are transported across the chamber on the surface of the drift tube. This process is due either to the electric field of his own space ion charge (which exceeds the beam space charge for low \( T \)) or to the ambipolar expansion of the created plasma (in case of the Debye radius determined by the plasma electron temperature being sufficiently small). The ion density per unit length is almost constant along the system in a quasi-stationary. The ion and electron distributions perpendicular to the beam velocity make the total electron density to be comparable to the ion density, i.e. the system can be considered as a quasi-neutral one. A rapid development of the beam instability at the short distance from the injection plate is observed in the plasma electron component. As a result of this process the dispersion of the beam distribution function by energy appears with the value near 1. This dispersion increases with the accumulated plasma density approaching the beam one. Far enough from the injection plate \( (Z = 0) \) the level of fluctuations decreases cause the large dispersion instability growth rate is small. The distribution function is common for all system electrons. Its low energy part corresponds mainly to the electrons created by an ionization process. The calculation shows that the electrons temperature is
\[ T_e = 0.2 \text{ mc}^2 \] that's why \( C_s = 0.045 \text{ c} \).

The beam instability is not observed if \( J_0 \ll 1 \) or the plasma electron concentration is below the beam concentration.

A simple hidrodinamico model for the ambipolar expansion of electrons and ions from the ionization region (taking the space distribution of the beam electrons and the temperature of the plasma electrons as the given ones) is able to realize stationary distributions of the electrons and ions created during the ionization process as well as the ion velocity profiles of expansion. These distributions depend on the parameter \( \alpha = \frac{J_0 C_s}{C_e} \), where \( C_s = \sqrt{\frac{T_e}{m_e}} \) is the ion sound velocity. The \( \alpha \) parameter is the product of the ionization frequency out the time of expansion with the ion sound velocity. For \( \alpha > 1 \) expanding ions at the distance of several radii from the beam axis receive the velocity some what higher than \( C_s \), i.e. they receive the energy some what higher than the plasma electrons temperature. The electron plasma density becomes equal or exceeds the beam density. The presence of plasma electrons in the beam can excite the electron-electron instability which development can increase the plasma electron temperature to a great extent and influence the distribution function of beam electrons.

In this numerical experiment the \( \alpha \) parameter was not fixed, i.e. the electron temperature was found after the computations. Using the computed density distributions the characteristic beam radii can be calculated. It makes possible to compare the computational results with results obtained by simple hidrodinamico model (see Fig. 1). The good data agreement allows to use this model for scaling. The realized modeling shows the ion acceleration with the current value below the critical one, occurs in radial direction up to the energy accumulated plasma electron temperature in a quasi-stationary mode. In the transient process the radial accelerated ions with the energies less than \( \frac{mc^2 J_0}{P_s} \) can be observed. It's due to the ion capture and acceleration by a uncompensated beam field with the subsequent ion liberation after the charge neutralization of the beam. Thus the beam with the current below the critical value injected into the gas doesn't generate ions with energies exceeding the electron beam ones.

Quite different process realized by a beam injection into the gas with the current being over the critical value. The computational results for a typical version with the parameters \( J_0 = 2, J_0 = 8, M = 100, R_T = 6, Z_T = 10, \Omega = 0.3 \) are shown on Fig. 2-4. The process starts with the pseudocathod creation. The space charge electric field, locking the beam, accelerates the ionization electrons back.
to the folk and repel them from
the volume. The ions are accelerated
in the direction of beam injection.
For the small $\Phi$ a quasi-stationary
ion flux is formed. The ions are ac-
celerated up to energies near the
beam electron ones. If the character-
istic time of ionization is longer
than the average time of ions leav-
ing the beam volume, i.e. the ion-
ization overtakes the ion evacuation
and the ions are able to neutralize
the beam charge. It makes the beam
possible to expand within the ion
cloud, supporting a pseudocathod at
its front, the necessary condition
this mode can be written for the
normalized concentration as follow-
ing:
$$\Phi \approx \frac{1}{\beta_0} \left( \frac{2J_0}{M} \frac{\gamma - 1}{(\gamma^{2/3} - 1)^{3/2}} \right)^{1/2}. $$
The ionization front moves together
with the pseudocathod. In case of ho-
mogeneous gas and constant injection
parameters the moving realizes with
a constant velocity. Until the pseudo-
cathod doesn’t move the ionization
process ions leave the volume by the
front boundary with the following
velocity:
$$V \approx \sqrt{\frac{2}{2}(\gamma - 1)S_0/M},$$
where $S_0$ is the distance between
the ion creation place and the pseudo-
cathod surface related to the
thickness of the pseudocathod ($0 < S_0 < 1$).
When the pseudocathod starts to mo-
ve, the ions can be divided on ref-
lected and transmitted ones depend-
ing on their creation place. The ra-
tio of the transmitted ions to the
ion total within the pseudocathod is
approximately equal to $\frac{M^{2}}{2/(\gamma - 1)} < 1$,
and they receive the velocity in the
range from 0 up to $B$. The rest of
ions - $1 - \frac{M^{2}}{2/(\gamma - 1)}$ is reflected
by the electric field of the moving
pseudocathod and accelerated up to
velocities from $2B$ up $B + \sqrt{B^{2} + 2(\gamma - 1)/M}$. Thus the moving pseudocathod creates
two groups of ions. The group of
fast ions with energies exceeding
the beam energy reaches the collector
the first; the second is the group
of slow ions with the velocity less
or equal to the pseudocathod one,
and the energies are less than the
beam electron one (Fig. 2).

For $\frac{M^{2}}{2/(\gamma - 1)} > 1$ there are no re-
lected ions and only the slow ions
group is present. The groups of fast
(1) and slow (2) ions in the pseudo-
cathod region one can see in the
phase picture in Fig. 3. The charge
of ions leaving the pseudocathod be-
hind is compensated by slow elec-
trons charge. The ion phase picture
in the pseudocathod region doesn't
qualitatively change in the range of
computations (about 6 beam radii).
The ion beam indicator has a di-
rectional diagram width about $\pm 15^\circ$,
which shows that the pseudocathod
electric field almost correspond to
the flat geometry. Computations show, that Cherenkov resonans ion acceleration by REB injection into gas with pressure gradient is possible; but very high accuracy of law $P(x)$ is needed. This accuracy was not realized in experiments in all appearance.

This interpretation explains why in real experiments the energy of accelerated ions increases with the growth of the neutral gas pressure up to the certain limit and for high pressures the acceleration mechanism disappears. The described acceleration mechanism given a complete explanation of both the measurements in a real experiment and the results of numerical modeling in 2D (and 1D for control) case. Thus the pseudocathod collective electric field is the main reason for the appearance of accelerated ions at the REB injection into gas.

As to experiments on the ion acceleration by low-voltage beam stationary injection into gas, the plasma accumulation by ionization process at low gas pressures can be stopped by an ambipolar diffusion at the plasma density level comparable with the beam density, as it was considered, and two stream instability is possible. In the case of non-relativistic beam (without strong self-focusing) the formation of structures with trapped particles at the nonlinear stage of this instability occurs. These structures are similar to the pseudocathod sequence. The energy of the ions created in those structure can exceed the electron injection one [12].

It's obvious, that by ensuring of the pseudocathod movement with a constant acceleration, which corresponds to its electric field, the increasing of the ion spending time within an active zone during the Cherenkov resonance is possible. Like in any other resonant accelerator the autophasing conditions should be fulfilled. It ensures the stable zone of ion capture and acceleration. Such conditions are fulfilled within an electric field decreasing with $Z$, which shouldn't be fast like in case of the monochromatic electron beam.

According to [7], the ionization wave with an increasing phase velocity produced by an external ionizer must be ensured. The question arises: will the pseudocathod follow the ionization wave at any velocity, will it be destroyed when leaving the injector, can front velocity reach relativistic value? Some value restrictions of the ionization wave velocity were given in Ref. [5]. It can be explained by the following arguments. The magnetic field of a self-focusing beam is characterized by its energy per unit length and is
proportional to the second power of the beam current. This field should be created by the beam propagating in the drift chamber. On the other hand the energy input as a beam kinetic energy is proportional to the first of the beam current. Thus the rate of the front movement should be lower for high currents. Since in a self-focusing state with a current being over the critical value the electrons move mainly in the transversal direction and the longitudinal component of the particle velocity is in average small (it became lower with the current increasing) the velocity of the pseudocathod couldn't be close to the light velocity. In experiments when the current value exceeds the critical one, the maximum obtained values of this velocities were not higher than 0.5-0.7 of the light velocity. Is this restriction a principal one?

To answer these questions a special investigation was performed by using 2D (in r and Z) nonstationary kinetic model in which the gas electric conductivity was determined either by external source or by the beam ionization effect.

Numerical experiments with currents being over the critical value show convincingly that the front velocity restrictions exist and the pseudocathod being far from the injection plane is not destroyed.

One of the most interesting result obtained by the numerical experiments is the phenomena of the flying pseudocathod (FPC). It appears that the beam with a current below the critical value, i.e. being incapable to form a pseudocathod in the quasi-stationary mode, forms a flying pseudocathod structure at the front when propagating in gas (Fig. 4). Contrary to the typical pseudocathod this structure appears only as a dynamical one and this is the reason to call it a FPC. The FPC can move with a velocity becoming closer to the light velocity with the ratio

\[ \frac{v}{c} = \left( \frac{\beta_0 - B}{B^2(1 - \beta_0 B)^2} \right) \]

(see Ref. [3]).

Numerical experiments with currents being over the critical value show convincingly that the front velocity restrictions exist and the pseudocathod being far from the injection plane is not destroyed.

The FPC doesn't form in any case, but in the conditions of the propagation limit velocity reaching. If the beam ionization effect is insufficient, a certain restriction for external ionizer appears: the plasma conductivity must have the
value of \(-c/4\pi L\); in the contrary case the PPC is destroyed and the self-focusing beam front moves very slowly. When the beam is able to generate the plasma conductivity with a necessary rate (for most gases it is satisfied with any current), the beam forms PPC moving with a limit velocity and independing of an external ionizer. The external ionizer can break the PPC mode, if the conductivity of plasma created before PPC, exceeds \(c/4\pi L\). In this case the external ionizer seems to be used as a stabilizing factor.

The estimations show (Fig. 6) that at the current of 10 kA and the electron energy changing during the 30 ns time interval from 1 MeV up to 5 MeV the protons with the energy of 1 GeV and acceleration length of 4 m can be realized.

The main difficulty of realization of the discussed possibility of the ion acceleration in the PPC field up to relativistic value of energy by REB injection into gas is the beam propagation instability linking with a curve deformation for example forbidden in our model. This question needs a supplementary investigation.

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AN INVESTIGATION OF THE MAGNETIC INSTABILITY OF ARCS

J. Mentel*
Allg. Elektrotechnik and Elektrooptik
Ruhr-Universität Bochum, FRG

Introduction

The physical laws which determine the behavior of an arc are well known /1/. And somebody else who has not special experience with arcs may think that an arc will be well determined by the properties of the gas and the electrodes, by the arc current, by a mass flow and by the boundary conditions.

But people who have experimental experience with arcs know that this is not true. They have observed for many times that a stationary arc well defined by boundary conditions changes into a chaotic discharge by small disturbances. As an example in Fig. 1 an unstable high current arc on a graphite cathode in nitrogen is shown. The arc is moving in front of the cathode with high velocity. The insufficient time resolved picture gives the impression of a fully turbulent discharge. Therefore in most cases special precautions are necessary to run a stationary stable arc.

In many applications arc instabilities cause problems for example in plasma processing or in arc discharge lamps /2/. But there are also cases in which arc instabilities can be helpful for example for arc quenching in a high voltage circuit breaker during current zero /3/.

Fig. 1: Picture of an unstable high current arc on a graphite cathode in N₂ taken with insufficient time resolution.

* The author appreciates support from the "Deutsche Forschungsgemeinschaft" grant Me 615/2, Me 615/6-2,3,4.
There are different reasons for the instability of an arc discharge. We can distinguish between

- electrode instabilities
- hydrodynamic instabilities
- magnetic instabilities.

Without special precautions we have a superposition of the various instabilities - as shown in the example in Fig. 1 - and a mutual amplification. The arc root instabilities are amplified by the magnetic field and perhaps also by hydrodynamic instabilities /4/.

But as we will see also a straight arc column without forced convection and protected against disturbances from the electrodes can become unstable. The only reason is the selfmagnetic field /5/. The effect of a selfmagnetic field can be enhanced by an external magnetic field in the direction of the arc axis /6,7/.

Experimental arrangement

To separate the magnetic instability from other effects we operate a long cylindrical low current arc with well-defined boundary conditions. The arc burns in a cylindrical water-cooled fused silica tube with an inner diam. of 20 mm. The arc length is 200 mm, the arc current ranges from 5 - 12 A. The electrodes are submerged in water-cooled copper tubes with an inner diam. of

Fig. 2: A low current hydrogen arc at atmospheric pressure in a water-cooled fused silica tube (earth magnetic field uncompensated) a) straight arc column at 8 A; b) helical arc column at 9.3 A.
9 mm to screen off the arc column from electrode effects /8/. For the detailed investigation H₂ was used.

Fig. 2a shows a cylindrical hydrogen arc at a current strength of 8 A. The light emitting core is just 2 mm thick. If we increase the current above a definite value - in our case 9.3 A - the arc is shaped to an helix as shown in Fig. 2b. The helix with a fixed amplitude is a new stable state. If we increase the current by a step function this stable state is achieved by a characteristic time constant. The helix can be right or left handed.

The complete experimental arrangement can be seen in Fig. 3. On the outer surface of the discharge tube a cylindrical coil is mounted by which an external axial magnetic field can be produced. The coil is powered with a fast current source.

The helix orientation and the helix amplitude is measured by a special electrooptical device. Fig. 3 shows the arrangement of the measuring heads. The device consists of two pairs of linear photodiode arrays arranged rectangular to the tube axis and one pair rectangular to the other. Onto them a magnified picture of the arc column is projected so that the x- and y-coordinates of two arc cross sections can be measured in two planes perpendicular to the tube axis.

The Helmholtz-coils provide for the compensation of the horizontal component of the earth magnetic field. Its vertical component is compensated by a small d.c. current through the cylindrical coil. More details of the ex-

Fig. 3: Complete experimental setup with the discharge tube, a cylindrical coil on the tube, Helmholtz-coils and a device for the measurement of the arc position.
Experimental set-up are given in /8/.

**Basic physical effects**

The force driving the deflection $\epsilon_c$ of the arc is the Lorentz force. A disturbance in the cylindrical current distribution sketched in Fig. 4 cannot be compensated by the gas pressure as in the cylindrical case. So a resulting force is acting transverse to the arc producing a mass flow $\rho \vec{v}_m$. This mass flow increases the arc deflection and by the deflection the force. The force can be enhanced by an axial external magnetic field which crosses with the deviation of the current density from the z-direction.

The deflection of the arc is counteracted by thermal effects which are shown in Fig. 5. On one hand the electrical field strength and therefore the electrical power input is higher at the inner side of the curved arc than at the outer side. This effect can be reduced by transparently emitted radiation.

On the other hand approaching the wall the arc is stronger cooled at the outer side than at the inner side of the curvature. This effect is increased by reabsorbed radiation.

![Fig. 4: Destabilization of the cylindrical arc by a disturbance of the cylindrical self magnetic field.](image)

![Fig. 5: Thermal stabilization of the cylindrical arc by an inhomogeneous electrical field and an inhomogeneous heat flow.](image)
Theoretical procedure

For a quantitative description of the arc instability the full set of equations has to be solved /5,8/:

the electromagnetic equations in an electrostatic approximation
the mass balance
the momentum-balance with the Lorentz force and the viscosity term
and the energy balance.

We have solved the equations by linear perturbation theory with an ansatz:

\[ A = A_0(r) + A_1. \]  

(1)

\( A_0 \) are the undisturbed cylindrical solutions. For the perturbation quantities \( A_1 \) we take in accordance to the helical shape of the unstable arc the subsequent form

\[ A_1 = \Lambda_1(r,k,t) \exp(i(\theta + k\lambda z)). \]  

(2)

\( r, \theta, z \) are cylindrical coordinates, \( t \) is the time. \( k = 2\pi/\lambda \) depends on the pitch \( \lambda \) of the helix. By this ansatz the perturbation quantities are separated in a \( r \)-dependent amplitude and in a periodic factor \( \exp(i(\theta + k\lambda z)) \).

Minus describes a right handed helix, plus a left handed helix. From this linear perturbation theory we get the stability limits and the growth rates - but not the helix amplitude.

For the moment we approximate the spatial dependent electrical conductivity in the arc by a channel of const. electrical conductivity \( \sigma_r \) with the radius

Fig. 6: Channel model for the electrical conductivity in the arc.

Fig. 7: Helical displacement of the arc.
If $T_c$ is the temperature for which the electrical conductivity in our channel model jumps from zero to a constant value the isotherme $T_c$ localizes the arc.

If we add to the temperature distribution of the cylindrical arc $T_0(r)$ the perturbation of the temperature $T_1$, this results in a displacement $\delta$ of the circular isotherme $T_c$.

$$T_0(r+\delta) + T_1(r+\delta) = T_c = T_0(r_c)$$  \hspace{1cm} (3)

$$\delta = \epsilon_c \cdot \exp i (\varphi + k z)$$  \hspace{1cm} (4)

$\epsilon_c$ is the helix amplitude

The displacement is shown in Fig. 7.

In a more general theory which is developed now the simple channel model is replaced by a multiarea model. In this case the arc position is described by a set of isotherms of a shape according to Eq. 4.

Results of theory

As a result of the perturbation theory we have found a linear differential equation for the helix amplitude $\epsilon_c$ /8/:

$$\epsilon_c \frac{\partial}{\partial r} \Delta_1 + \epsilon_c \frac{\partial}{\partial \varphi} \Delta_2 + \epsilon_c \frac{\partial}{\partial z} \Delta_3 + \epsilon_c \frac{\partial}{\partial r} \Delta_4 = 0$$  \hspace{1cm} (5)

$\Delta = \frac{\mu_0}{\eta E}$ is called Maecker's number and is the characteristic dimensionless number of the stability problem.

$$\frac{B_a}{B_s} = \Delta$$  \hspace{1cm} \text{and}  \hspace{1cm} \frac{B_s}{2\pi r_c} = \frac{\mu_0}{\eta E}$$

is the ratio of an external axial magnetic field $B_a$ to the maximum value of the selfmagnetic fields $B_s$.

$$\tau = \frac{\rho_a h_a \Delta^2 \pi}{1 E}$$ is the time constant of the instability. It is determined by the ratio of the energy content of the arc to the power input per unit length.

$\rho_a h_a$ enthalpy per unit volume at the arc axis

$\mu_0$ permeability of free space
$\eta$ viscosity
$I$ arc current
$E$ electrical field strength in the column

$$\Delta_{p} = \Delta_{p} (r_c/R, k)$$

are determinants which depend on the ratio $r_c/R$ of the channel radius $r_c$ to the tube radius $R$ and on the pitch of the helix $\lambda = 2\pi/k$. They represent:

\begin{itemize}
  \item $\Delta_1$: destabilization by the selfmagnetic field
  \item $\Delta_2$: thermal stabilization
  \item $\Delta_3$: destabilization by an external magnetic field
  \item $\Delta_4$: thermal inertia
\end{itemize}

Discussion of the solution and comparison with the experimental results

a) If we put in the $\varepsilon_c$ equation of the helix amplitude:

$$\varepsilon_c = 0; \quad FM = 0 ,$$

we get from it the marginal value of Maecker's number:

$$M_{k_c} = - \frac{\Delta_{1} (r_c/R, k)}{\Delta_{1} (r_c/R, k)}$$  \hfill (6)

Fig. 8: Marginal Maecker number in dependence on the normalized pitch $\lambda/2\pi R$.

Fig. 9: Stability limit curves for the Maecker number for different ratios of the arc radius to the tube radius $r_c/R$. 
Fig. 8 shows the course of the marginal Maecker number in dependence on the pitch \( \lambda \) for a special normalized channel radius /5/. The curve divides the \( \lambda -Mk \) plane in a region with positive growth rate-of the instability and into a region in which the corresponding disturbances are damped out. A characteristic of the curve is its minimum for a special pitch. We begin with a stable arc and approach the stability limit from below by increasing the Maecker number. This can be done for example by an increase of the arc current. Then at first the minimum of the marginal curve is reached. The abscissa of the minimum gives therefore the pitch of the instability in its starting point. The experimental pitch agrees perfectly with the theoretical result (\( \lambda_{cr} = 3.84 \) R). The Mk-value of the minimum determines the absolute stability limit of the cylindrical arc and is the critical Maecker number \( Mk_{cr} \).

The position of the marginal curves in the Mk-diagram depends on the ratio of the arc radius to the tube radius \( r_c/R \) /9/. As is shown in Fig. 9 the curves are displaced to higher values of Mk with increasing \( r_c/R \). The ratio \( r_c/R \) increases with the invested arc power \( L \).

From Mk and the minima of the marginal curves \( Mk_{cr} \) the arc current was determined in dependence on the tube radius for which the wall stabilized hydrogen arc becomes unstable /9/. The resulting curve shown in Fig. 10 agrees well with the experimental points.

b) As a next step we investigate the influence of an external axial magnetic field on arc stability:

For \( r_c = 0 \)

and a given Maecker number Mk we can evaluate a marginal external magnetic field \( FM_G \) which just destabilizes the arc:

\[
FM_G = \left( -\frac{A_\lambda}{A_t} \right) \left( 1 - \frac{Mk_G}{Mk} \right)
\]

The results for \( FM_G \) for different arc currents are shown in Fig. 11 /7/. Negative values of \( FM_G \) occur for unstable arcs. In this case the marginal field has the opposite direction as the axial component of the helix field. The minima of the curves are the critical values of the external magnetic field \( FM_{cr} \).

For \( FM_{cr} > 0 \) and \( FM > FM_{cr} \) the arc is destabilized.

For \( FM_{cr} < 0 \) and \( FM < FM_{cr} \) the arc is stabilized.

But this situation cannot be maintained for a longer time since a small disturbance causes a flip over of the helix orientation - e.g. from a right
to a left handed helix - so that the external magnetic field and the axial component of the selfmagnetic field get the same direction.

Fig. 12 shows the measured and calculated critical axial field in dependence on the arc-current \( I_a \). The agreement is good. It is remarkable that the critical magnetic field amounts only some Gauss. The measurement of the stabilizing negative values is not possible.

c) Growth rate

In a further step we investigate the growth rate of the instability. For the helix-amplitude from Eq. 5 the subsequent general expression can be derived:

$$
\epsilon_c = \epsilon_0 \exp\left[-\frac{\Delta_+}{r\Delta_2}(Mk - Mk_G)\int_0^t 1 - \frac{FM(t)}{FM_G} dt\right]
$$

(8)

The external magnetic field is taken time-dependent.

For \( FM=0 \)

we get for the growth rate of the instability the simplified expression:

$$
\Omega_0 = \frac{1}{r}( - \frac{\Delta_+}{\Delta_4})(Mk - Mk_G)
$$

(9)

In Fig. 13 the growth rate \( \Omega_0 \) for different arc currents in dependence on \( \lambda \) is shown \( /7/ \). All disturbances will be damped out if for our experimental

![Fig. 10: Current strength at the inception point of the instability in dependence on the tube radius R for a H\(_2\)-arc.](image)

![Fig. 11: Marginal external magnetic field of an hydrogen arc for different arc currents in dependence on the pitch \( \lambda \).](image)
conditions the arc current is less than 9.3 A. For higher arc currents the
growth rate curves have a positive maximum at a certain pitch. The maximum
value should be found also experimentally.

A correct measurement of the growth rate is rather difficult. The arc must
be brought suddenly from a stable to an unstable cylindrical state for
example by a current step. The time constant by which this new cylindrical
state is achieved competes with the growth rate of the instability.

But from the general expression for the helix amplitude follows that it can
be confined to small values by a time varying external magnetic field.
We have used the current source for an external magnetic field and the elec-
trooptic helix measuring device to stabilize the cylindrical arc by an
automatic control loop /10/. By switching off the loop we were able to
measure the growth rate. The results will be shown later together with other
results. For a const magnetic field

\[ F_M = \text{const} \neq 0 \]

we get for the growth rate the modified expression:

\[ \Omega_B = \Omega_c \left( 1 - \frac{F_M}{F_M^G} \right) \]  \hspace{1cm} (10) \]

For \( M_k < M_k^G \) and without an external magnetic field the growth rate is

Fig. 12: Critical external magnetic
field for a \( H_2^- \)-arc in de-
pendence on the arc current.
R=10 mm; p=1040 hPa.

Fig. 13: Growth rate of the helical
instability in dependence
on the pitch \( \lambda \) for a \( H_2^- \)
arc. R=10 mm; p=1040 hPa.
negative $\Omega_c < 0$ and $\text{FM}_G > 0$.

$$\text{FM} < \text{FM}_G - \Omega_B < 0$$

But applying a supercritical B-field:

$$\text{FM} > \text{FM}_G - \Omega_B > 0$$

the growth rate becomes positive.

For a supercritical magnetic field the growth rate can be measured relatively simple by applying a rectangular pulse of this field onto a stable arc.

Fig. 14 shows theoretical growth rate curves in dependence on $\lambda$ for three different parameter sets. The measured points demonstrate that the expected maximum values are observed indeed. Unfortunately the critical pitch does not agree as well with the theory as the growth rate itself /8/.

Fig. 15 shows the maximum values of the growth rate dependent on the arc current for an external magnetic field of $B = 10$ G, 5 G and also for the case without a magnetic field /8/. The agreement of the experimental points with the theory is pretty good. The points for zero magnetic field are measured by switching off the magnetic field of the automatic control loop stabilization.
Conclusions

We have shown that the magnetic instability is one of the sources of arc instability.

We have developed a quantitative theory of the magnetic instability and we have checked the theory by experiments under well defined conditions.

But the magnetic instability is not only of general interest. Enhanced by an external magnetic field it has found a special technical application in an arc spinner interrupter in which the arc is dismembered by the magnetic instability before current zero /11/.

Further development-investigation of the helical state

In the moment we investigate aspects of the magnetic instability which are connected with a phase transition in dissipating systems /12/.

For that purpose we have measured the helix amplitude at first in dependence on the arc current without an external magnetic field. The result is shown in Fig. 16. The curve indicates a square-root dependence \( \varepsilon \propto \sqrt{1-I_{cr}} \) which is typical for second order phase transitions with symmetry breaking. The right handed helix is labelled by a positive and the left handed helix by a negative sign of the amplitude.

We have also measured the helix amplitude in dependence on an external magnetic field. The result is presented in Fig. 17. Also in this case we have a square root relation. But the curve splits up in two branches, one for the right handed and the other for the left handed helix.

Fig. 16: Helix amplitude measured in dependence on the arc current at a H$_2$-arc.
The gap between them reduces with increasing arc current. For $I > I_c$, one branch moves over the other one so that an hysteresis curve is produced. A measured hysteresis curve is shown in Fig. 18. The flip over from one branch to the other occurs statistically. By the magnetic field the second order phase transition is converted to a first order phase transition.

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Mechanism and Elementary Processes in High Pressure Pulsed Laser

Tomoo Fujioka

IRI LASER LABORATORY

Industrial Research Institute, Japan

1201, Takada, Kashiwa-shi, Chiba-ken, 227, Japan

Introduction

Since Jabon's invention of He-Ne Laser in 1961, gas lasers have operated in very wide frequency range, from microwave to 15.2 nm in soft X-ray.\(^1\) The highest output powers recorded till now are 2.2MW in CW\(^2\) and 5kJ/pulse in pulsed mode\(^3\), both from single resonators. Many kinds of applications of gas lasers have realised both in the military and in the industry.

TEA laser invented by Beaulieu in 1969\(^4\) increased drastically the output energy of pulsed gas lasers, which operated in the discharge of the atmospheric pressure. The preionization techniques using the corona\(^5\), uv\(^6\), or X-ray\(^7\) made it more efficient.

The raregas monohalide excimer lasers invented by Ewing and Brau\(^8\) in 1975 need intrinsically the high pressures and the high input powers. So the technologies developed for the TEA CO\(_2\) lasers could apply well for them. The much effort have been paid for the development of raregas monohalide lasers and they have progressed quite well, but they are still not enough to be applicable for the industrial uses.

Since the discovery of transverse mode locking in He-Ne laser in 1968\(^9\), the author have worked in the field of gas lasers, including the invention of X-ray preionization\(^7\), the development of the highest power of pulsed gas laser (HF)\(^3\), etc.

From the standpoint of a laser scientist in this paper, the author will describe what are not known, what kinds of researches should be done, what kinds of inventions should be done, etc., in order to make the pulsed lasers, especially rare gas monohalide lasers, more useful.
The raregas monohalide lasers pumped by the electron beam

For KrF, XeCl, and XeF the kinetics are well known now, and the theoretical data can be compared with the experimental.

In the case of the electron beam pumping, the energy of the injected electron beam is absorbed in the gas. The absorbed energy are divided into the ionization energy and the metastable energy of the gases with the ratio of $1/0.33$, though the thermalization process have not known physically. The ratio is correct in the experiment and also in the Monte-Carlo simulation. The addition of the small amount of halogens don't affect to the ratio.

Many papers have been published concerning the analysis of electron beam pumped raregas monohalide lasers, but the author believe the paper by F. Kannari et.al.\textsuperscript{10} should be the most reliable. They re-checked the important rate constants, and the mixing of KrF(B,C) states and the V-T relaxation of KrF(B) were included.

The gain constant $g_0$ and the absorption constant $\mathcal{L}_a$ measured by AVCO\textsuperscript{11} are compared with the theoretical analysis by Kannari et.al in tables 1 and 2. The discrepancies between them are in 15%. In table 3 the theoretical analysis are compared with the experiments of some other laboratories, and the discircipancies are within 10% though the pulse shape of the electron beam is rectangular in the analysis.

The temporal behaviors of the gain and the output power etc. in the analysis fit well with those in the experiments. In Fig.2 the output powers

![Simplified Kinetics Diagram](image-url)
### Table 1: Comparisons of Small-Signal Gain (%/cm) between Our Code Prediction and Experimental Result Reported by Klimek et al. (see Ref.17)

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### Table 2: Comparisons of Small-Signal Absorption Coefficient (%/cm) between Our Code Prediction and Experimental Result Reported by Klimek et al. (see Ref.17)

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<thead>
<tr>
<th>( J_{eb} [\text{A/cm}^2] )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>( P_2 [%] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.6</td>
<td>0.2</td>
<td>0.6</td>
<td></td>
</tr>
<tr>
<td>0.6</td>
<td>0.7</td>
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<tr>
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<td>0.2</td>
<td></td>
</tr>
</tbody>
</table>

### Table 3: Code Prediction of the KrF Laser Oscillator

<table>
<thead>
<tr>
<th>Pulse Width (ns)</th>
<th>Gain Length (cm)</th>
<th>Total Pressure (atm)</th>
<th>Excitation Rate ((\text{MW/cm}^3))</th>
<th>( n_{\text{int}} ) (Energy)</th>
<th>( n_{\text{int}} ) (Energy)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ewing et al. [58]</td>
<td>50</td>
<td>50</td>
<td>1.5</td>
<td>0.7</td>
<td>9</td>
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<tr>
<td>Ewing et al. [58]</td>
<td>50</td>
<td>50</td>
<td>2</td>
<td>1.2</td>
<td>9</td>
</tr>
<tr>
<td>Tisone et al. [5,6,7]</td>
<td>50</td>
<td>40</td>
<td>1.3</td>
<td>1.8</td>
<td>13</td>
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<tr>
<td>Tisone et al. [5,6,7]</td>
<td>50</td>
<td>40</td>
<td>3.3</td>
<td>7</td>
<td>10</td>
</tr>
<tr>
<td>Edwards et al. [8]</td>
<td>60</td>
<td>35</td>
<td>2</td>
<td>1.5</td>
<td>11</td>
</tr>
<tr>
<td>Jacob et al. [20]</td>
<td>600</td>
<td>100</td>
<td>1.7</td>
<td>0.18</td>
<td>9(power)</td>
</tr>
</tbody>
</table>
Theoretical Analysis

The lasing output from E-beam pumped KrF laser. Kr/F\(_2\) = 99.5/0.5. Transmission of output coupler is 40%.

The efficiency of the theoretical limit

\[ \eta = \frac{h\nu}{\langle \varepsilon w \rangle} \]

is 19.2% for Ar/Kr/F\(_2\) mixture, though 21% for Kr/F\(_2\) mixture, and also because more electric power can be poured into Kr/F\(_2\) mixture even at the lower pressure by the reason of higher z-number of Kr.

The kinetics in Ar/Kr and Ar/Xe mixtures are well known. But those in Ar/Ne mixture are not well known.

The fitness between experiments and theories indicate that the kinetic data are correct. All of the theoretical analysis till now were in one dimensional. When the three dimensional simulation code is developed, more accurate comparison between experiments and theories will be done.

The ion beam pumped laser is attractive because very high power is poured without magnetic field. But the cross section of the excitation of rare gas by the collision with ions, the ionization cross section, etc. have hardly been known. And the phenomena at very high intensity excitation rate such as 100 MW/cm\(^3\), the recombination process at high electron number density for example, are also not well known.

The differences between electron-beam and TE discharge pumpings of pulsed gas lasers.

In the electron-beam pumping, the impedance change in the laser medium does not have influences on the injected e-beam energy \( e \). it is electrically separated from the circuit. So, the pressure and the mixing ratio of the medium can be widely selected. As there is no instability problem in the

As in the case of Fig. 2, Kr-rich or Ar-less mixtures for KrF lasers have been studied recently\(^{12}\), because...
electron-beam pumped laser medium, and the excitation of over 10MW/cm³ atm is attainable.

The pulse width of electron-beam pumped laser is limited by the diode short. The current $i$ of the electron beam is

$$i = \frac{4}{9} \varepsilon_0 \sqrt{\frac{2e}{m}} \sqrt{\frac{3}{2} (d-ut)^{-2}}$$

where $d$ is anode-cathode separation, $u$ is the drift velocity of ions. As $u$ is 1-2 cm/µsec the pulse width is shorter than µsecs.

Fig. 3 Comparison of Discharge Pumping and Electron Beam Pumping on Gas Lasers

In the case of the discharge the load of the electric circuit is the laser medium itself, and it is usually difficult to choose the best medium condition because of the restriction both from the circuit and the medium. The single connection point between the circuit and the laser medium also make it difficult to match the impedances of them in the discharge, on the other side two connection points make it easier in the electron beam, as shown in Fig.3.

The discharge pumping has the following difficulties
i) The uniform grow discharge is difficult throughout the large volume of high pressure.
ii) The impedance of the discharge temporally change from infinit at the start of the discharge to less than 1Ω.
iii) The impedance at the quasi steady state is as low as less than 1Ω.

iv) The plasma instability will occur when the input energy is increased.

The pre-ionization is used to solve i)5-7), and the spiker-sustainer system is proposed to solve ii)18), in which two electric circuits, one for the breakdown and the other for the sustainer, are parallelly connected to the laser medium. To solve iii), the discharge region can be divided and circuits is parallelly connected to the each divided laser medium.

The use of short input pulse seems to solve iv), but the reduction of the pulse width is inherently difficult.

The electron-beam seems to be better than the discharge. However, as the life of the thinfoil is short even with the current density of less than 500A/cm² and the repetition rate of the electron-beam is low, the electron-beam is usually not superior than the discharge for the practical use, though very high energy per pulse is obtainable with higher intrinsic efficiency.

The comparison between the theoretical analysis and the experiments in the discharge pumped raregas monohalide lasers.

In the former paragraph it is shown that the theoretical analysis of electron-beam pumped KrF laser well fit to the experiment. The situation is similar in XeCl. These mean the kinetics in KrF and XeCl lasers are roughly correct, though they may be exactly real.

The kinetics in the discharge pumped excimer lasers are the same as in the electron beam pumped ones, so that the theoretical analysis must be also fit to the experiment in the discharge. However it doesn't usually do well.

In Fig.4 and 5 the analysis and the experiments both by Keio Univ. are shown, in which the discharge volume is 450 cm³, the input density 3.5MW/cm³ through PFL with X-ray pre-ionization.

The voltage at the beginning of the discharge don't fit between the analysis and the experiment as shown in Fig.4. This discrepancy may attributed to the stray inductance, which is rather ambiguous. The laser output power fit well as shown in Fig.5, though the discripancy in the input power can be seen due to that of the voltage.

The experiments by Watanabe et.al.13) using UV pre-ionization are
The pre-ionization has a great influence on the characteristics of the high pressure pulsed lasers, so the author mentions it in the next paragraph.

The relation between the pre-ionization and the output of the discharged gas lasers.

The pre-ionization is inevitable in the high pressure pulsed gas lasers, and the electron injection, UV, and X-ray are used for it.

In the X-ray pre-ionization invented by the author et al., the number density of electrons is not so much as that in UV, but it can pre-ionize uniformly the wide volume with the changeable number density and pre-ionized region.

Compared with the analysis by Keio Univ. in Fig. 6. The output energy in the experiment, 9.3J, is not so different from that of the analysis, 8.5J, though the
The output energy and pulse width of KrF discharged laser increased with $\sqrt{n_e}$ as in Fig. 7, $n_e$ is the electron number density, as shown first by Sumida et al.\textsuperscript{14} and later by Alcock et al.\textsuperscript{15}

When the pre-ionization restrict in the part of laser medium, the lasing occur only in the pre-ionized region as shown in Fig. 8. This effect first observed in KrF by Sumida et al., and later also in XeCl by Alcock et al. The pre-ionized electron is moved by the electric field, ionizing the gas molecules in the cone-shape as shown in Fig. 9. However, as the velocity of the electrons in the excimer discharge is nearly $10^6$ cm/sec, they move only by 1mm in the 100 nsec of the discharge. In 100 nsecs the discharge can not attain to the real steady state, and the dependence of the output power on the number density
of electrons seems reasonable because the output must be dependent to the number of the cones. It seems real intuitively but this effect has not been analysed theoretically.

The region of the discharge, the lateral width of the lasing, etc. change with the changing ratio of rare gases, which are also not theoretically clear. The lateral width of the lasing is usually narrower than the apparent discharge width, which strongly influence on the amount of the impedance for the analysis.

Not only the spacial width of the lasing, but also the temporal width of the lasing can be influenced by the preionization. Taylor et al. succeeded to get 1.5 μsec length of XeCl pulse using the uniform preionization by KrF laser light.16)

We can say the discharge pumped laser has not understood well as such a simple phenomenon as the pre-
ionization effect is not analysed.

The theoretical analysis of HgBr laser considering the spacial distribution of the pre-ionized electron indicates the strong dependence of the laser output on the pre-ionization, as shown in Fig.11. 17)

Such analyses should be done also in KrF or XeCl.

Kinetics data which are not clear now

i) The accuracy of the collision crossections.

The measured crossections of the collisions are usually coincident by paper by paper, within 20-40% in the momentum exchanges, 10-20% in the ionizations, and 20-40% in the excitations. They should be more accurate.

ii) The ion recombination due to three body collisions.

The rate of the recombination due to three body collisions must decrease in the $10^{13}$-$10^{15}$/cm$^3$ regions when the plasma shielding effect is included, 19) which has not been measured experimentally. The velocity of
the reaction, such as

\[ \text{Kr}^+ + F^- + \text{Ar} \rightarrow \text{KrF}^* + \text{Ar} \]

at 10^{14}/\text{cm}^3 ion number density and 2 atm Ar, decrease by 40%, according to the theory.

Though the ion shielding effects don't change largely the excimer production efficiency, according to the analysis by Obara et al., they should be experimentally clear.

iii) The velocity of the mixing in B and C states, and the relaxation between vibrational states in KrF, XeCl, etc.

As the energy differences, for example, \( \Delta E_B, C = 80\text{cm}^{-1}, \Delta E_V = 327\text{cm}^{-1} \)

\( E_0 = 4 \) in KrF, are comparable to the kinetic energy of the room temperature (\( \approx 20\text{m}^{-1} \)), these velocities should be rapid. But they could have strong influences on the energy extraction efficiency, especially in the amplification of the short pulses. Some of the measurements have been published, such as Obara et al.'s, in which they measured V-F relaxation velocity and B-C mixing velocity from the relation between fluorescence depletion and the laser power, but much more measurements should be done.

iv) The absorption species

The non-saturable absorbers, such as Kr\(_2^*\) and Kr\(^*\) in the Kr rich mixture, have the strong effect on the extraction efficiency of raregas monohalide lasers, though the saturable absorbers don't. They have not yet been measured accurately.

v) Ne buffer mixtures

In the Ne-diluted mixture for ArF laser, the main reactions for Ne\(^*\) relaxations are,\(^{20,21}\)

\[ \text{Ne}^+ + F^- + \text{M} \rightarrow \text{Ne}^* + F + \text{M} \]

\[ \text{NeF}^* \rightarrow \text{Ne} + F + \text{hv} (108\text{nm}) \]

The branching rations between these reactions are not known.

vi) F\(_2\) laser

Many things have not known in F\(_2\) laser.

The proposed production reactions of F\(_2\)(D\(^1\)) are,
The reaction rates, branching ratios between the above reactions, etc., have not measured.

Two papers were published concerning the radiative life time of \( F_2^+ \), 3 ns in one \(^{20}\) and 30 nsec in the other paper. \(^{22}\)

The identification of the absorption species, their absorption coefficients, etc. have not also measured.

The future prospects of raregas monohalide lasers

In order to apply the raregas monohalide lasers widely for the industrial uses, at the first, the life of the circuit must be longer. That of the switch must be more than \( 10^{10} \) shots. But also, the output and the efficiency should be better by the effort of the experimenter.

According to the recent analysis by Ohwa and Obara the high efficiency can be obtained by choosing the proper gas mixing ratio, gas pressure, and excitation rate. \(^{23}\) As a example, 12.5% of intrinsic efficiency can be obtained at 3.5 MW/cm\(^3\) of 3 atm. which is three times larger than the highest experimental record, 4.4% by Long, Jr. \(^{18}\) Even with the capacitor transfer circuit, high efficiency, for example 3.3% as shown in Fig.12, is prospected.

The characteristics of the raregas monohalide lasers will be improved by the efforts of the scientists in the ionized phenomena.
Acknowledgements

The author would like to express his hearty thanks to Prof. M.Obara of Keio Univ. for his preparation of the data for this paper.

References

1. Introduction

Discharges for continuous lasers like the very well known helium-neon laser or the noble gas ion lasers traditionally use the positive column part of a low-pressure gas discharge as the active laser medium. The primary process to produce the active species is in a first step the electron-impact excitation or ionization of one species which in turn can transfer its energy to a second species which is then the lasing particle species. In the positive column the electron energy distribution has a certain shape which is determined by the longitudinal field in the (usually narrow) tube. Through this confinement alone many lasing species cannot be excited in ordinary low-pressure discharges, at least not in single-collision events by electrons.

A low pressure discharge has, however, not only the positive column as exciting and luminous region. There are parts of the discharge, especially near the cathode, which can also be used for excitation of laser active particles. The most promising part is the so-called negative glow which is separated from the cathode of the discharge by a dark space, the cathode sheath. In the negative glow ions are produced by electrons which have been accelerated through the cathode sheath in a comparatively strong field on the order of some kV/cm because the entire voltage drop between cathode and negative glow (which has a space potential of about anode potential in the absence of a positive column) occurs in this space. The consequence is an electron energy distribution which on the one hand has a high-energy tail where part of the electrons still has an appreciable (directed) energy from the sheath, but on the other hand also an increased low-energy proportion because the negative-glow plasma is a cool quasineutral
plasma in which the field strength is much lower than in the positive column. It should therefore be possible to reach higher energy states (multiple ionization + excitation in one step, e.g.) as well as populate states in which the low-energy electrons play a role (population of excited states by recombination etc.) in the negative glow.

The only drawback to use this region of the discharge is its limited extension. Whereas the positive column can be made several meters long, the negative glow is usually only a few centimeters long. This is only true in a longitudinal (tube-like) structure. To make longer negative glows, a hollow cathode may be employed. A hollow cathode is a tube of metal which serves as the cathode of a glow discharge. The positive column is usually suppressed, a wire along or near the center of the tube serving as an anode. In this case the negative glow may spread along the whole length of the tube, sustained by fast electrons streaming into the glow column radially from the wall. Such a tube like hollow cathode is well suited as a laser tube if some requirements are fulfilled. A summary on the properties and geometrical design principles is found in refs. 1 - 4.

In addition to the excitation conditions which are different between positive column and negative glow, the geometry of the hollow cathode arrangements poses some geometrically enhanced features ("Hollow cathode effect") which could be used for special laser applications. The main part of this paper will mainly deal with these features of the hollow cathode discharge.

2. Elementary processes relevant for the excitation of laser states

Among others, there are the following elementary processes which are used or might be used for the excitation of laser states in continuous laser discharges:

Excitation transfer, e.g.

\[ \text{He}^1S_0 + \text{Ne} + \text{He}^{1}S_0 + \text{Ne}^3S_2 - \Delta E \]

\[ \text{Ne}^* + \text{O}_2 + \text{Ne}^{1}S_0 + \text{O} + \text{O}^*(3p^3P) + \text{K.E.} \]

Electron impact excitation

\[ (np)^6 + e^- + \text{KE} + (np)^5 \text{ms} + e^- \]

\[ \text{md} + e^- \]
Charge neutralization

\[ A^+ + B^- \rightarrow A + B + \text{KE} \]
\[ (\text{HeNe})^+ + e^- \rightarrow \text{He}^+ + \text{Ne}(2s) \]
\[ \text{Me}^+ + 2e^- \rightarrow \text{Me}^* + e^- + \text{KE} \]

charge transfer + excitation

\[ \text{He}^+ + \text{Cu} \rightarrow \text{Cu}^{4*} + \text{He} \]

Penning ionization

\[ \text{He} 2^3S + \text{Cd} \rightarrow \text{He} + \text{Cd}^{4*} + e^- \]

Resonant excitation - energy transfer

\[ \text{He} 2^3S + \text{Kr}^+ \rightarrow \text{He} + \text{Kr}^{4*} \]

Endoergic charge transfer excitation

\[ X^* + Y + \text{K.E.} \rightarrow X + Y^{4*} \]

electron impact ionization + excitation

\[ e^- + X \rightarrow X^{n+1} + (n+1)e^- \]

In the following, characteristics and features of the hollow cathode discharge will be investigated, especially concerning their usability for these elementary processes as laser excitation mechanisms.

3. Laser relevant properties of hollow cathode discharges.

a) Electron energy distribution.

The electron energy distribution function differs in both the density of fast electrons streaming in from the wall of the cathode and the density of the very slow, "plasma" electrons from the distribution function of a positive column. Generally both parts are higher in the hollow cathode discharge, favoring the population of high-energy states by fast electrons as well as the population of excited states by recombination /5,6/.
b) Geometry effects:

Talking mostly about cylindrical hollow cathodes, there are the following properties which were observed in hollow cathodes:

b.1. "Pendulum" electrons.

At low pressure electrons starting from the cathode walls can traverse the negative glow more than once, if they lose little energy in collisions. At too low pressures, however, the positive energy of the electrons leaving the surface leads to their absorption on the opposite wall if there are no collisions in the volume. This seems to be a criterion of the low-pressure operating limit of hollow cathode discharges /7/.

b.2. Density "singularity" on the axis.

Electrons released from a cylindrical hollow cathode and accelerated by a radial field toward the axis must have a density increase toward the axis of the cylinder (mathematically a singularity with infinitely high density proportional to 1/r with r → 0). Only the very fast electrons that have never undergone any collisions show this effect. It is most easily studied by measuring the density distribution of multiply charged ions mass-spectrometrically /8/ or light-spectroscopically /9/. Excited highly charged ions show the density distribution of the next higher charge state in the ground state.

Two examples for highly charged krypton ions are shown in Figs. 1 and 2. Since for optically emitting states the excitation cross section increases with energy and the excitation rate with energy and density, the geometrical density increase of the fastest electrons plays a role in the excitation of laser states in ionic lasers. The effect is especially important in high-voltage hollow cathode arrangements as those proposed by K. Rózsa /1/.
Fig. 1. Radial density distributions of the individual ions in the negative glow of the hollow cathode discharge (r = 10 mm). The densities are not drawn to scale. Discharge pressure 0.046 Torr, discharge current 6 mA.

Fig. 2. Radial intensity profiles of the Kr I, Kr II and Kr III lines in a Kr hollow cathode discharge (radius 1 cm, length 4 cm). Discharge pressure 0.07 Torr, discharge current 15 mA.
b.3. Slow plasma electrons

The plasma electrons may be useful for excitation of laser states by recombination. The population of excited states by dielectronic recombination of helium ions in a helium hollow cathode discharge at pressures around 4 torr has been shown to play a role near the axis /6/. Inversion, however, is only possible for e.g., metal ions which must be introduced into the discharge, thereby affecting the energy distribution /5/. Radially accelerated ions sputter neutral metal particles from the cathode wall, which in turn are transported into the interior of the discharge. Penning ionization and charge transfer are important excitation mechanism for metal ion laser states. A number of metal lines have been shown to lase in such hollow cathode arrangements /1/, /2/. Cd II, Zn II, I II, Hg II, As II and TI II lines are some examples.

b.4. Ion-impact excitation

There are two possibilities for ion impact excitation in hollow cathode lasers:

The ion-atom or ion-molecule reaction of ions with neutral partners in the negative glow where both partners have essentially thermal energies, e.g.

\[
\text{He}^+ + \text{Cu} \rightarrow \text{Cu}^{++} + \text{He}
\]

and the exoergic or endoergic charge-transfer excitation, e.g.

\[
\text{He}^+ + \text{Xe} \rightarrow \text{Xe}^{+*} \left(^{4}D_{5/2}\right) + \text{He} + 0.63 \text{ eV};
\]

\[
\text{He}^+ + \text{N}^{(4S)} \rightarrow \text{N}^{+} \left(2p^3 3D_j^o\right) - 1.4 \text{ eV}
\]

where the ions must have appreciable energies. The former process is presumably used in metal-vapour lasers of the type He-Cu, He-Ag, He-Au for the excitation of metal-ion laser states in the ultraviolet region.

The latter excitation may only be useful in the sheath part of the discharge: Here the ions can have energies in the range of some eV up to maybe 100 eV. The charge transfer excitation cross section for endoergic processes has generally a shape which can be characterized by a threshold near the value of the endoergicity, a steep increase to a maximum value.
around 50 - 100 times the threshold energy and a slow decline \cite{10}. Similar curves are obtained for exoergic ion-atom collisions. Ions having energies of about 100 eV should be able to excite Xe II lines in an appreciable intensity in the sheath of a low pressure hollow cathode discharge. That this is so is visible from Fig. 3: here, the excitation of the $\text{Xe}^+ 4D_{5/2}$ state emitting the 1048 - 1052 Å line is shown in a pure xenon discharge, where only electron impact excitation is possible, and in a helium discharge containing 3% of xenon. In the latter case, the electron impact excitation is still dominant around the axis, in the middle of the negative glow, but now additional maxima appear in the region of the sheath or "dark space" which in the light of the line now appears as "bright" \cite{9}. Many examples in noble gas - discharges containing traces of other atomic and molecular gases \cite{11} show that the ion-impact charge exchange excitation

Fig. 3. Radial intensity profile of the XeII lines 1048 + 1052 Å in
a) pure xenon, b) a mixture of 3% Xe and 97% He. The peak around 8 mm in He-Xe is due to XeII excitation by He$^+$ ion impact. Discharge current 15 mA, total pressure a) 0.12 Torr, b) 0.3 Torr.
peaks in the sheath if the endoergicity is 0.1 - 1 eV. An impressive result is the excitation of the N II 1084-1086 Å line in helium-nitrogen discharge where the electron impact excitation in the negative glow plays virtually no role as compared with the charge-exchange excitation in the sheath (Fig. 4). Probably reaction of helium ions with atomic nitrogen into the $^3D^0_j$ state emitting 1084-86 photons is the dominant process here. The endoergicity is 1.4 eV which is just appropriate to yield peak emission for 100 eV helium ions /12/.

Fig. 4. Radial scan of the intensity of the NII 1084 + 1086 Å line ($2p^3^3D^0_j$) across the hollow cathode discharge. Radius of the hollow cathode: 1 cm, length 4 cm, discharge pressure (He + 4% N$_2$): 0.46 torr, discharge current 15 mA, resolution 20 Å. Glow edge at 6 mm.
b.5. Negative ions.

Negative ions in hollow cathode discharges were shown to be found at particular places inside the hollow cathode discharge in oxygen and hydrogen discharges /13,14/. They have their density maximum near the glow edge, well separated from the density maximum of the positive ions. Obviously the potential distribution in the hollow cathode discharge forms a deep well for the negative ions between the axis (where the positive ions are kept by a small well) and the sheath (steep voltage increase toward the wall). Positive-negative ion recombination which might be used for laser excitation thus might take place in a hollow cylinder near the glow edge. No attempt has hitherto been made to explore laser action by this process.

4. Conclusions

Continuous laser discharges are studied now for more than 20 years, with increasing effort during the last 10 years. Despite the variety of the excitation processes, a surprisingly small number of design principles have reached technical maturity. Among the least practically applied structures are hollow cathode discharges, although a number of laboratories has undertaken basic studies and shown the feasibility for a number of excitation schemes. The present paper has shown that even now a number of excitation mechanism could be explored which have not been used hitherto and may give promising results in wavelength regions which are usually not easily approached in conventional designs. For the application of these schemes, however, new design geometries must be developed. For endothermic charge transfer excitation in the sheath of a cylindrical hollow cathode, e.g., a hollow laser beam would result which is undesirable. Concentration of the sheath region into a filament structure is desired there. Similar principles may be wanted for recombination or charge neutralization lasers. Despite numerous excitation mechanism which may be used in hollow cathode configurations for laser action, the practicable design including lifetime considerations, resonator structures etc. needs still a lot of effort from practical laser research and development laboratories.
5. Acknowledgements

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In arc-heated chambers with longitudinal or transversal forced air cooling of the electric arc, turbulent heat and mass transfer exerts a substantial, and sometimes determining influence on the form of the arc and its energetic characteristics. Turbulence may either occur during the dynamic interaction of a gas flow with the arc column, i.e. with the current-conducting zone, or appear in the current conducting zone itself due to the possibility of the existence of magnetohydrodynamic instabilities.

There are a number of theoretical and experimental studies [1-4] which consider different types of electric-arc discharges in a turbulent flow and discuss the mechanism of turbulence. The physics of its occurrence in the presence of the electric discharge is extremely complicated. This seems to account for the fact that so far there is no satisfactory model of the turbulent arc and the number of experimental investigations in this field is very limited.

The present work attempts to elucidate the simplest case, i.e. the electric arc burning in a cylindrical channel in the developed region of a gas flow. The interest to such longitudinally cooled arcs is stimulated by several facts. First of all, it is the availability of an extensive class of electric-arc heaters with the longitudinally cooled arc which are used in industry. Besides, the sum of the experimental data available shows that the appearance of turbulence in the current-conducting zone is determined by the turbulent boundary layer formed on the channel wall and propagating to the axis. In other words, the nature of turbulence in this case is purely hydrodynamical and its other possible mechanisms (e.g., magnetohydrodynamical) do not seem to be of particular importance. Initial turbulence arising at the entrance of the flow into the channel can be either removed or reduced to a minimum by damping devices and other methods.

In a cylindrical channel with the longitudinally cooled arc, three flow regions can be observed, i.e. initial, transitional and developed tur-
bulent flow regions (fig. 1).

The initial region can be characterized by the axisymmetric time-stable form of the arc column and by the electric-field intensity which is practically constant along the latter. The heat transfer in the arc column itself is essentially determined by radiation transport at sufficiently high currents (> 100 A). The heat flux attaining the channel wall is determined by the arc-column radiation practically completely.

The transitional region can be characterized by the appearance and increase of downstream oscillations of the arc column as a whole and by the destruction of the axisymmetric character of the column due to the penetration of turbulent fluctuations into the current-conducting part of the flow. All this determines the electric-field intensity increase along the length of the transitional region and the increase of the heat flux into the channel wall. The transitional-region length in the presence of the electric arc amounts to 5-10 gauges in all, whereas in the case of a common gas or liquid flow its length amounts to a few tens of gauges.

Fig. 1. Schematic diagram of flow development in a cylindrical channel with an arc (above). Voltage distribution of electric field and structure of arc column along the length of the channel (below).
In the developed flow-region the electric-arc intensity does not vary with respect to length. The arc form acquires a complex structure varying in time and space. However, as will be shown later, the deviations of the arc in the direction of the periphery are limited to the radius which is substantially smaller than the radius of the channel. The heat flux into the channel wall (the other parameters of the discharge being constant), after having attained a certain maximum value corresponding to the zero value of the local thermal efficiency also changes only slightly.

The above characteristic of the gas flow development in a channel with the arc is illustrated by fig. 1 (flow diagram). Fig. 1 shows changes in the electric-field intensity along the length of the channel and a change in the arc form in different flow regions, respectively. The experiments were performed in the air. The intensity of the electric field, \( E \), was measured by the washer technique. The variations in the free aperture were followed by means of high-speed photoregistration with the time resolution of \( 1 \times 10^{-6} \) s.

Processing of photoregistrograms allowed us to determine fluctuations of the optically revealed arc form in the radial direction. The value of these fluctuations turned out to be at the level of 4-5%. Velocity fluctuations in the vicinity of the channel axis in a developed turbulent flow are rather isotropic and their value lies in the same range. Thence one can conclude that fluctuations in the arc form are due to velocity fluctuations determined by gasdynamic turbulence.

In the developed turbulent region the heat transfer in the current-conducting part of the arc is determined by turbulent heat-conductivity, and the model according to which one can calculate the volt-ampere characteristic sufficiently enough is given below. For the region described the heat flux on the channel wall is a sum of the convective heat flux \( Q_c \), determined by turbulent heat transfer, and the radiation heat flux \( Q_r \). The complete heat flux to the wall may be assumed to be an additive value, i.e. \( Q = Q_c + Q_r \), which allows us to easily enough determine the convective component. Experimental data processing showed that convective heat
Fig. 2. Curves for a developed turbulent channel flow: a - dependence of Stenton number on Reynolds number in a developed turbulent region; $I = 50-200 \text{ A}$. b - radiation intensity distribution of turbulent air arc, continuum $\lambda = 393 \text{ nm}$, $d = 2 \times 10^{-2} \text{ m}$.  
1, 2, 4 $Re = 3, 2 \times 10^4$ $I = 50,100,140 \text{ A}$;  
3, 5 $Re = 7,45 \times 10^4$ $I = 100,140 \text{ A}$;  
c - radial distribution of turbulent viscosity coefficient $\epsilon$, and spatial correlation coefficient of longitudinal velocity fluctuations $R(r)$, for a developed turbulent tube flow.

transfer in a developed turbulent flow region with the arc can be described to a great accuracy, by the convective heat transfer equation for turbulent flow under common conditions, i.e. the influence of the arc is practically lacking.

Fig. 2a shows the dependence of the Stenton number, $St = \frac{Q_e}{\rho u h}$, on the Reynolds number, $Re = \frac{\rho u d}{\mu}$, which is satisfactorily described by the equation $St = 0.021Re^{-0.2}$ typical of common turbulent heat transfer in tubes. Here $\rho$ is the density, $u$ is the velocity, $h$ is the enthalpy, $\mu$ is the viscosity, $d$ is the channel diameter. All the values in $St$ and $Re$ are average with respect to mass.

Such a character of the convective heat transfer on the channel wall should naturally be determined by the structure of the entire flow. Let us determine the size of the relation of the current-conducting zone or the
boundary of the current-conducting part deviation under the influence of turbulence during the time which is substantially greater than the characteristic time of the fluctuation motion. To this effect, we registered, with the help of a prism spectrograph, the intensity distribution of the continuum radiation at the wavelength of $\lambda =393$ nm along the channel radius. The radial distribution of the radiation was recorded during $8$ s, and the characteristic time of the fluctuation motion being $10^{-4}-10^{-5}$ s. The results of these measurements are presented in fig. 2c. Within the investigated range of arc currents and Reynolds numbers, the registered deviation radius is not beyond the scope of $r = \frac{2r}{d} \leq 0.5$. Let us attempt to relate the obtained deviation value (i.e. the averaged value of the current-conducting arc zone) to the characteristic features of the developed turbulent tube flow. Fig. 2c shows distributions along the tube radius of the dimensionless turbulent viscosity coefficient $E_T$ and of the spatial correlation coefficient $R(r)$ characterizing the turbulence structure. In this case it is a measure of existence of large-scale rolls $\left[5\right]$.

Comparison of the curves given in figs. 2b and 2c allow us to conclude that the current-conducting arc zone in the developed turbulent flow region lies in the near-axis region of the tube. The dimensions of this zone in the radial direction are limited to the area of existence of large-scale rolls and are not beyond the maximum position of the turbulent transfer coefficient. Further, up to the tube channel wall, the structure of the flow hardly differs substantially from the structure of a common turbulent flow. This very fact determines the convective heat transfer law corresponding to the heat transfer in the common turbulent flow. Therefore one can affirm that the stabilization mechanism of the electric arc near the axis of the cylindrical channel is determined by the structure of the turbulent flow.

Among all the theoretical models of the turbulent electric arc, the
simplest model which conforms best with experiment is a "channel" data model suggested by the authors of ref. [7]. It concerns the coincidence of calculated and experimental volt-ampere characteristics within a wide range of the turbulent-arc parameters. The gist of the model (fig. 3a) is as follows. The turbulent flow is divided into two areas, i.e., the core and the laminar sublayer of thickness $d_L$. The turbulent heat-conductivity $k_T$ of the core is much greater than the molecular one $k$, by virtue of which the temperature and velocity gradients are assumed to be equal to zero. The flow core is the current-conducting zone with the constant temperature $T_0$. The temperature and velocity gradients in the laminar sublayer are assumed to be constant and the sublayer itself is a non-conducting zone and its thickness is found from its stability criterion [8]. Radiation energy transfer is not taken into account. In this case, in calculating the volt-ampere characteristics of the turbulent arc we get a simple analytical ex-

Fig. 3. Comparison of calculated and experimental volt-ampere characteristics of a discharge; a - model of "channel" turbulent arc. b - argon: $P = 0.1$ MPa, $d = 1 \cdot 10^{-2}$ m, $x - I = 50$ A; $P = 1.1$ MPa, $d = 0.7 \cdot 10^{-2}$ m, $+ - I = 100$ A, $\Delta = 200$ A, $- - 300$ A. c - air: $I = 50-500$ A, $P = 0.1-2$ MPa, $d = (1-3) \cdot 10^{-2}$ m, $x - Re = (3-7) \cdot 10^4$; $c - Re = 1 \cdot 10^5$; $\Delta = 5 \cdot 10^5$. 
Comparison of figs. 1, 2b and 3a shows that the "channel" model of the turbulent arc and the real picture of the physical processes in the electro-discharge chamber are rather different. The arc temperature $T_e$ calculated according to the "channel" model for those parameters which are shown in fig. 3b ranges within $6 \cdot 10^3 < T_e < 10^4$ K in all. The radiation flux $Q_r$ calculated according to this model per unit length for the spectrum region $\Delta \lambda = 200-2000$ nm is much smaller than that measured in the experiment. E.g., in the case of the air, at $d = 2$ cm, $Re = 6.4 \cdot 10^4$, $I = 140$ A the experimental value $E = 27.8$ W/cm, $Q_r = 60$ W/cm, whereas the calculation according to the "channel" model yields $Q_r = 4$ W/cm.

Having determined the rectangular temperature profile (as in fig. 3a) from the condition $I = \frac{\pi}{4} \frac{\partial^2}{\partial T^2} GE$ and $Q_r = \frac{\pi}{4} d_T^2 \delta$ on the basis of the well-known function for the conductivity coefficients, $\delta$, and the volume radiation $\delta$ ($\Delta \lambda = 200-2000$ nm) of the temperature, we will determine the averaged diameter of the current zone $d_T$ and the temperature by the successive approximation method. For the above conditions this estimate yields $T_e = 12000$ K and $d_T = 0.183 \frac{d}{2}$. The obtained value of is close to the dimensions of optical heterogeneities (see fig. 1).

It should also be noted that the characteristic length of turbulent mol determined as $L = \int \frac{R(r)}{d} d' \ (\text{see fig. 2c})$ for a tube [5] amounts to $L = 0.14 \frac{d}{2}$.

The similarity of the volt-ampere characteristics according to the "channel" model and obtained in the experiments does not seem to be arbitrary and reflects the physics of the process. Ref. [7] points out that in case when heat conductivity is dominant, the dimensions of the conducting zone do not affect the volt-ampere characteristic of the discharge. Besides, it is possible that the heat transfer between the turbulent current structures and the non-conducting zone is determined by the thermal resist...
tance of the thin layer similar to the viscous sublayer on the wall where the molecular heat conductivity of the latter is dominant.

In general the totality of the above data gives us a rather phenomenological idea about the turbulent arc. However, these data may be a basis for real estimates of the processes in high-temperature turbulent flows.

One of the important moments in turbulent-arc physics may be radiation-turbulence interaction which manifests itself in the fact that the radiation heat transfer between turbulent molra and the radiation of high-temperature molra lead to an equalization of temperature fields, reduction of temperature fluctuations, respective reduction of turbulent energy transfer and redistribution of electric-field intensity.

As an example of numerical calculations of the heat transfer in the flow of the electric-arc argon plasma in a cylindrical channel without swirl, one can show the influence of radiation on the turbulent characteristics of the medium and on the intensity of the electric field. To this effect, into the system of equations describing the stationary electric arc in a turbulent flow as

\[ \rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial r} = -\frac{\partial P}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r} \left[ \nu \left( \frac{\partial u}{\partial r} \right)^2 \right], \]  

\[ \frac{\partial}{\partial x} (\rho \rho u) + \frac{\partial}{\partial r} (\rho \rho v) = 0, \]  

\[ \rho u \frac{\partial T}{\partial x} + \rho v c_v \frac{\partial T}{\partial r} = \frac{1}{r} \frac{\partial}{\partial r} \left[ \nu \left( \frac{\partial T}{\partial r} \right)^2 \right] + \sigma B^2 - div \vec{E} \times \vec{B}, \]  

\[ P = \rho RT, \]  

\[ j = \sigma E + \sigma \vec{E} \times \vec{B}, \]  

\[ G = G_0 \rho \int_0^r \omega u r dr. \]

we introduce, in accordance with [9], the turbulent Prandtl number

\[ Pr_t = \left( \frac{\nu}{\nu} \right) \left[ \frac{1}{2 \beta Pr} - \sqrt{\left( \frac{1}{2 \beta Pr} \right)^2 \left( \frac{\nu}{\nu} \right)} \right]^{1/3}. \]

In relation (7), we used, as the Prandtl criterion, its efficient value determined by taking into account radiation:
Here $\alpha$ is the temperature-conductivity coefficient, $\alpha_\nu$ is the spectral absorption factor, $B_\nu$ is the spectral intensity of the blackbody, $I_\nu$ is the spectral intensity of the radiation, $\nu$ is the frequency and $\alpha, \beta, \Lambda, \nu_\tau$ are empirical constants.

The following empirical formula for steady-state tube flows is accepted as an expression to calculate the turbulence scale:

$$\ell/R = 0.14 - 0.08 (r/R)^2 - 0.06 (r/R)^4$$

To more correctly determine the interaction effect, the radiation component in the energy equation was calculated taking into account two-dimensional radiation transfer [10]:

$$\text{div} \frac{\mathbf{E} \xi}{\delta} = \int \left\{ 4 \alpha_\nu B_\nu - \alpha_\nu \int \alpha_\nu B_\nu \, r' \int \exp[-\tau(s)] \, d\nu \, dr' \, \frac{dx}{s^2} - 
$$

$$- \alpha_\nu \int \int \exp[-\tau(s_{\nu})] \left( R - r \cos \psi \right) R \, d\nu \, dx \right\} \, d\nu$$

where $s$ is the coordinate along the optical beam, $x$ is the longitudinal coordinate, $\psi$ is the angle, $\tau$ is the optical thickness, $I_{c\nu w}$ is the spectral intensity of the wall self-radiation.

The spectral absorption factor $\alpha_\nu$ was calculated for the lines and the continuous spectrum.

In the wall region the turbulent viscosity factor was found from the relation suggested by Deissler [11]

$$\mu_\tau / \mu = 0.154 uy [1 - \exp(0.0154 uy)] , \, 0 < \tau < 50$$

where $\tau = y (\tau_w / \omega_w)$, $y = \frac{R - r}{\omega_w}$, $\omega_w = \mu_w (\frac{\partial u}{\partial r})_w$ and in the flow core from the Reichardt formula [12]:

$$\mu_\tau / \mu = 0.133 \nu [0.5 + (\frac{r}{R})^2] \left[ 1 - (\frac{r}{R})^2 \right]$$
To study the influence of radiation on turbulence it is impossible without taking into account fluctuations of the electrodynamical quantities.

Temperature fluctuations in electric arc burning in turbulent flow result in growing of electron density fluctuations which, in their turn, influence the transfer coefficients and the conductivity in particular. The value of the electric-field intensity, with allowance for temperature fluctuations, is determined, according to [14], by the following expression:

\[ E = \frac{d}{\delta t} \left( 1 + \frac{v_{ic}}{v_{ic}} \right) \left[ \exp\left( \frac{v_{ic}^2}{2} \right) - 1 \right] ^{\frac{1}{2}}, \]

where \( E \) is the electric-field intensity, \( \delta t \) is the frequency of collisions of an electron with an atom and \( v_{ic} \) is the ionization potential. The system of equations (1)-(6) was solved by the pass method in combination with the iteration technique.

The analysis of the calculated results showed that the value of the turbulent heat conductivity coefficient, \( k_{tr} = \frac{\mu_p \sigma_p}{Pr} \), diminishes under the influence of the interaction of radiation with turbulent energy transfer. In case when the current in a 1·10^{-2} m dia. channel is 250 A and the flow rate is 3 m/s, the decrease in the turbulent heat conductivity amounts to \( \approx 12\% \) (Fig. 4).

The electric-field intensity (Fig. 5) calculated without taking...
into account conductivity fluctuations (curve 1) grows monotonously along
the length of the channel and the value $E$ determined with allowance for
temperature fluctuations (curve 2) shows a pronounced maximum in the tran-
sitional flow region (10 gauges). While passing to the developed flow, the
turbulent energy transfer, temperature fluctuations and level of the elec-
tric-field intensity decrease as a result of radiation. The obtained cal-
culated values are in agreement with the results of ref. [13] for the
cooled arc without swirl and with the experimental values for smaller di-
ensions of the channel [14].

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the interaction of radiation with turbulent temperature fluctuations


Formation of Large Electrostatic Potential Due to High-Speed Plasma Flow Along Magnetic Field Lines in Space Plasmas

Haruichi Washimi
The Research Institute of Atmospherics, Nagoya University, Toyokawa 442, Japan

1. Introduction

Formation of energetic electrons of several to ten KeV in auroral flares or in solar flares is one of the most challenging problems in space plasma physics. In this paper the formation of large-scale and large-amplitude electrostatic potential along magnetic field lines is discussed. In auroral plasma the potential increases over a very long distance of about several $10^4$ km along the magnetic field lines between the plasmasheet of the magnetotail at the equatorial region and the topside ionosphere at the polar region. It is considered from observations that the potential increases very slowly near the plasmasheet and abruptly at about several to ten $10^3$ km altitude near the topside ionosphere where the density is of the order of $10^2$/cc.

Recently there have been many works on the double layer both by experiment and by computer simulation. In most of them, the large electrostatic potential difference is applied between anode and cathode, or some special velocity distributions of charged particles, such as an energetic electron beam, are assumed for the initial conditions and then the potential formation at the middle part of plasma have been discussed. But in space plasma these potential differences or specific velocity distributions seem not to be causes but results. Therefore most of the studies by experiment and by computer simulation so far done are not directly applicable to the space problems.

One of the promising models for the potential formation in flare region may be the magnetic mirroring model. It has been shown in a simplified model that the discrepancy of the pitch angle distributions between ions and electrons results in a potential difference along the field lines between the plasmasheet in the equatorial region and the topside ionosphere in the polar region (Alfvén and Fälthammar 1963). A similar result has been obtained by Persson (1963). A self-consistent analysis of the potential has been studied (Chiu and Schulz 1978) under a condition of quasi-neutrality. Assuming an anisotropic distribution for the electrons at the plasmasheet and taking into account the ionospheric ions and electrons as well as trapped electrons, the potential profile between the plasmasheet and the topside ionosphere was estimated. However, their self-consistent analysis seems to have several difficulties in explaining observations both on the energetic electrons of several to ten KeV as well as on the potential profile which sharply increases very near the topside ionosphere.
Recently the Petcheck-type first reconnection which accompanies high-speed plasma flow of the order of Alfvén speed almost along the field lines at the plasmasheet has been confirmed by computer simulation (Sato 1979). Subsequently Serizawa and Sato (1984) have studied the potential difference due to the plasma flow by estimating ion and electron influxes under a currentless condition. They showed that the potential difference, $\psi_{SS}$, is

$$
\psi_{SS} = \frac{1}{e} \frac{W_0}{1 + T_{O1}/T_{Oe}}.
$$

where $T_{O1}$ and $T_{Oe}$ are temperatures of ions and electrons, respectively, and $W_0$ is the kinetic energy of the flow at the plasmasheet. Ions have the main part of the high-speed flow energy at the plasmasheet, but eq. (1) shows that in the course of the potential formation, electrons are accelerated and gain $\varepsilon\psi_{SS}$ of the system energy. For typical parameters at the plasmasheet both in the terrestrial magnetosphere and in the solar atmosphere (Fig. 1), we can expect the flow speed to be driven by a reconnection process to the order of several $10^3$ km/s, which corresponds to a flow energy of the order of 10 keV.

By putting adequate electrons which are trapped between magnetic and electrostatic potentials, the potential profile has been discussed by Washimi and Katanuma (1985). This paper is based on their work. In section 2 the basic process of the potential formation due to high-speed flow is discussed. In section 3 the method of analysis is shown. The potential difference is discussed in section 5 and the potential profile is discussed in section 6.

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Fig. 1 (a) Terrestrial magnetosphere and (b) solar atmosphere.
2. Basic Process of Potential Formation due to High-Speed Flow

As is well-known, particles which move toward a strong field region along the field lines with large pitch angle are reflected by the mirror effect which tends to decrease the density of the particles, but on the other hand, particles of small pitch angle are not reflected but can go into the region of strong field which tends to increase the density. In general, the density of a given species is a function of the mirror ratio, $\gamma$. For particles with an isotropic velocity distribution, the reflection and the collection effects are just balanced and the density is constant along the magnetic field lines. For particles with a highly shifted distribution in the parallel direction, only the collection effect works and the density increases with $\gamma$ when $\gamma$ is not large. But because the pitch angle of each particle increases with $\gamma$, the density is saturated at some value when $\gamma$ is large. Because of the great difference in the thermal speeds of ions and electrons of almost the same temperature, the fast plasma flow is composed of ions of highly shifted distribution and electrons of almost isotropic distribution. Thus the density of ions tends to increase with $\gamma$ while the density of electrons tends to be constant. But due to the condition of quasi-neutrality, a potential is formed. This potential reflects some of ions and accelerates electrons so that the quasi-neutrality is satisfied.

The equation of motion for a particle along a field line is,

$$m_j v_j \frac{dv_j}{dx} = - \frac{dB}{dx} - e_j \frac{d\phi}{dx}, \quad (\text{for } j = i \text{ and } e) \quad (2)$$

where $m_j$ is the mass, $v_j$ the velocity, $B$ the magnetic field, $\phi$ the potential, $\mu_j = (1/2)mv_j B$ the magnetic moment and $dx$ the line element. Alfvén and Fälthammar assumed a simple relation, $(dv_{i\parallel}/dx)/v_{i\parallel} = (dv_{e\parallel}/dx)/v_{e\parallel}$ as a condition of quasi-neutrality, which results in the relation between $\phi$ and $B$,

$$\frac{d\phi}{dx} = K \frac{dB}{dx} \quad (3)$$

where $K$ is,

$$K = \frac{1}{e} \frac{\nu_e W_{i\parallel} - \nu_i W_{e\parallel}}{W_{i\parallel} + W_{e\parallel}} \quad (4)$$

Here $W_{j\parallel} = (1/2)mv_{j\parallel}^2$ and $W_{j\perp} = \nu_j B$. Eqs. (3) and (4) show the important result that if $W_{i\parallel}/W_{e\parallel}$ is not equal to $W_{i\parallel}/W_{e\parallel}$, that is, if the pitch angle distributions are not the same, the potential is formed.

They further considered that $K$ is invariant and estimated that the potential difference, $\Phi_A$, is

$$\Phi_A = \frac{1}{e} \left[ \frac{W_{i\parallel} W_{e\parallel} - W_{e\parallel} W_{i\parallel}}{W_{i\parallel} + W_{e\parallel}} \right] (\gamma_e - 1) \quad (5)$$
where suffix 0 denotes the values at the plasmasheet and $\gamma_\perp$ is the mirror ratio at the topside ionosphere normalized by $B_0$. The potential difference given by this expression increases with $\gamma$ which results in an unphysical value for large $\gamma$. Actually, in view of eq. (2), the total energy,

$$\varepsilon_j = W_{j\perp} + \nu_j B + e_j \phi,$$

is invariant but $K$ is not. The measure of anisotropy $(\nu_e W_{e\perp} - \nu_i W_{i\perp})$ expressed by eq. (4) decreases and tends to zero with the increase of $\gamma$, which leads to a saturated potential for large $\gamma$. We show that, from the condition, $K = 0$, the revised expression of $\phi^{AF}$ obeyed by eqs. (3) and (4) is found to be

$$\phi^{AF} = \frac{1}{e} \left( \frac{\nu_e \varepsilon_i - \nu_i \varepsilon_e}{\nu_i + \nu_e} \right).$$

This expression is found to be consistent with eq. (1) for the high-speed plasma flow where $\varepsilon_i \gg \varepsilon_e$.

In the case of high-speed plasma flow, $W_{i\perp}$ can be considered much greater than $W_{i\|}$, $W_{e\perp}$ and $W_{e\|}$ at the plasmasheet. Then $K = \frac{(\nu_e - \nu_i)(\varepsilon_e - \phi)/(\varepsilon_e - \phi)}{e}$. By substituting this expression into eq. (3), we have the asymptotic behaviour of $\phi$ for large $\gamma$,

$$\phi = \phi^{AF} \left[ 1 - \exp\left( - \frac{(\nu_i + \nu_e)^2}{\nu_i (\varepsilon_i + \varepsilon_e)} B_0 (\gamma - 1) \right) \right].$$

In real plasma, as is discussed in the following sections, in addition to the incoming ions and electrons from the plasmasheet, trapped electrons and ionospheric particles must be taken into account.

3. Method of Analyses

The profile of the steady state potential is analysed with both quasi-neutrality and currentless conditions. Let us assume axisymmetric configuration, for simplicity, between the plasmasheet ($x = x_0$) where the magnetic field is very weak and the topside ionosphere ($x = x_\perp$) where the field is strong. Between $x_0$ and $x_\perp$ we can expect a monotonically increasing function of the potential (Fig. 2). The total potential, $\nu B + \phi$, for ion is also monotonic. For this case, as is shown in Fig. 3, we must take into account three kinds of ions, i.e., passing ions which come down from $x_0$ into $x_\perp$, reflected ions which come down from $x_0$ but are reflected before $x_\perp$, and ionospheric ions which come up from $x_\perp$ to $x_0$.

For the same profile of the magnetic field and the electrostatic potential, total potential, $\nu B + \phi$ for electron is no longer a monotonic function, due to the negative charge of the electrons. Thus in addition to the passing, reflected and ionospheric electrons, trapped particles should be also taken into account (Fig. 4). While the ionospheric ions are accelerated by their total potential and come up from $x_\perp$ to $x_0$, the ionospheric electrons
are reflected by their total potential back toward $x_i$ because the thermal energy of the ionospheric particles is much too low in comparison with the potential energy.

In our system, the distribution of incoming ions, $f_{i0}$, is assumed to be a shifted Maxwellian with drift velocity, $V$, i.e.,

$$f_{i0}(v_h, v_\perp) = N_0\left(m_i/(2\pi T_0)^{3/2}\right)\exp\left[-m_i\left((v_h-V)^2+v_\perp^2\right)/(2T_0)\right], \quad \text{(for } v_h \geq 0)$$

$$= 0, \quad \text{(for } v_h < 0)$$

and the distribution of incoming electrons is a half-Maxwellian with no drift velocity,

$$f_{e0}(v_h, v_\perp) = n_e(m_e/(2\pi T_0)^{3/2})\exp\left[-m_e(v_h^2 + v_\perp^2)/(2T_0)\right], \quad \text{(for } v_h \geq 0)$$

$$= 0, \quad \text{(for } v_h < 0)$$

where $n_e$ is specified so that ion and electron fluxes are the same, i.e.,

$$\int_{-\infty}^{\infty} (f_{i0} - f_{e0}) v_\perp dv_\perp v_h dv_h = 0,$$

which results in

$$n_e = (N_0/2)\left[\text{erf}\left((m_i/2T_0)^{1/2}V\right) + 1\right].$$

Some of these particles

![Diagram](image)

**Fig. 2** Schematic profiles of the magnetic intensity, $B$, and the electrostatic potential, $\phi$.

**Fig. 3** Components of ions in (a) $x$-$x$ space and in (b) $x$-$y$ space.
pass into the topside ionosphere and the others are reflected before the topside. The background electrons have a loss-cone distribution,

\[ f_{eb}(v_h, v_\perp) = n_b (m_e/(2\pi T_0))^{3/2} \exp\left(-m_e(v_h^2 + v_\perp^2)/(2T_0)\right). \] (10)

Because the loss-cone angle \( \theta_0 = ((e\phi_L/T_0)^{-1})/\gamma_L \)^{1/2}, is negligibly small, \( f_{eb} \) is almost Maxwellian. Hence these electrons are not the cause of the potential formation. Neither do these electrons have a net current. \( n_b \) is specified so that the quasi-neutrality is satisfied. The distributions of the ionospheric ions and electrons at the topside ionosphere (\( x=x_L \)) are Maxwellian with low temperature \( T_L \).

If trapped electrons are absent, the potential sharply increases up to the maximum potential very close to the plasmasheet and is constant over a wide region of the system up to the topside ionosphere. The density difference associated with the incoming ions and electrons is partially canceled only by the ionospheric electrons in this case. Though the formation process of trapped electrons is a future problem, it may be reasonably considered that the passing particles induce backscattered electrons in the ionosphere which are trapped by the total potential. Because the trapped electrons suppress the potential hump at the front of the steep increase, the position of the steep increase shifts to the ionospheric side with the increase of trapped electrons. Thus the distribution of trapped electrons may be specified as followings;

\[ \text{ionospheric electrons} \]
\[ \text{trapped electrons} \]

(a) passing electrons

\[ x_0 \]

reflected electrons

(b) \( \varepsilon-\mu \) space

Fig. 4 Components of electrons in (a) \( \varepsilon-x \) space and in (b) \( \varepsilon-\mu \) space.
\[ f_{et}(v_u, v_\perp, \gamma) = N_t(\gamma)(m_e / (2\pi T_e))^{3/2} \exp\left(-\left[\frac{\nu_u^2 + \nu_\perp^2}{2} - 2e(\phi - \phi_e)\right] / (2T_e)\right) \]
\[ N_t(\gamma) = N_g\left(\frac{T_e / T_g}{3/2[1 - \tanh((\gamma - \gamma_\perp) / \Delta\gamma)] + 1}\right) \]

where \( T_e \) is the effective temperature of the trapped electrons, \( \gamma_e \) the position of the temperature change from \( T_e \) to \( T_g \) and \( \Delta\gamma \) the width of the temperature change, respectively. The parameters \( N_g, T_0 \) and \( T_e \) in this paper are prescribed as 1/cc, 1 KeV and 0.1 eV, respectively. The mirror ratio, \( \gamma_\perp \), between \( x_0 \) and \( x_\perp \) is assumed to be \( 10^4 \).

Before going to the analysis of the potential profile, the potential difference between \( x_0 \) and \( x_\perp \) is discussed in the next section by using a currentless condition only.

4. Potential Difference

In general, the potential difference derived by the high-speed plasma flow given by eqs.(8) and (9) is dependent on the potential profile. For a special profile, i.e., a potential which sharply increases very near \( x_0 \) and then remains constant up to \( x_\perp \) (Fig. 5(a)), electron flux becomes maximum.

Fig. 5. Two limiting cases for the potential profile, (a) and (b), which correspond to the minimum and maximum potential differences, respectively.
under the given conditions of the high-speed flow. Because of the currentless condition, ion flux also becomes maximum which corresponds to the minimum potential difference. On the other hand, for the opposite limit of the potential profile, i.e., the potential remains zero from \(x_0\) until just before \(x_k\), where it sharply increases (Fig. 5(b)). For this potential, electron flux is minimum, thereby the ion flux is also minimum, which corresponds to the maximum potential difference.

Fig. 6 shows the relation between flow velocity or flow energy and the potential difference. The parameter \(N_k\) is selected to be \(10^2/\text{cc}\). Thus it is found from this figure that when \(V\) is of the order of 1000 Km/s and \(N_k/N_0\) is not so large, the potential difference is enough to have energetic electrons of the order of several KeV at the topside ionosphere. Because \(\mu B_{k}\) is much greater than \(e\phi\) for electrons of the order of 1 KeV, the actual potential difference is almost equal to the minimum potential in Fig. 6 independent of the potential profile, except in the special case where the potential profile is nearly equal to the case in Fig. 5(b). It is also found that the minimum potential is approximately identical to the relation given by eqs. (1) and (6). But, as is shown below, this relation holds only when \(N_k/N_0\) is not large.

Because the outflow of ions coming up from the topside ionosphere into the plasmasheet is greater for denser \(N_k\), the
Fig. 7  Relation between the density at the topside ionosphere and the potential difference when $V$ is 1000 km/s and $N_0$ is 1/cc.

Fig. 8  Potential Profiles for (a) $\gamma_t = 1$, (b) 10, (c) $10^2$ and (d) $10^3$, respectively, under the conditions $\Delta \gamma = 50$ and $T_t = 200$ eV.
net inflow of ions decreases with the increase of $N_L$. Therefore, to keep the currentless condition, as is shown in Fig. 7, the potential difference decreases with the increase of $N_L$ and becomes zero at a critical density of the topside ionosphere, $N^c_L$. The value of $N^c_L/N_0$ is $6.5 \times 10^3$ under the condition that $V$ is 1000 km/s. This result is consistent with observations that the potential is formed at high altitude in the polar region where the ionospheric density is low enough, such as of the order of $10^2$/cc.

5. Potential Profile

The potential profile depends on parameters, $\gamma_t$, $\Delta \gamma$ and $T_t$ in eq. (11). It is shown in Fig. 8 that the position of the steep increase of the potential is shifted toward the topside ionosphere with the increase of $\gamma_t$ for the fixed values of $\Delta \gamma = 50$ and $T_t = 200$ eV. When the topside ionosphere ($\gamma = \gamma_0 = 10^4$) corresponds to $2 \times 10^3$ km altitude, the position where $\gamma = 10^3$ is about $10^4$ km altitude for the dipole magnetic field.

As is shown in Fig. 9, the increase of the potential is more sharp for small $\Delta \gamma$. When the effective temperature of the trapped electrons, $T_t$, is

![Potential profiles](https://via.placeholder.com/150)

Fig. 9 Potential profiles for (a) $\Delta \gamma = 50$, (b) 100 and (c) 200, respectively, under the conditions $\gamma_t = 10^3$ and $T_t = 200$ eV.
less than $\simeq 100$ eV, the sharp potential increase near the plasmasheet is not suppressed enough, but for $T_t \gtrsim 200$ eV, the potential is suppressed over a range $1 < \gamma < \gamma_t$, which is shown in Fig. 10. In view of Figs. 8, 9 and 10, it is found that the set of parameters, $\gamma_t \gtrsim 10^3$, $\Delta \gamma \approx 10^2$ and $T_t \gtrsim 200$ eV, results in an appropriate potential profile which compares well with observations. The profiles of the total density, $N_{\text{total}}$, and the density of the trapped electrons, $N_{\text{trap}}$, are shown in Fig. 11. The difference between $N_{\text{trap}}$ and $N_{\text{total}}$ is due to the densities of the passing and reflected electrons. Because the ratio of these densities is $\approx n_e/n_b \approx 0.1$, the ratio of the density of passing electrons and $N_{\text{total}}$ is about $10^{-2}$ which also compares well with observations. The current due to the passing electrons is $0.6 \mu A/m^2$.

In summary a revised magnetic mirroring model discussed here can explain very well the basic properties of the energetic electrons in auroral flares, not only in the potential difference, but also in the potential profile and the density structure together with the position of the aurora at the high altitude. Observationally, in addition to the large scale profile of the

![Potential profiles for different temperatures](image_url)

**Fig. 10** Potential profiles for (a) $T_t = 100$ eV, (b) 200 eV and (c) 300 eV, respectively, under the conditions, $\gamma_t = 10^3$ and $\Delta \gamma = 50$. 
The thick line means the total density and the thin line does the density of the trapped electrons.

potential (Frank and Ackerson 1971), the microscopic structures of the potential have been found (Mozer et al. 1979). These microscopic structures seem, in our opinion, to be a consequence of excitation by the energetic electrons which are accelerated initially by the large scale potential discussed here. In such a turbulent state, the effective collision frequency is expected to be high enough for having the trapped electrons.

Acknowledgements

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References

Mozer, F. S., Cattle, C. A., Tenerin, M., Torbert, R. B., Von Glinski, S.,
PHYSICAL ASPECTS OF PLASMAS FOR PLASMA-CHEMICAL APPLICATIONS

F.J. de Hoog, D.C. Schram

Department of Physics, Eindhoven University of Technology,
P.O.Box 513, 5600 MB Eindhoven, The Netherlands.

1. Introduction

During the last ten years interest in the use of plasma for chemical applications has increased considerably. Polak gave reviews at ICPIG X (1971, Oxford) and ICPIG XIII (1977, Berlin). Suhr (1973, Prague), Fauchais and Rakowitz (1979, Grenoble), Molinari (1983, Düsseldorf) reviewed developments in the modelling of plasmas used in chemical processing based on an increasing insight in the relevant physical processes. The expansion of this field is illustrated by the ever growing attendance at the biennial International Symposium on Plasma Chemistry, the last of which took place recently in Eindhoven. The aim of this paper is to define the field of plasma chemistry and stress certain recent developments in the physical understanding of various chemical plasmas. It is impossible to be complete. For a complete survey one might consult the Proceedings of the ISPC VII [1,2].

The applications can be classified into the following three classes: volume chemistry, surface modification, heating and acceleration of (particulate) matter. The advantages of plasmas are many. In most cases use is made of the non-equilibrium character of the distribution of reactants and reaction products, which offers possibilities for unique reaction channels with large reaction rates. Especially in thermal plasmas, where the degree of ionization is high, large specific energies can be present. The heating of reactants by electric fields adds to the methods of equilibrium chemical processing in the gas phase the full spectrum of discharge modes and plasma configurations. Electron-, gas- and substrate temperatures can be different and can cause a strong difference in the vibrational temperature of participating molecules with respect to their translational temperature. Of course limitations are present. Plasma dynamics is determined by the extra constraints imposed by the Maxwell equations. Dissociative excitation and ionization require energies from 5 eV to 10 eV which may have a negative effect on overall energy efficiency. Use of low pressure discharges can lead to small gas efficiencies and may be only of interest for specific reactions. An important point is that because of the complicated physical situation in a chemical plasma modelling and interpretation of phenomena on a fundamental basis is difficult. Partly this is due to lack of data on cross sections for the relevant processes, but also, by the rapid development of the field a lot of research is concluded at an empirical level.

2. Field definition

To give an idea of the size of the field we review in table I plasma chemistry applications as they have been presented July 1985 at the VIIth ISPC. This review is certainly not exhaustive, but indicates general trends.
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**TABLE I**: A choice of the various plasma chemical applications as discussed at the VII<sup>th</sup> ISPC, Eindhoven 1985 [1].
Roughly speaking the field can be split up in several ways. Volume processes tend to be based on high density thermal plasmas, with a degree of ionization larger than 1% and large specific energy inputs of the order of 1 eV/molecule, which sets the conditions for high efficiencies and high yields. Here inductively coupled arcs, DC and AC arcs are used. Processes are mainly synthesis, pyrolysis, but these types of discharges are also used for heating, melting and acceleration of particulate matter.

Surface modification is usually based on the use of low density plasma, with a degree of ionization much smaller than 1%, but an energy input for each incoming particle in the order of 30 eV/molecule. The use of energy in this case is inefficient. Electron temperatures (≈ 1 eV) are much larger than the temperature of the ambient gas (and the substrates). When thermal loading of reactantia and products or walls and substrates should be minimized this kind of plasma mode is usually chosen. Here DC glows, capacitively coupled RF discharges and microwave discharges are utilized.

We should note here that in surface modification of substrates for IC-technology the capacitively coupled RF discharge has gained a dominant position. This is for a great part due to the specific plasma property of generating a negative potential of the substrate vs. the plasma. This bias potential causes ion enhanced surface processing at much larger rates than when no bias is present. In etching this phenomenon brings about anisotropic etching which makes it possible to prevent undercutting of micromized structures. In the second part of this presentation we would like to focus in on these kind of discharges.

It would be worthwhile to discuss here whether the energy- and gas-efficiency of high density thermal plasmas could be utilized in surface modification at the same time maintaining low substrate temperatures. A possibility is to make use of passive plasmas. In this case the plasma production and the plasma utilization should take place separately. Provisions have to be made to expel the plasma from the plasma source and to direct it to the surface to be treated. Such a solution has two advantages: the constraints on the plasma near the surface are less stringent as there is no demand to generate plasma at that location. Secondly, only the power in the particle beam is transferred to the surface. At the Vth International Conference on Ion and Plasma Assisted Techniques (München, May 1985) a number of applications based on this principle have been presented. Here we would like to present such a plasma source base on a magnetized hollow cathode plasma [3,4].

\[
\begin{align*}
\text{gas} & : \text{argon} \\
\text{cathode} & : \text{Ta Ø 6 mm} \\
\text{pressure} & : 0.27 \text{ Pa} \\
\text{magn. field} & : 0.2 \text{ T} \\
\text{arc current} & : 50 \text{ A} \\
\text{arc length} & : 1.3 \text{ A}
\end{align*}
\]

**FIG.1**

**FIG.1** : Characteristics of a Magnetized Hollow Cathode Source (from ref. [3,4]).
In Fig. 1 a schematic set-up is shown. The plasma source (index s) is fed with a total gas flow \( \dot{N}_g \), which is ionized within the cathode tube and is propelled into a vacuum chamber which is continuously pumped. Mass conservation requires at the hollow cathode orifice

\[
\dot{N}_s = \int (n_p w_p + n_n w_n) 2\pi r dr,
\]

where \( w_p, w_n \) are transport velocities and \( n_p, n_n \) are particle densities of ions and neutrals respectively. Because the medium expands into vacuum the transport velocity and the effective area in the expanding beam increase. The power consumed is used to produce the plasma, to raise the temperature and to accelerate the resulting plasma.

To analyze this process we have to consider the overall momentum balance

\[
(n_p + n_n) w_p (w - \bar{v}) = -\bar{v}_p - \bar{v}_n + j_x B
\]

in which we have ignored the electron contributions to the inertia and viscosity terms. If we neglect the latter for simplicity and note that \( j_x B \) is small compared to the pressure gradients, we can see that eq. (2) suggests acceleration to sonic velocities provided substantial pressure gradients are created.

In the hollow cathode arc, plasma is produced and heated to temperatures in the eV range. Power densities are in the range \( 10^{10} \, \text{W/m}^3 \) which leads up to substantial ionization. Because of heating and ionization the pressure increases leading to expansion into the low pressure volume with sonic or even supersonic exit velocities.

FIG. 2:
On axis velocities of ions and neutrals from a MHCA in argon at various distances from the cathode tip (from ref. [4]).

FIG. 2
In Fig. 2 one can see for plasma and neutral densities at the orifice of about \( 5 \times 10^{20} \, \text{m}^{-3} \), the particle velocities along the direction of the flow. When using the ion-momentum balance a consistent picture of the measured values of ion forward velocity, rotational velocity, electron temperatures and particle densities can be built up [4]. Close to the cathode we can compare the outgoing ion flux with the
incident gas flux. The results are depicted in Fig.3, where we see that at sufficiently low gas fluxes and sufficiently high currents gas efficiencies close to 1 are reached.

**FIG. 3:** Ionized fraction (α) of mass effusing at 10 mm from cathode tip as a function of mass throughput at various currents (from ref. [4]).

This transformation of an ingoing gas flow into a high velocity, hot ion flux with excellent gas efficiency could be used in surface processing and has in fact been applied to the deposition of titanium oxide [5]. Another example of the above principle is the use of a cascade-arc to generate the effluent plasma beam. The pressure in the cathode arc is higher than in the hollow cathode and an even more intense particle beam is observed. Measurements of the temperature and of the velocity of disturbances indicate that again the plasma dynamics can be described by the conservation laws. Sonic velocities are predicted and observed [6]. Plasma ion density is estimated to be $10^{20}$ m$^{-3}$ and exit velocities have been found to be $10^4$ A m/s in which A is the mass number. High gas efficiencies can be obtained. Certainly in the field of highly efficient, high yield application more developments are to be expected. Rusanov [7] mentioned in a review paper the possibilities of electron beam generated plasma. Generally speaking be pointed out the similarity between laser plasmas and chemical plasmas. The development and modelling of the latter can benefit largely from the laser field. Also the use of intense short wave radiation for the generation of chemical plasmas is being considered [8].

3.1. Aspects of the R.F. glow discharge

In the remaining part of the paper we will focus on the R.F. glow discharge which has gained such a prominent role in surface modification, both for deposition and etching. It will be impossible to give a complete overview of the development of this field. Within this short presentation we will discuss recent developments along lines laid out by the formulation of a number of topical problems:

1) The biasing of electrodes and/or substrates.
2) The energy distribution of electrons.
3) Surface chemistry.
4) Discharge modelling.
3.2. Electrode biasing

In etch and deposition plasmas in most cases use is made of plasmas between two planar and parallel electrodes. In a D.C. coupled case the electrodes have the same negative bias with respect to the plasma. It can be noticed that either through A.C. coupling of the R.F. voltage or by the use of an isolated substrate attached to one of the electrodes it is possible to disturb this balance and to control the D.C. bias by a proper choice of the area of the plasma-electrode interfaces and the voltage of the R.F. source. In this way it is possible to control sputtering rates [9] and to bring about anisotropic etching of substrates and prevent undercutting, as it is experienced in anisotropic wet etching. Etching of micronized structures in this way became possible with etch rates up to $10^4 \text{ A/min}$ [10] in $\text{SF}_6-\text{O}_2$ on silicon. Of course one should take care that the ion bombardment does not affect the selectivity of the etch process and that thermal loading of the wafer is limited. Therefore the physical mechanisms underlying this bias-phenomenon will be discussed here. Godyak [11] discussed an equivalent circuit which makes it possible to describe the electric behaviour of the R.F. discharge in a simple way. This has been elaborated upon by Keller et al [12].

In Fig. 4a this circuit, which clearly shows the rectifying properties of the discharge, is depicted. The sheaths developed between plasma and electrode are represented by capacities. The continuous ion current leaving the plasma leaks through

$$\text{FIG.4a}$$

the resistances and the fact that the wall potential can not be larger than the plasma potential is taken care of by the diodes. If one assumes that $C_2 > C_1$ the average D.C. bias across each of the sheaths is

$$V_g - V_1 = V_g - V_2 = C_2/(C_1 + C_2)v,$$

where $v$ is the R.F. amplitude.

If one puts a large blocking capacitor $C$ in series next to $C_2$, then the charging of the plasma should take place through both diodes (Fig. 4b) and

$$V_g - V_1 = \frac{C_2}{C_1 + C_2}v, \quad V_g - V_2 = \frac{C_1}{(C_1 + C_2)}v.$$

The conclusion is that the larger drop develops in front of the sheath with the

$$\text{FIG.4b}$$
smaller capacity. In fact
\[
\frac{V_g - V_1}{V_g - V_2} = \frac{C_2}{C_1},
\]

In practice the capacities are non-linear circuit elements. They can be calculated using Langmuir sheath theory and it is clear that not only the electrode area but also the sheath current density plays an important role. Based hereupon Koenig and Maissel [13] found
\[
\frac{V_g - V_1}{V_g - V_2} = \left(\frac{A_2}{A_1}\right)^4,
\]
where \(A_1, A_2\) are the interface areas. The large power dependency suggests that to change the sheath potentials only a minor unbalance in the electrode area is necessary. This point which is very essential in the design and control of etch and deposition systems has been checked experimentally by several authors. Coburn and Kay [14] showed that the power law tends to follow \((A_2/A_1)^n\). Horwitz [15] following Godyak pointed to the various discharge modes possible in the domain studied. It is possible to have discharges where the capacitive impedance plays a dominant role (e.g. Ar, 13.5 MHz) and those where the resistive currents are dominating. In the first case charge can be stored in the discharge and only the current averaged over one period should be zero. In the second case instantaneous charge conservation is required. From Horwitz interpretation of Coburn's results it can be seen that the Koenig and Maissel law does not hold for both cases considered. Coburn's results are sustained by recent data by Köhler et al [16] who measured the energies of ions coming out of an argon plasma (Ar, 13.5 MHz, 20 mTorr) having crossed the sheath. They concluded that in a D.C. coupled case the voltage drop ratio scales with the inverse of the area ratio, but that in an A.C. coupled case a larger power (= 2) is observed. Recently we developed an expression, based on a Langmuir sheath model and instantaneous charge conservation which explains the results of Köhler for the measured bias voltage \(v_c\). From this model it appears that
\[
v_c = V \cos \left(\frac{A_1}{A_1 + A_2} \pi\right) + \ln \frac{A_1}{A_2}.
\]
From the same model it is found that
\[
\left(\frac{V_g - V_1}{V_g - V_2}\right) \sim (A_2/A_1)^3
\]
which is in view of the data by Horwitz [15] and Köhler [16] a satisfactory result. To solve this problem in a more satisfactory manner a combination of both approaches should be followed. Both the conduction currents and the dielectric currents should be taken into account at the same time and results should be checked in a standard experimental situation.

3.3. Electron energy distribution

The basis for all modelling of discharges with low electron densities as they are
encountered in plasma surface processing is the electron energy distribution. They define both the transport properties of the plasma and the production rates of ions and excited particles. In molecular plasmas where chemical reactions occur with large rates the determination of this distribution is much more complicated than in monoatomic classical discharges. As Polak [17] already pointed out the superelastic vibrations $v$ deexcitation of molecules can be a very effective source of electron heating in the lower energy range. Therefore the distribution of vibrational states of the dominant molecule plays an important role. More general this implies that the solution of the Boltzmann equation for the electrons is embedded in the solution of a system of master equations of the relevant reaction products in their various states. The presence of a flowing gas complicates the problem even further. It is worthwhile to mention the work by the Bari-group which made a concentrated effort in modelling the decomposition of carbonmonoxide and the development of the EED in CO$_2$ and N$_2$ discharges [18,19]. Since these approaches are very complicated the programme sketched above has been hardly executed. Only isolated aspects have been subject of study. Margenau [20] was the first to show that in a homogeneous plasma without electron-electron interaction and inelastic collisions the EED can be written as

$$f(v) \sim \exp \left(-\frac{m v^2}{kT} - e \frac{E}{M} \frac{v}{m v} \right),$$

where $v$ is electron velocity, $T$ gas temperature, $M$ and $m$ are heavy particle and electron mass, $v_m$ is the momentum transfer rate and

$$E_e = E_0 \sqrt{\frac{v^2}{2v_m} \left(\frac{v^2}{v_m} + \frac{v^2}{m v_m} \right)},$$

where $E_0$ is the amplitude of the R.F. field with circular frequency $\omega$.

When $\omega < v_m$ and $\sqrt{\frac{v^2}{m v_m} + \frac{v^2}{m v_m}}$ the Davydov-Druijvesteijn distribution evolves but for $v_m$ constant a quasi-Maxwellian is found. The average energies are

$$\langle e_{DD} \rangle \sim \frac{\sqrt{\frac{2}{m}} e E_0 \lambda}{v_m},$$

$$\langle e_{QM} \rangle \sim \frac{e E_0}{6m v_m^2 \lambda}.$$ 

Here $\lambda$ is the mean free path.

When $\omega > v_m$ because of the stochastic heating mechanism also a quasi-Maxwellian is found with average energy

$$\langle e \rangle_{SH} \sim \frac{e E_0^2}{3m \omega^2}.$$ 

Eletskii and coworkers [21] pointed out that in R.F. discharges where $\omega > v_m$, assuming constant $v_m$, the average gain between collisions amounts to $e E_0^2/m v_m^2$ and it takes $M/m$ collisions before a new electron fits into the EED. When loss processes like diffusion and/or inelastic collisions dominate the average energy of the distribution should scale like
where $V_D$ is the electronic diffusive loss rate. This occurs when $\omega < \frac{M}{m} v_D$. Note that in this case $<\epsilon>$ is proportional with the square of the gas density. This reasoning can be extended to the lumped inelastic loss rate $V_{in}$. It is clear that both diffusive and inelastic losses processes tend to decrease the average energy.

To maintain a discharge the value of $E_0$ should be adapted.

Calculations on R.F. discharge EED-s in a spatial homogeneous case solving the time dependent B.E. including inelastic processes have been published recently by Winkler et al [22], albeit only in the monoatomic gas neon. Their results show clearly under what conditions in the EED modulation effects appear. For electrons with energy below the excitation threshold in that part of the EED modulation effects appear when

$$\omega < \frac{m}{M} v_D.$$  

In the high energy part where the energy loss rate is simply $V_{in}$ the EED will not follow the modulation for

$$\omega > v_{in}.$$  

In a standard situation for planar R.F. glow discharges at 13.5 Mhz at low pressures

$$\omega \simeq \frac{v}{m} \quad \text{and} \quad v_m \simeq 10 v_{in},$$

one may expect to be at the edge of the on-set of field modulation.

Two recent interesting approaches should be mentioned here. Starting from the idea that the electrodes in a planar reactor are bombarded by ions of 100 eV or more. Garscadden [23] assumed that secondary electrons with a very high energy should added to the B.E. as a source term at high energies. These electrons are very effective in carrying out inelastic collisions because of their energy but also because of confinement and multipactor effects. In this respect the influence of these electrons is similar to fast electrons in a hollow cathode glow discharge. This idea has been checked in a mixture of silan and hydrogen. Numerical calculations showed that the number of fast vs. slow electrons could be $10^{-4}$.

Kushner used a Monte Carlo method in calculating the EED in a planar R.F. glow discharge [24]. He also included secondary electrons emitted from the electrodes acting as an $e$-beam and found a solution in both energy and space in one dimension; in Fig.5 one can see some of his results for CF$_4$, the electrode spacing is 3 cm, the gas pressure 50 mTorr and the R.F. frequency 13.56 Mhz. It can be seen that the electron energies in CF$_4$ are fairly high compared with common knowledge about electron energies in glow discharges of 1-3 eV. However, the same author states that his data are backed up by experiments [25]. Also in the fairly shallow planar geometry and at the relatively low pressures used, diffusion effects may have a large influence on the plasma balance, necessitating a higher electron temperature. That those effects may be strong has been shown by Vallinga et al [26]. He solved the B.E. in the standard situation in a CF$_4$ etching plasma (13.5 Mhz; 50 mTorr, electrode spacing $\sim 10^{-2}$ m). Here again $\omega v_m < 1$. Instead of a Fourier expansion in time, a multiple
FIG. 5:
Typical computed result for the average electron energy in CF$_4$ in a planar discharge. Electrode spacing is 3 cm, gas pressure 50 mTorr (at 500 K), R.F. frequency is 13.6 Mhz.
Curve a is without, curve b with secondary electron emission (from ref. [24]).

FIG. 5

time scale formalism was used. The approach included elastic collisions and excitation, ionization, dissociative attachment and vibrational excitation. The spatial dependency was also taken into account. From the particle balance it was possible to obtain expressions for the breakdown and steady state values of the effective field strength E$_e$. In Fig.6 calculated values of E$_e$/p are shown as a function of the CF$_4$-pressure.

FIG. 6:
Breakdown ($\Lambda = 1$ cm) and maintenance ($\Lambda = 10$ cm) values of the reduced effective field in a R.F. plasma in CF$_4$ (13.6 Mhz) as a function of pressure (from ref. [26]).

FIG. 6

In Fig.7 the EED at E/p of 12 V/cmTorr is shown. From the calculations it was also possible to explain some features in the experimental data of the average electron density in a planar R.F. plasma in CF$_4$ taken by Bisschops et al [27] (see Fig.8). One can see that at pressures in the order of 20 Pa the electron density per unit of input power displays a clear maximum. It is however not possible to ascribe this maximum to the transition pressure where $\omega = \sqrt{\frac{e}{m}}$ and where the electron heating
FIG. 7: Calculated electron energy distribution compared with standard distributions of the same average energy (from ref. [26]).

FIG. 8: Experimental values of electron density in quasi-planar R.F. discharge in CF₄ as a function of pressure at various input powers (from ref. [27]).

mechanism is changing character. For CF₄ this point should be at a pressure of 0.02 Torr. We noticed however that at the experimental value of 0.2 Torr the mean free path of an electron becomes of the order of the thickness of the glow; lowering the pressure will certainly have effects on the electron heating mechanism. This point is sustained by experiments in argon where no such effect has been found within the experimental range of pressures. This may be explained by the fact that momentum transfer cross sections are one order of magnitude higher in Ar than in CF₄. Conclusively we may say that the work on electron energy distributions in low pressure etching plasmas is proceeding slowly. Possibly only the implementation of large scale computer efforts, on a scale as in laser or fusion plasma modelling will generate real breakthroughs.

3.4. Surface processes

In low pressure plasma etching the main point is the removal of surface material with good anisotropy and selectivity. Here we will sketch the viewpoints presently discussed on the various surface mechanisms. We will focus in on the controversy about physical sputtering and chemical sputtering discussing the results of some experiments. The spontaneous etch rate of a substrate can be drastically increased when exposed to fluxes of energetic particles. Ion bombardment appears to be the most important process, although electron beams and lasers are also used. Usually the
process is subdivided into three steps [28].

The first step is chemisorption of the particles taking part in the etching process. The next step is the formation of the reaction product. The third step is the desorption of the reaction product. E.g., first in a F-Si system F\(_2\) is chemisorbed (when F-containing molecules are used this chemisorption has to be dissociative). After formation of the saturated product (SiF\(_4\)) desorption of the molecule follows, although it is known that SiF\(_2\) is also a reaction product [29]. The presence of a discharge may influence all three steps.

I. - The formation of radicals from the plasma increases the chemisorption on surfaces which appear inert to the parent molecule.
   - The ion bombardment may enhance the access to the surface of etchant species by preventing the blockage of the surface by deposition of unsaturated species and polymerization.

II. The formation of reaction products is a complex phenomenon.
   - Winters [28] considers this step to be similar to the oxidation of a surface. The difference is that halogen compounds usually are volatile. Anion and kation formation generate electric activity which assist the kinetics of the reactants in the surface layer. Ion bombardment may supply energy for the activation of those reactions producing weakly bound molecules desorbing into the gas (chemical sputtering).
   - Flamm et al [30] stress the possibility of damage induced by the ion bombardment resulting in active sites at the surface and several monolayers below where the product formation can take place at higher rates (damage induced chemical reaction, RIE).

III. Ion bombardment may assist the desorption rate of adsorbed reaction products by sputtering. This process is known as chemically enhanced physical sputtering.

It is clear that in any etching process, depending on the composition of the etching gas and the character of the substrate one or more of the mechanism indicated above may dominate. It is in this specific field that only slow headway is being made and that controversial interpretations of the scarce experiments is evident. An interesting example of this phenomenon will be discussed here.

In order to find definite answers to questions in this field experiments should be well defined. The etching of silicon by chlorine under simultaneous bombardment of argon ions has been investigated recently in beam experiments by several groups. Recently Sanders et al [31] and McNevin et al [32] addressed the question whether physical sputtering or chemical sputtering is the dominant mechanism in the etching of silicon. Both experiments were similar albeit that in one of the experiments the substrate temperature could be varied. A new feature in both these experiments was the use of ion beam modulation techniques in order to analyze the time dependency of the etching process. Sanders found that the dominant product was SiCl\(_2\) and SiCl. Interesting is that the products show an energy distribution which is definitely superthermal (Fig. 9) and the authors drew the conclusion that physical sputtering plays a dominant role in the Si(Cl, Ar\(^+\)) process. On the other hand McNevin found the
dominant product to be SiCl₄ and observed a definite time delay for the reaction to get started after the ion beam has been switched on. Thus suggest an ion-enhanced chemical reaction, where the incident ion provides energy to the surface and this energetically excited surface then has an enhanced reaction rate to form the SiCl₄ products. These experiments using quite sophisticated techniques give a good idea what the state of the art is in discriminating the various surface reactions. A lot of progress has been made in the experimental approach, but controversies still flare.

3.5. Etch plasma modelling

In the last couple of years several groups have tried to set up plasma models of R.F. discharges at conditions comparable to etch- and deposition conditions.

A number of studies have concentrated specifically on discharge dynamics. From these the recent work Tachibana et al [33] should be mentioned. They studied a discharge in methane, compounded a complicated discharge model based on a manifold of coupled rate equations and checked their model by measuring the EFD, the dissociation of CH₄ by an absorption technique and the deposition rate of carbon films.

In Fig.10 the concentrations of the various reactants vs. power input as they emerge from the model is shown. From the deposition rate it was concluded that the active species for the deposition should have a density of 10¹¹ - 10¹² cm⁻³. When the sticking probability of CH₂ is close to unity this radical certainly is the one responsible for the deposition. If however the sticking probability of CH₃ is lower, it was concluded that CH₂ is the molecule responsible for the deposition. From a study like this it can be seen how complicated it is to model a plasma which is really etching. Apart from the homogeneous reaction also heterogeneous processes should be taken into account and problems like reactor loading and flow dependency should be faced.

Recently Edelson and Flamm published a one dimensional plug flow reactor model of flowing CF₄ etching Si. They did not calculate the electron energy distribution in a self consistent approach but instead plugged in a plasma density and an electron energy from experiment.

A cylindrical packet of gas is flowing in the downstream direction entering and passing the active plasma region and an afterglow region. The corresponding one-dimensional equations for the production and loss of the various species can be
treated as an initial value problem. The authors took into account 16 gas phase reactions, 11 surface reactions, ambipolar diffusion of electrons, positive and negative ions, surface recombination and polymerization. Rate constants were put in that were known from literature, although measuring conditions may not be matching. In some cases rates from homolog reaction or only order of magnitude estimates have been put in. In Fig.11 we can see the development of the various concentrations along the distance coordinate for a CF$_4$ plasma containing Si. (Fig.11a Charged species, Fig.11b Neutral species). It is noteworthy to stress the important role
C₂F₆ which is being formed from CF₃. The comparison with an experiment consisting of downstream sampling of neutral species with a mass spectrometer was only partly satisfactory [35]. A full quotation from the paper by Edelson and Flamm may sum up properly the present state of modelling of plasmas used for plasma chemical applications.

"The results show that many key processes are very poorly known from a quantitative standpoint. Major uncertainties arise from the transport of radicals and ions, which has been included in the mechanism in an approximate manner, with rate coefficients which are only crude estimates. Improvements to the model will most certainly require a more detailed representation of transport in what is essentially a three dimensional problem, - - - - - - - - - - - - - -
Expansion of the gas phase chemistry model does not seem to be warranted, however the surface chemistry needs better quantifications".

4. Final remarks

In this presentation we only were able to give a glancing survey of the physical problems encountered in the applications of plasma to chemical processing. Here in this field of exploding size so many disciplines meet that one can hardly speak of a cross- or interdisciplinary science, but of a discipline in itself. Workers have to be able to understand if not to speak the language of physical chemistry including surface physics, surface chemistry, atomic and molecular physics but also that of plasma and discharge physics.

A noteworthy trend is that applications seem to be running ahead of more fundamental studies. This certainly does not contribute to the scientific status of the field. In this respect plasma physicists have an obligation of staying in touch with the applications, their main task being the improvement of diagnostic methods and the design and use of numerical codes.

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References


The basic properties of electron holes, ion holes and double layers are reviewed, including the role these electrostatic phase space structures are playing in the nonlinear behaviour of driven bounded plasmas.

1. Introduction

Electron holes (EH), ion holes (IH), and double layers (DL), the objective of this report, constitute a subclass of nonlinear electrostatic modes, often referred to as BGK modes [1]. Generally speaking, they are saturated states of two-stream unstable collisionless plasmas in which saturation is provided by particle trapping. Representing states far away from thermodynamic equilibrium, these structures are found in current carrying and voltage driven plasmas as well as in plasmas driven by beam particle injection or by wave launching.

Although some of them have been known for a long time [2], dating back until 1929 when Langmuir analyzing his experiments, inferred on the existence of DLs, renewed interest came up only recently mainly due to the improved access to these entities in laboratory and numerical experiments. Also new analytical material is available supplementing the experimental data.

Trapping, of course, implies that these structures are not amenable to macroscopic descriptions like MHD or other fluid descriptions. It is the Vlasov picture which has to be invoked.

In the language of modern nonlinear dynamics these structures appear to be attractors and are generated rather independently of the details of initial and boundary conditions. Provided that the excitation mechanism is sufficiently strong, they will come up inevitably and can last sufficiently long to affect the characteristic properties of a plasma, e.g. its dynamical evolution.

The situation is in some sense analogous to conventional fluid dynamics, where the Bénard cells, having received a great deal of attention, play a similar role. It is well known that a horizontally layered fluid, heated from below, becomes structured by the appearance of convection rolls, when the Rayleigh number exceeds a certain threshold value. This analogy can be strengthened by noticing that in certain approximations both media behave like incompressible 2-D fluids. The Vlasov description for one space dimension takes place in the two-dimensional phase space. Due to Liouville's theorem the two-dimensional phase space fluid is incompressible, and it is an
easy matter to cast the equations governing the motion of an electron phase-space fluid, the Vlasov equation and Poisson's equation into the equations for a 2-D incompressible ordinary fluid.

In the following, I shall discuss some of the topics of this field whose growing attractiveness is reflected by two symposia that took place in 1982 and 1984 at Roskilde and at Innsbruck, respectively. The proceedings of these Symposia [3, 4] are sources of further information as well as several review articles [5, 6, 7].

Section 2 is devoted to an experiment which played a key-role in the development of the field of holes. An analysis of EHs, IHs and DLs is represented in the subsequent sections. The evidence of these structures in computer simulations and laboratory experiments is reported in Section 6, whereas dynamical transient properties are addressed in Section 7.

2. The Risø-experiment on electron holes

As mentioned, great progress in the understanding of electrostatic structures has been achieved experimentally by LYNOV et al. [8], at the Risø-Institute in Denmark. Fig. 1 shows the schematic set-up of their experiment. It consists of a single-ended Q-machine in which a collisionless plasma is produced by surface ionization on a hot tantalum cathode. Radial confinement is obtained by a homogeneous magnetic field ($\omega_{ce} \gg \omega_{pe}$). A surrounding cylindrical brass tube with a slit acts as a wave guide. A short negative voltage pulse on the left part forces the enclosed electrons to leave and to enter the target plasma where they constitute an effective, two-stream like perturbation. The plasma response to this excitation was measured by axially

![Scope

Fig. 1: Schematic set-up of the Risø-experiment. After [8].

![Time-dependence of $\phi$ at equidistant positions. After [8].

Fig. 2: Time-dependence of $\phi$ at equidistant positions. After [8].
movable Langmuir probes. Fig. 2 shows the time-dependence of the negative electrostatic potential recorded at various positions. Two distinct structures of opposite polarity are seen. A fast moving negative potential pulse and a positive, slower potential pulse. The latter structure being associated with a density depression is the aforementioned EH. It is remarkably stable. The fast pulse, on the other hand, decreases in amplitude and spreads. It could be identified as a Gould-Trivelpiece soliton which is governed for small amplitudes by a Korteweg-de Vries (KdV) equation. The width of the soliton is broader than that of the hole despite its larger amplitude. This is somewhat surprising because the hole should be broader if it belonged to the class of KdV-solitons, too. Whereas the velocity of the soliton is several times larger than the electron thermal velocity $v_{te}$, and corresponds to the Gould-Trivelpiece mode, the hole velocity is of the order of $v_{te}$ only or even less. Argued from a linear basis, it is in the velocity range where one would expect strong electron Landau-damping and hence, a complete suppression of the hole structure. This does, however, not correspond to the real observation. Note also a slight asymmetry of the hole increasing with time. Furthermore, it is experimentally found that the hole but not the soliton vanishes when the pressure is increased. In addition, in cases where two holes are created, the coalescence of both to a single hole could be seen. These features prove experimentally the non-soliton property of EHs and indicate that the hole is something else.

Unfortunately the device did not allow measurements of the electron distribution function $f_e$ to get further information. However, the authors could perform a particle simulation which was adapted to their laboratory experiment. The numerical results are presented in the next two figures.

In Figs. 3, 4, three structures are seen in a certain frame moving to the right: the fast moving soliton, the hole, which is almost standing in this frame, and a new structure (later on being referred to SEADL) moving to the left. As seen by the phase space pattern the soliton accelerates and decelerates the whole electron fluid and is, hence, a macroscopic phenomenon. But the other two structures, as one easily recognizes, are of different type. For both, the distribution remarkably deviates from Maxwellian. The hole has a vortex-like structure in phase space (remember the analogy to fluid dynamics mentioned in the Introduction) being characterized by a deficit of slow (deeply trapped) electrons within the structure. The third structure represents a monotonic potential transition (DL) and is associated with a two-stream-like distribution at maximum potential, the two branches of which unite on the low potential side. The latter structure is the most unstable one and shows the onset of a transition into a hole.

Before discussing the EH properties analytically, it should be noted that this sort of trapping vortices has been observed many times in computer runs simulating two-stream unstable situations [9 , 10], etc. They have also been found in the simulation of plasmas interacting with launched large ampli-
tude waves as, for example, in the Raman [11] and Brillouin [12] back scattering or in the lower hybrid heating [13]. We, therefore, may state that trapping vortices or holes are common structures in plasmas driven by distinct excitation mechanisms.

3. **Theory of electron holes**

Theoretically [14, 15], electron holes sharing these properties have been explained as equilibrium solutions of the Vlasov-Maxwell equations, as will be shown next. These equations reduce in the electrostatic, 1-D limit to

\[
\begin{align*}
[ v_0 \phi_x - \phi'(x) \phi_v ] f_e(x, v) &= 0, \\
\phi''(x) &= \int dv f_e(x, v) - 1.
\end{align*}
\]

Eqs. (1), (2), are formulated in the wave frame \((x - v_0 t + x)\) where the structure becomes stationary assuming that it propagates with a velocity \(v_0\) in the laboratory frame. Ions are treated as a constant neutralizing background. The spatial coordinate \(x\), the velocity \(v\), and the electrostatic potential \(\phi\) are normalized by the Debye length \(\lambda_D\), the electron thermal velocity \(v_{te}\), and \(T_e/e\), respectively, where \(T_e\) is the electron thermal energy in the
unperturbed medium which is assumed to be in thermal equilibrium. The latter implies the boundary conditions
\[
\phi, \phi' + 0, \quad f_e(x, v) = c \exp \left[-\frac{1}{2} (v + v_o)^2 \right]
\]
as \( |x| \to \infty (c = (2\pi)^{-1/2}) \).

The method of solution consists in solving (1) in terms of the constants of motion where use is made of the global form of \( \phi \) (which is bell-shaped here), and inserting this solution into (2) which is solved then for \( \phi \) [14 - 17].

A solution of (1) consistent with (3) is given by
\[
f_e(x, v) = \left\{ \begin{array}{ll}
c \exp \left[-\frac{1}{2} (\sigma \sqrt{v^2 - 2\phi} + v_o)^2 \right] & E_e > 0 \\
c \exp \left[-\sigma \frac{\phi}{2} (v^2 - 2\phi) - \frac{v_o^2}{2} \right] & E_e \leq 0
\end{array} \right.
\]
where \( E_e = v^2/2 - \phi \), and \( \sigma = \text{sgn} v \) are the constants of motion of free electrons (\( E_e > 0 \)). Trapped electrons are represented by the second line in (4), for which \( E_e \leq 0 \). The distribution function (4) at a fixed position within the structure is illustrated qualitatively in Fig. 5. Note that the distribution function is continuous everywhere, especially at the border of

trapped particles which is given by \( |v| = \sqrt{2\phi} \) (dashed line). The population of trapped particles is reduced when \( \beta \), the trapped particle parameter, turns out to be negative. Hence, \( f_e \) given by (4) is of vortex-type when \( \beta \) is sufficiently negative. The phase space trajectories are shown qualitatively in Fig. 6 for a bell-shaped electric potential.

Integration of (4) yields the electron density \( n_e \) which depends on the parameters \( v_o \) and \( \beta \) (for the explicit form, see [16]). Poisson's equation (2) then becomes

Fig. 5: The electron distribution function in velocity space.

Fig. 6: The particle trajectories in phase space in the vicinity of an EH.
In (5) we have introduced the "classical potential" $V$ (SAGDEEV potential) which is depending on $\phi$, $v_o$, and $\beta$.

The integrated form of (5) reads

$$\frac{\phi'(x)^2}{2} + V(\phi; v_o, \beta) = 0,$$

where $V(0; \beta) = 0$ is assumed. There is a unique correspondence between the electric potential $\phi$ and the "classical potential" $V$, as illustrated in Fig. 7, for a bell-shaped potential having a maximum value $\psi$(amplitude).

![Fig. 7: The correspondence between electric and "classical potential".](image)

Two conditions are necessary for the existence of a solution:

1) $V(\phi; v_o, \beta) < 0$ in $0 < \phi < \psi$,  

2) $V(\psi; v_o, \beta) = 0$.

The second condition is called the nonlinear dispersion relation because it implicitly determines the hole velocity $v_o$ in terms of $\psi$ and $\beta$.

![Fig. 8: The range of existence of EH solutions in the parameter space.](image)

An exploitation of the range in the parameter space $[15]$ for which genuine EH solutions exist, is shown in Fig. 8, the range of existence lying between the curves $v_o = 1.3$ and $v_o = 0$. Obviously, EHs need a negative $\beta$ for their existence. This as well as the finite value of $v_o$, $0 < v_o < 1.3$, is in accordance with the experimental observation. There is apparently no limitation in the hole amplitude $\psi$. 

$$\phi''(x) = n_e(\psi; v_o, \beta) - 1 - \frac{\partial V}{\partial \phi}. \tag{5}$$
In the small amplitude limit, $\psi << 1$, the whole structure can be resolved analytically. The electron density becomes

$$n_e(\phi; v_o, \beta) = 1 - \frac{1}{2} Z^r \left( v_o / \sqrt{2} \right) \phi - \frac{4 b}{3} \phi^{3/2} + \ldots,$$

where $Z^r$ is the real part of the plasma dispersion function

$$(Z(z) = \pi^{-1} \int \frac{dt \exp(-t^2)}{\sqrt{t-z}}),$$

and $b$ is given by the expression

$$b = \pi^{-1/2} \exp \left( - v_o^2/2 \right) \left( 1 - \beta - v_o^2 \right).$$

The nonlinear dispersion relation (7b) becomes

$$- \frac{1}{2} Z^r \left( v_o / \sqrt{2} \right) = \frac{16 b}{15} \phi^{1/2},$$

which can be solved for $v_o$:

$$v_o = 1.305 \left( 1 - \frac{16}{15} b \phi^{1/2} + \ldots \right).$$

This shows that the EH is a nonlinear descendant of the slow electron acoustic mode which in the long wave-length limit is defined by the linear dispersion relation $\omega = 1.305 k v_{te} [14, 18]$. The "energy law" (6) with $V$ given by

$$V(\phi; v_o, \beta) = \frac{8}{15} b \phi^2 \left( \sqrt{\psi} - \sqrt{\hat{\phi}} \right),$$

can be integrated to yield

$$\phi(x) = \psi \text{sech}^2 \left( \sqrt{\frac{b}{15}} \frac{\psi}{\phi} x \right).$$

The condition (7a) implies $b > 0$ from which follows $\beta < -0.71$. Hence, $\beta$ must be sufficiently negative. The potential form (13) deviates from that of the KdV soliton which is given by

$$\phi(x) = \psi \text{sech}^2 \left( \sqrt{\frac{3}{6}} \frac{\psi}{6} x \right),$$

in two respects: in the power of the sech and in the scaling of the argument. The EH's width, $\Delta_{EH} \sim \psi^{-1/4}$, is less than that of the KdV soliton, $\Delta_{KdV} \sim \psi^{-1/2}$, for comparable amplitudes, in agreement with the observation.

The evolution equation, of which (13) is a stationary solution, reads

$$\phi_t + 1.305 \left( 1 - 2 b \sqrt{\psi} \right) \phi_x - 1.305 \phi_{xxx} = 0.$$

This equation does not belong to the class of integrable differential equations. Hence, the coalescence of two EHs propagating with comparable speed such that the interaction time exceeds the bounce time of electrons in the superimposed potential well [19], seems to be a natural event in the class of solutions of (15).

The electron density is diminished at the center, $n_e(\psi) = 1 - \frac{\mu}{15} b \psi^{3/2}$, and it is clear that collisions with neutral particles introduced by an increase of the gas pressure will fill up the trapped electron region. The existence condition, $\beta < -0.71$, is then no longer met, and the hole - but not the soliton - will disappear.
The third structure which is identified as a DL based on the slow electron acoustic branch [20, 21] moves with a velocity given by \( v_0 = 1.305 \left(1 - \frac{8}{3} b \psi^{1/2}\right) \) which is smaller than that of the corresponding KH. Thus, it appears that even details of the experimentally observed structures can be explained analytically.

If the radial boundedness of the plasma is taken into account [14], the nonlinear dispersion relation (10) is essentially modified by an extra term \( K^2 \) on the left-hand side, where \( K = 2.4/R \) and \( R \) is the normalized plasma (cylinder) radius. This implies that \( R \) has to satisfy \( R > 5 \) for an EH to exist, a requirement which is, of course, satisfied in the experiment.

Recently a fuller investigation of the existence of EHs in the finite amplitude region in radially bounded plasma has been made by LYNOV et al. [22]. Including also waterbag distributions, they essentially arrive at the same results, in cases where a comparison is possible. Their general results show that for non-vanishing \( K \) there is a limit for the allowed amplitude, which tends to infinity as \( K \rightarrow 0 \). For \( K > 0.5 \) only extremely low amplitude EHs can exist. As expected, there is also an upper limit for the speed \( v_0 \leq v_{0MAX}(K) \).

![Fig. 9](image)

The range of existence in the full parameter space is indicated in Fig. 9 (shaded area), where the hole width \( \Delta_{EH} \) is plotted versus the amplitude for the Maxwellian-like distributions. For larger amplitudes, the hole width is seen to increase with increasing amplitude, in contrast to its small amplitude behaviour. The EHs extracted from a particle simulation (circles) are found to lie entirely in the area admitted by the theory. For an individual hole, however, a discrepancy up to 40 per cent between the observed width and the
calculated one was also found.

Despite the non-soliton property of holes it is instructive to associate "particle" properties with an EH [23, 19]. For this reason, we generalize the electron density (8) in the small amplitude limit:

\[ n_e = 1 + \beta^{-2} \phi + \int \phi (\bar{v} + O(\phi^2)), \]  

(16)

where \( \beta^{-2} = \frac{1}{P} \int f'_o (v) dv \), and \( \bar{v} \equiv f_t (\sqrt{2 \phi - v^2}) - f'_o (v) + \frac{1}{P} f'_o (v) \phi \).

P means principle value, \( f_t \) is the trapped electron distribution, and \( f'_o \) stands for the unperturbed free electron distribution, \( \bar{v} \) is, as we know, negative because of the lack of trapped particles. The integration in (16) has to be taken over the trapped range. One immediately checks that (16) reduces to (8) if the distributions (4) are inserted.

With (16) Poisson's equation becomes

\[ -\phi'' + \beta^{-2} \phi = -\int \bar{v} \phi (\bar{v} + O(\phi^2)), \]  

(17)

The second term in (17) can be interpreted as a shielding term. It is produced by free electrons and becomes most effective for large distances (\( \phi \rightarrow 0 \)), where the charge density \( \rho_{EH} \) vanishes. In view of \( \bar{v} < 0 \), \( \rho_{EH} \) is positive. It is at maximum at the core of the hole. Defining charge, mass, momentum, and energy of the hole:

\[ (Q, M, P, T) \equiv \int_{\infty}^{+\infty} \int_{\infty}^{+\infty} f (x, v, v^2 + \phi) \bar{v} \phi, \]  

(18)

we get \( Q = -M > 0 \). An EH can thus be interpreted as a cloud of positively charged particles embedded in an electron fluid which acts as a dielectric medium. A dc-electric field in one direction gives rise to a movement of the electrons into the opposite direction carrying with them the positively charged cloud. This "wrong" direction for the motion of the cloud is compensated by assigning a negative mass to it.

We close this section by mentioning the possibility of asymmetric EHs, Fig. 10, which can be excited in plasmas with different asymptotic states, e. g. in a double plasma device [24]. A slight asymmetry was already noticed in the EH produced in the laboratory experiment, Fig. 2.

Fig. 10: Three diagrams characterizing an asymmetric EH. After [24].
where the reflection of electrons caused the asymmetry.

4. **The ion hole and related phase space structures**

In view of the previous sections we can easily infer on the existence of IHs [25, 15].

The Vlasov-Poisson system

\[
\left[ \partial_t + \mathbf{v} \cdot \nabla + \frac{q_s}{m_s} \mathbf{E} \cdot \nabla \right] f_s = 0 \quad s = e, i, \tag{18}
\]

\[
\mathbf{V} \cdot \mathbf{E} = 4 \pi \sum_{s=i, e} q_s \int \mathbf{d}^3 \mathbf{v} f_s, \quad \mathbf{E} = - \nabla \phi, \tag{19}
\]

is seen to be essentially invariant with respect to the transformation \((\phi, - e) \rightarrow (- \phi, + e)\). Hence, a hump in \(\phi\) describing an EH goes over to a negative dip representing an IH, the characteristic features of which are shown in Fig. 11. There is not a one-to-one correspondence between both holes, because the "passive" species is treated differently in both cases. For an EH the ion density was assumed to be unity, for an IH we may generally assume that \(n_e\) is of Boltzmann-type, \(n_e = \exp(\phi)\). This introduces some characteristic changes in the properties of IHs. The most remarkable ones are

\[
\begin{align*}
\text{i) } & \frac{T_e}{T_i} > 3.5, \\
\text{ii) } & \left[ \frac{e\phi_{\text{min}}}{T_e} \right] \leq 1, \tag{20}
\end{align*}
\]

which must be valid for ion holes to exist. Condition i) says that the electrons must be sufficiently hot which is the same requirement as for ion acoustic waves. The second condition implies that the depth of the potential energy is limited by the electron thermal energy.

![Fig. 11: Three diagrams characterizing an IH. After [7].](image)

Fig. 12 shows the existence diagram for IHs. We have the same situation as before, i.e. IHs exist only when the trapped ion parameter is sufficiently negative. The ion distribution is of vortex-type. The IH velocity \(\mathbf{u}_i\), which this time is normalized by the ion temperature \(T_i\), is of order unity and,
hence, very small. IHs are more or less standing structures.

For small amplitudes we get \[ u_0 = 1.305 \left[ 1 + \frac{T_i}{T_e} - \frac{16}{15} b \psi^{1/2} \right], \] (21)
where \( b \) follows from (9) if \( v_0, \beta \), are replaced by \( u_0, \alpha \). Eq. (21) tells us that IHs are based on the slow ion acoustic branch \[ [16, 18, 25]. \]

Again an asymmetric version of a hole exists \[ [7, 24, 26, 27] \] shown in Fig. 13 together with the corresponding phase space plots.

The quasi-particle interpretation of an IH can be summarized by
\[ -\phi'' + (1 + \lambda^{-2})\phi = \int f \, dv \equiv \rho_{IH} < 0, \] (22)
where \( f \) and \( \lambda \) are defined in analogy to (16). The unity stems from the linear term in the Taylor expansion of \( \exp (\phi) \). An IH can be understood as a negatively charged cloud embedded in an ion fluid, having a negative charge and a negative mass: \( Q = M = \int \rho_{IH} \, dx < 0. \)

Two other important collisionless systems should be mentioned which develop similar structures. The first one are stellar systems, the second one particle accelerators and storage rings.

The distribution of stars is governed by the Vlasov equation in which the gravitational field \( \mathbf{E} \) follows self-consistently from Poisson's equation:
\[ [3_t + \mathbf{v} \cdot \nabla + \mathbf{E} \cdot \mathbf{a}] f = 0, \] (23)
\[ \nabla \cdot \mathbf{E} = -4\pi G \int d^3 v f. \] (24)
\( G \) being the universal gravitation constant. Comparing (23), (24), with (18), (19), resp. (22), for ion holes, we conclude that due to the minus sign in (24), we get a bunch-
ing of trapped particles rather than a deficiency. Hence, an H is the pendent to a galaxy where only trapped particles are present (no shielding!). The Coulomb repulsion of ions in the former corresponds to the gravitational attraction of stars in the latter; the two-stream instability is replaced by the Jeans instability [28].

The circulating particles in a storage ring or particle accelerator represent another Vlasov system [29],

$$\partial_t f + v \partial_x f + \frac{F}{m} \partial_v f = 0,$$

where $x$ is the longitudinal coordinate along the tube, $v (\equiv v_e)$ is related to the radius, and $F$ is the electric force determined self-consistently. The effective mass of a particle is denoted by $m$. It is determined by the radial variation of the guide magnetic field and turns out to be proportional to $\frac{dw}{dE}$, where $w$ is the circulation frequency, and $E$ the relativistic longitudinal energy. The latter expression can be positive or negative depending on whether $E$ is below or above a critical value $E_c$, called the transition energy. When $m$ is positive ($F < E_c$), a hole can be observed in the charged particle beam [30]. It is caused by two-stream instabilities. If, on the other hand, $m$ is negative ($E > E_c$), the beam is observed to clump, the density becomes modulated forming clusters. The underlying instability is called the negative mass-instability. The repulsive Coulomb force acts on particles with negative mass in the same manner as the attractive force acts on positive masses. Hence, to a positive-mass system with a deficiency of particles there corresponds a negative-mass system with an excess of particles. This duality principle is sometimes called mass-conjugation-theorem [31]. Saturn's rings and holes in the rings constitute another dual system [28].

5. **Double layers**

The by far most important and most known structure we are discussing in this Review, is the DL for which numerous original and review papers have been presented dealing with experimental [2, 32 - 48], theoretical [5, 7, 20, 21, 24, 25, 49 - 58], and numerical [59 - 68] investigations. A DL is defined as a monotonic transition of the electric potential connecting smoothly two differently biased plasmas. This is achieved by a dipole-like charge distribution, as shown in Fig. 14, where the electrical and classical potential are drawn together with the total charge density, $\delta n = n_i - n_e$, for a single ionized plasma. According to POISSON's equation, $\delta n = - \delta \phi$, a positively

![Fig. 14: The electrical and classical potential of a DL. The dotted line represents the dipole-like total charge density.](image-url)
charged layer gives rise to a region of negative curvature of $\psi$ and vice versa, and hence, two oppositely charged layers are needed to build-up the DL structure. Again, trapped particles must be involved, as can be seen by a simple counter argument, namely, if only streaming (i.e. non-reflected) particles would be present, the spatial constancy of each current, $n_j u_j = \text{const.}$, $j = i, e$, would imply that the required asymptotic charge neutrality cannot hold simultaneously on both sides of the DL structure. Due to the different acceleration each species experiences in the DL, the densities are affected differently: The density of ions (electrons) injected from the high (low) potential side decreases (increases) with decreasing potential. Therefore, if the densities are equal on one side, they have to differ on the other side, and charge neutrality cannot be established there.

As in the discussion of solitons or solitary wave solutions, one has to specify on which branch of normal modes DL solutions are looked for. DLs based on different branches turn out to differ in the phase space topology. A DL in its simplest form, ignoring magnetic field [58], or geometry [68] effects, and taking into account a minimum set of distributions only, is given by one of the three structures shown in Fig. 15.

![Diagram of DL types](image)

**Fig. 15**: Three types of DLs. The second (third) row represents the ion (electron) phase space, whereas the various densities are plotted in the 4th row. After [21].

The first column represents the proper DL called the strong or beam-type DL. Its main characteristic features [5, 7, 24, 25, 54] are listed as follows:

- there are four distributions involved which are separated in phase space,
- the drifting species described by modified shifted Maxwellians given by
eq. (4a), enters the DL region with finite velocity called the BOHM-criterion for DLs, $v_o \geq v_o c (|u_o| \geq |u_{oc}|)$ for electrons (ions), where $v_{oc}$ ($u_{oc}$) is of order unity,

- the trapped species must be sufficiently strong $(\alpha, \beta) \geq (\alpha_c, \beta_c)$ which are $O(1)$,
- it exists in the finite amplitude regime only, $\psi > \psi_c = O(1)$, (i.e. there is no corresponding linear model),
- the LANGMUIR "condition" [49], which relates the electron and the ion current in a definite way ($j_e / j_i = \sqrt{m_i / m_e}$), is not a necessary requisite but seems to be preferred by stability arguments,
- this type of DL is usually subject to various two-stream instabilities yielding to hf ($\xi$)-turbulence on the high (low) potential side which superimposes, usually in a non-destructive manner, the coherent DL structure.

We note in parenthesis that the most general expressions for the two BOHM-criteria are given by $V''(0) < 0$ for the electrons, and by $V''(\psi) < 0$ for the ions, respectively, where $V(\psi)$ is the classical potential. It is this type of DL which is usually excited in a strongly driven plasma, and which gives rise to the observed strong particle acceleration.

The second column in Fig. 15 shows the so-called slow electron acoustic double layer (SEADL) [56, 21]. It is based, as the name says, on the slow electron acoustic mode and does have a small amplitude limit. Its most striking feature is the "tuning fork" configuration of the particle distribution in the electron phase space. Favourable conditions for its existence are hot ions (including $T_i = \infty$ which represents immobile ions [21]), and a less dense plasma on the high potential side. As seen immediately, a SEADL is a descendant of EIIs. Since both demand the same prevailing conditions, it is not surprising that they can be generated simultaneously in a plasma (see Sect. 2 and Figs. 3, 4).

Like in the IH case a simple exchange of the species leads to the slow ion acoustic double layer (SIADL). The tuning fork configuration is now found in the ion phase space. There is an upper limit of the amplitude $\psi \leq \psi_c$. It is this DL structure which can exist under current-free conditions [55]. As in the IH case, the electrons have to be sufficiently hot.

It is worth mentioning that the first self-consistent analytic expression for a DL at all was given for the last two DLs in the small amplitude regime by KIM [56]:

$$\psi(x) = \pm \frac{\psi}{4} \left[ 1 \pm \tan h (\kappa x) \right]^2. \quad (\kappa = \sqrt{\psi}) \quad (26)$$

More recently, BIJARBARUA and GOSWAMI [69] were able to get DLs on the ordinary ion acoustic branch by allowing a two-temperature electron distribution, verifying a conjecture made by TORVEN [70]. Both compressive and rarefactive DLs can be obtained, dependent on the temperature and density ratio of hot and cold electrons. A compressive (rarefactive) DL is defined
by a decrease (increase) of the density and the potential in the direction of propagation and is generally subsonic (supersonic). The corresponding solitons [71] in contrast, require reversed conditions and are both supersonic.

I close this section with two remarks. Firstly, DLs are, in general, as abundant as solitons, and only slight modifications are needed to cast a soliton solution into a DL solution [72]. Secondly, the DLs based on the ion branches are a manifestation of an old idea of SAGDEEV and others [73], concerning the necessity of reflected ions and represent the first self-consistent solutions of what was called earlier a laminar collisionless electrostatic shock [74].

6. Experiments on holes and double layers

Numerical simulations and laboratory experiments performed in the last decade have revealed many aspects about the existence and the possible generation mechanism of phase space structures. Only a few of them shall be mentioned here. Since we have already treated in some length EHs we will concentrate here on IHs and DLs.

One of the first numerical observations of phase space structures which are related to (asymmetric) IHs and SIADLs was made by SAKANAKA [75], simulating the interaction of an ion beam injected into a plasma, a process which was studied experimentally and numerically also in Ref. [76]. The simulation reveals that the electric field at the ion beam front is distributed such that the leading beam particles will be accelerated whereas the succeeding particles will be slowed down. This gives rise to a region exhibiting a tuning-fork configuration corresponding to a SIADL, followed by a quiet heated region where the beam has merged in the background plasma, and by a region with one or more IHs which accomplish the merging of the two-stream unstable beam. IHs and its asymmetric version were, furthermore, seen in simulations treating current-carrying plasmas [77 - 81]. An electron drift with velocity as low as $v_d = 0.5 v_{te}$ [79, 80], the lower limit depending on de-
tails of the numerical conditions, gives rise via the ion acoustic two-stream instability to the local excitation of an IH which due to electron reflection on the negative potential dip becomes asymmetric. Details of this scenario were described in Ref. [24]. An example of this type of simulations is given in Fig. 16a, b, showing a plane asymmetric IH in two dimensions (Fig. 16a), the growth of which was, in this case, triggered by an imposed density depression, and a plane SIADL (Fig. 16b), respectively. Both turn out to be weakly transversally unstable [21, 83]. In the laboratory, asymmetric IHs and SIADLs have been generated by FUJITA et al. [45], when an applied positive potential was switched-off suddenly, by CHAN et al. [82], when electrons were abruptly injected, and by SEKAR and SAXENA [46], when a steadily injected e-beam was modulated by a negative step potential. An important observation made in these simulations and real experiments are the strong current reduction at the moment when the asymmetric IH becomes excited, and the possible transition of the latter into a SIADL (see Fig. 16b) or even into a strong DL [81]. This observation seems to play a key role in the understanding of more involved laboratory experiments in which a negative resistance and an associated current disruption are found. I should mention here the investigation of LUTSENKO et al. [36, 84] in a straight high current, low pressure discharge, who found 1) the formation of a DL at a point where the plasma density is depressed, ii) the generation of intense beams of electrons and ions in the space-charge region, iii) the appearance of a high resistance which was not "anomalous", and iv) intense microwave emission at the instant when the high-current beam is formed. The authors suspect that this "volume mechanism" limiting the current may as well be the cause for the current disruptions observed in high-current toroidal discharges or in a plasma focus.

Similar conclusions concerning the mechanism of DL formation were recently drawn by TORVEN et al. [48] experimenting with a triple plasma device. A spontaneous current disruption caused by electron reflection on a negative potential dip (NPD) was found to be the triggering mechanism of a DL. However, as described also by other authors [85–89], the NPD need not be an IH. It was stretched in this experiment and, thus it was more quasi-neutral-like. This controlling function of the NPD could be studied in more detail, if in addition to the applied voltage $U_o$ an inductance $L$ was introduced in the external circuit of the device. If $U_o$ was above a critical value, periodic current disruptions correlated with DLs were seen. The inductive over-voltage produced by the disrupted current was several hundreds of volts over the plasma ($T_e = 8$ eV, $n_e = 10^{16}$m$^{-3}$), and was found to be concentrated at the DL that maintained as long as the circuit was able to produce the over-voltage. The energy stored initially as magnetic energy in the inductance was transferred to particle energy in the DL. In this series of experiments the DL was periodically switched on and off, and was more or less standing. There exist, however, situations in which the DL is found to be in motion giving rise to a periodic phenomenon called potential relaxation.
oscillations, the concern of our last paragraph.

7. Potential relaxation oscillations

The oscillatory phenomenon to be discussed here has been observed in many experiments [87, 90 - 97] in which the finite length of the system is of crucial importance. A typical arrangement is a plasma diode (single ended Q-machine) consisting of a grounded plasma source and a positively biased collector plate terminating the plasma. If the applied voltage is sufficiently large, low frequency oscillations of typically 1 - 10 kHz are seen. Fig. 17 shows the space-time behaviour of the electrostatic potential of IIZUKA et al. [87], the evolution of which being initiated by an expanding plasma. One readily recognizes two main phases within one cycle of about 400 μs. The first one is characterized by a strong propagating DL which is accompanied by a broad NPD on its low potential side, the second one by a fast increase of the potential in the whole column shortly after the DL has reached the anode. The collected target current is sawtooth-like in time with the decaying phase during the presence of the DL. The oscillation period is correlated with the transit-time of the DL which moves with approximately 2 - 3 times the ion sound speed. This propagation velocity is determined by the speed of the expanding plasma on the low potential side, enabling the DL to satisfy the two BOHM criteria [98]. The second, more rapid phase, is due to an instability of the electron-rich sheath which is formed at the anode after the arrival of the DL. The electrons in this sheath and in the column are quickly lost, and the resultant positive space charge gives rise to an increase of the space potential because the ions cannot respond on this fast time scale.

BURGER [99] who investigated this phenomenon in connection with thermionic converters as well as BRAITHWAITE and ALLEN [100] argue that there exists a second dc-state which is adapted by the system after a rearrangement of the electrons, occurring on the fast electron time-scale during which the ions "are virtually frozen".

The formation process of the DL itself could be resolved numerically by Refs. [97, 101] in a plasma diode simulation in which the particles are emitted with equal temperatures. It is found that the DL is preceded by an EH which propagates with \(v_{te}\) through the plasma and which provides after its arrival at the collector the change in phase space topology necessary for the build-up of the DL. The EH, the original vortex-like structure of which is
shifted in phase space and is subsequently cut by the anode, leaving behind low energy electrons on the anode side and accelerated electrons which join continuously with fresh low velocity electrons provided by the source. The former constitute the trapped electrons, the latter the free streaming (un-trapped) electrons in the DL. The ions, on the other hand, are accelerated by the EH hump-potential in such a way that a high energy component moving towards the cathode survives, forming the free ions in the DL configuration. The trapped low velocity ions are produced by the source.

In other words, in this situation the EH takes over the role of the IH to "trigger" a strong DL in a bounded plasma by a mechanism which is quite different from the IH generation mechanism (see also [102, 103]). The NPD is then merely a result of the ambipolar expansion of the plasma provided by the source into the tenuous plasma giving rise to a small, time-varying modification of the DL profile. The expanding electrons diffuse ahead of the ions and produce the NPD acting as a barrier (virtual cathode) for low energy electrons.

The oscillation frequency of the PKO turns out to be independent of the applied voltage [97]. Thus this phenomenon bears many characteristic features of the constant frequency oscillations noticed by ENRIQUES et al. [91] and investigated by ALLEN and co-workers [92]. They were observed always when the dc applied voltage was roughly equal to that required to produce dc current saturation. The dc current-voltage characteristics in the presence of oscillations has a negative slope corresponding to a differential negative resistivity provided that the electron density is high enough. Further references in which a correlation between DL generation and negative resistance was pointed out are given by [43, 104 - 107]. KNORR [108] discussed the negative resistance and the associated hysteresis effects in DL carrying plasmas with regard to THOM's (cusp) catastrophe theory [109].

Two comparisons should be made before closing this section. Current relaxation oscillations of similar type have been reported in solid state physics and in electro-negative gas discharges.

The first ones are the well-known GUNN-oscillations [110] in semi-conductors having two conduction bands with different energy minima (e.g. n-type GaAs). These oscillations are characterized by a repetitive propagation of an electric field pulse (DL) seen during the phase of current limitation and associated negative resistance. This pulse is maintained by electrons in the lower (upper) conduction band with a high (low) mobility in analogy to the free (trapped) species. If the applied dc field exceeds a threshold value, electrons may tunnel into the higher conduction band where, supported by collisions, the conduction drift velocity is reduced and therewith the current.

A second analogue observation has been made by SABADIL [111]. He got periodic current oscillations in the positive column of an oxygen discharge similar to the GUNN oscillations. The dissociation of O₂ by electron attach-
ment into $O^{-}$ and $O$ leads to heavy negative ions which play the role of conduction electrons having the larger effective mass. This dissociation process requires a minimum electron energy of 4.4 eV which must be delivered by the external field. If the concentration of these negative ions $N_-$ exceeds that of electrons $N_e$ by a certain amount ($N_-/N_e \approx 20$), a dipole space charge-layer propagating periodically from the cathode to the anode is seen, the so-called T-layer. The frequency of the oscillation decreases monotonically with increasing distance of the electrodes. T-layers could also be observed in CO$_2$ discharges [112].

8. Conclusions

Many aspects of localized electrostatic structures which are akin to those discussed in this Review have not been touched. I should at least mention some of them: holes and double layers in the ionosphere and in the solar corona, electrostatic barriers as end plugs in tandem mirrors, U-shaped DLs, DLs and holes propagating obliquely to a magnetic field, sheaths and DLs at electrodes, ionization dependent structures, and many others.

Here we have concentrated on the simplest possible way to excite and to describe collisionless phase space structures appearing as volume structures in a plasma. Although the analytical properties derived are strictly valid in equilibrium situations only, scenarios could be developed describing qualitatively dynamical features of more involved transient processes. The reason is that these structures, as said in the Introduction, possess the property to attract deviating time-dependent solutions very similar to the self-similar solutions in other areas of nonlinear physics. From this follows that the true time evolution of a given dynamical system can often be divided into separate time intervals each of which being governed by one or more excited nonlinear steady-state structures, even if the whole system is changing appreciably for a long time. Of course, the regions connecting these intervals in which the dynamical evolution is more violent, are out of the scope of a steady-state treatment and, therefore, globally valid descriptions are hardly to get.

A typical example has been presented in the last section where the main phase in the oscillatory behaviour of a voltage driven plasma diode was characterized by the presence of a propagating DL. In this case laboratory and numerical experiments yield essentially identical results despite the fact that many idealizations had to be made in the numerical model (e. g. neglect of boundary and geometrical effects, of magnetic field- and external circuit effects, etc.). This demonstrates that in this case the main physics was not affected by these simplifications. It appears then that in such situations only "a few degrees of freedom" are effectively used by the plasma. In this sense the phase space structures discussed in this Review are a manifestation of this reduced albeit nonlinear dynamical behaviour. It seems to be beneficial to consider them as basic elements in the description of more complex
driven plasmas far away from thermodynamic equilibrium.

In view of the novel results obtained in recent years by the common efforts of experimentalists and theoreticians, one may think of other phenomena to be attacked on similar lines. An example are the current disruptions observed in high-current low-pressure discharges.

Note added in proof. In a recent linear turbulent heating experiment of INUZUKA et al. [113], similar to that of Ref. [36, 84], strong DLs have been recorded coincident with a NPD and a low value of the heating current.

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A review is given on plasma production of metal vapour arcs in vacuum. Based on experimental results, plasma acceleration, structure of cathode spots and models of spot formation are discussed. In particular, the role of surface cleanness is underlined. In this way a coherent though not complete picture of vacuum arcs is obtained, but also controversial problems like current density or time constants of cathode spots are analysed.

1. Introduction

Between two metal electrodes in vacuum, electrical arcs can be ignited which burn with relatively low voltage, and which have virtually no upper limit of the current (while there is a lower limit of a few amperes). As stated by Becker[1] the common term vacuum arc is paradoxical in itself - if there is vacuum, there is no arc, if there occurs an arc, there is no longer vacuum. While this is merely a problem of semantics (actually the arc is fed by plasma emitted from hot spots on the electrode surfaces, thus the term metal vapour arc would be more appropriate), the arc exhibits also physical paradoxes that are not yet satisfactorily solved.

A review on vacuum arcs is confronted with these problems as well as with their controversial interpretation in the literature. Therefore, simplifications cannot be avoided in order to obtain a concise representation. Furthermore, lack of space requires several restrictions. The current range will be limited to a few 100 A, i.e. to the region where only few arc channels (cathode spots) exist simultaneously. Sufficiently clean surfaces will be preferred (as these are present e.g. in vacuum switches) thus reducing the variety of possible effects. Finally, phenomena caused by magnetic fields will not be considered.

2. Arc plasma

When starting in ultra high vacuum, with gas-free electrodes, and with moderate currents, there is no plasma production far from the cathode (within the gap or at the anode). All plasma originates from mobile point sources at the cathode surface, called cathode spots. As demonstrated by Becker[1], the amount of neutral vapour at the spots is small, being probably produced by side effects (evaporating droplets or emission from the wake of the moving spots). Therefore the degree of ionization of the active arc plasma is not far from unity. Several authors[4–7] studied the plasma ions far from the spot. They found abundance of multiply charged particles, and a high velocity directed from the spot towards the anode in the form of a cosine distribution. Table 1 (after Kittel[7]) shows typical values for Cu, 100 A.
Table 1

<table>
<thead>
<tr>
<th>Ze (1.6 x 10^-19 As)</th>
<th>v (m/s)</th>
<th>E/Ze (V)</th>
<th>V_b (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.77</td>
<td>1.6 x 10^4</td>
<td>48</td>
<td>21</td>
</tr>
</tbody>
</table>

Ze - charge, v - directed velocity, E/Ze - kinetic energy per charge, V_b - arc burning voltage.

According to table 1, the ions have more directed energy than corresponding to the applied voltage. Because of this high velocity, surrounding walls receive considerable positive currents when biased negatively (at cathode potential or below). This has been established by Kimblin [9-13] who found the wall ion current I^+ is a constant fraction of the total arc current I for all investigated metals,

\[ I^+ = f I, \quad f = 0.08-0.1. \quad (1) \]

The fraction f does not depend on the wall radius, thus the ion current density j^+ must vary as r^-2, r being the distance to the spot. Since j^+ = Zev, also the plasma density n is proportional to r^-2. Because of equation (1) n is also proportional to the arc current I. Finally, nv~cosθ (cosine distribution), θ being the angle to the surface normal at the spot. The proportionality constant c is obtained by integrating j^+ over the wall surface which is in line of sight to the spot. Using equation (1), the result is

\[ nr^2/Icosθ = c \approx 2f (π Zev)^{-1}. \quad (2) \]

With the values from table 1, we have

\[ c \approx 10^{13} \text{ m}^{-1}\text{A}^{-1}. \quad (3) \]

Hence, Kimblin’s measurements allow general conclusions on the plasma density outside the spot.

A similar result can be obtained from the cathodic erosion rate, i.e., the ratio of mass loss at the cathode to the current-time integral. Because of mass conservation, the mass flow \( \dot{m} I \) must be equal to \( m_1 \int n v dP \), where the integral is taken over a hemispherical surface around the spot (\( m_1 \) - ion mass). Using the relation

\[ n = c \frac{1}{r^2} \cos θ, \]

the integration results in \( c = k(π v m_1)^{-1} \). Inserting experimental values for \( k \) (5 x 10^-5 - 10^-4 g/As [11]), we obtain \( c = (0.9 - 1.9) \times 10^{13} \text{ m}^{-1}\text{A}^{-1} \), in agreement with relation (3).

This result implies large plasma densities at small distances r. Fig. 1 shows the electron density of a carbon arc plasma measured by Langmuir-microprobes (Ivanov et al. [14]). In the current range 30-100 A the measuring curve can be approximated by \[ nr^2 = 1.8 \times 10^{15} m^{-1}. \] This is consistent with equation

\[ n = 1.8 \times 10^{15}/r^2 \]

Fig. 1: Electron density n of carbon arc plasmas as a function of distance r to the spot [14].
for 2:1, v = 10^4 m/s, θ = 0. The relation holds down to r = 15 µm, corresponding to densities up to 10^15 m^-3. If an extrapolation to smaller distances were allowed, the density would be still higher (see section 3).

The electron temperature near the spot was found to be 1 - 2 eV [14,15]. In view of the high ion velocities, this is rather low. Consequently, there must be processes that accelerate the plasma ions to energies ≈ 50 eV (the great difference between the thermal and the directed energy of the ions is the reason why we neglected the thermal velocity when deducing equation 2).

Plyutto, Ryzhkov and Kapin [14], who first reported detailed measurements on the ion energies, explain the acceleration by the gradient of electron pressure p in the expanding plasma. This gradient equals a force -gradp/n = -kTe^-1 gradn = -kT grad ln n which is balanced by a corresponding electric field. The integration from a distance r_1 to a distance r_2 yields a voltage kT e^-1 ln n(r_1)/n(r_2). Since there are gradients of pressure and density towards the cathode as well as towards the anode, the associated voltage drop forms a potential hump as indicated in fig.2.

On an analogous base, Moyzhex and Nemchinskii [16] suggest a gain of kinetic ion energy

E_{kin} = (Z+1) kTe ln n(r_1)/n(r_2). (4)

Experimentally, a potential hump could not be demonstrated; therefore it must be situated close to the cathode (r_1 < 0.1 mm). Assuming n ~ r^-2, Z ~ r_2/r_1 = 10-100, equation (4) results in a voltage drop of (7-14)kT/e < 30 V. Thus, the potential hump can only explain a part of the ion energies.

Lyubimov [17] assumes another gas dynamic phenomenon: The arc current I heats the electrons of the expanding plasma, and the electron transfer momentum to the ions. There is a voltage drop AV_p in the plasma (in fig. 2 the region r_2 - r_1). The power IAV_p is not completely randomized, a part IAV being converted into directed energy of the plasma jet. The plasma mass is εI t, thus with the directed velocity v the kinetic jet energy is 0.5εI tv^2 = IAV t (ε - erosion rate, t - time). Therefore we have 0.5 v^2 = AV/εt. The accelerating voltage of an ion with mass m_1 is then

V_1 = 0.5(Ze)^-1 m_1 v^2 = (m_1/Z_e ε t) AV. (5)

The factor m_1/e ε t = (I/e)/(t/m_1) signifies the ratio of the electron number to the ion number. So the acceleration process is the more effective the higher the relative number of electrons in.
Equation (5) yields a voltage $V_1$ of $(4...40)\Delta V$. Together with the potential hump this will be sufficient in order to explain the ion energies. This agrees with Miller[6] who analysed the interrelation of ion charge and ion energy and who concluded that both mechanisms are working. Also Harris[18] included both processes in his flow theory of the arc. For Cu, 80 A, he calculated ion energies of 45 eV and multiplicities $Z = 1.77$, in excellent agreement with the experiments (table 1). Thus, the ion acceleration seems to be sufficiently understood.

The randomized part of $\Delta V_p$ leads to an increase of the plasma temperature. Far from the spot, electron temperatures up to 7 eV have been found in [14], in independence on the arc current. Similarly, Goldsmith and Boixman [19] report about 6 eV near the anode of an Al-arc, and Drouet [20] concluded 5.4 eV for the anodic region of a Cu-arc.

Thus far, the time played no essential role, our arguments were stationary. However, Mesyats and co-workers[21] explain the ion velocities by explosive surface processes. From experiments with exploding wires[22], it is known that the matter is ejected with velocities of $(1-2) \times 10^4$ m/s due to overheating the surface. This would yield the measured ion energies if the explosions were sufficiently frequent. The relevance of explosive processes will be discussed in section 5.

3. Spot structure

The real size of the cathode spot is decisive for plasma production, electron emission, erosion rate, energy balance and other important processes. At present it is one of the most controversial parameters.

At currents $> 100$ A, there exist several spots simultaneously [23]. If there are no anode spots (compare section 6), the cathode spots can be considered as sufficiently independent from each other, so we discuss single spots that are typical for currents of 10-100 A.

At first Farrall[24] raised clearly the problem of a possible substructure of a seemingly single spot. Harris [25] suggests every spot may consist of an array of internal cells. He assumes the cells are subjected to repelling forces due to dipole interaction, and to attractive forces due to magnetic interaction, so an equilibrium distance between the cells is established. A similar conception has been already published by Sena[26].

With clean surfaces, it is difficult to decide whether a spot consists of internal cells. Fig.3 shows a magnified view of an arc of 80 A running from a trigger electrode over the surface of a clean copper cathode.
(open shutter photograph). The smallest discernible spot structures have diameters of 20-40 µm. The larger structures were probably build up successively in this time-integrated picture by the spot movement, i.e. they do not correspond to a real spot. With a better time resolution (framing time ≤ 1 µs) it can be seen that such spots on clean surfaces split sometimes into 2-3 fragments, but these join again quickly after that. This may be an indication of an internal structure where the repelling forces dominate only momentarily.

But if the arc is fed by surface films, the cathode spot is unambiguously structured. Such films may be contaminants on metallic substrates or metal films on dielectric substrates. The latter have been thoroughly investigated by Kesaev [27]. Fig. 4 shows an erosion track of the Kesaev-type with an arc of 50 A, 2 ms, burning on an Al-film on glass (the location where the aluminum is removed appears black in the figure). In contrast to fig. 3 there are numerous branches. The repelling forces seem to dominate.

Kesaev measured the maximum current of a cell as a function of film thickness. The thinner the film the smaller is the cell current (and, consequently, the more branches occur). For copper, the maximum current tends to saturate at values below 4 A (the corresponding film thickness being < 1 µm). Therefore, with massive cathodes one would expect about 10 internal cells at 50 A arc current, but since they do not separate, they cannot be discerned, neither in spot photographs (fig. 3) nor in the remaining surface traces. Fig. 5 shows a spot track due to a 50 A arc on clean stainless steel. It consists of overlapping craters with an average diameter of 10 µm. The width of the whole track varies between 10 µm and 30 µm. This is due to a chaotic (successive) displacement of the spot centre, and to an...
occasional branching. But the essential melting structure is probably the crater. In any case, the track width represents an upper limit of the melting zone. Outside the track, the surface shows no sign of melting and erosion. Since evaporation is possible only within the molten surface area, we have three characteristic limits for the radius of the plasma source - about 15 \( \mu \text{m} \) according to the maximum track width, about 5 \( \mu \text{m} \) according to the crater, and a value below 5 \( \mu \text{m} \) according to the hypothetical internal cells. In the literature, there are more detailed data on the crater radius \( r_c \) [28,29]. They can be summarized by the empirical formula

\[ r_c = r^w \exp (I/I^w), \]  

with \( r^w = 1.7, 1.2, \) and 3.8 \( \mu \text{m} \), and \( I^w = 83, 33, \) and 46 A for copper, molybdenum and stainless steel, respectively.

Inserting in equation (2) a radius \( r=15 \mu \text{m} \) (according to the track width) we obtain a plasma density at the spot edge of about \( 2 \times 10^{24} \text{ m}^{-3} \) for \( I=50 \text{ A} \). According to equation (6), this current produces crater radii of about 3 \( \mu \text{m} \) on clean Cu-cathodes. Inserting the value we obtain about \( 6 \times 10^{25} \text{ m}^{-3} \). An inner structure would reduce the available area further on, resulting in still higher densities.

As indicated in fig.4, with cathodes consisting of surface films the influenced area is much greater. Similarly, with contaminated cathodes the visible spots have diameters of about 100 \( \mu \text{m} \) [15]. But if the spot is stretched by a transverse magnetic field, it can be seen that the really affected area is smaller. This is shown in fig.6 [30] where several arc spots produced parallel traces on a Mo-surface covered by an oxide film of a few nm. Each trace indicates the existence of internal fragments. The magnetic field (1.4 T) prevented the fragments to diverge (in contrast to fig.4). The track width is again around 10 \( \mu \text{m} \), like in the case of clean surfaces. But now it consists of tiny craters with diame-
ters ≤ 1 μm that are dispersed over the active surface. Obviously, many of them were formed simultaneously. Sometimes there occurred transitions from this pattern to the concentrated crater train like in fig. 5. In this case the track width is reduced to about one third, and the movement appears to be more chaotic.

Fig. 7 gives a detail from fig. 6. Apparently, the sources of metal vapour are even more restricted than with clean surfaces. But it is possible that the arc plasma is also fed by non-metallic surface contaminants (oxides, hydrocarbons) which are removed from the area between the craters. Perhaps the small craters correspond to the internal fragments of a clean surface spot.

In any case, the maximum size of the source of metal vapour plasma is about the crater diameter, i.e. a few μm. This signifies a high plasma density (equation 2), and consequently a considerable power input to the surface due to ion bombardment.

4. Current density

In the literature, the current transfer from the cathode into the arc gap is interpreted mainly by electron emission. It is often assumed that the region where the current passes the surface coincides with the area of plasma production. This results in very high current densities because of the small size of the molten zone. For copper, 50 A, the emission current density would be about 10^{12} A/m^2 [28].

Such values are disputed by Rakhovskii and co-workers [31-33], because the conductivity of the plasma adjacent to the spot does not suffice. The voltage drop in the plasma ball at the cathode surface is [32]

$$\Delta V_r = \varphi_{pl} I/2\pi r_0,$$

(7)

where \(r_0\) - spot radius, \(\varphi_{pl}\) - plasma resistivity. Calculating \(\varphi_{pl}\) according to the theory of Spitzer [34], one obtains about 10^{-4} \Omega m. This theory is not well adapted to our problem, because the arc plasma is already non-ideal (e.g. the Coulomb logarithm for electron - ion collisions in \(\leq 2\)). But probably the order of magnitude of \(\varphi_{pl}\) will be still correct. Using therefore the values \(\varphi_{pl}=10^{-4} \Omega m, I=50 A,\) and \(r_0 = r_c = 3 \mu m,\) we obtain \(\Delta V_r = 265 V.\) In general, such a voltage does not appear at the electrodes. Puchkaryov and Frolovskovskii [35] have shown in an elegant experiment that the discharge can exist at voltages below 50 V also on a nano-second time scale, therefore eventual voltage transients from the outer circuit cannot explain the general current conduction.

But internal voltages may occur that help to maintain the current density. This is related with the problem of ion acceleration (section 7). E.g. a potential hump as caused by the gradient of electron pressure (equation 4) should be especially high at the lateral spot edge, because the plasma density decreases rapidly in thin direction (as indicated by the cosine distribution of the plasma ions). A further effect is stressed in a recent paper by Daybelge [36]. As the plasma expands laterally in the magnetic field of the arc current, additional voltages are induced (like in a moving conductor) with a maximum again at the spot edge. Nevertheless, these internal voltages will hardly exceed 100 V.
Several other phenomena (enumerated in [37]) may provide for high current densities, too. In particular, explosive processes may contribute. Already a sufficiently fast (non-stationary) emission of metal may create a high particle density such that there is a quasi metallic conductivity. As it will be shown in the next section, craters can be formed within a time $\tau \leq 10$ ns. If a sufficient fraction of the crater volume is transformed into plasma, we obtain a density up to $10^{28}$ m$^{-3}$ near the surface, close to the degeneration limit. Similarly, equation (2) indicates a density after plasma acceleration up to $10^{26}$ m$^{-3}$. Because of mass continuity, the density must be still higher before acceleration of the plasma, i.e. directly at the surface. Thus we would expect the occurrence of a very dense plasma layer with high conductivity. Magnified spot profiles as in fig.3 seem to confirm the existence of a thin bright layer at the surface (arrow B) which differs from the remaining plasma region (the corresponding step in the density would favour the potential hump).

However, this is still speculative. Apart from the early studies of Hermoch [38] where the electrode zone of plasma jets in air has been investigated, no satisfactory experimental work has been done with respect to the plasma dynamics of spots at clean surfaces in vacuum, and with sufficient temporal and spatial resolution ($10^{-6}$ s and $10^{-5}$ m, respectively). The existing theories on the complicated flow dynamics of the near surface plasma (e.g. Harris[18], Schrade et al. [39], Daybelge [36]) cannot substitute such experiments.

If the arc current passed the cathode surface through a broader area than the zone of melting and evaporation (which is definitively limited), the discussion on the current density would be unnecessary. This is possible if (i) there is sufficient electron emission outside the craters, or (ii) at the cathode the current consists mainly of plasma ions. Rakhovsky [15] suggests stationary electron emission in the moderate electric field beneath the plasma (space charge field). From vacuum breakdown experiments it is known that field dependent electron emission occurs already at $10^6 - 10^7$ V/m (Latham [40], Halbritter [41]). But the electrons come from very narrow surface areas with orders of magnitude typically of $10^{-14} - 10^{-16}$ m$^2$. Therefore currents $> 0.1$ A would imply again high densities at the emission centres. Case (ii), i.e. no essential electron emission at all, would mean that the integral of the current density $j^+$ of the plasma ions over the cathode surface equals the arc current. In order to estimate this current, we put $j^+ = 0.4 \frac{Zen}{v_1^2} = (2 kT_e/m_i)$ like in the theory of Langmuir probes. Using for the density $n$ equation (2) and omitting there the cosine (the most favourable case), the integration yields $I^+/I \approx 0.1$, i.e. the ion current $I^+$ to the cathode is only about 10% of the total arc current $I$, as far as the region outside the crater is concerned.

Also the experiments indicate a relatively low ion current fraction at the cathode. In [42] arc spots were driven over a slit in a Cu-cathode. Before the spots crossed the slit only the plasma current reached the second cathode half, amounting to 10-20% of the arc current. Only when the spots passed the slit, the current at the second
half rose to 30 s. From the rise time and the spot velocity, a spot current density of about $10^{11}$ A/m$^2$ could be estimated.

Consequently, we must either postulate an unknown process of electron emission which takes place at cold, extended surfaces beneath the plasma, or we must admit the possibility of the high current densities.

5. Spot models and crater formation

The present section deals with surface processes that determine plasma generation and spot behaviour. Since measurements are extremely difficult in this region, we begin with theoretical models as a guide.

Assuming that both electron emission and plasma production are taking place at the same location, most of the stationary theories explain the current transfer by thermo-field emission, and the plasma generation by evaporation with subsequent ionization, e.g. [1, 43-46]. This depends on the surface temperature which is determined by Joule heating (due to the emission current), emission cooling, and ion impact heating. The space charge field of the ions contributes to the emission process. Thus, there are positive and negative feedbacks of non-linear processes, controlled by poorly known material parameters. Therefore, simple analytical solutions are not to be expected. Ecker [1, 46] has overcome the inherent uncertainty of the description by using necessary limits instead of exact equations. In this way existence areas can be calculated as indicated in Fig. 3. There are two possible areas, called 0-mode and 1-mode. Because the area...
spot traces indicate clearly a temporal development. After a certain time
the spot extinguishes, and a new spot
is formed at a new location. The arc
leaves the molten surface zone. It
does not drill a hole into the cath-
ode, although this could be energetically
favourable.

Several authors suggest the first
step of a spot cycle consists of surface
explosions [47-50]. Sometimes it
is assumed that the whole arc spot is
nothing else than a sequence of explo-
sions [51,52]. Generally speaking, an
explosion in ejection of matter on a
faster time scale than by evaporation.
In particular the ignition has explosive
features. In this stage the energy source is Joule heating of sur-
face irregularities. The latter term
means chemical structures with con-
taminated surfaces (leading to dielec-
tric surface breakdowns), or geomet-
rical structures (micropoints) with
clean surfaces. While the former must
be replenished by outer processes
after consumption by the arc, the lat-
ter are reproduced by the spot itself
(see below). The rise rate of the en-
ergy input into these structures can
become very great, so that the lattice
temperature of the solid cannot follow
immediately the temperature of the elec-
tron gas (which is heated first). Typi-
ical relaxation times have the or-
der of magnitude of 10 ps [53]. Experi-
mentally, about 30 ps have been found
for Cu [54]. But also in less extreme
cases, a rapid energy input cannot be
balanced by the evaporation rate that
is limited at least by the sound ve-
locity [55]. So the surface will be
locally overheated, followed by explo-
sive ejection of matter. This will be
particularly severe when the tempera-
ture maximum is achieved in the bulk
below the surface [56,57]. A further
feature of explosive processes is that
the presence of ionized matter in front
of the surface enhances greatly the
electron emission ('explosive emis-
sion' [58]), because the electron space
charge is diminished. Furthermore the
emitting surface grows continuously.
So a positive feedback between power
generation and plasma generation is es-
tablished. Finally, in view of the nar-
row space (nm) and the short times (ps),
a classical description may become in-
adequate. A quantum mechanical treat-
ment has been attempted in [60].

The explosion provides for the ini-
tial plasma and for the necessary cur-
rent flow. After this stage, Ecker's
existence diagram can be already em-
ployed, but a temporal development re-
mains. The area of plasma production
grows, therefore the current density
(if related to this area) decreases.
If this were not so, a run-away situ-
ation would occur, as shown by Hantsche
[61], because the resistivity of the
heated metal increases with temperature
(thus more Joule power is released).
The expansion of the molten crater vol-
ume prevents this effect, leading fi-
nally to a situation where the heat
losses dominate (mainly due to emission
and to heat conduction). Such a scenario
is employed by Daalder [62], and by Lit-
vinov, Mosyats and Parfyenov [63]. The
latter suggest a narrow temperature
profile within the crater area. The
very hot inner zone with radius $r_s$
emits the electrons, therefore the Joule
heat is generated only there. This area
is surrounded by a cooler zone with
radius $r_c$ where the metal is still liq-
id, but no emission occurs. Fig.9
shows $r_s$ and $r_c$ as a function of time.
The experiments seem to support the shorter times. Fig. 10 shows a crater on molybdenum formed within 10 ns [29]. An analysis of the chaotic sp. movement yielded \( r^2/\tau \approx 10^{-3} m^2 \) [64] (on the supposition that arc tracks consist of successively formed and overlapping craters as in fig. 5). This is compatible with the result of Lavinov et al., if we put \( r = r_s \).

However, if we adopt the model of these authors, we are confronted with extreme surface temperatures (in the beginning in excess of \( 10^4 \) K), and with very large current densities. The model starts with values \( > 10^{14} \text{ A/m}^2 \) and ends at some \( 10^{13} \text{ A/m}^2 \) when the spot is fully developed. It was already difficult to understand densities of \( 10^{12} \text{ A/m}^2 \) (section 4), now the problem is multiplied.

In [59-62] a mechanism has been proposed which explains the short formation time with less extreme current densities. The initial dense plasma (created by surface explosions) expands with a velocity of about \( 10^4 \text{ m/s} \), covering a surface area with radius \( r = 2.55 \) \( \mu \text{m} \) after about 0.5 ns. Ions are accelerated from the plasma towards the surface (possibly aided by voltage transients at ignition - Farrall [2]) and ion bombardment produces new plasma desorption of surface films. With clean surfaces, a thin surface layer with thickness of \( \leq 0.1 \) \( \mu \text{m} \) is melted and partly vaporized. Considering the extreme load of the surface atoms - with an ion current density of \( 10^{10} \text{ A} \) - each atom is hit by an energetic metal ion (20-50 eV) within a time of about 300 ps - the upper surface layers wi
be easily removed (so the term evaporation may not be very adequate). From the ejected matter new plasma is formed which replaces the trigger plasma. Because of the rapid energy transfer, heat conduction into the solid can be neglected. The power deposited into a given volume by this ion impact heating surpasses the Joule heating power in the explosion centre, if the heated surface layer is sufficiently thin. This layer will be rapidly consumed or pushed away by the plasma pressure (the latter has been already suggested by McClure [68]). The ions gain energy in the cathode fall and probably also in the region of the potential hump. So sufficient power is available in order to attack new surface layers. In this way the melting front proceeds into the interior faster than by heat conduction (like drilling a hole by a laser beam). Thus, like Daalder [62] we suggest the active spot area corresponds approximately to the melting zone, but like Litvinov et al. we assume formation times of a few nanoseconds.

The acceleration of the liquid surface metal leads to formation of droplets and micropoints at crater rims as shown in Fig. 11. Fursey [56] suggests that the micropoints are pulled out by the electric field. These structures are essential for the survival of the arc in the case of clean surfaces, because they serve as a source of new explosions. Mesyats [69] proposed the following mechanism: if a sphere is formed at the top of a micropoint (Fig. 11), the collected plasma current will be constricted at the neck (which connects the sphere with its holder) to a high density. The Joule heat generated at this location will provoke an explosion.

The repeated formation of such surface structures guarantees the existence of the arc. It explains the spot displacement by a length of about \( r_c \). Finally, it creates a latent substructure which may be the cause for spot division. In contrast to contaminated surfaces, these explosion sites are attached to the molten pool, so a tendency to diverge is weak (compare Section 3).

Various interesting ideas can be mentioned only shortly. Zharinov and Sanochkin [70] explain rapid crater formation by the action of the surface tension. Daybelge [36] describes the nucleation of droplets in the expanding plasma near the surface due to the cooling of the plasma. Alexander [71] suggests the occurrence of a kind of ion recycling at the surface, because the sputter efficiency increases with temperature [72]. This would alleviate the maintenance of the spot plasma. But more experimental experience is needed in order to judge the relevance of these processes.

Fig. 11: Micropoints (Cu-cathode) [29].
6. The anode region

With moderate currents the anode serves only as a passive collector of electrons without active emission. In general there is no smooth transition of the plasma potential to the potential of the anode, because the plasma currents are not matched necessarily to the arc current. Since the directed plasma velocity \( v = 1-2 \times 10^4 \text{ m/s} \) is small in comparison with the thermal electron velocity \( v_e = (kT_e/2\pi m_e)^{1/2} \approx 4 \times 10^5 \text{ m/s} \), we have for the saturation current density of the electrons \( j_e, s = e n v_e \). With large anodes the corresponding electron current exceeds the arc current. Therefore a negative voltage drop \( \Delta V_a \) develops at the anode in order to repel the superfluous electrons. The anode current density \( j_a \) is then
\[
 j_a = j_e, s \exp(-\Delta V_a/kT_e).
\]

The anode fall \( \Delta V_a \) is indicated in fig. 2 by the dashed curve 1.

With a flat anode, radius \( R_a \), located at a distance \( L \) from the cathode, and with a plasma density \( n \) according to equation (2) where the plasma is assumed to emanate from the centre of the cathode surface, the integration over the anode surface yields an anode current \( I_a \)
\[
 I_a = \frac{\pi n v_e \Delta V_a}{(1+L/R_a)^2}.
\]

Putting \( I_a/L \approx 1 \text{ A}^{-1} \), we obtain \( e \Delta V_a \approx kT_e \) for \( L/R_a \approx 1 \). In this case, irrespective of the arc current the voltage drop corresponds to the plasma temperature near the anode (5-7 eV). Consequently, the voltage drop \( \Delta V_p \) caused by the current in the plasma (fig. 2) does not influence very much the burning voltage \( V_b \); the latter being near the cathode fall \( V_c \).

However, for \( L/R_a > 3 \) the saturation current from the plasma becomes inferior to the arc current. Now a positive voltage drop \( \Delta V_a \) develops in order to draw more electrons from the plasma (fig. 2, curve 2). This voltage increases with the current and may assume considerable values as illustrated in fig. 12 [73].

The figure indicates a new process: when \( \Delta V_a \) surpasses about 50 V, it may become energetically more favourable to produce anode vapour. An anode spot is developed which delivers new plasma, thus enhancing the plasma current and reducing the voltage drop (and consequently the burning voltage, fig. 12). The concrete cause for the formation of anode spots may be due to local density fluctuations, possibly influenced by cathodic explosions [20]. Another reason may be the plasma heating by the arc current [14], because Fig. 12: Arc burning voltage as a function of arc current for Cu [73]. a) \( L/R_a = 3.8 \), b) \( L/R_a = 1 \). c) \( L/R_a \ll 1 \). L-gap length, \( R_a \) - anode radius.
there is a positive feedback between plasma temperature and current density—a local increase of the current leads to locally higher plasma conductivities.

7. Concluding remarks

In recent years considerable progress has been achieved in understanding the dynamics of cathode spots which influence also the plasma in the gap and near the anode. The main point is the non-stationary surface erosion and the plasma production having momentary explosive features. Because of the extreme physical conditions in the spot, the controversy on current density and time constants is not yet definitively settled. Almost entirely obscure is the plasma within the spot. Future experimental and theoretical work should tackle preferentially this difficult task.

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Brown Boveri Research Center, CH-5405 Baden, Switzerland

1. INTRODUCTION

1.1. GENERAL

Switching devices, known as circuit breakers, are used to protect networks for the transmission and distribution of electrical energy from damage by isolating the defective parts in the event of a short circuit [1-4]. An arc, initiated by contact separation, provides a medium which can be transformed rapidly from a conducting to an insulating state and which forms a conducting bridge between the parting contacts until they have reached a separation sufficient to insulate the network parts. In today's high technology era the use of arcs is still the most economic and reliable means of switching and it is unlikely that an alternative will be found in the foreseeable future.

The extinction of arcs is achieved by different methods depending on the network voltage. At lower voltages, the arc resistance, and hence the arc voltage is increased by lengthening the arc, by cooling and/or by splitting into a number of series arcs so that the arc cannot be maintained by the circuit voltage [5]. The extinction medium is normally air. This method is no longer applicable at higher network voltages because the arc voltage cannot be increased to values comparable to the network voltage. In this case, current interruption is possible only by the rapid removal of the charge carriers when the arc current goes through zero. An event that occurs periodically in AC-networks but must be artificially induced in DC-networks. Deionization takes place either by volume recombination or by neutralization at surfaces. Volume recombination occurs in high pressure arcs such as those in SF₆, air or oil. Neutralization at surfaces of the enclosure and electrodes occurs with metal vapor arcs in a vacuum ambient (vacuum arcs) [6].

The treatment of the physical phenomena during the dielectric recovery period will be restricted to high pressure arcs of high voltage breakers. Vacuum arcs were discussed in detail by Juettner [7]. Axially blown arcs widely used in switchgear (Fig.1) are especially suited for a detailed discussion. Their properties are well defined. SF₆ gas is choosen as the interruption medium. Due to its high specific heat at low temperatures [9] and high electron rate [8] it has a superior interruption capability and breakdown strength in comparison with other gases.

Many of the processes that will be described also occur in other interrupting media or in other types of arcs such as in systems applying magnetic fields, i.e. cross flow arcs.
SF$_6$-metalclad substations with breakers for 550 kV. In high voltage transmission networks, breakers are needed to interrupt currents up to 50 kA or more, at voltages in excess of 800 kV. The metal housing is at ground potential and the high-voltage conductors are enclosed in an SF$_6$ gas atmosphere at a pressure of several bars. The interrupter units of the breakers are contained in the vertical parts of the housing. In order to cope with the high voltages, several units are connected in series for each phase. Nevertheless, the voltage across the open ends of one unit recovers to peak values of several hundred kV within a few hundred microseconds. Smaller breakers are designed to operate at the distribution level at significantly lower voltages.

1.2. AXIALLY BLOWN ARC

The principal arrangement of an interrupter unit with an axially blown arc is shown in Fig. 2. The strong axial gas flow resulting from the imposed axial pressure gradient has two functions.

First, it restricts the arc to the central region, thereby preventing nozzle damage.

Second, and more important, the axial flow results in two cooling mechanisms, convection cooling and cooling due to turbulent mixing [10]. Convection cooling, which results from the outflow and replacement of the plasma, is alone insufficient to yield a rapid reduction of the gas temperature and charge carriers near current zero. The enforced cooling provided by turbulent mixing is needed in addition [10]. Under the influence of the axial pressure gradient, the plasma at the axis of the system, with a lower mass density, is accelerated to higher velocities than the heavier cold gas.
3O3

Tiller /3/ investigated the radially free full circle arc in Ar-gas stabilized by the self- and external magnetic fields. J. Blass /24/ proposed a method by which the whole set of projections for the layer of cascade arc can be recorded with one interferometric step.

Co-operative CT group in Novosibirsk has studied many of electrical, optical and gas-dynamical properties of a non-stationary turbulent arc plasma by 6-channel emission tomograph /25, 26/. Density and temperature fields within two-jets plasma device have been restored in the experiment /27/.

b/ Tokamaks, stellarators.

Reconstruction algorithm was worked out for purposes of local phase measurements of the plasma density within "tokamak-3" /28/. A successful realization of fast polychromatic recording of four projections for a selected cross section was achieved in the Lawrence Laboratory on the "Tormac-4" /29/. Local distributions of spectral line profiles of hydrogen and helium were established, allowing further reconstruction of the density and temperature fields. Similarly to the already mentioned experiment of A. Plessl /22/, the authors of Ref. /30/ obtained on the "Alcator-A" tokamak up to 18 f-n-shaped projections of X-ray emission from a plasma rotating with a period \( T = 0.456 \text{ msec} \). Soft X-ray tomography has been developed /31/ in order to observe magnetic islands structure in the "JIPP T-II" tokamak plasma.

Techniques for the reconstruction of 2D images from projections obtained on "PLT" tokamak were described by N.R. Sauthoff and S. von Goeler /32/. In the papers /33, 34/ numerical simulation of CT problem was realized with application to tokamak devices "ISX-B" and "JET" respectively. In the paper /35/ cited above similar reconstruction procedure with isolines of special form referred to stellarator plasma is proposed. CT on the base of ellipsometry principles with several viewing angles available in tokamak is suggested in /35/.

Diagnostic experiments on "Pulsator" /36/, "Tokamak-10" /37/ and stellarator from Lebedev's Institute /38/ can be regarded though with some reserve, as tomographic studies. Since it was not possible to obtain more than two projections in these experiments because of space limitations, the authors were forced to use a definite parametrization of the problem. Numerical calculation of "actual" radial profile of ion temperature from "measured" energy spectra of charge-exchanged neutrals in "PULSATOR" tokamak also has been possible due to strong a priori restrictions /39/.

As Prof. R.J. Bickerton emphasized in his invited lecture at ICPIS-XVI /40/, "our knowledge of tokamak behavior still tends to be more botanical or empirical in nature rather than deeply physical". There is a hope now that consistent attacks of the problem by CT techniques will change the situation radically.

c/ High-temperature laser-produced plasmas.

The feasibility of obtaining 3D tomographic reconstruction of the X-ray emission in laser imploled targets with an array of pinhole cameras has been demonstrated by G.N. Nilenbo et al. /41/. Only four two-dimensional projections were used. The mathematical details of the procedure and the results of computer modeling are analyzed in the paper /42/.

The single pinhole camera, however, has low signal-to-noise ratio especially for a weak radiation source because of its small opening area. Fresnel zone plate (FZP) camera is much brighter and may provide a radiation collection solid angle four to six orders of magnitude greater than a pinhole camera.
surrounding it, resulting in a large radial difference between the axial velocities of the hot and cold gas. As a consequence, a shear layer with high vorticity is built up at the arc boundary [10]. The free shear layer becomes unstable in the region of the nozzle throat and transition to turbulent flow occurs. For undisturbed inflow the velocity difference is too small and therefore no turbulence is developed upstream; the plasma between the nozzles forms a well defined cylindrical column (Fig. 3). Nevertheless, turbulence develops in the upstream region as well during the dielectric recovery period [11] after current interruption. This peculiarity will be discussed later in connection with the temperature decay.

Recent investigations [12] of turbulent shear flows have shown that in addition to small scale structures such flows contain large scale eddies that play a significant role in the dynamics of the mixing process at the interface of the hot and cold fluid. These large structures entrain fluid from well within the adjacent layers and together with the action of the smaller eddies, this leads to efficient mixing and radial transport of...
Differential interferogram of the decaying arc at current zero showing a well defined cylindrical arc column upstream between the nozzles inlets. The diameter is still about three millimeters at that location. Great care was taken to make the inflow symmetrical and undisturbed and therefore practically free of turbulence and vorticity [11].

The main result of turbulent mixing is to reduce the high temperatures quickly. In addition, the convective mechanism plays an important role in removing the mixed gas from the region between the contacts.

As the current approaches zero and the ohmic energy input drops simultaneously, the plasma outflow results in a shrinking of the arc diameter. At current zero the diameter is smallest in the region of the nozzle throat. As a result the cooling due to turbulent mixing is most effective in this region [10, 13]. Nevertheless, the total resistance of the residual arc channel at this time is still of the order of 100 ohms (Fig. 12). As the high voltage of the network increases from zero at a rate of a few
A post arc current of few amperes is driven through the arc channel. The resultant electric power input counteracts the energy-loss mechanisms. Depending on which way the balance tips, this leads to either thermal reignition or interruption of the power current flow. This time interval from I = 0 to definite current interruption is called the thermal regime and normally lasts several microseconds.

In case interruption occurs, the dielectric recovery phase follows and the resistance increases dramatically due to recombination and attachment processes among the electrons and the heavy species. Meanwhile the transient recovery voltage continues to build up across the decaying arc channel. However due to the low conductivity, the current becomes so small that the electric energy input can be ignored and decay of the residue occurs freely without any further energy input. The breakdown strength steadily increases to its final constant value at room temperature. On the other hand, if the cooling does not occur fast enough, the residual hot gas is overstressed and breaks down. In this case arc reignition occurs and the breaker fails. A measurement of the increase of breakdown strength with time, the so-called recovery characteristic of a model breaker is shown in Fig. 4.

![Figure 4](image)

**Fig. 4:** Measurement and calculation (solid curve) of recovery characteristic of a model breaker with fixed contacts. The arc was ignited by exploding a wire. The time scale begins at current zero. The measurement was performed by applying voltage pulses of high enough value to induce breakdown at different delay times after current interruption. The term recovery refers to the lower end of the scatter band. The "fast-slow-fast" behavior is typical of dielectric recovery for undisturbed inflow. At voltages above the dot-dashed line breakdown occurs between the nozzle faces outside of the residual plasma channel.
2. DIELECTRIC RECOVERY PERIOD

2.1. FLUID MODEL - BASIC EQUATIONS

Because of the strong interaction due to the high gas density the plasma states can be modeled as a fluid in which all species have the same kinetic temperature. Under these conditions the governing equations for one volume element of the freely recovering plasma (without electrical energy dissipation) are the conservation equations for mass ((1a) or (1b)), momentum (2) and energy (3) [16-18]. In addition, an equation of state, normally the ideal gas law is used.

\[
\frac{\partial \rho_s}{\partial t} + \nabla \cdot (\rho_s \mathbf{v}_s) + \nabla \cdot (\rho \mathbf{v}) = 0
\]  

\[
\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \rho + \rho (\nabla \cdot \mathbf{v}) = -\nabla p + \nabla \times ((\nu + \eta_t)(\nabla \times \mathbf{v}))
\]

\[
\frac{\partial \mathbf{v}}{\partial t} + \rho (\mathbf{v} \cdot \nabla) \mathbf{v} = (\mathbf{v} \cdot \nabla)p + \nabla \cdot ((\kappa + \kappa_t)\nabla T)
\]

\[
\eta_t = 0.5\rho \varepsilon \Delta v
\]

\[
\kappa_t = \rho c_p \varepsilon \Delta v
\]

\[
\rho = \frac{\bar{\rho}}{\rho_s} (m_s n_s)
\]

\[
h = \frac{\bar{h}}{\rho_s} (m_s n_s e_s) + \frac{p}{\rho}
\]

- \(\rho_s\): mass density of species "s"
- \(\rho\): total mass density
- \(\Gamma_s\): net mass production rate per volume of the species "s" due to the different chemical reactions
- \(n_s\): particle density of the species "s"
- \(m_s\): mass of the species "s"
- \(\bar{v}\): gas velocity (mass-average)
- \(\mathbf{v}_s\): diffusion velocity of species "s" (difference of species-average velocity to mass-average velocity \(\mathbf{v}\))
- \(p\): gas pressure (isotropic)
- \(T\): gas temperature
- \(h\): enthalpy
- \(c_p\): specific heat
- \(e_s\): internal energy of the species "s"
- \(\eta\): coefficient of viscosity
This set of equations describes not only the temperature decay of the axially blown arc but is valid for decaying high pressure arcs in general.

When the temperature decay rate is small compared to the chemical reaction rates the concept of local thermodynamic equilibrium can be used and the concentration of the various species "s", as well as the material and transport properties can be calculated from statistical considerations [9, 18, 19]. In this special case the temperature decay is given by the total mass balance (lb) in conjunction with equation (2) and (3).

However, when the plasma temperature decay rate becomes comparable to or faster than some reaction rates, a state of chemical nonequilibrium develops. Generally this requires the mass balance for each of the different species "s" (1a) in conjunction with equation (2) and (3).

The influence of turbulent mixing results in additional mass diffusion and an increase of exchange of momentum (viscosity) and energy (heat conduction). The effects of turbulent and thermal particle diffusion are described by the third term on the right hand side of equation (1a). The radial transport of momentum and energy of the turbulence can be modeled by the phenomenological Prandtl mixing length hypothesis [21] (equation (4) and (5)). Viscous dissipation and energy transport by radiation can be neglected.

The turbulent heat transfer, which is an order of magnitude greater than that due to thermal conduction, should be large at low temperatures in order to provide an effective cooling during recovery. This requires the product of the mass density $\rho$ and specific heat $c_p$ to be as large as possible (equation (5)) which occurs when dissociation or recombination take place. $SF_6$ meets these requirements extremely well and is superior to other gases in this respect [9].

Under conditions of local thermal equilibrium the species composition is determined by equilibrium mass action laws and is a function of temperature and pressure only.

In general however, the species composition is retarded and determined by the set of equations (1a), (2) and (3). The terms $\Gamma_s$ in equation (1a) are the net mass production rates due to the different chemical reactions [16]. The production rates depend on the rate coefficient and of the species concentrations involved. The rate coefficients alone are unique functions of the gas temperature because the different excitation temperatures are equal to the kinetic temperature of the gas.

If the species densities $n_s$ deviate from equilibrium conditions, the
mass density of the gas $\rho$ (equation (6)), the enthalpy $h$ (equation (7)),
the coefficients for viscosity and heat conduction also change. However,
the turbulent coefficients $\eta_t$ and $\kappa_t$ dominate, so that one can in principle
neglect the influence of changes in $\eta$ and $\kappa$. The material functions $\rho$ and $h$
appear to be shifted to lower temperatures in comparison with their
equilibrium values due to the retardation.

The pressure distribution is in general determined by the geometry and
the imposed pressure difference [11]. The calculation starts with the tem-
perature distribution at current zero. These temperature values have to
measured or calculated using a different model.

2.2. SPECIES COMPOSITION - TEMPERATURE DECAY

In general a direct solution of the set of differential equations
(1a), (2), and (3) is not feasible because the treatment of this large
number of equations is too difficult. Besides this, there is a lack of
knowledge about the turbulent mass exchange and not all rate coefficients
for the different chemical reactions are known. To circumvent this problem,
an iterative approach can often be used. The calculation is first performed
with the assumption of local thermodynamic equilibrium. The result is a
good approximation of the gas temperature decay for the quasi-steady case
in which the plasma cools down very slowly. On the basis of this equili-
 brium temperature decay the retardation of species composition and the re-
combination can be estimated. Such an estimate of the variation of species
composition with time at the stagnation point of an axially blown arc is

Fig. 5: Calculated variation of the various species of decaying arc resi-
duals at the stagnation point as a function of time based on equi-
librium temperature decay. M is the total particle density [22].
given in Fig. 5. Immediately after current interruption recombination of $S^+$ with electrons, mainly by three-body-recombination is observed. High densities of $S^+$ and $F^-$ ions are also preserved during this period. The formation of $SF_6$ sets in rather abruptly in the time interval between 50 and 100 $\mu$s. This occurs at temperatures of about 1200 K, retarded a few hundred degrees Kelvin (Fig. 6), [23].

![Graph showing particle densities of $SF_6$ as a function of temperature](image)

**Fig. 6:** Particle densities of the equilibrium composition of $SF_6$ as function of temperature $T$ at a pressure of 3 bar [23].

The relaxation and its influence on material functions can be modeled by shifting the material functions to lower temperatures and the more rapid formation of molecules by a reduction of the temperature interval at which the release of internal energy occurs. By repeating the calculation using the modified material functions, a good estimate of the temperature decay can be obtained for temporal relaxations that are not too extreme [15].

The temperature decay at the stagnation point of the model breaker of Fig. 2 calculated using the iterative method is shown in Fig. 7. The temperature decays very rapidly and flattens out after about 40 $\mu$s. After about 130 $\mu$s the decay rate again increases. A calculated spatial temperature distribution during the first 50 $\mu$s is shown in Fig. 8. Upstream, between the nozzles, the temperatures are still high. However the cylindrical column has shrunk to a diameter of about one millimeter due to the axial outflow. Downstream of the nozzle throat the temperatures have fallen to a very low value due to the effective turbulent mixing.
Fig. 7: Calculated temperature decay at the stagnation point of the model breaker of Fig. 2 [15].

Fig. 8: Calculated spatial temperature distribution during the first "fast" recovery phase. The origin of the coordinate system corresponds to the stagnation point in the center between the nozzles (Fig. 2). Gas flow is along the z-axis. The radial coordinate is perpendicular to this direction. The nozzle throat of 10 mm diameter is located at z = 10 mm [15].
Differential interferogram of the decaying arc at the beginning of "slow" recovery phase showing the development of turbulence around the stagnation point of the nozzle system (Fig. 2) [11].

The rapid temperature decay at about 40 μs results from the development of turbulence in the upstream region as well (Fig. 9). The reason for the onset of turbulence observed in differential interferograms is still unclear [15]. The development starts locally at the nozzle inlet at the edge of the inner core and then spreads over the entire column. The generation and persistence of turbulence even near the stagnation point could perhaps be explained by a stability analysis; however, with the present state of knowledge, this is rather difficult. Possibly, the sudden heat release due to delayed recombination during this phase contributes to this effect.

The release of internal or dissociation energy is also responsible for the slowing down of the temperature decay. The effect is more pronounced because the recombination process occurs at lower temperatures at which
Convection cooling has also decreased. The rate of temperature decay increases again when most of the SF₆ molecules have been formed.

The temperature decay is sensitive to the radial extent of the hot-gas column at current zero and to the axial pressure profile. On the other hand, the pressure and initial temperature value at the stagnation point have little influence [33].

2.3. BREAKDOWN PROCESS - CRITICAL FIELD STRENGTH

To transform the resulting temperature curve into a recovery characteristic a criterion to characterise the breakdown process is needed.

The location of the first ionization depends on the applied electric field strength and on gas the density and composition at the different points. Apart from the thermal region and a short transition region after current interruption, breakdown is initiated by an avalanche process [24] in which new electrons are created. In a dielectric breakdown the applied electric field imparts sufficient kinetic energy to the electrons between collisions to ionize molecules or atoms. Streamer discharges develop from avalanches when the number of electrons is increased to such a value that the electric field around the avalanche head is mainly determined by space charges. In most instances a complete breakdown of the gap occurs through a subsequent leader discharge [24]. In many gases electrons are attached to heavier species forming negative ions in an extremely rapid process whereby the number of ionizing electrons is reduced [8]. An exponential increase of the electron number, an avalanche and a subsequent streamer discharge occur if the net ionization coefficient, the difference between ionization and attachment, is greater than zero [25]. For SF₆, the net ionization coefficient changes from negative to positive values at a reduced field strength value of 360 10⁻¹⁷ V cm⁻², which corresponds to 89 kV/(cm*bar) at room temperature. This value is a material property and is called the critical field strength.

For the discussion of the variation of the dielectric strength with temperature it is assumed that the hot gas is in thermal equilibrium. Below a temperature of 1500 K only small amounts of SF₆ have dissociated (fig. 6). The critical field strength is constant up to these temperatures because the dissociative and nondissociative electron attachment exhibit opposing trends [26] and the ionization coefficient is unchanged at these low temperatures. This behavior is confirmed by measurements of the breakdown strength in the wake of a cross flow arc (triangles in Fig. 10) [27].

Rothhardt et al [28] measured breakdown values normalized by the mass density in a shock tube experiment. They also found that the normalized breakdown strength was unaffected by temperature up to 800 K and dropped to 80% of this value in the range of 800 K to 2000 K. Beyond this point, the normalized breakdown strength drops linearly to very small values within an interval of 900 K. The data of Rothhardt et al. renormalized by particle density are shown by the dotted line in Fig. 10. This renormalization is
Fig. 10: Critical electric field strength of hot SF₆ under equilibrium conditions normalized by the value at room temperature. The limits of the gas composition were calculated by Ruchti [23] (Fig. 6). The triangles (Schade [27]), the dot (Schade [29]) and the dotted curve (Rothhardt et al [28]) are experimental data. The lower bound on the critical field strength, shown by the thick dot-dashed line, is estimated from calculations of Hayashi [30]. The values for atomic fluorine result from a calculation of Hayashi [30] (thin dot-dashed line) and an estimate of Brand [31] (shaded field).

Based on the dependence of mass density on temperature as calculated by Ruchti [23]. The dot in Fig. 10 is a measurement in the wake of a rotating arc [29].

At temperatures higher than about 3000 K SF₆ consists mainly of atomic fluorine for which only calculated values are available. Hayashi [30] used the Boltzmann equation and obtained a critical field strength value for pure atomic fluorine about 6% of that for SF₆ at low temperatures. This is in good agreement with an estimate of Brand [31] (Fig. 10). The dielectric strength is reduced because the energy transfer due to inelastic collisions in atomic gases is much lower than that in molecular gases, and consequently electrons under the influence of the applied electric field can reach the energy necessary for ionization at a lower field strength. In addition, the attachment cross section of atomic constituents is significantly lower.

The dielectric properties of the different constituents in the transition region from SF₆ to atomic fluorine and sulfur are unknown. However,
Hayashi [30] has been able to calculate the critical field strength of mixtures of SF$_6$ with atomic fluorine. A lower bound on the critical dielectric strength in the transition region is obtained if one assumes that SF$_5$ has the same dielectric properties as SF$_6$ and that molecules containing less than five fluorine atoms have the same properties as atomic fluorine and simulates the hot, dissociated SF$_6$ by a mixture SF$_6$ and F only. The thick dot-dashed line in Fig. 10 results from applying the calculations of Hayashi. The agreement with the experimental data of Schade [27, 29] is surprisingly good if we consider the additional contribution of SF$_4$ and SF$_2$ which have their maximum densities at the indicated temperatures. The fact that the dielectric strength of SF$_6$ is not significantly reduced up to temperatures of about 2000 K has also been confirmed by observations during the development of switchgear equipment. The shock tube measurements of Rothhardt et al [28] predicts much larger reductions.

At temperatures of about 2500 K the densities of positive and negative ions reach values of $10^{11}$ cm$^{-3}$ and the applied electric field is distorted by the formation of space charges. Finally, at temperatures of the order of 3000 K to 4000 K ion and electron densities are so high, that essentially electrical conduction occurs. The breakdown process is then governed by ohmic heating and energy loss rather than by dielectric breakdown. This thermal breakdown may be distinguished from streamer breakdown at lower gas temperatures by the fact that much longer times are needed, due to the heating process involved. In the case of dielectric breakdown ionization occurs over a very small time interval and during avalanche and streamer formation the gas temperature is unaffected by the ionization process. For dielectric breakdown much higher field strengths are required.

In summary, the critical dielectric strength of SF$_6$ for equilibrium conditions drops sharply at temperatures above about 2000 K at which the main dissociation takes place. For the case where equilibrium does not exist, it can be concluded from experiments that the value of critical dielectric strength should be within 20 per cent of that of SF$_6$, once the percentage of SF$_5$ or SF$_6$ formed is greater than about 30%.

2.4. RECOVERY OF THE BREAKDOWN STRENGTH

Calculated temperatures can now be transformed into recovery characteristics and compared with experimental results (Fig. 4). The overall agreement is quite good. In particular, the three typical phases fast-slow-fast are well predicted by the theory.

During the first rapid increase of the breakdown voltage the high temperature region around the stagnation point is still electrically conducting. The resistance of this channel section is estimated from experiments to be about one megohm which is several orders of magnitude lower than the downstream resistance [11]. When a high voltage pulse with the waveform shown in Fig. 11 is applied across the gap no substantial current flow occurs because of the large downstream resistance. However, space
Fig. 11: Measured currents I during the first "fast" recovery phase. The full lines ((3) in Fig. 11a; (2) in Fig. 11b) denote currents produced by voltage pulses (dashed line) with amplitudes well below breakdown. The voltage pulse begin (t = 0) is at different times after current zero (Fig. 11a: curve 1: 5 μs, curve 2: 9 μs, curve 3: 12 μs; Fig. 11b: curve 1: 14μs, curve 2: 20 μs). The dot-dashed line ((1) in Fig. 11a) shows predischarges with return to capacitive behavior. The dotted curves ((1) in Fig. 1a and 1b) are currents in case of breakdown at different times. When the voltage has reached a constant value the current reduces to a few μA, corresponding to the high resistance downstream (Fig. 12). The measurements were performed on a model breaker specially designed for this purpose [11].

Charges are formed at the end of the conducting zone; this build-up of charge carriers results in a measurable transient current flow (Fig. 11) proportional to the derivative of the applied voltage [11].

Due to the build-up of space charges the applied electric field is
Fig. 12: Total gap resistance $R_g$ of the model breaker of Fig. 11, measured after current zero [11].

distorted as shown Fig. 13. Because the high temperature region is shielded, the breakdown is determined by the conditions in the high field strength region at the end of the conducting channel. The dielectric strength increases rapidly up to the time at which the gas temperature decay slows down (Fig. 7).

Fig. 13: Estimated electric field strength distribution along the axis of the model breaker of Fig. 2. The origin of the $z$-axis corresponds to the center of the nozzle system. Curve (1) represents the space charge free field determined by the nozzle contour. The reduction of the electric field around the stagnation region during the first "fast" recovery phase (curve (2)) results from the build-up of space charges [15].
From this moment the conditions at the stagnation point determine breakdown because charge carriers are no longer available to shield this zone against the high applied electric field. The breakdown strength is now proportional to the gas density at that point. The scatter in the measured breakdown voltage is in all likelihood caused by processes which prevent the first streamer from developing into a complete breakdown. This hypothesis is supported by experimental observations. Fluctuations in the gas density caused by turbulence are by themselves probably too small to cause the observed experimental scatter of breakdown values.

3. CONCLUDING REMARKS

We can conclude that if a fast recovery or a high interruption capability is needed, cooling due to turbulent mixing is necessary in addition to convection cooling. The turbulence can be self-generated, as in the case of the axially blown arcs, or externally produced, for instance, by the application of magnetic fields [32]. The controlled flow conditions of a model experiment do not always prevail in actual circuit breakers and a well-defined cylindrical arc column does not exist at current zero between the nozzle inlets. Perturbations of the gas inflow can trigger the onset of turbulence which then builds up to its full intensity already in the upstream region at current zero. Consequently, the temperatures decay much faster than in model experiment, especially in the stagnation point region, and the breakdown values immediately after current zero are higher [11, 15, 33]. Nevertheless, the physical phenomena responsible for the dielectric recovery are the same as those in the model experiment.

The high temperature regions are shielded by space charges. Ionization starts at that location where the local reduced electric field reaches the critical values.

The ideal interruption medium should have a low energy content at high temperature, a high specific heat at low temperatures to ensure an effective turbulent energy transfer and a high dielectric breakdown strength. SF₆ meets these requirements very closely.

The aim of this paper was to demonstrate the present state of our knowledge concerning dielectric recovery of arcs in switchgear. Considerable progress has been made. We now have a good understanding of the dominant physical phenomena and are able to predict the recovery of switching devices for technical applications within reasonable limits [34]. Scaling laws have been established enabling a comparison between experiments or extension to new geometries [33, 34].

The understanding of the physical phenomena has allowed the development of a new generation of switchgear, the so-called self-extinction breakers. These create a blast to extinguish the arc by heating the gaseous interruption medium SF₆ with arc energy [35, 36]. An optimum balance between sufficient heating and small reduction of the dielectric strength had to be found.
From the scientific standpoint, however, a lot of open questions remain. First, there is a lack of basic data, for instance rate coefficients and data on the dielectric strength of the decomposition products of SF$_6$ and of other dielectric media. In addition, different processes such as the unexpected development of turbulence in the stagnation point need to be explained in detail.

Fig. 14: Differential interferogram of the upstream region (Fig. 2) during the first "fast" recovery phase showing a thin core within the residual hot gas channel [11].

The fact that open questions still exist is demonstrated very well by a differential interferogram of the hot gas during the first recovery phase (Fig. 14). An extremely thin core is observed within the column of the hot gas. It has not been possible to identify the physical property responsible for this change in refractive index. We assume that it is related to complex species recombination, but are unable to verify this fact.

We hope that we have been able to convince you that research in connection with switching devices for high voltage networks is not only of great practical importance but also very challenging from the scientific point of view.

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COMPUTERIZED TOMOGRAPHY OF FLAS.A
IN LABORATORY AND SPACE
N.G. Preobrazhensky
Institute for Pure & Applied Mechanics, Siberian Branch
of the USSR Academy of Science, Novosibirsk, 630090, USSR

Introduction.
In recent years image reconstruction from projections, or computerized tomography (CT) has been one of the most widely applicable techniques of signal processing. CT has had a profound impact on all branches of basic and applied sciences, especially in medical imaging.

The 1979 Nobel Prize in medicine (A.K. Cormack, G.N. Hounsfield) has been awarded for work on CT. The 1982 Nobel Prize in chemistry (A. Klug) has also been awarded on image reconstruction; this time for restoration of viruses, nucleoprotein complexes and macromolecules from electron micrographs.

During the same period the tomographic ideas have found application also in non-destructive evaluation and testing, solid state physics, geophysics (including ocean surveillance and atmospheric research), aero- and hydrodynamics, plasma physics, radioastronomy and many other fields. In the present paper I shall describe briefly the peculiarities and possibilities of CT methods in plasma diagnostics.

CT attacks a set of inverse problems characterized by the common trait that the desired quantities are the values within a field but measurements can be made only through the field. More often one extracts the cross section of an inhomogeneous object (e.g. plasma). Images can be stored in the form of stack of two-dimensional (2D) matrices or equivalently, as a three-dimensional (3D) matrix of numbers.

Conventional imaging techniques are based on the fundamental methods of reconstruction from projections as first introduced by J. Radon /1/. A number of more tractable reconstruction procedures have been developed including the, "onion peeling", polynomial expansions, Fourier synthesis, filtered back-projection etc /2/. Such algorithms assume a complete set of accurate measurements of the projections in order to reconstruct an artifact-free image.

In plasma physics as in many other fields of natural science the measurements fail to accurately present a complete set of projections. The inadequacy of the measurements can include noise, insufficient data, screening effects, finite limitations, various nonidealities, poor temporal resolution etc. These inadequacies inevitably result in a variety of distortions of the reconstructed image and create serious obstacles for a wide range of CT methods in plasma research.

Nevertheless, the number of successful applications illustrating the rich prospects and unique properties of plasma tomography is quite large now and it seems useful and in proper time to discuss the principle of the method and the possibilities of it.

Basic concepts and formulas.
From a mathematical point of view the procedure of tomographic reconstruction is a typical problem of integral geometry /3/ usually formulated as follows. Let \( f(x) \) and \( g(x,y) \) be sufficiently smooth functions, defined, respectively,
in n-dimensional \((z \in \mathbb{R}^n)\) and \((n+k)\)-dimensional spaces. \(K(y)\) is a family of \(k\)-dimensional manifolds in \(\mathbb{R}^k\), where \(k < n\). If a function

\[
f(y) = \int W(x,y)g(x)d\sigma\quad (1)
\]

is assumed to be known, with \(d\sigma\) being the measure element on \(K(y)\), then the problem of integral geometry is to find out the intercommunication of various properties of functions \(g, f, W\) and manifolds \(K\).

Conventional CT problem is to find \(g(x)\); in the most cases \(n=2\) or \(j, k=1\) or 2, \(W=1\) and the manifold \(K\) are points on a rectilinear or curvilinear trajectory. More complicated situation arises when one has to take into account interdependence of \(K\) and \(g\).

In classifying of plasma tomography problems it is convenient to distinguish three large groups of such problems according to increasing complication of their solution.

**a/ Linear CT problems with strong a priori restrictions.**

In this case the trajectories of the rays penetrating the plasma in a selected plane are assumed strictly rectilinear, and the a priori information concerning the unknown function \(g(x)\) is usually of the following nature.

In the simplest case the functional form of \(g(x)\) can be given, and then it remains only to determine several unknown parameters appearing in the problem. A preliminary analysis of the information content for this problem (calculation of Fisher matrix) and parameter search (the maximum likelihood technique) are well developed and widely used procedures. A lot of examples referring to the problems in plasma physics are given in Ref. /4,5/.

Further, the shape of the curves on which the unknown 2D distribution \(g(x)\) attains a constant value (isolines) can be assumed known. The simplest and most fully investigated case is that of isolines in the form of concentric circles, when the object is inhomogeneous only in the radial direction, and to restore \(g(x)\) (here \(x\) is the radial variable) only a single projection \(f(y)\) is needed. Equation (1) with \(W=1\) degenerates in this case to the classical Abel equation

\[
f(y) = \int_{0}^{y} (y-x)^{-1/2} f(x)dx\quad (2)
\]

having for a continuously differentiable function \(f(y)\) the single continuous solution

\[
g(x) = \frac{1}{\pi} \int_{0}^{x} \frac{x}{x-y} (y-x)^{-1/2} f(y)dy\quad (3)
\]

A large number of examples of physical problems leading to the Abel equation were considered in the review article /6/. It is worthwhile to note that if an axially symmetric plasma is self-emitting, eq. (2) is valid only in the limit, when self-absorption of radiation and fluctuating shifts of axis can be neglected. In the more general case /7/ the solution of the problem evolves into a stage of deconvolution and series of successive inversions of the Abel type.

The a priori restrictions on the function \(g(x)\) can also be related to the symmetry properties of the plasma, expressed not in terms of the given configuration of isolines, but directly in the language of group theory. A typical example from high-temperature plasma diagnostics and solid state plasma physics is a Compton scattering tomography in 3D momentum space.

Quite generally stated tomographic problem with isolines was studied in Ref. /8/, where the isolines are described by a given set of arbitrary convex closed curves without self-intersections.
b/ Linear_CT problems with weak a priori restrictions.

Higher in the level of complications is the class of 2D and 3D tomographic problems in the literal sense of the word, when the object is asymmetric, parametric representation or shapes of isolines are not employed and the (weak) a priori restrictions induced on $g(x)$ are related only to stability of the solutions. The path of ray, as formerly, assumed to be given and rectilinear.

It is important to recall that the existence of a simple inversion formula like (3), seeming at first glance to solve the problem completely, in fact does not at all imply that the error in the restored field $g(x)$ will be of the same order as the error in the recorded projection $f(y)$. In reality the problem belongs to a class of ill-posed (unstable) problems /9/, and any algorithm of numerical reconstruction of $g(x)$ requires regularization, i.e. the use of complementary restrictions on the function $g(x)$, consistent with random measurement errors in $f(y)$.

Turning again to the initial eq.(1) we put in it $W=1$ and get $\{L(y)\}$ we consider the family of all possible hyperplanes in n-dimensional space. In this formulation the problem of seeking $\varphi(x)$ was solved by one of the founders of integral geometry J. Radon, in 1917 /1/. Fig.1 explains the results of the radon inversion in the two-dimensional case.

Let $L$ be a ray intersecting the plasma, with $s$ the distance measured along it, $O$ the origin of some coordinate system, $\mathcal{F}$ the angle between the base line $OM$ in the selected plane and the perpendicular dropped from $O$ onto $L$, $p$ the shortest distance from $O$ to $L$, and $\vec{n}$ the unit normal determined by the same angle $\mathcal{F}$. In this notation eq.(1) must be replaced by

$$f(p,\vec{n}) = \int g(\vec{r}) \, ds$$

where $\vec{r}$ is the 2D position vector turned with respect to $Ox$ by angle $\mathcal{F}$. As shown by Radon

$$g(\vec{r}) = \frac{V^2}{2\pi^2} \int_{-\infty}^{\infty} f(p,\vec{n}) \ln |p-\vec{n}| \, dp$$

$$= \frac{I}{2\pi^2} \int_{-\infty}^{\infty} \frac{\partial^2 f(p,\vec{n})}{\partial p^2} \ln |p-\vec{n}| \, dp$$

$$= -\frac{I}{2\pi^2} \int_{-\infty}^{\infty} \frac{\partial f(p,\vec{n})}{\partial p} \frac{dn}{\ln |p-\vec{n}|}$$

$$= -\frac{I}{2\pi^2} \int_{0}^{\pi} \frac{f(p,\vec{n}) \, dn}{(p-\vec{n})^2}$$

(5)

After an appropriate regularization one can use any of expressions (5); the inner integrals are understood in the sense of Cauchy principal value.

3D inverse radon transform can be written in the form:

$$g(\vec{r}) = -\frac{V^2}{4\pi^2} \int_{0}^{2\pi} f_n(\vec{r}, \vec{n}) \, d\Omega_n$$

$$= -\frac{I}{4\pi^2} \int_{-\infty}^{\infty} \left[ f_n(\vec{r}, \vec{n}(p)) \right]_{\vec{n}=\vec{r}/n}$$

(6)

Fig.1

Definition of the variables used in formulas (4) and (5). Explanations are given in the text.
Here \( \bar{r} \) is the general 3D position vector and angular integral extends over \( 2\pi \) ster. Note that formulas (6) are somewhat simpler than their 2D counterparts (5), since the convolution with \( \ln |p| \) is not required in 3D case.

Reconstruction of 2D emissivity distribution for an optically dense plasma was considered in /10/ in this case \( \mathcal{W} \) in (I) and is a function found from a complementary experiment.

c/ Tomographic problems for curvilinear trajectories

The transition to curvilinear ray trajectories, which can be primarily caused by taking into account refraction of the plasma under consideration, gives rise to new, more complicated and quite diverse classes of CT problems. In a number of applications (e.g., in geophysics) the form of trajectory is assumed to be known /11/; unfortunately, it is not the case for plasma physics as a rule.

Axially symmetric optical inhomogeneities, as well as inhomogeneities in a planar layer due to strong beam refraction were investigated by a number of authors, see e.g., /12/. Curiously enough, despite the ray bending under conditions of axial symmetry, one again obtains the Abel integral equation, though the variables occurring in it have a totally different meaning than in the case of rectilinear trajectories.

When the shape of the curvilinear trajectories is not known a priori and the object is asymmetric, even in the simplest approximation of geometric optics we encounter a nonlinear CT problem. In the Institute for Pure & Applied Mechanics, Novosibirsk, the following iterative algorithm has been worked out and then approved in a number of numerical experiments.

Let \( \mathcal{R}_o \) be an operator of conventional and \( \widehat{\mathcal{R}} \) of generalized (for curvilinear trajectory) Radon transform. Let further \( g \) be a correction to the refractive-index \( n_0 \) of the medium and \( g_0 \) a zero approximation for correction sought from condition

\[
\mathcal{R}_o g_0 = \mathcal{R}^{-1} \mathcal{R}_o \mathcal{R}_o g_0 \quad (7)
\]

Here the function \( f \) is recorded interferometrically, and the resulting phase distribution is referred to as the projection. If operator \( \mathcal{E} \) is known, then

\[
f_i = \mathcal{R}_o (\mathcal{E}_i + n_0) - \mathcal{R}_o n_0 \quad (8)
\]

In eikonal approximation one can write

\[
(\nabla s)^2 = (\mathcal{E} + n_0)^2 \quad (9)
\]

\[
2(\nabla n, \nabla s) + \nabla^2 s = 0
\]

where \( A(x,y,z) \) and \( S(x,y,z) \) are wave amplitude and phase, respectively. The system (9) is solved by the explicit differences scheme and its stability is determined as usual with the help of Courant-Friedrichs-Levy condition. Calculation of ray trajectory from (9), i.e., the solution of direct problem, defines the structure of the generalized Radon transform operator \( \mathcal{E} \).

Corrections \( g_i \) at each step can be calculated in the following way:

\[
g_i = \mathcal{E}^{-1}_m \left( f_i - f_{i-1} \right) \quad (10)
\]

where inverse modified Radon operator is written in shorthand as

\[
\mathcal{R}_m^{-1} = \mathcal{R}^{-1} \mathcal{H}_y \mathcal{D}_y \quad (11)
\]

\( \mathcal{H}_y \) is the operator of differentiation with respect to the first variable \( y \) of a function of two real variables; \( \mathcal{D}_y \) is the Hilbert transform operator relative again to the first variable \( y \). \( \mathcal{R}_m \) is a modified back-projection operator:

\[
\mathcal{R}_m f = \int_{0}^{2\pi} f [\mathcal{R}_o (\mathcal{E}_0 (y))] \, dy \quad (12)
\]
where \( p_0(y) \) denotes the intersection on the line of projection recording.

Comparison with the well-known Cha-Vest algorithm /I3/ proves the better convergence of the iterative process described above. A particular case of algorithm when \( R = R_m \) was considered in Ref./I4/.

The situation in the case of strong refraction is much troublesome. There due to the effect of virtual rays intersection /I5/, as well as effects like Maxwell's "fish eye", the danger exists to lose the uniqueness of the CT solution.

Though within the eikonal approximation one often succeeds in avoiding several crude artifacts of refraction nature, the resolving power of the restored image can remain inadmissibly low. Further progress is related to taking into account the diffraction effects, i.e. to solutions of the full wave equation or the Helmholtz equation. These solutions are usually found with the help of Born or Rytov approximations /I6/. The consistent theory of the corresponding CT problems is only starting to be developed.

It must be stressed that the classification suggested above has by no means to be considered as encompassing all conceivable types of plasma CT problems. For example, one can extend straightforward the usual 2D tomographic concept to 3D case by involving time \( t \) or frequency \( \nu \) as "third variable". As a result, chronotomography or spectrotomography /I7/ comes into existence. The extension of tomographic principles to the well-known methods of Thomson or Compton scattering could make it possible to reconstruct directly the distribution function of a gas or plasma in 6-dimensional phase space. A wide variety of CT problems arises as strong coherent fields are employed under conditions of occurrence of nonlinear effects such as self-focusing, self-diffraction, parametric instabilities, formation of solitons, nonlinear Landau damping etc. Unfortunately, however, the formulations of such inverse problems are not yet supported by necessary experimental material.

A short survey of applications.

In this paper no attempt has been made to summarize comprehensively an extremely wide field of CT experiments in plasma physics. Descriptions of some techniques and analysis of results can be found in several recent reviews /2,5,6/. Practically only a short guide to references and some remarks can be given below.

a/ Asymmetrical arc discharges and plasma jets.

Axial symmetry is substantially destroyed by the application of an external magnetic field, the interaction of a plasma with the flow of the incoming gas or with some solid obstructions (including walls, probes, electrodes etc).

One of the first really tomographic research was a study /I8/. D.M. Benenson et al. /I9,20/ investigated the effects of velocity, current and transverse magnetic field upon temperature distribution within cross-flow arc discharge. N. Sebald /21/ fulfilled the measurements of the temperature and flow fields of the magnetically stabilized cross-flow \( N_2 \) arc.

A. Flessl /22/ caused the arc itself to rotate, which under the assumption of stationary discharge in a rotating reference system made it possible to gather quite simply the required number of projections. The 2D temperature field of the gas was determined from the \( H_\alpha \) line with the deviation of the plasma state from LTE being taken into account; in this experiment a complicated structure of the flow was made evident, including several vortices and stagnation points.
of equivalent resolution. This and other advantages of FZP were demonstrated by experiments used the "Cyclops" laser-target irradiation facility at Lawrence Livermore Laboratory /43/. Comprehensive mathematical framework for FZP technique is provided in the review article /44/, where a lot of additional references is given.

But FZP camera inevitably produces the artifacts in the reconstructed image /45/. The uniformly redundant array (URA) has been introduced by E.E.Fenimore and T.W. Cannon /46/ as artifact-free array for the coded aperture imaging. This URA camera has been for the first time applied to the CO$_2$ laser driven compression experiment /47/. The camera has tomographic capability as FZP but superior to it in some respects because the system point spread function of the URA camera can be made practically the delta function. URA camera has been successfully applied to take an X-ray image on a cannonball target /48/; the tomographic capability of it was falsely improved by applying the iteration method.

It is worthwhile to mention also the results of co-operative USSR-USA research on numerical processing of interferograms from strongly inhomogeneous phase objects /49/ and CT investigation of small dense targets made by groups from Lebedev's Institute (Moscow) and from Novosibirsk /50/. Analysis of a novel non-linear 3D CT algorithm for laser-produced plasma study is given in the paper /51/.

d/ Space studies, astrophysics.

One meets a great variety of approaches, assumptions, algorithms and objects of observation in this CT field. Half a century ago V.A.Ambartsumyan has solved the so called Edington problem: to restore the spatial velocities distribution for the stars near the Sun from a set of radial (ray) velocities (for a review see the paper /32/). Mathematically the problem is rather simple: it is a 3D Fourier transform of a spherically symmetric function in the velocity space. Similar case concerning the inner structure of globular star clusters was considered by E.L.Kosarev /53/; an example referred to the cluster of flare stars in the Pleiades was discussed.

The study of K.N.Bracewell /54/ also should be regarded as a pioneer one: it contains investigations of UV emission of solar plasma, measured by the method of strip sums which are purely tomographic in their mathematical framework. To the same category also belong studies on lunar radar ranging /55/ and on the interpretation of data obtained on a radioheliograph /56/. Important overview of the mapping of radio emission from celestial objects is given by K.N.Bracewell /57/.

In many cases the scientists applying CT techniques to space plasma physics were forced to resort to rather strong a priori restrictions, and only isolated attempts are known of reconstructing the inner structure of truly symmetrical objects. For example, in studying of the solar corona the standard assumptions made are of central or axial symmetry /58/, the possibility of describing the electron density distribution function by a small number of parameters /59/, the correctness of calculating of 3D magnetic field distribution from 2D (photosphere) data by using the Laplace equation /60/, etc.

Recently L.D. Altschuler summarized the results on reconstruction of the global-scale 3D solar corona /61/. The magnetic field and electron density distributions were restored. However, it was necessary to manipulate with projections obtained successively during half the period of
the Sun rotation around its axis, i.e. during 14 days.

C.G. Fesen and P.L. Hays /62/ described a technique which inverts satellite airflow data producing volume emission rates as functions of altitude and position. The CT inversion is applied to the data obtained when the spacecraft spins in the orbital plane.

Somewhat specific, but undoubtedly promising, is a method of studying the 3D spatially inhomogeneous structure of planetary nebulae from the planar pattern of its isophots /63/. Coded apertures have made significant imaging contributions in the fields of X-ray astronomy: e.g. the structure of the Vela supernova remnant /64/ and the emissivity from Crab nebula /65/ have been investigated. Various CT approaches connected with the occultation experiments /66, 67/ also are of interest for astrophysics.

Conclusion.

I hope the basic considerations and the examples of application given above show you the significance of computerized tomography in plasma physics nowadays and the new real possibilities in the near future. The impression emerging from a survey of CT literature of recent years is that this interdisciplinary field is presently undergoing a rapid expansion in various directions. The result of the world wide research efforts is that CT of plasma in laboratory and space which only several years ago was in its infancy provides now a reliable and very informative diagnostic tool permitting the deeper understanding and closer control of plasma processes.

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1. INTRODUCTION

For both steady-state and pulsed Townsend discharges in a uniform electric field $E$, the inter-electrode gap contains three regions of interest.

(a) The cathode (non-equilibrium) region. Here electrons retain some memory of the initial conditions. The extent of this region can be estimated by requiring that electrons starting from rest should fall through a potential difference of the order $2\bar{E}/e$ where $\bar{E}$ is the mean energy at equilibrium. Since $\bar{E}/e \approx D/\mu$ this potential difference corresponds to a distance of $2D/\mu E = 2D/W$, where $D$ is the diffusion coefficient, $\mu$ is the electron mobility and $W$ the electron drift velocity. This can only be regarded as a very rough approximation since each transport parameter or reaction rate will approach their steady-state values in quite different ways. For example, the drift velocity approaches its final value in the order of one momentum transfer collision time when the energy dependence of the momentum transfer cross-section gives a constant mean free time. On the other hand the onset of ionization (or other processes with even higher threshold energies) may occur only after electrons have fallen through a potential difference many times greater than $\bar{E}/e$ for small $E/N$ values. Keeping these comments in mind, an approximate value for the extent of the non-equilibrium region will be given by $\lambda_L z = 1$, where $\lambda_L = W/2D_L$, $z$ is the distance from the cathode and $D_L$ is the longitudinal diffusion coefficient.

(b) The equilibrium region. The behaviour of an electron swarm can now be described using constant transport parameters. The hydrodynamic model of swarm or steady-state discharges is applicable and the energy distribution function depends on both $E/N$ and gradients of the electron concentration $n$. 
(c) The anode-boundary layer. In this region large changes in the electron concentration occur in a distance comparable to the energy exchange length $1/\lambda_L$ and the hydrodynamic model is no longer satisfactory. The mean energy increases since absorption at the anode reduces the number of electrons diffusing back in the field direction. These electrons would normally lose energy to the field and contribute to the number of lower energy electrons in the energy distribution.

There are still many interesting, although difficult, problems to be addressed in relation to the boundary layers and even in the equilibrium region. There have been few experimental investigations concerning the internal structure of swarms, and indeed these topics have been characterized by simulation rather than experimentation. In this paper some optical observations of Townsend discharges will be presented which indicate that these problems are amenable to further experimental study. Only the cathode layer and the equilibrium region will be discussed, although the techniques could be applied to the anode boundary region.

2. THE CATHODE NON-EQUILIBRIUM REGION.

This region is characterized by a spatial and/or temporal dependence of the energy distribution function for a given group of electrons leaving the cathode. Of the various phenomena associated with spatial variations in the energy distribution perhaps the most striking are the Holst-Oosterhuis luminous layers observed for steady-state Townsend discharges in atomic gases [1] - [6]. The layers are most distinct over an intermediate range of $E/N$ values for a given gas. In this range the main features of the layers can be explained by electrons gaining energy from the field with a relatively small energy loss by recoil in elastic collisions until they reach the threshold of an electronic excitation level. Electrons then lose most of their energy in an inelastic collision producing a region of excited atoms (and luminosity) with a corresponding decrease in mean energy. Since the mean energy is spatially dependent, the constancy of electron flux (in the absence of ionization or attachment) cannot be achieved by a uniform electron concentration except in the idealized case of a constant mean free time for electron momentum transfer. Consequently the spatial variations in luminosity and mean energy are accompanied by variations in electron concentration as shown schematically in Figure 1. The changes in electron concentration can be so large that particular care should be taken when inferring excitation frequencies from measurements of the light intensity.
Figure 1. A schematic representation of the formation of luminous layers in a gas with one excitation level having a threshold energy $e \nu_a$. The relative positions of the maxima in mean energy, light output and electron concentration depend upon the energy dependence of the collision cross-sections.

For low $E/N$ values, electrons undergo sufficiently many elastic collisions to produce broadening of the local energy distribution through recoil energy losses and at the same time the mean energy is reduced. The luminous layers become progressively weaker and more diffuse as $E/N$ is reduced. At large $E/N$ the presence of ionization and the sharing of energy between the two outgoing electrons reduces the strong correlation between the electron energy and axial position and again the layers become less distinct and reduced in number.

This simple description of layer formation is substantially modified when more than one excitation level is considered. This is demonstrated in Figure 2 for two levels with threshold energies $e \nu_a$ and $e \nu_b$. There is structure evident in each layer which should be observable using spectral analysis of the light output.

Hayashi [7] has carried out detailed simulations for helium using a number of inelastic states and his results demonstrate these variations in the spatial excitation of different excited states.
Simulations of pulsed Townsend discharges also indicated that the layered structure should be seen within isolated swarms even though the spatially integrated characteristics of the swarm (e.g. mean energy, centre of mass drift velocity and higher moments) have substantially reached their steady-state values [8], [9].

The non-equilibrium region is more complex in molecular gases since there are a large number of electronic, vibrational and rotational energy loss channels. There is a paucity of experimental data for molecular gases although Fletcher [5] has observed the decrease in the layer structure of luminosity when a few percent of N₂ is added to Townsend discharges in He. Simulation studies [10] for SF₆ indicate the existence of strong spatial variations in energy and attachment rate in that gas. This is attributed to the low threshold energies for attachment and vibrational excitation and the large energy gaps to the first electronic excitation threshold.

In N₂ however, various simulations have shown little indication of oscillations in discharge parameters - some approaching smoothly towards their equilibrium while others may "overshoot" the equilibrium value. This remnant oscillation is not different in kind to the Holst-Oosterhuis layers but only in character as a consequence of the multitude of inelastic processes. In particular the large vibrational cross-sections of N₂ in the vicinity of 3eV means that many electrons starting from the cathode with small energies undergo one or more collisions with vibrational excitation of the ground
state before they reach the first excitation threshold. A detailed Monte-Carlo simulation [11] of the time evolution of the energy distribution at high $E/N$ values clearly showed the ebb and flow of electrons through energy space and the rapid spreading of the distribution function away from the "discrete" spectrum characteristic of the atomic gases. This simulation suggested the possibility of observing some degree of oscillatory behaviour by injecting electrons with energies above 5eV for which the ground state vibrational cross-sections have negligibly small values. After gaining additional energy from the field, electrons lose energy by a variety of inelastic electronic transitions giving again a "quasi-discrete" spectrum although broadened by the vibrational and rotational structure of the electronic states. Many of the electrons return to energies above the maximum in the ground state vibrational cross-sections so that further oscillations in mean energy and excitation rates would be expected.

Figure 3. Experimental arrangement for observation of the light output from Townsend discharges.

Using the experimental arrangement shown in Figure 3, the light output from steady-state Townsend discharges in $N_2$ was measured in the non-equilibrium region using the slit collimators [12]. The electron source was placed 1mm behind a hole in the cathode using acceleration voltages > 5 volts. Gas pressures were sufficiently low (0.2-0.3 torr) to ensure that very few electrons undergo inelastic collisions before entering the drift region. The light emission from the (0,0) band of the second positive system at 337.1nm is shown as a function of the additional potential fallen through (i.e. equivalent to axial position) in Figure 4(a) for $E/N = 330$Td and an injection energy of 8eV. A
Monte-Carlo simulation was also carried out [13] using the cross-sections given by Tagashira [14] and 8eV initial energy. The excitation rate to the $C^3\Pi_u$ state was determined although it should be noted that this cross-section set makes no allowance for the vibrational structure of the C-state. The theoretical results shown in Figure 4(b) reproduce the general features of the experimental values but differ significantly in detail. Although contact potentials produce some uncertainty in the injection energy and the potential scale for the experimental results, it is clear that modifications to the assumed cross-sections are required.

NITROGEN $\frac{E}{N} = 332$ Td (3371)

Figure 4(a). Measured radiation in the (0,0) band of the second positive system in $N_2$.

Figure 4(b). Monte-Carlo simulation of excitation to the $C^3\Pi_u$ state.
The energy distribution function as determined by the Monte-Carlo simulation is shown in Figure 5 for an axial position where electrons have fallen through a total potential of 17.5 volts (including the 8 volt acceleration potential. A spread of initial energies of approximately 0.25eV was included in the calculations). The absence of any electrons with energies between 12eV and 17.5eV follows from the absence of inelastic collisions until the threshold of the $A^3\Sigma_u^+$ state is reached at 6.17eV provided that the injection energy is sufficient to ensure that vibrational collisions can be neglected until the first inelastic collision has occurred.

Experiments of this type should be valuable in the determination of cross-sections, particularly near the threshold energy of the excitation process. The analysis and interpretation of the results would be simplified if radiation from several states were recorded, although quenching of long-lived states would be required to retain the spatial dependence of the excitation rates. Since very few inelastic collisions are involved in these experiments, it is likely that the differential scattering cross-section would be important whereas isotropic scattering has been assumed in the Monte-Carlo results presented here. Clearly, further work is needed to obtain the full potential of this type of investigation.
3. **The Steady-State Region.**

When a group of electrons leave the cathode and pass beyond the non-equilibrium region described in the last section, the energy distribution for the group (and any progeny in the presence of ionization) reaches a steady state except at very large values of $E/N$ for which electron run-away occurs. The electron concentration in a swarm can be described in terms of constant transport parameters and reaction rates [15]. However this does not imply that the energy distribution is spatially uniform and in this hydrodynamic regime the distribution function can be expanded in a concentration gradient expansion (for cylindrical symmetry),

$$f(r,z,e,t) = n(r,z,t)g_0(e) - \frac{1}{\lambda L} \frac{3n(r,z,t)}{3z} g_1(e) + \ldots$$

where the factor $1/\lambda L$ is included to make $g_1(e)$ independent of pressure and of the same dimensionality as $g_0(e)$. Only these first two terms of the expansion will be considered in the following discussion.

In a steady-state Townsend discharge this can be written as

$$f(r,z,e) = n(r,z)[g_0(e) - \frac{1}{\lambda L} \frac{3n}{3z} g_1(e)]. \tag{1}$$

For a one-dimensional discharge, or integrating (1) over all radial positions, with $n(z) = n_0 \exp(\alpha x z)$,

$$f(z,e) = n(z)[g_0(e) - \frac{\alpha x}{\lambda L} g_1(e)]. \tag{2}$$

However in an experiment such as that shown in Figure 3, $\frac{1}{n} \frac{3n}{3z}$ is not constant and the normalized energy distribution function varies throughout the discharge region. Experiments have been carried out to illustrate this spatial dependence of the energy distribution by observing the ratio of spectral intensities in different regions of a steady-state discharge [16]. Using a gas mixture of He:N$_2$:CO$_2$:CO = 54:34:6:6, transport parameters were first measured by observing the spatial and temporal distribution of photons emitted from a pulsed electron swarm [17]. These data allow the electron concentration to be calculated for a steady state discharge, although the results can only be regarded as approximate since the boundary regions are not properly accounted for in this approach. The results are shown in Figure 6(a) for $E/N = 300$Td at a pressure of .373 torr and an electrode separation of 6.0 cm. The corresponding
values of the parameter $\frac{1}{\lambda_L n} \frac{\partial n}{\partial z}$ occurring in equation (1) are shown in Figure 6(b).

Figure 6(a). A contour diagram of the electron concentration for a steady-state discharge with a point source.

These results show that $\frac{1}{\lambda_L n} \frac{\partial n}{\partial z}$ varies appreciably with radial position, although care must be taken to ensure that $\lambda_L z > 1$ if the non-equilibrium region is to be avoided.

Transverse scans of the radiation emitted from the steady-state Townsend discharge were made with the linear collimator shown in Figure 3 and using single photon counting techniques. It is desirable to monitor two states with markedly different excitation thresholds to maximize changes in the intensity ratios which result from small changes in the energy distribution. The (0,0) band of the second positive system at 587.1nm and the (0,0) band of the first negative system at 391.4nm of N$_2$ were chosen with threshold energies of 11eV and 18.75eV respectively. After Abel inversion of the transverse scan data, the radial dependence of the excited state population of the upper level of each band could be determined. In order to remove uncertainties in calibration of the photon detection sensitivity the profiles for each band were normalized to the radially integrated photon count rate.
From equations (1) and (2), this normalized count rate $R_1$ for a given band is given by

$$R_1 = \frac{n(r,z)}{n(z)} \left[ \nu_{10} - \frac{1}{\lambda L} \frac{\partial n}{\partial z} \nu_{11} \right] / \left[ \nu_{10} - \frac{\alpha T}{L} \nu_{11} \right],$$

where

$$\nu_{10} = \left( \frac{2}{m} \right)^{\frac{1}{2}} \int_0^\infty Q_1(\varepsilon) e^{\frac{1}{2} \mu_0(\varepsilon)} e^{\varepsilon} d\varepsilon,$$

$$\nu_{11} = \left( \frac{2}{m} \right)^{\frac{1}{2}} \int_0^\infty Q_1(\varepsilon) e^{\frac{1}{2} \mu_1(\varepsilon)} e^{\varepsilon} d\varepsilon$$

and $Q_1(\varepsilon)$ is the cross-section for excitation of the upper level of this transition.

If $\frac{1}{\lambda L} \frac{\partial n}{\partial z} \nu_{11} \ll \nu_{10}$, then

$$R_1 = \frac{n(r,z)}{n(z)} \left[ 1 - \left( \frac{1}{\lambda L} \frac{\partial n}{\partial z} - \frac{\alpha T}{L} \nu_{11} / \nu_{10} \right) \nu_{11} \right].$$

Comparing these normalized rates for both bands, we obtain

$$\frac{R_1}{R_2} = 1 - \left( \frac{1}{\lambda L} \frac{\partial n}{\partial z} - \frac{\alpha T}{L} \nu_{11} / \nu_{10} \right) \frac{\nu_{21}}{\nu_{20}}.$$

If the states designated 1, 2 are taken to refer to the $C^3\Pi_u$ and the $B^2\Sigma^+$ states respectively it would be expected that $\nu_{21} > \nu_{11}$ since the "tail" of the distribution function is more sensitive to small changes in the mean energy. Then

$$R_1/R_2 = 1 + k \left( \frac{1}{\lambda L} \frac{\partial n}{\partial z} - \frac{\alpha T}{L} \right) \frac{\nu_{21}}{\nu_{20}},$$

with $k$ a positive constant, and the results shown in Figure 6(b) indicate that $R_1/R_2$ would be less than unity in the central regions of the discharge and greater than unity in the outer regions. Experimental values of $R_1/R_2$ are shown in Figure 7 for $z = 2.5 \text{cm}$ where the electron concentration has a saddle point at $r = 0$ (of Figure 6(a)). Using the experimental value of $R_1/R_2$ at this point together with the previously measured values of $\alpha L/\lambda L$ we find $k = 2.0 \pm 0.2$ for $E/N = 300 \text{Td}$ in this gas mixture.
Although several theoretical evaluations of the terms in a gradient expansion of the distribution function have been published, these have been used mainly for evaluation of the so-called time of flight, pulsed Townsend and steady-state Townsend transport parameters. As far as the author is aware only the time of flight parameters have been measured. The results described above concerning the spatial distribution of excitation rates show that this method can be used to obtain information about the individual terms in the gradient expansion of the distribution function.

4. CONCLUSIONS

Previous studies of light emission from Townsend discharges have concentrated on the evaluation of transport parameters and reaction rates for ionization and excitation which have been spatially integrated. However only time of flight transport parameters are obtained and excitation rates cannot be regarded as absolute unless an independent measure of the electron concentration is made. A reasonable estimate of the electron concentration can be obtained by measuring the anode current, but this can only be approximate in the presence of ionization since a satisfactory treatment of the boundary conditions has not been fully developed.

Nevertheless, the results presented here show that many features of the energy distribution function can be obtained using the optical observation method in both the boundary and steady-state regions of the discharge. In particular the concentration gradient expansion of the distribution function can be studied in some detail. Further work is required to establish the conditions for which this expansion is valid.

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      (1923) Phil. Mag. 46, 1117.


   (Hayashi has carried out a simulation with a more detailed set of excitation cross-sections - personal communication).


     (see also reference [8]).


EXCITED STATES IN $N_2$ REACTIVE PLASMAS WITH APPLICATION TO SURFACE PROCESSES

A. Ricard

Laboratoire de Physique des Gaz et des Plasmas, Bâtiment 212, Université Paris-Sud, 91405 Orsay Cédex, France, UA 73 (CNRS)

Abstract

The spectroscopic diagnostics is a powerful in situ and non intrusive method of analysis of reactive plasmas. Gas mixtures with nitrogen are employed for surface treatments such as metal nitriding and coating. Emission and absorption spectroscopy with high power tunable gas lasers have been applied to $N_2$, $N_2$-$H_2$ and $Ar$-$N_2$-$H_2$ glow discharges to study the $N_2(X,V)$ ground state vibrational distribution (CARS spectroscopy and rare gas actinometry) and to determine the spatial distributions of atomic and molecular excited states such as $N^+$, $N_2$, $H$ and metal atoms of the target. Correlations are given between the relative densities of the plasma excited states and the surface characteristics.

Introduction

Reactive plasmas with $N_2$ are commonly used for metal surface treatment such as surface nitriding and coating. In surface nitriding, the plasma is a glow discharge in $N_2$-$H_2$ mixtures where the sample to be nitried is connected as a cathode. In surface coating, the plasma must satisfy the two functions of evaporating a metal source and activating the gaseous products to obtain a specific compound on the substrates. An example of surface coating is given with $T_i$ N deposition on steel substrates by using an $Ar$-$N_2$-$H_2$ reactive plasma. Such plasma treatments are employed in the engineering field for tool hardening and resistance to corrosion.

In surface treatments by plasmas, there is a poor understanding of plasma chemistry processes such as composition and spatial distribution of excited states, energy (temperatures) of the active species, specific reactivity of neutral species and ions. The present paper is focused on $N_2$ excited states in plasmas for surface treatments. Low pressure plasmas ($< 5$ Torr) are concerned in glow discharge regime. In these conditions, neutrals and ions are mainly excited by electron collisions inside the plasma volume.

First, the excitation of $N_2$ states in a glow discharge column in flowing $N_2$ is analysed in discharge and post-discharge regimes. Electron distribution function and excited state populations have been calculated in connection with M. Capitelli in Bari [1]. The vibrational distribution of $N_2(X,V)$ which are the most populated species in $N_2$ glow discharges has been especially studied. The calculated values are compared to first experimental results of $N_2(X, V \leq 15)$ distribution obtained by CARS spectroscopy with the ONERA group [2].

Then excited states in $N_2$-$H_2$ glow discharges for metal surface nitriding are analysed by emission spectroscopy. A special focus is given on rare gas actinometry used to connect excited state and ground state relative populations.

Finally, the first results on excited states in $Ar$-$N_2$-$H_2$ plasmas for TiN deposition are given.

These studies are undertaken to correlate plasma and surface characteristics in view to control the plasma deposition process.
1) Vibrational distribution of \( N_2(X,V) \) ground states in a glow discharge and in a post-discharge of flowing \( N_2 \)

The vibrational distribution of \( N_2(X,V) \) in the discharge has been calculated by solving a system of vibrational master equations coupled to the Boltzmann equation for the electron energy distribution (e.d.f.) [1]. A result is shown in Fig. 1 for a residence time of \( 10^{-3} \text{ sec} \). The characteristic vibrational temperature \( T_1 \) \( (N_1/N_0 = \exp - \Delta E_1/k_0 \) is \( \theta_1 = 4400 \text{ K} \) (full line in Fig. 1).

Measurements of \( N_2(X,V) \) vibrational distribution have been undertaken by CARS with the ONERA group. The experimental arrangement is in Fig. 2. A Yag-Nd laser with a dye cell is used to separate two laser beams with frequencies \( \nu_1 \) (5320 A - 80 mJ) and \( \nu_2 \) (variable in the red ~ 6000 A - 3 mJ - \( \delta \nu_2 = 0.05 \text{ cm}^{-1} \)). These two laser beams crossed the discharge tube as shown in Fig. 2. The discharge tube was with six electrodes allowing creation of discharges and post-discharges of several lengths (from 20 cm to 120 cm) in flowing \( N_2 \). The \( N_2 \) pressure can vary from 1 to 5 Torr, the discharge current from 10 to 100 mA.

The two laser beams interact with the \( N_2(X,V) \) molecules to produce an anti-Stoke signal as shown in Fig. 3 with the frequency \( \nu_3 = 2\nu_1 - \nu_2 \). The CARS signal intensity \( (I_c) \) is detected by using a dichroic lens to eliminate \( \lambda_1, \lambda_2 \) and by a spectrometer as shown in Fig. 2. We have the following relation [2]:

\[
I_c = \nu_3^2 (\Delta N^V_{V+1})^2 I_1^2 I_2
\]

where \( I_1, I_2 \) are the two laser \( (\lambda_1, \lambda_2) \) intensities and \( \Delta N^V_{V+1} = N_{V+1,J} - N_{V,J} \) with \( \Delta J = 0 \pm 2 \).

The experimental results of \( N_2 (X, V < 14) \) vibrational distribution found for a discharge of 40 cm in length (residence time \( 6 \times 10^{-3} \text{ sec} \) are given in Fig. 1. It can be seen an over population of \( N_2 (X, V > 5) \) in comparison to the Boltzmann distribution. This over-population is the result of an efficient \( V - V \) pumping up effect as given by the relation:

\[
N_2 (V) + N_2 (W - 1) \rightarrow N_2 (V - 1) + N_2 (W)
\]

The pumping up mechanisms are effective in time scales of about 0.1 sec in the present discharge conditions.

The \( V - V \) vibrational transfers produce also a strong vibrational excitation of \( N_2 (X,V) \) in post discharge conditions which are combined to superelastic electron collisions to produce a strong coupling of \( \theta_1 \) vibrational temperature and "\( T_e \)" electron temperature in times of the order of \( 10^{-3} - 10^{-2} \) sec [1]. In times greater than \( 10^{-3} \) sec, vibration-translation (\( V - T \)) destruction processes for \( N_2 (X,V) \) and diffusion and recombination loss terms for electrons are effective so that \( \theta_1 \) and "\( T_e \)" are decreasing.

CARS measurements in discharge and post-discharge conditions are given in Fig. 4. These results clearly demonstrate that the \( N_2 (X,V) \) populations remain highly populated in late post-discharges (\( \Delta t \sim 10^{-2} \) sec). The vibrational temperature was slowly decreasing from...
4500 K in the discharge to 3000 K in the late post-discharge.

2) Excited states in \( \text{N}_2 - \text{H}_2 \) glow discharges for metal surface nitriding

Plasma treatments are conducted with a glow discharge where the sample to be nitrided is used as a cathode. A discharge voltage is applied to a \( \text{N}_2 - \text{H}_2 \) gas mixture (1 - 5 Torr) introduced into a reactor to perform the plasma nitriding at a chosen cathode temperature (840 K for a steel surface). The emission spectroscopy has been set up to analyse the plasma excited species in operating conditions of plasma nitriding. The actinometry method has been used to determine the main excitation processes in the plasma glow near the cathode. The \( \theta_1 \) vibrational temperature of \( \text{N}_2 (X) \) ground state has been deduced.

2.1) The nitriding device

The experimental device is shown in Fig. 5 (for more details, see ref. 3). The best steel nitriding conditions of the steel cathode have been found as the following: total pressure 2.5 Torr, discharge current 1 - 5 mA cm\(^{-2}\) and cathode temperature 550 °C. In these conditions, an abnormal glow discharge is produced whose the negative glow near the cathode was analysed by emission spectroscopy using a HRS Jobin-Yvon grating spectrometer (resolution \( \delta \lambda = 0.2 \ \text{Å} \) in the visible range (3000 - 9000 Å). The plasma light was collected through pin-holes (diam. 2 mm), 1 cm apart, along the reactor wall, allowing analysis of the negative glow axial distribution near the cathode dark space.

2.2) Excited species in \( \text{N}_2 - \text{H}_2 \) plasmas

The identified species are reported in table I with the optical transitions, the characteristic spectral lines and the corresponding excitation thresholds. The \( \text{N}_2^+ \) first negative bands are the most intense spectral lines. The relative intensity distribution of \( \text{N}_2^+ \), \( \lambda = 3914.4 \ \text{Å} \) is shown in Fig. 6 versus the axial distance upward the cathode. The \( \text{N}_2^+ \) band intensity first increases out the dark space (about 1 - 2 mm) to reach a maximum value and then quasi-exponentially decreases up to about \( z = 8 \ \text{cm} \) (see Fig. 6). In \( \text{N}_2 - \text{H}_2 \) mixtures, the \( \text{N}_2^+ \) lines remain the most intense lines even with 90% \( \text{H}_2 \).

2.3) The \( \text{N}_2^+ \) rotational temperature

The R-Branch \((\Delta N = +1)\) of \( \text{N}_2^+ (\text{B}^2\text{g}^+, \nu' = 0 \rightarrow \text{X}^2\text{g}^+, \nu'' = 0) \), \( \lambda = 3914.4 \ \text{Å} \) is given in Fig. 7, for 40% \( \text{H}_2 \) in a 2.5 Torr \( \text{N}_2 - \text{H}_2 \) mixture. For a Boltzmann distribution of the \( \text{N}_2^+ (\text{B},0) \) rotational levels, the \( I_N \) rotational intensity (\( N \) lower rotational level) is given by the following relation [4]:

\[
\log \left( \frac{I_N}{I_{N+1}} \right) = -\frac{B_0}{kT_R} \frac{\hbar c}{N+1} (N+2) \tag{3}
\]

where

\[
\frac{B_0 \hbar c}{kT_R} = \frac{3}{R(K)}
\]
It has been found that the experimental rotational spectra such as in Fig. 7 follow
the Boltzmann relation (3) and that the rotational temperature $T_{N_2^+}$ can be determined.
Results are shown in table 2 for several $N_2$ percentages at constant voltage (500 V) and
total pressure (2.6 Torr). The spectra correspond to the maximum intensity of the negative
glow as shown in Fig. 6. The cathode temperature $T_K$ measured using a thermocouple is also
reported in table 2. It has been found a good correlation between $T_{N_2^+}$ (measured at axial
distance $z = 1 - 2$ cm) and $T_K$ for all the $N_2 - H_2$ mixtures except for pure $N_2$ where $T_{N_2^+}$
was always found higher than $T_K$. In $N_2 - H_2$ mixtures, the $N_2^+$ rotational temperature in the
negative glow near the cathode appears to be a reliable diagnostics of the cathode surface
temperature.

2.4) The gas rare actinometry method

The main limitation of the emission spectroscopy method is that the detected excited
states are not always simply related to the corresponding ground states. In out of equili-
brium plasma at low gas temperatures, the electron collisions are the main excitation pro-
cesses. Also by using a rare gas impurity, as previously described for etching plasmas
[5,6], the main kinetic process can be deduced in favourable conditions.

By using first a 2 % Argon impurity in the $N_2 - H_2$ nitriding plasma, it can be observed
in Fig. 8 that the excitation threshold for $Ar I$ 7504 Å (13.5 eV) is about the same than for
$N_2$ 2$^\text{nd}$ positive, 3371 Å (11.1 eV). Moreover, the excitation cross sections have similar
shapes for the two species. Consequently, if direct electron excitations are the main pro-
cesses for Ar (4p) and $N_2$ (C) states, the $N_2$, 3371 Å over Ar, 7504 Å intensity ratio must be
directly proportional to the $N_2$/Ar ground state density. It is really what it is observed
in Fig. 9 for axial distance $z = 1$ cm above the cathode.

Then by using a 2 % Neon impurity, the ionic state $N_2^+$ (B,0) (18.7 eV) can be compa-
red to the Ne (3p) (18.6 eV, cf. Fig. 8). In Figure 10, the variation of $\nu_{N_2^+} (B,0)$
$I (N_2^+ 3914 \text{ Å})$
$I (Ne 5852 \text{ Å})$
is reported as a function of $[N_2]/[Ne]$ for axial distance $z = 1$ cm. The total
destruction frequency $\nu_{N_2^+} (B,0)$ : radiative and quenching by $N_2$ and $H_2$ has been calculated
[7].

It can be observed in Fig. 10 that the intensity ratio is proportional to the ground
state density ratio for $N_2 - H_2$ mixtures with $H_2$ upper than 10 %. Consequently, since the
Ne and Ar excited states are directly populated by electron collisions (the quenching of
argon and neon metastable states by the $N_2 - H_2$ main gases is efficient), the $N_2^+$ (B) and
$N_2$ (C) states are also directly populated by electron collisions at the beginning of the
negative glow near the cathode fall.

At distances $z > 1$ cm, the intensity ratios deviate more and more from linear vari-
tions to reach quadratic shapes as shown in Fig. 11 for $I_{N_2}/I_{Ar}$ at $z = 1.7$ cm for
$H_2 < 50 \%$. When the $H_2$ percentage is growing, the cathode fall depth increases from about
1 mm in pure $N_2$ to 3 mm in pure $H_2$. The border between cathode fall and negative glow moves
apart the cathode with the $H_2$ percent so that the $I_{N_2}/I_{Ar}$ variation with $[N_2]/[Ar]$ remains
linear at $z = 1.7$ cm for $H_2 > 50 \%$ (cf. Fig. 11). In this case, direct electron excitations
are still the main kinetic process.
2.5) The vibrational temperatures in $N_2 - H_2$ plasmas

Direct electron collisions for excitation of $N_2^+ B$ and $N_2 C$ states can be used to determine the vibrational temperature of $N_2 (X)$ ground state. The $\theta_{vib}$-vibrational temperatures are reproduced in Fig. 12 for $N_2 - H_2$ discharges (2.7 Torr, 500 volts). The $\theta_2 C$ and $\theta_{1 B^+}$ temperature are given by the following relations (7):

$$\frac{[C, 2]}{[C, 0]} = \exp \left[ - \frac{5650}{\theta_2} (K) \right]$$ (4)

$$\frac{[B^+, 1]}{[B, 0]} = \exp \left[ - \frac{3418}{\theta_1} (K) \right]$$ (5)

where the brackets are for excited state densities.

The ground state $\theta_{1 X}$ temperature has been calculated by using the Franck-Condon factors connecting the $N_2 (C)$ and $N_2^+ (B)$ states to the $N_2 (X)$ ground state. $\theta_2 C$ has been chosen instar $\theta_1 C$ as giving more precise values of $\theta_{1 X}$ (7).

It can be observed in Fig. 12 that the $N_2 C$ and $N_2^+ B$ vibrational temperature give about the same value of $\theta_{1 X}$ confirming that direct electron collisions occur at $z < 1 \text{ cm}$. The $\theta_{1 X}$ temperatures are in the range 1500 - 2000 K, higher than the cathode temperature and varying in the same way with the $H_2$ percentage.

3) Excited states in $Ar - N_2 - H_2$ reactive plasmas for $Ti_N$ deposition on steel surfaces

The emission spectroscopy has been used to analyse the excited states of Ar, $N_2$ and $H_2$ in plasmas pulverising $T_i$ for $Ti_N$ deposition on steel surfaces.

3.1) The experimental set-up

The plasma reactor and the spectroscopic diagnostic is indicated in Fig. 13. The plasma reactor is a stainless steel chamber of 45 cm in diameter and 90 cm high. A titanium emitter $K_c (T_i)$ is located at the center. It is a circular array (5.2 cm in diameter) of eight titanium rods (1 cm in diameter). A plain steel substrate (S) was located at a distance $z = 16 \text{ cm}$ from the $T_i$ array. A loop electrode (A) (10 cm in diameter) was located between $K_c$ and $S$ (for must details see ref. 8). The spectroscopic diagnostics rested on a rotating table allowing a spatial distribution study between $K_c (T_i)$ and (S) with a precision of $\pm 0.5 \text{ cm}$.

3.2) Excited species in $Ar - N_2 - H_2 - T_i$ plasmas

Selected observed spectral lines are listed in table 3 for $T_i$, $H$, $N_2$ and $Ar$. A large number of $Ar$, $A^+_r$ and $T_i$ spectral lines were emitted in the spectrum between 300 and 500 nm which complicated the identification of $T_i^+$ and $N_2$ emissions. For $T_i^+$, the $T_i II 3759 \AA$ has been chosen. For $N_2$, the $N_2 (B^3 \Pi_g - A^3 \Sigma_u^+)$ band head at $\lambda = 6623 \AA$ was the
only one appearing in a free spectral range and sufficiently strong to be detected. The \( N_2 \) 2\(^{\text{nd}} \) positive and \( N_2^+ \) first negative systems although more intense was hidden by the Ar and \( T_1 \) emissions. When the loop electrode (A) was connected to a positive voltage \( V_A < 100 \) volts all the spectral line intensities were observed to increase between \( K_c \) and \( S \) (cf. Fig. 13 and ref. 8) by a factor varying between 1.3 and 1.7 (\( V_A = 90 \) Volts). In Fig. 14, the intensities of Ar, \( N_2^+ \), H and \( T_1 \) spectral lines are reported as a function of position between \( K_c \) and \( S \). The Ar and H excited atoms decrease with about the same spatial distribution. This decrease is enhanced for the \( T_1 \) excited states. But the \( N_2 \) (B) excited states keep a constant density value over about 10 cm before to decrease near the substrate. The main consequence of these spatial distributions is that the density ratio of the \( N_2^+ / T_1 \) excited states increase from \( K_c \) to \( S \). Noting can be deduced for the corresponding ground states since the kinetic processes for \( N_2 \) and \( T_1 \) excitation have not been elucidated. Nevertheless, nuclear analysis of the substrate coating has shown that the coating composition varied continuously from \( T_1 \), \( N_2 \), close to the \( T_1 \) emitter, to stochiometric \( T_1 \), \( N_2 \) farther away [8]. Such a variation in fact follows these of the \( T_1 \) and \( N_2 \) excited states.

**Conclusion**

Plasma analysis using optical spectroscopy has been applied to reactive plasmas with \( N_2 \) for surface treatments. The excited species have been identified in the negative glow from the target to the substrate. The spectral line intensities can be used to control the steady state conditions for a given process. In some experimental conditions, for example with \( N_2 \) - \( H_2 \) plasmas for steel nitriding, the rotational distribution of \( N_2^+ \) 1\(^{\text{st}} \) negative spectral lines (band head 3914.4 \( \text{Å} \)) can be identified to the gas and cathode temperatures allowing a quantitative control of the nitriding process.

To relate the excited state to the ground state population, the kinetic analysis of electron excitation must be accurately undertaken. The actinometry method using a rare gas impurity can be used to detect the main excitation channel and to correlate excited to ground states. Some results have been obtained for \( N_2 \), at the end of the cathode fall (beginning of the negative glow), allowing a determination of the \( N_2^+ (X) \) ground state vibrational temperature.

The absorption spectroscopy is a powerful method giving quantitative results on the active specie populations with good resolution (< 1 mm) in the whole target - substrate space. The incoherent resonance absorption has been successfully applied to rare gas metastable but is limited to visible resonance line. The coherent absorption using power dye lasers can be extended to non resonant absorption. CARS spectroscopy allows a determination of vibration al distribution of homonuclear molecules as for \( N_2 (X,V) \) presented in this paper. Multiphoton absorption can be investigated to detect species whose excitation thresholds are over 10 eV as for \( H, N, O \) [9].

Emission and coherent absorption spectroscopy can be combined to study the kinetic processes in the reactive plasmas and to extend the application field of the emission spectroscopy to the ground states analysis.
Table 1 - Most intense spectral lines observed in N$_2$ - H$_2$ negative glow discharges with a steel cathode (2.6 Torr, 500 V).

<table>
<thead>
<tr>
<th>Species</th>
<th>Transition</th>
<th>Spectral line (nm)</th>
<th>Excitation threshold (eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>N$_2^+$</td>
<td>B$^2\Sigma_u^+ \rightarrow X^2\Sigma^+$</td>
<td>391.4 ($v' = 0 \rightarrow v'' = 0$)</td>
<td>18.7</td>
</tr>
<tr>
<td>N$_2$</td>
<td>C$^2\Pi_u \rightarrow B^2\Pi_g$</td>
<td>337.1 ($v' = 0 \rightarrow v'' = 0$)</td>
<td>11.1</td>
</tr>
<tr>
<td>N$_2$</td>
<td>B$^3\Pi_g \rightarrow A^3\Pi_u$</td>
<td>380.4 ($v' = 0 \rightarrow v'' = 2$)</td>
<td>7.4</td>
</tr>
<tr>
<td>NH</td>
<td>A$^3\Pi \rightarrow X^3\Sigma^-$</td>
<td>456.0 ($v' = 0 \rightarrow v'' = 0$)</td>
<td>3.7</td>
</tr>
<tr>
<td>H</td>
<td>Balmer series</td>
<td>656.3 (H$_\alpha$)</td>
<td>12.1</td>
</tr>
<tr>
<td>N</td>
<td>2p$^2$3p $\rightarrow$ 2p$^2$3s</td>
<td>746.8</td>
<td>12</td>
</tr>
<tr>
<td>N$^+$</td>
<td>2p3p $\rightarrow$ 2p3s</td>
<td>568.0</td>
<td>20.66</td>
</tr>
<tr>
<td>Fe</td>
<td></td>
<td>561.6</td>
<td>5.54</td>
</tr>
</tbody>
</table>

Table 2 - Cathode surface temperature ($T_K$) and rotational temperature of N$_2^+$($T_{N_2^+}$) for several N$_2$ - H$_2$ mixtures (2.6 Torr, 500 V).

<table>
<thead>
<tr>
<th>H$_2$ (%)</th>
<th>$T_K$(K)</th>
<th>I(mA)</th>
<th>$T_{N_2^+}$(K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>720</td>
<td>62</td>
<td>870</td>
</tr>
<tr>
<td>10</td>
<td>830</td>
<td>90</td>
<td>810</td>
</tr>
<tr>
<td>20</td>
<td>810</td>
<td>90</td>
<td>810</td>
</tr>
<tr>
<td>30</td>
<td>790</td>
<td>85</td>
<td>810</td>
</tr>
<tr>
<td>50</td>
<td>740</td>
<td>75</td>
<td>770</td>
</tr>
<tr>
<td>60</td>
<td>720</td>
<td>70</td>
<td>700</td>
</tr>
<tr>
<td>70</td>
<td>680</td>
<td>60</td>
<td>660</td>
</tr>
<tr>
<td>90</td>
<td>550</td>
<td>38</td>
<td>550</td>
</tr>
</tbody>
</table>
Table 3 - Relevant spectral lines of species in the Ar - N₂ - H₂ - Ti plasma.

<table>
<thead>
<tr>
<th>Species</th>
<th>Transitions</th>
<th>Spectral lines (nm)</th>
<th>Excitation potential $E_s$ (eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ti</td>
<td></td>
<td>521.0</td>
<td>2.43</td>
</tr>
<tr>
<td>Ti⁺</td>
<td></td>
<td>375.9</td>
<td>3.91</td>
</tr>
<tr>
<td>H</td>
<td>H(n=3 → n=2)</td>
<td>656.3</td>
<td>16.6(H₂)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>12.1(H)</td>
</tr>
<tr>
<td>N₂⁺</td>
<td>B $^2S_u^+$, v=0 → X $^2S_g^+$, r=0</td>
<td>391.4</td>
<td>18.7</td>
</tr>
<tr>
<td>N₂</td>
<td>B $^3S_g$, v=6 → A $^3S_u$, v=3</td>
<td>662.3</td>
<td>8.5</td>
</tr>
<tr>
<td>Ar⁺</td>
<td>3p⁵(4p → 4s)</td>
<td>696.5</td>
<td>13.3</td>
</tr>
<tr>
<td>Ar⁺</td>
<td>3p⁴(5s → 4p)</td>
<td>376.5</td>
<td>22.5</td>
</tr>
</tbody>
</table>

References


Vibrational distribution of $N_2(X,V)$ ground state in discharge regime. Th.: calculations with $p = 5$ Torr, $T_O = 500$ K, $E/n_0 = 6 \times 10^{-16}$ Vcm$^2$, $n_e = 10^{11}$ cm$^{-3}$, $\theta_1 = 4400$ K and residence time $t = 10^{-3}$ sec. CARS-Exp.: Experimental results by CARS absorption with $p = 4$ Torr, $T_O = 550$ K, $E/n_0 = 6 \times 10^{-16}$ Vcm$^2$, $n_e = 5 \times 10^{10}$ cm$^{-3}$ and residence time $t = 6 \times 10^{-3}$ sec.

Experimental set-up for $N_2(X,V)$ measurements in discharge and post-discharge regimes. Tube radius $R = 1$ cm, distance between electrodes (nickel cylinder $\varnothing 1$ cm): 40 cm.

CARS detection of $\Delta N_V^{V+1} -$ Anti-Stoke signal of frequency $\nu_3 = 2\nu_1 - \nu_2$.

Vibrational distribution of $N_2(X,V)$ in discharge and post-discharge regimes.

Disch : $p = 2$ Torr, $T_O = 550$ K, $E/n_0 = 5 \times 10^{-16}$ Vcm$^2$, $n_e \sim 5 \times 10^{10}$ cm$^{-3}$, $\theta_1 = 4500$ K, $t = 10^{-2}$ sec.

Post-Disch : time $\Delta t = 10^{-2}$ sec. after discharge (above conditions) $\theta_1 = 3000$ K.

Experimental set-up for steel surface nitriding: A anode, K steel cathode (dia. 3 cm), O hole (dia. 1 cm) and pin-holes (dia. 2 mm), L optical lens, PM photomultiplier, P picoammeter, R recorder, Gas flow 1-5 l/h STP, $p = 1-5$ Torr.

$N_2^+(\lambda = 3914$ Å) intensity in log. scale versus axial distance $Z$ upward the cathode (K). Conditions : $N_2$ 2.6 Torr, 770 V, 160 mA, $Q = 2$ l/h STP, $T_K = 850$ K.

Rotational spectra of R-branch of $N_2^+$, $\lambda = 3914$ Å, $\delta \lambda = 0.2$ Å, 540 V, 90 mA, $N_2$ 1.6 Torr, $H_2$ 0.9 Torr, $T_K = 850$ K.

Energy levels of Ne, $N_2$ and Ar concerned by actinometry measurements.

Intensity ratio of $N_2$, $\lambda = 3371$ Å ($I_{N_2}$) and Ar I 7504 Å ($I_{Ar}$) versus $[N_2]/[Ar]$ density ratio at the negative glow maximum, ($Z = 1$ cm).

Intensity ratio of $N_2^+$, $\lambda = 3914.4$ Å ($I_{N_2^+]})$ and Ne I 5852 Å ($I_{Ne}$) versus $[N_2]/[Ne]$ density ratio at the negative glow maximum. $\omega_{N_2^+(B,0)}$ is the $N_2^+(B,0)$ quenching rate.

Intensity ratio of $N_2$, $\lambda = 3805$ Å ($I_{N_2}$) and Ar I 7504 Å versus $\% N_2$ and $[N_2]/[Ar]$ density ratio for two distances upward the cathode.

Characteristic vibrational temperatures of $N_2^+(B)$ : $\theta_1^B$, $N_2(C)$ : $\theta_2^C$ and $N_2(X)$ : $\theta_1^X$. $T_K$ : cathode temperature.
Fig. 13

Experimental set-up for T_{1}N deposition on steel surface. \( K_{c}(T_{1}) : (T_{1}) \) emitter array of eight \( T_{1} \) rods (rod 1 cm dia.) array 5.2 cm dia., 15 cm height), (S) stainless steel substrates, \( \phi \) loop electrode of 10 cm dia. (rod 1 cm dia.) with a cooling water, (O) pyrex window, (P,M) photomultiplier.

Fig. 14

Ar, N_{2}, H and \( T_{1} \) spectral line intensities versus axial distance \( Z \). H : H_{\lambda} 6563 Å, Ar, 6965 Å, N_{2}, 6623 Å and \( T_{1} \), 5210 Å. Gas mixture 0.82 Ar - 0.10 H_{2} - 0.076 N_{2}.

Electrical parameters: \( V_{E} = -1150 \) V, \( I_{E} = 4.3 \) A, \( V_{A} = +90 \) V, \( I_{A} = 6 \) A, \( V_{S} = -200 \) V, \( I_{S} = 0.55 \) A.
Fig. 5

Fig. 6
Fig. 7

Fig. 8

Fig. 9

Fig. 10
NON-DEBYE PLASMA THEORY
A.S. Kakljugin, G.E. Norman
Moscow Radiothechnlcal Institute of the USSR Academy of Sciences
125083 Moscow, USSR

A review is given of the physical ideas of non-Debye plasma theory based on: a) an analysis of plasma electric-field fluctuation statistical properties [1], b) distinction between free, bound (atomic) and localized states according to the plasma electron energy spectrum [2], c) treatment of localized states by means of the disordered structure theory concepts [2-4], d) use of the Ebeling virial functions for calculation of the pair scattering-states contribution [5]. There is discussed an analytic expression for the correlation energy which takes into account short-wave fluctuations and describes the formation of the short-range order in plasma [1].

A soft gap in non-Debye plasma electron-energy spectrum between free and bound states is shown to exist [2, 3, 6]. The theory is applied to the thermodynamic properties calculations. The results obtained are compared with the density expansion [5], the computer simulation [7] and experiments [8, 9].

1. Plasma model
1.1. Plasma electric fields

Fluctuations of these fields determine both the correlation contribution in non-Debye plasma properties and electron localization. The theory of screening, dielectric properties, free charges correlation, localized electrons motion and fluctuations of the potential must be self-consistent.

The spectral density of plasma electric-field fluctuations $\delta E$ is related with the longitudinal dielectric permittivity,

$$( \delta E, \delta E)_{q} = 4 \pi T [1 - \varepsilon^{-1}(q)]$$

where $T$ is temperature. Therefore one can relate $\varepsilon(q)$ with the polarization energy

$$U_0 = -(T/2)(2 \pi)^{-3} \int dq \ [\varepsilon(q)-1]^2 / \varepsilon(q)$$

as well with Fourier-spectrum of the screened potential $V_{\ast} = 4 \pi q^{-2} \varepsilon^{-1}(q)$. A continuous-medium approximation is used for calculation of $\varepsilon(q)$ for Debye plasma. This approximation is valid for $q < q_0$, where $q_0 l_0 \approx 1$, $l_0$ is the characteristic length of the physical infinitesimal volume. The value of $l_0$ must be smaller than the length of the fluctuations which should be taken into account and greater than that to be smoothed.

There are several types of charge density fluctuations in plasma. Their corresponding lengths are: the mean distance $a \approx n^{-1/3}$, the screening
radius \( r_c = \varpi^{-1} \) and the path of kinetic relaxation \( l_i \), here \( n_n e = n_1 \) is mean electron (ion) density, \( \varpi^2 = 8 \pi n_e e^2 \beta \), \( \beta = 1/kT \). Since \( \varepsilon(q) = 1 + x^2/q^2 \) and \( x < x^{-1} \ll 1 \) in Debye plasma, in this case only \( q \approx x \) contribute in \( V_q^+(\varphi) \) and \( U_0 \) and one can define \( l \) in such a way that \( x \ll l \ll x^{-1} \) or \( x \ll q_0 \ll n^{1/3} \). Thus, the continuous-medium approximation for \( \varepsilon(q) \) is correct and all long-wave fluctuation contributions are taken into account.

With increasing non-ideality parameter \( \gamma = \beta e^2 (n_e + n_i)^{1/3} \) or decreasing Debye number \( N_D \), non-Debye conditions is reached when \( r_o < a \) or \( n^{1/3} < x \). The fluctuations for this case have been investigated in [1]. For small \( q \ll q_0 \approx n^{1/3} \) continuous-medium approximation is valid. These wave vectors correspond to distances exceeding \( n^{-1/3} \), i.e. the interactions for such range are weak in comparison with the thermal energy. One can conclude that correlations in this range of \( q \) are still of Debye character (curve section \( abc \) in Fig. 1). At fluctuations with \( q > q_0 \) one should consider point charges. From the Kotelnikov theorem applied to this case it follows that the fluctuation amplitudes are negligible and \( \varepsilon(q) = 1 \) for \( q > n^{1/3} \) (section \( abcd \) in Fig. 2). The vertical jump \( AB \) at \( q = q_0 \) is a rough approximation for the intermediate section \( bc \).
The estimation of $q_0$ may be made by analogy with that of the Debye temperature for the lattice heat capacity which gives $q_0 = 2(n_e + n_i)^{1/3}$.

Moreover, a correlation function $K(r)$ for localized electrons which do not contribute in screening were obtained for Debye

$$K(r) = T^2(8\pi_0)^{-1} \exp(-2r)(1 - x r/3)$$

and non-Debye plasmas

$$K(r) = 4x e^2 m_r(x^{-2} \sin x + x^{-1} \cos x + s(x)),$$

where $x = q_0 r$. The evolution of $K(r)$ with decreasing $N$ are shown in Fig. 2.

1.2. Electron localization

There exist electronic states which are intermediate between discrete and continuous energy spectra of plasma. These states can be considered neither as belonging to a certain ion, nor as states with infinite motion. They are many-electron and many-ion localized states and should be treated in the frame of quantum theory of disordered structures [2-4]. The localized states do not contribute to the ideal plasma properties but become very important in non-Debye plasma.

For the following consideration it is necessary to estimate two values: the overlap integral $J$ which determines the probability of electron tunneling between neighboring ions and energy interval $\Delta$ of the nearest states which can form a cluster. The potential relief fluctuations $U$ were considered in section 1.1.

The overlap integral for the states with equal principal quantum numbers $s$ and other zero quantum numbers can be used to estimate $J$ [2]. A calculation yields the function $J_s(R)$ where $R$ is the distance between the ions. Replacing the variable $s$ by $R_s = -m e^2/h^2 s^2$ and substituting $R = n^{-1/3}$ we get the function $J(R_s)$ for different electron densities.

For an isolated atom $\Delta = \Delta_1 = m e^2/h^2 s^3$. One can use $\Delta = \Delta_1$ for plasma when $\Delta_1 > \Delta_F = 3 e a_0^2 s^2$ (Stark splitting). Here $a_0$ is the Bohr radius, $F$ is a mean plasma microfield. $\Delta = \Delta_2 = \Delta_1/s$, if $\Delta_1 < \Delta_F$ [2]. The transition from $\Delta = \Delta_1$ to $\Delta = \Delta_2$ was approximated by a vertical jump at Inglis-Teller value ($\Delta_1 = \Delta_F$).

The values of $J$ and $U$ are compared for Anderson localization model and $U$ for Lifshitz one. The values of $J(R_s)$, $U$ and $\Delta_F$ depend on the plasma density differently and the three cases can be distinguished.

The intersection point of $J(R_s)$ with $\Delta (R_s)$ for the ideal plasma $-\varepsilon_0$ is situated in the region where $\Delta = \Delta_2$ (Fig. 3a). Both the Anderson $J < U$ and Lifshitz $4J < \Delta$ localization criteria hold for $R_s < -\varepsilon_0$ and electron spectrum consists of atomic states. The Inglis-Teller merging exists as well. The electrons are localized according to Anderson but become delocalized according to Lifshitz, for $R_s > -\varepsilon_0$. 
where $\Delta/4 < j < U$. An electron state transforms from one-ion to many-ion one but its wave function is space-confined by range of the long-wave micropotential fluctuations. The electron is localized in this fluctuation according to Anderson and is delocalized in the fluctuation volume according to Lifshitz.

For non-Debye plasma (Fig. 3b) the intersection point of $\mathcal{J}(E)$ with $\Delta(E)$ is shifted to the vertical section of $\Delta(E)$ and the value of $E_0 = E_{IT}$ is defined by the Inglis-Teller formula. This case differs
from the previous one that the Inglis-Teller merging vanishes. Discrete non-overlapping atomic levels lie at $E_S < - \varepsilon_0$. The states of electrons which are localized in long-wave micro-potential fluctuations exist over the region $E_S > - \varepsilon_0$ where $U > J > \Delta/4$.

The case in Fig. 3a transforms to the case in Fig. 3b at densities $n_e + n_i \approx 0.5 \times 10^{18} \text{ cm}^{-3}$. The jump from $\Delta_1$ to $\Delta_2$ is only an approximation and virtually an intermediate range exists.

At $n_e + n_i > 0.5 \times 10^{21} \text{ cm}^{-3}$ the intersection of $\mathcal{H}(E_S)$ with $\Delta(E_S)$ occurs on the line $\Delta = \Delta_1$ (Fig. 3c). For $E_S > - \varepsilon_0$ delocalization takes place both according to Anderson and to Lifshitz. This energy range is a precursor of a valence or liquid-metal conductance bands.

The motion equation for localized electrons in non-Debye plasma may be approximately reduced to the equation for brownian anharmonic oscillator [4]. Characteristic frequencies of localized electrons oscillations turn out to be close to the plasma frequency and its overtones. It may be shown that no more than one electron is simultaneously localized in each long-wave micro-potential fluctuation.

### 1.3. Electron energy spectrum

Now the estimation of the electron density of states in the whole range of energies will be presented for the case of non-Debye plasma which corresponds to Fig. 3b. A simple model may be proposed to obtain effective density of states which involves both one-particle and many-particle states [3]. The plasma micro-potential relief is approximated by the set of wide shallow randomly distributed potential wells. The well depth is equal to $\varepsilon_0$ and its diameter is assumed to be of the same order of magnitude as the screening radius. An electron-electron interaction is approximated by a constant $2 \varepsilon_b$ if the distance between electrons is less than diameter of the well either by zero if that distance exceeds the diameter. The average spacing between the well edges is of the same order as the well diameter and one can assume that electrons localized in different wells do not interact. Therefore only those localized electrons which occupy the same well contribute to the interaction energy which is equal to $2 \varepsilon_b$ for each such pair.

A calculation of the density of states $G(E)$ is fairly sophisticated even for the described simple model and have been performed in [3]. The results are shown in Fig. 4. The solid curve represents the density of state calculated in single-electron approximation in the whole energy range from atomic states ($E < - \varepsilon_0$) to free states ($E > 0$). It was proved [3] that $dG(E)/dE$ in single-electron approximation for any micro-potential relief is always positive. Only many-electron effects result in the formation of the energy gap. The dashed curve corresponds to the more realistic case $\varepsilon_0 < \varepsilon_b$ (cf. section 1.2) and point-dashed curve to $\varepsilon_b < \varepsilon_0 < 2 \varepsilon_b$. The single-electron approximation remains valid for $E < - \varepsilon_0$ and $E > 0$. 


One may state that it is impossible to give a unique definition of \( G(\varepsilon) \) in the case which includes electron-electron interactions since presentation of many-electron interaction energy in the form of the sum of single-electron energies is rather arbitrary. However it was proved that the existence of an energy gap in \( G(\varepsilon) \) does not depend on this arbitrariness. The form of this expression of interaction energy only affects the gap profile.

The existence of this gap is due to the fact that because of the repulsion energy \( 2\varepsilon_p \), localization of the first electron prevents or even forbids the localization of the second electron etc. The repulsion yields the decrease in the number of states in the well compared with the single-electron case. Since in the range of the gap \( G(\varepsilon) > 0 \) this gap was called a soft one by analogy with a similar effect in doped semiconductors.

2. Thermodynamics

2.1. Free energy

The concept of the gap in plasma electron energy spectrum allows to propose an unambiguous cutoff procedure to avoid divergency of the atomic partition function and to write the free energy \( F \) as a sum of contributions from different parts of electron energy spectrum

\[
\beta V^{-1} = f_a + f_{ei} + f_q + f_c + f_l.
\]

Here \( V \) is volume, \( f_a \) is contribution from the ideal gas of atoms in the ground and excited states. Free states contribute as ideal electron-ion gas \( (f_{ei}) \), quantum effects in pair scattering \( (f_q) \), and correlations \( (f_c) \). The localized electron contribution is \( f_l \).

The number of electrons and ions which are bound in atoms coincide. But there are no localized ions because the period of ion plasma oscillations exceeds the time of change in the ion energy. Therefore all ions which are not bound in atoms should be referred to as free ions. But the plasma model with unequal numbers of free electrons and ions is inconvenient. As the first approximation at \( \varepsilon_p \)'s calculation we shall set these numbers equal \( (n_e = n_i = n) \).

Let us proceed to calculation of various \( f \) in eq. (2). Expressions for \( f_a \) and \( f_{ei} \) are conventional. Note that atomic partition function \( \sigma \) in

\[
f_a = n_a \left\{ \ln \left[ \sigma^{-1} n_a (2\pi \hbar^2 /\mu)^{3/2} \right] - 1 \right\}
\]

is cut off by the energy gap \(-\varepsilon_p\). Here \( n_a \) and \( \mu \) are the atom density and mass. The expressions for \( f_q \) may be simplified for the low-temperature plasma if Born parameter \( \xi = (2m\beta)^{1/2} e^2/\hbar \) essentially exceeds unity [5],

\[
f_q = \pi^{3/2} \xi^{-1} \beta^{3/2} e^6 \pi^2 \]

2.2. Correlation contribution

From the dielectric permittivity discussed above follows the expression for the non-Debye plasma correlation energy [1]

\[ U_c = -\pi^2 (2\pi)^{-2} \arctg \left[ 2(n_e + n_1) \frac{1}{3} \frac{1}{\varepsilon - 1} \right]. \]

At \( n^{1/3} \gg \varepsilon \) \( \arctg \) is equal to \( \pi/2 \) and we obtain for \( U_c \) the Debye limit. At \( n^{1/3} \ll \varepsilon \) or \( \gamma \ll 1 \) one can replace \( \arctg \varepsilon \) by \( \varepsilon \) and obtain an expression

\[ U_c \approx -(2\pi) \varepsilon^2 (n_e + n_1)^{4/3} \]

which formally coincides with the Madelung energy and does not depend on temperature.

For the evaluation of \( f_c \) one should integrate the thermodynamic equation \( U_c \, d\beta = d(\beta f_c) \) from 0 to \( \beta \). As correlations vanish at \( T \to \infty \) the final result is (\( \gamma = \pi \varepsilon \))

\[ f_c = \frac{2}{3\pi^2} (n_e + n_1) \left[ \ln \left( 1 + \gamma - 2\gamma^3/2 \right) \right]. \] (5)

2.3. Localized electrons contribution

In order to exclude from consideration the density of localized electrons a following derivation may be developed. First, the contribution of localized electrons in thermodynamic potential \( \Omega_1 \) is determined in the frame of grand canonical ensemble. Then we put \( f_1 = \Omega_1 / TV \) and replace the electron chemical potential in the right side by \( -\partial F / \partial N_e \).

The obtained integral equation is reduced to the closed definition of \( f_1 \) after neglecting the term \( \partial f_1 / \partial N_e \). For conditions considered this term can be shown to be the smallest one in \( \partial F / \partial N_e \).

The derivation of the grand canonical partition function of localized electrons is described elsewhere [4]. The main assumptions are that the number of localized electrons in one well does not exceed unity and that the potential relief consists only of the wells and the humps, i.e. that there are neither small wells inside the humps nor small humps inside the wells. The calculations were carried out for a system of identical wells, for a mixture of wells of various depth and for the wells with continuous distribution of depth values. The final result is given by the equation

\[ f_1 = -3\pi^{-4} (n_e + n_1) \ln \left\{ 1 + 2\pi \frac{1}{3} \frac{1}{2} \frac{3}{2} \varepsilon \right\} \]

\[ \times \exp \left[ \left( \frac{3}{2} \frac{1}{2} \frac{3}{2} \varepsilon \right) \right] + (2/3) \pi^2 \left( \ln \left( 1 + \gamma - y \right) - 3y^2 \arctg \gamma^{1/2} \right) \left[ 1 - \exp \left( -1.22 \frac{4}{5} \frac{1}{2} \right) \right] \] .

(6)
2.4. Energy, pressure, ionization equilibrium

Now we can insert (3)-(6) into (2), calculate derivatives of the resulting expression with respect to temperature, volume and numbers of particles and obtain formulas for energy, pressure $P$ and equation of the ionization equilibrium. The results of calculations for $P$, interaction energy $\Delta E$ and lowering of the ionization potential $\Delta I$ are shown in Fig. 5

![Graphs showing energy, pressure, and ionization potential](image)

for two values of the Born parameter: solid lines $1 - \xi = 6$ and $2 - \xi = 4$. The dashed line represents numerical results for the effective pair potential model computed by Monte Carlo method [7]. The model incorporates the pair quantum effects and many particle Coulomb interactions.

The Monte Carlo method allows us a direct calculation of screening, density fluctuations and classical localization. The gap effect was taken into account via thermodynamic perturbation theory, and the temperature dependence was neglected. Both approach [7] and the present theory consider the same effects. However, in approach [7] the plasma model is approximate while the computations are accurate and the results are numerical. In the present theory the plasma model is more refined, the results are analytical while the calculations are approximate. A satisfactory agreement between the solid and dashed lines corroborates the reliability
of the results.

Let us consider P-V-T and P-V-H equations of state, where H is enthalpy. The results for caesium plasma are shown in Fig. 6a for V=200 cm$^3$/g and in Fig. 6b for 100 cm$^3$/g. Here the solid line was calculated using the present theory and the shaded region represents experimental results [8]. A comparison of experimental (exp) [9] and theoretical (th) results for argon plasma in two state points are given below.

<table>
<thead>
<tr>
<th>P (kbar)</th>
<th>T (K)</th>
<th>V, cm$^3$/g</th>
<th>H, 10$^{10}$ erg/g</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.63</td>
<td>23500</td>
<td>39.7±4</td>
<td>37</td>
</tr>
<tr>
<td>5.00</td>
<td>21000</td>
<td>10.5±1</td>
<td>10</td>
</tr>
</tbody>
</table>

The agreement both for Cs and Ar is satisfactory and has been achieved through a cut-off of atomic partition function at the lowest gap energy and the effective repulsion in the expressions for P, ΔH and ΔI.

2.5. Comparison with density expansion

A rigorous expression is an expansion with respect to the total densities of charged particles $n_{oe} = n_{oi} = n_0$ [5]

$$\rho FV^{-1} = n_{ei}(n_0) - n_{ce}n_{oi} K(T) - \sum_{a,b} B_{ab} n_a n_b$$

$$- \frac{3}{2} K^3(n_0) / 12\pi + \alpha(n_0)^{5/2} \ln n_0.$$  (7)

Here the first, second and third terms are an ideal-gas, a bound-state ($K(T) = 8 \pi^{3/2} e^6 \rho^{3/2} \sigma / \xi^3$) and scattering - states contributions, respectively, and the fourth one is the Debye correlation energy. The densities $n_0$ incorporate both free and bound charges, and therefore the expansion (7) is only valid for a fully ionized plasma. To extend the range of validity of eq. (7) it is necessary to transform it into expansion with respect to $n_e$, $n_i$ and $n_a$. A correct procedure for this purpose has not yet been known.

The density expansion for pressure contains the contribution from atoms in an explicit form. For this reason it seems to be more preferable than eq. (7). However, when starting from pressure expansion it is necessary to reconstruct $F$ by integration with respect to $V$. Because of a change in the ionization degree with changing $V$ a procedure for such integration has not yet been developed as well.

A fugacity expansion yields a more realistic picture of atoms formation. But only those terms of this expansion have been calculated which are sufficient for a description of the low-density limit. The fugacity expansion terms have no evident physical meaning. This shortcoming makes
impossible the extension of the fugacity polynomial to higher densities.

Thus, the rigorous approaches yield no expressions which are valid for a partially ionized non-Debye plasma. There exist only semiphenomenological approaches; see [1-5, 7, 10] and references herein.

Leaving alone the problem of a rigorous definition of bound states let us compare eq. (2) with eq. (7) in the limit of full ionization and $N^2 \rightarrow \infty$. The ideal-gas contributions coincide. A cut-off due to $E$ becomes exponentially unessential, and $\sigma$ in eq. (3) transforms in Plank-Larkin partition function from eq. (7). The $f_q$ contributions are the same both in eq. (7) and in eq. (4). Thus, the contributions sums $f_a + f_{ei} + f_q$ are identical in both eqs. (2) and (7) in the limit considered. An expansion of $f_o$ with respect to $\gamma$ is given by

$$f_o = -(h/3)\pi^{1/2} \gamma^{3/2} - 1.5 \gamma^2 + \ldots$$

The first term is a Debye one from eq. (7). The second term is of the $n^{5/3}$ order and is lacking in eq. (7). The point is that eq. (7) only includes long-wave correlation, renormalization of density of charged particles from $n_{e0}$ to $n_e$ and quantum corrections. The eq. (5) takes into account the short-wave correlations as well, and in the limit $N^2 \rightarrow \infty$ it should contain terms defined by the ratio of characteristic lengths of long-range and short-range correlations $\zeta / n^{1/3} \sim n^{1/6}$. The lack in such terms is a shortcoming of eq. (7). The $n^{3/2} + \kappa/6$ terms reveal formation of the short-order in non-Debye plasma.

The eq. (6) for $f_1$ is only valid for a definite range of non-zero values of $\gamma$ [2,4]. Therefore, the expansion of $f_1$ with respect to $\gamma$ is deprived of any sense, contrary to eq. (8). Another expression describes the $f_1$ contribution in the limit $\gamma \rightarrow 0$. Not writing it down we only mention that $f_1$ decreases with $\gamma$ faster than any exponent of $n$ and, consequently, it is not contained in eq. (7).

From the above discussion it follows that $n^{3/2}$ and $n^2$ contributions coincide in eqs. (7) and (2). The $n^{5/2}$ ($\ln n + c$) contributions have not been written down explicitly in eq. (7), cf. [5]. From their physical meaning one can see only a partial correspondence of these terms with $f_q$, $f_o$, or $f_1$. However, the $n^{5/2}$ contribution itself becomes significant only for those densities where higher terms must also be considered.

Finally, we summarize the main results of this section. In the semiphenomenological approach developed in this paper the bound, localized and free states are introduced from an analysis of the electron energy spectrum. The qualitative approach is also a basis to define and calculate various contributions to the free energy. A loss in rigour is compensated by resulting enlargement in the validity range of eq. (2)-(6) compared to any expression of the type (7). The resulting gain comes from the ab initio definition of bound, free and localized states, partial summation of higher
terms with respect to n when calculating $f_0$, including $n^{3/2} + k/6$ contributions, cutting off $\varepsilon$ at $-\varepsilon_0$, consideration the electron density of states, calculation of the localized electrons contribution. One can also add in eq. (2) terms of charge-neutral, neutral-neutral and other interactions.

3. Other properties

The existence of the soft gap in electron spectrum not only effects the thermodynamic properties. The effect is especially significant for optical properties of non-Debye plasma. Since this question was discussed in [6, 11-14, 2] we report here only the main results: a) the spectral lines series does not merge smoothly in the corresponding photocontinuum but terminates, and there is a transparency window between the lines and the photocontinuum; b) photocontinuums corresponding to the levels which are in the range of the gap vanish and are not compensated by an increase in free-free transitions; the resulting bleaching (brightening) being of the order of ten and more in certain spectral intervals; c) the integral characteristics, such as Rosseland mean and total energy emitted, change by several times. These conclusions are corroborated by observations of plasmas for various chemical species. At the present time considerable efforts are directed to hydrogen for which generation of non-Debye plasma is difficult and the effects discussed have not yet been observed.

The effect of the gap on electrical conductivity was considered in [15].

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PULSE BREAKDOWN IN UNIFORM ELECTRIC FIELDS

E. E. Kunhardt
Weber Research Institute
Polytechnic University
Farmingdale, New York 11735

I. Introduction

Over the last few decades there has been and continues to be an interest in the phenomenon of the growth of ionization of a gas in a uniform electric field, and the subsequent breakdown of the insulating properties of the gas. Early investigations of this phenomenon were conducted primarily, in non-attaching gases at low pressures, using DC electric fields. Townsend [1] formulated a criterion for the breakdown of the insulating properties of the gas under such conditions. This criterion can be expressed (in non-attaching gases) as follows [2]:

\[(\omega/\alpha) (e^{ad-1}) = 1\]  

(1)

where \(\omega\) is the generalized coefficient for secondary electron production, \(\alpha\) is the Townsend primary ionization coefficient, and \(d\) is the electrode separation. Equation (1) implies that breakdown occurs when the secondary processes, resulting from the traversal across the gap of a primary electron and the corresponding \(e^{ad-1}\) secondary electrons, are sufficient to regenerate the primary electron at the cathode. When this occurs, the current becomes self-sustaining.

In the derivation of Eq. (1), only "single-particle" processes were considered. That is, processes that involve the interaction of a primary particle (such as an electron, an ion, or a photon,) and a target (a ground state neutral, or the cathode). "Collective" processes (those involving the interaction of a number of particles, such as space-charge effects) have been assumed to be unimportant.

Using the physical interpretation of Eq. (1) to explain the breakdown of a gap under pulsed conditions, it would be concluded that the breakdown delay time, from the application of the pulse to the gap (and taking the statistical delay to be zero), is proportional to the time constant of the relevant cathode-related secondary processes. For a 1 cm gap, at atmospheric pressure, these time constants are of the order of \(10^{-5}-10^{-7}\) sec. In early investigations of the breakdown of atmospheric pressure, 1 cm gaps, subjected to voltage pulses with amplitude above the DC breakdown voltage, \(V_s\), breakdown delay times of the order of \(10^{-7}\) sec or less were observed [3]. These values were found to be too short to be associated with any secondary cathode processes. As a consequence of this and other observations (such as the apparent independence, at high pressures, of \(V_s\) on the cathode material [4], and the filamentary nature of the ionized channel [5]), the validity of the Townsend theory (the physics associated with Eq. (1)) under these conditions (high pressure gas, and/or applied voltages above \(V_s\)) was questioned [4-6].

A number of models were introduced to explain breakdown under pulsed, overvolted (applied voltages above \(V_s\)), high pressure conditions (see next section). It should be noted that some of the discrepancies between the breakdown experiments and the Townsend theory were later resolved [2]. Moreover, the domain of validity of the various models has changed as better understanding of the processes has been achieved [7-10]. For a review of the historical development of breakdown physics, the reader is referred to [11] and [12]. A consequence of this historical development is that DC and pulsed breakdown in gases have been treated essentially as separate phenomena.

In the discussion of pulsed breakdown, the approach taken in this paper is to emphasize the basic breakdown processes (single particle and collective) and their contribution to the development of breakdown. In general, these processes are common to both DC and pulsed breakdown, and their roles in both cases are very similar. From this perspective, no difference can be made between DC or pulsed breakdown. However, since our interest is in breakdown under pulsed conditions, examples of how the basic processes contribute to the development of breakdown will be restricted to such conditions.
A discussion of the various models of pulsed breakdown is given in the next section. In these models, a number of processes that are fundamental to breakdown in general (in addition to those introduced in the Townsend theory) have been introduced. Because of its relative importance and since it has been a source of controversy, one of these processes (the streamer) is discussed in more detail in Section III. In Section IV, the various collective and single-particle processes are used as a basis for discussing a number of generic pulsed breakdown situations, that includes the nano-second regime. Concluding remarks are given in Section V.

II. Breakdown Under Pulsed Conditions

A number of models have been proposed to explain the breakdown of a gap under pulsed conditions. The more significant features contained in these models are the introduction of: a) collective processes (for example, space-charge effects, and streamers) [4-6], b) circuit effects [8,9], c) electron kinetic effects [10,13], and d) cathode flares [14]. These models are discussed below.

a) Streamer Models

The streamer model was proposed, independently, by Meek [6], Raether [5], and Loeb [1]. Actually, these models were initially intended to explain breakdown (D.C. or pulsed) of high pressure (atmosphere) gaps.

Meck's objectives [6] were to obtain a modified form of Eq. (1) which would be compatible with the experimental observations at high values of pd. Raether and Loeb directed their attention to the observed short time lags and to the filamentary character of the breakdown.

In reality, theirs [4-6] was more than one model [15]. However, the fundamental concept used by these investigators in the development of their models was common to all. This concept was that at a certain stage in the development of a single electron avalanche, the space-charge field and photolionization of the gas in the interelectrode space became the more important mechanisms that influence the further development of the avalanche, and indirectly, the breakdown of the gap.

The main difference between the models lay in the stage at which these mechanisms became operative. For Meek and Loeb, this occurred when a single avalanche reached the anode, whereas for Raether, it occurred when the avalanche was somewhere in the middle of the gap. From this point on, the models invoked different arguments to explain the breakdown of the gap. However, they all used photolionization of the gas surrounding the avalanche as the principal mechanism.

According to Raether [5], on its way to the anode, the avalanche reaches a critical stage (determined by the number of electrons in the avalanche) such that the space-charge field became significant (compared to the external field). The secondary electrons generated just ahead of the avalanche by photolionizing radiation produced in the avalanche, are located in a region of high field due to the enhancement (above the applied field) caused by the avalanche space-charge field. In this region of high field, these photo-electrons multiply efficiently, generating a space-charge cloud which rapidly grows to the dimension of the parent cloud, but at a position closer to the anode. The progress is repeated continuously to the anode boundary. This progression, due to the space-charge field and photolionization, is called a streamer. This process is discussed in more detail in the next section. Once the anode is reached, Raether argues, a similar process begins to occur at the cathode end of the parent avalanche. The photo-electrons generated in this region are accelerated towards the avalanche, extending the ion sheath of the parent avalanche toward the cathode. Loeb and Meek only considered the cathode directed streamer. That is, they assumed that the avalanche becomes critical just in front of the anode.

What determines the critical stage of an avalanche? Meek arbitrarily chose it to be when the radial field due to the avalanche is equal to the external field. Raether, on the other hand, defined the critical stage from experimental observations of avalanche development in a cloud chamber. He observed that streamers developed when

$$\alpha x_{ct} \sim 20$$  (2)

where $x_{ct}$ is the average position of the avalanche when it becomes critical. Using Eq. (2), he calculated a formative time for breakdown. He argued that since the streamer propagation velocity is high (of the order of speed of light), the ionizing fronts quickly reach the electrodes. Subse-
quently, the weakly ionized, filamentary, channel that bridges the gap quickly becomes a highly conducting channel leading to the collapse of the voltage. In this case, the formative time is basically the time, \( t_{cr} \), it takes an avalanche, started by a single electron, to become critical. This time is given by [5]

\[
 t_{cr} \sim \frac{x_{cr}}{w_e} \sim 20/\alpha w_e
\]

where \( w_e \) is the electron drift velocity in the applied field. From Eq. (3), it is found that the number of electrons in an avalanche when it becomes critical is:

\[
 N_{cr} = e^{\alpha x_e}
\]

and using Eq. (2)

\[
 N_{cr} \sim 10^8
\]

Equations (3) and (4) assume that the space charge field has no effect on the propagation and growth of the avalanche (see next section). Note that this number is only of significance at atmosphere pressures, where the corresponding charged-particle density is \( \sim 10^{14} \text{ cm}^{-3} \). Both, total number and density of charged particles, determine the condition for which space-charge effects are significant.

It is interesting that the criterion for the streamer onset, Eq. (2), is consistent with the breakdown criterion introduced by Schumann in 1923 [10]. He used an integral form of Eq. (2) to define a condition for the breakdown of the gap. That is,

\[
 \int_0^x \alpha(x) dx \sim 10^{18}...20
\]

This expression is also applicable when the field is non-uniform.

Further support for the streamer model of breakdown, as the above models have come to be known, came from experiments by Fletcher [17]. The time lags he measured seem to agree with those obtained from Eq. (3). However, the validity of using this equation for his experimental conditions has been questioned by many [8,0]. It appears that the initial number of electrons in the gap was of the order of \( 10^4 \), a number too large to be consistent with Eq. (3) which requires a single initial electron.

Similar types of experiments were carried out by Felsenthal and Proud [18]. They explained the results by using a modified pulsed-microwave breakdown model. The modification in essence returned them to Eq. (3) in the frame of a moving avalanche.

**b) Other Models of Pulsed Breakdown**

A dynamic Townsend theory was developed by Davidson [19]. It was a time dependent description of breakdown taking cathode processes into account, but neglecting collective processes. This formulation was used to describe the time evolution of breakdown for voltages near and slightly above the D.C. breakdown voltage determined from Eq. (1). Although this analysis is highly mathematical, it provides a complete, self-consistent, description of breakdown in this situation. Since the fundamental physics has already been discussed in Section 2, no further details will be given.

In reference to Fletcher's experiments, Dickey [8] pointed out that a voltage collapse may also have been due to space-charge motion in the gap and not to the bridging of the gap by a conducting channel. This motion induced a current in the external circuit, and thus, a voltage is generated at the load. Since the supply voltage was constant, the voltage across the gap collapsed. Representing the motion of the charge carriers by an equivalent current source, and using a transmission line model for the gap region, Dickey obtained an expression for the voltage across the gap as a function of time. For Fletcher's experimental conditions, the values for the breakdown delay time obtained from this expression were in agreement with the experimental results.

Using similar ideas as Dickey, Mesyats et al. [9,20] developed a model for pulsed breakdown which recognized the importance of the initial number of electrons present in the gap when the voltage was applied. They differentiated between a single-electron initiation and multiple-electron ini-
Avalanche. In both cases, the fundamental concept is that of an avalanche chain. That is, an avalanche is assumed to stop growing due to the decrease in total field brought about by its own space charge field. Then, new avalanche develops from the head of the old one which undergoes the same evolution. This way a chain of avalanches is formed. In the case of multiple-electron initiation, the current generated by the many chains is determined to cause breakdown as in Dickey's analysis. In the case of single-electron initiation, a chain is said to cross the gap forming a highly resistive channel across the gap. However, due to the photo-electric effect at the cathode, additional chains are formed, thus reaching the same condition as in multiple electron initiation. The time delay to breakdown (neglecting the statistical delay) is thus longer for single electron initiation than for multiple electron initiation. They also observed this result experimentally [9]. It should be noted that although, in general, the growth of an avalanche as a whole decreases with time due to space-charge effects, the electrons at the front of the avalanche are continuously growing. Thus, the evolution of the weakly conducting filament across the gap occurs in a macroscopically continuous fashion (see next two sections).

Kunhardt and Byszewski [10] introduced a model for highly overvolted breakdown. In this model, the energy distribution function for electrons in the advancing avalanche is assumed to have two components: fast electrons and slow (thermal) electrons. The fast electrons can run away from the avalanche. This happens because the effective retarding force on an electron moving through a neutral gas decreases with increasing velocity in the case of electrons possessing sufficiently high energy (i.e., $u > 3 - 5 \epsilon_i$, where $\epsilon_i$ is the ionization energy). The energy threshold for these runaway electrons is determined by the electric field strength and the pressure and properties of the gas.

Once these fast electrons leave the avalanche, most of them no longer meet the runaway condition and begin to slow down. That is, they are "thermalized." This is due to the fact that they enter a region of decreasing field ahead of the avalanche, and the energy they gain from the field along their trajectory is not enough to overcome the losses. The trapping distance of these electrons is a function of their initial energy. Thus, these seed electrons, which are continuously emitted from the avalanche, multiply at various distances from the parent avalanche, rapidly extending the avalanche space charge towards the anode. On the cathode side, the photoelectric effect is assumed to be the primary mechanism for generating secondary electrons which are subsequently accelerated towards the high-field region of the parent avalanche.

As the amplitude of the applied field is increased, eventually a regime is reached in which runaway electrons are abundant [13]. The spatial structure of the breakdown channel in this case is diffuse. Experimental evidence of these runaway electrons exist [21,22]. These experiments, however, were not time resolved so that it is not known when during the breakdown runaway electrons were produced. Note that, when the (average) field at the cathode is $> 10^5 \text{V/m}$, the formation of cathode flares [10] may have to be considered in the description of gap breakdown. These flares are formed by the explosive heating of cathode micro-protrusions due to field emission currents, and the subsequent formation of a micro-plasma at the cathode surface. This phenomenon can be taken into account by re-defining the initial state of the gap (see next section).

### III. Streamers

In the previous section, the concept of streamers has been introduced. This phenomenon has received considerable attention (subsequent to Rarther’s work), both experimental [23-27], theoretical [27-31,40,41], and computational [32,33,54] and has been the subject of much controversy. The central issues have been: a) the existence of ionizing radiation, and b) whether the luminous fronts that have been observed to propagate away from the evolving avalanche, at speeds higher than the local mean-velocity at the avalanche boundaries, are streamers or luminous phase effects [2]. Due to space limitations, the various experimental and theoretical observations that have been made regarding streamers cannot be presented here. The reader is referred to the above references and to the review article [31] (and references therein) for further details.

A number of mechanisms have been proposed to explain the dynamics of streamers (anode and cathode directed). These mechanisms fall into two general categories: 1) those associated with the distribution in velocity of the electrons in the avalanche (electron drift [31], electron pressure [3b], and high energy electrons [10]), and 2) photoionization [4-6].
Since the validity of macroscopic models to describe the dynamics of streamers is always in question (see Section V), we carried out a kinetic investigation of streamer formation and propagation in an attempt to clarify their properties. That is, we have ascertained the role that the electron velocity distribution and photolization play in the formation and propagation of streamers and the properties of streamers shortly after formation; namely, speed of propagation, diameter, and density contours. The results can also be used to determine the range of application of a moment description (i.e., fluid equations). The existence of the photo-electrons ahead of the avalanche produced by photolizing radiation from the avalanche has been a subject of debate. However, both the presence of impurities, and of electrons with sufficient energy to excite energetic states that can produce photolizing radiation [36] cannot, at present, be disregarded. In our simulations for nitrogen, we have used the data of Penny and Hummert [37] to determine the production of ionizing radiation and photo-electrons in nitrogen (with impurities). It should be emphasized that this data has been taken as a model for these processes.

We have used a Monte Carlo technique to simulate the dynamics of the electron population in 6-dimensional phase-space, taking into account the space-charge electric field, self-consistently. For the time scales of interest (nanoseconds), the ions can be considered immobile. The space-charge electric field is computed assuming cylindrical symmetry. Details of the computational techniques have been presented elsewhere [38]. The physical problem investigated can be described as follows: consider two infinite parallel plate electrodes with nitrogen, at a density N, filling the inter-electrode space. A pulse of electrons is released from a small area of the cathode. Simultaneously, a voltage greater than the DC breakdown voltage is applied to the electrodes. The results to be briefly discussed here are from simulations carried out at E/N of 300 Td and 1000 Td, with nitrogen densities of 2.45x10^{18} cm^{-3}, and a single initial electron. This is the classical, single electron avalanche breakdown situation. The two values of E/N were chosen to illustrate the role of high energy electrons in the propagation of streamers. The evolution of the velocity distribution, rate coefficients, and transport parameters of the avalanche, for the time prior to the critical stage, are discussed elsewhere [39].

The critical stages of the avalanche and the development of anode and cathode directed streamers are shown in Fig. 1 for E/N = 300 Td. In this figure, contours of constant electron density are shown in r-z space. In the early stages (Fig. 1a), the avalanche is nearly symmetric and disc-shaped (since the longitudinal diffusion coefficient, D_L, is smaller than the transverse, D_T). As the total field at the head of the avalanche increases (due to the space-charge field), the photo-electrons produced ahead of the avalanche rapidly multiply and extend the boundary of the avalanche towards the anode (see Fig. 1b). It is interesting to note that when the anode directed streamer becomes apparent, the total number of electrons in the avalanche is 1.1x10^8, remarkably close to Raether's experimental results [5]. Moreover, at high pressures, the growth of the avalanche is nearly exponential up to the critical stage, even though the space-charge field significantly alters the uniform field conditions. The decrease in the ionization growth of an avalanche resulting from the reduction in the total field inside the avalanche is partially compensated by the concomitant increase at the front of the avalanche where the field is high. This compensation increases with pressure, so that at atmospheric pressures it is nearly complete.

A short time later (Fig. 1c), photo-electrons created at the rear of the avalanche initiate the cathode directed streamer. The propagation of the streamer is due to the rapid growth of the secondary (photo-electron) avalanches which serve to extend the density front. The concentration of the electrons in the secondary avalanches at the cathode side is ~10^{14} cm^{-3}, which is an order of magnitude higher than at the anode side. The process of streamer propagation is statistical since it depends on the few photo-electrons generated. These results support the view held by Dawson and Winn [40] and Gallimberti [41] regarding streamer propagation.

At the time the streamers form, E/N in the body of the avalanche is 134 Td. This value is 3% higher than the E/N at which DC breakdown occurs for the same gap conditions. This is the lowest E/N which has been observed in the simulations. This is a surprising result since traditionally the avalanche-streamer transition is assumed to occur when the field inside the avalanche goes to zero [31]. The body of the avalanche is in this aspect similar to the positive column of a glow discharge. That is, the value of E/N on axis is sufficient to keep ionization active to make up for
any losses. In these simulations, since the ions do not move, the electrons losses in the early stages of streamer formation are due to the radial expansion of the avalanche. For the later stages of development, the E/N in the body increases as the axial separation of the regions with net charge-density (i.e., the streamer fronts) increases, while the net charge in the layers stays constant (see Fig. 5 in next section).

The cathode-directed streamer under these conditions is observed to have a diameter of approximately while the anode-directed streamer has a diameter of \( \text{cm} \) (see Fig. 1c). These values increase with decreasing pressure. From plots similar to Fig. 1, we can determine the velocities of the anode and cathode directed streamers. The position on axis of the \( \text{cm}^3 \) density contour at the anode side of the avalanche is shown in Fig. 2 as a function of time for all the cases considered (see below). Also shown in the figure are the velocities of the corresponding anode directed streamers. For the conditions and duration of these simulations (in particular, no boundary effects), the cathode-directed streamer propagates at a slower velocity than the anode-directed streamer. The anode-directed streamer propagates in the early stages at nearly twice the velocity of the cathode-directed streamer. The values of the velocities are consistent with experimental values \([23,42]\). The effect of the lifetime of the state producing the photolonlzlng radiation on the propagation of the streamer was investigated using conservation (moment) equations (see next section). Lifetimes, \( \tau \), equal to 0, 0.1, and 2.6 nsecs were used. It was observed that as \( \tau \) increases, the density profiles at the front of the avalanche become steeper in the field direction; however, there are no significant changes in the radius or speed of propagation of the streamer.

To illustrate the role of kinetic effects, simulations have also been carried out (using the Monte Carlo method) neglecting photoionization. In this case, the cathode-directed motion of the avalanche has not been observed. Under these conditions, electron pressure effects do not propagate the cathode-directed streamer. Also, the anode-directed front was observed to propagate at a lower velocity (see Fig. 2). However, the velocity was a factor of 1.7 higher (at \( p = 283 \text{ Torr} \)) than the drift velocity in the enhanced field. This indicates that electron pressure plays an important role when photoionization is absent. Thus, in gases where the photoionization yield is negligible, this effect determines the propagation of the avalanche front towards the anode. The dynamics of propagation, in either case, are very similar. That is, it is due to the avalanche growth of a few
electrons found ahead of the avalanche. When the photolionization yield is low, these electrons are the result of the properties of the distribution at the front of the avalanche and are found closer to the avalanche. Because of this, the density profile in the absence of ionization is steeper than when photolionization is included. The field enhancement at the front is thus greater in the first case. This has significant consequences at higher E/N as discussed below. At E/N = 300 Td in nitrogen, electrons in the avalanche do not gain sufficient energy to "runaway" from the avalanche. That is, no high energy electrons are observed just ahead of the avalanche. Thus, high energy electrons do not play the equivalent role of photo-electrons in the propagation of the anode-directed streamer at moderate values of E/N. Runaways have been observed in nitrogen when the local value of E/N exceeds 1500 Td [43]. At an applied E/N = 1000 Td and in the absence of photolionization, the field at the head of the avalanche exceeds 1500 Td. Runaway electrons are observed and the velocity of the anode front increases considerably (see Fig. 2). However, when photolionization is included, the density profiles are less steep and the enhanced field at the head is below 1500 Td. Runaway electrons are not observed and the front propagates at a lower velocity (see Fig. 2). Thus, for E/N values of up to 1000 Td in nitrogen (where photolionization is important), runaway electrons are not observed. The electron energy distribution is well behaved in velocity space (i.e., no runaway electrons). Thus, it is possible to model, for lower values of E/N, the dynamics of avalanches and streamers with a proper set of fluid equations. When the energy distribution changes rapidly with position and time (as occurs, for example, near the streamer fronts when the photolionization is small), a proper set of fluid equations is difficult to obtain. In this case, non-local effects (when the average properties at (r,t) depend on the properties at (r',t')) due to large gradients in the local mean energy and velocity may have to be included (see Section V). At higher E/N, a kinetic treatment is necessary to properly account for the high energy electrons. When photolionization is present (as in the case of nitrogen), electron kinetics determines the fine structure of the streamer fronts. In this case, the propagation of the streamers is due to the avalanche growth of the photo-electrons produced in the enhanced fields near the streamer boundaries (as first suggested by Raether, Loeb, and Meek). Electron pressure effects observed for the anode-directed streamer in the absence of photolionization, are masked by the fact that the diffusion length (due to density and mean energy gradients) is smaller than the radius at which photo-electron are produced. However, in the absence of photolionization, these effects dominate. It is possible that for some gases, both processes may be equally important in propagating the anode streamer. The properties of the streamers in nitrogen presented here from the simulations at a background density of 2.45x10^{19} cm^{-3} are consistent with experimental observation of luminosity tracks [23,42].

IV. Breakdown Under Generic Gap Conditions

At present, a comprehensive quantitative theory of pulsed breakdown is not available. The difficulty in arriving at such theory stems from the fact that, besides single-particle processes (such as impact ionization, attachment, and secondary cathode processes), collective processes (such as space-charge effects, streamers, gas heating, and multi-step ionization) must also be taken into account in the formulation of such theory. Note that streamers are here considered as fundamental, collective processes that, along with the other processes (single-particle and collective) contribute to the development of pulsed breakdown. The term streamer implies a physical (collective) process that involves photo-ionization, space-charge fields, and impact ionization. Streamers are a subset of a more general class of ionizing potential waves [44]. The subset is defined by the fact that the gas ahead of the ionizing front is composed of neutral particles.

Percent overvoltage (amount above DC breakdown) has often been used as an indicator for the regime where streamers play a major role in the development of pulsed breakdown. This practice, however, is misleading. Recall from section I, the breakdown criterion (Eq. 1), namely $\frac{\omega}{\alpha} e^{ad} \simeq 1$. If $\omega/\alpha$ is small ($\leq 10^{-8}$), $e^{ad}$ must be large ($\geq 10^{6}$) in order to satisfy the criterion. In this case, Eq. 1 is not valid and collective processes would have to be taken into account already at 0% overvoltage; that is, at D.C. This is the case, for example, in SF$_6$ at high pressures [45]. Thus, Townsend breakdown (Eq.) and D.C. breakdown are not, in general, equivalent. As mentioned in the introduction, Townsend breakdown implies that only single-particle processes need to be considered for arriving at a breakdown condition. Thus, in general, single-particle and collective processes have to be taken into account in describing both DC and pulsed breakdown. Except at
very high applied fields (where runaway [10], and cathode flares [14] effects have to be considered,) there is no difference between DC and pulsed breakdown as far as physical processes (single-particle and collective) that need to be considered. The relative importance of the various processes in the development of breakdown is very dependent on the gap conditions. This is reflected, for example, on the spatial characteristics of the ionized channel.

In Fig. 3, a diagram is shown of a typical experimental set up for investigating pulsed breakdown. Information concerning the breakdown of the gap and the processes contributing to its development is obtained from: a) current and voltage oscillograms from probes located before the gap and at the load [12], and b) optical records of the light emitted by the discharge [46-48]. From the oscillograms, the breakdown delay time and the spark current and voltage can be determined (since the system is a transmission line [12]). From the optical records, more quantitative information can be obtained regarding the processes that contribute to breakdown. Due to the very low

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**Fig. 2.** Axial position of the density contour with $10^{12}$ particles/cm$^3$ as a function of time for a) $E/N = 300$ Td and b) $E/N = 1000$ Td.

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**Fig. 3.** Diagram of an experimental set up to investigate pulse breakdown. The u-v source is used to generate the initial distribution of electrons.
emission, it is difficult to obtain optical information from the very early stages of breakdown. Recently, Pfelffer and colleagues [49] and Byszewski and colleagues [50] have made significant contributions in this direction.

In pulsed breakdown experiments, the gap conditions are given by: 1) initial charge distribution, 2) type of gas, 3) gas pressure, 4) gap dimensions, and 5) applied field. It is assumed that the rise-time of the voltage pulse applied to the gap is nearly zero. The domain of gap conditions obtained from these five “parameters” is infinitely large. It is thus useful to define a finite number of “generic” conditions, and to understand how breakdown proceeds from them. The results from these generic problems may be extrapolated to other situations.

The initial charge distribution defines two classes of generic conditions. These are: 1) distributions whose space-charge field is significant (for example a plasma), and 2) those whose space-charge field may be neglected. Distributions of the first type may be produced with a high power laser or by a cathode flare. The discussion to follow, focuses on distributions of the second type. This type of distribution may be grouped into three classes: a) when the total (initial) number of electrons is sufficiently large (and relatively low density) that breakdown can be described by Dickey’s model [8], b) when the spatial extent and density of the electron distribution leads to essentially 1-dimensional streamer fronts (see Fig. 7), and c) when electron distribution is localized and near the cathode. This includes the single electron condition. Quantitative understanding of how the early stages of breakdown develop from these conditions has been achieved by numerical simulations [51-54]. The reader is referred to these papers for more details.

The simulation of the development of breakdown from a given initial distribution, using a kinetic description (see Section III), is very time consuming. Thus, an alternate description, using moment equations [55] is, from this point of view, desirable. The question arises as to the number of moments that need to be used to describe the development of breakdown, and how to obtain the various parameters that appear in their equation evolution. When the momentum and energy equilibration time and distance are small compared to any macroscopic scale variations of the system, then a single moment description (continuity equation for the various particle densities) is satisfactory. This is the case when the photolonization yield is sufficiently large (for a given applied field) that the space/time variations of the particle distribution satisfy the above criterion. In this case the evolution of breakdown can be determined from the following set of equations:

\[
\frac{\partial n_e}{\partial t} + \nabla \cdot n_e \mathbf{v}_e = \alpha n_e |\mathbf{v}_e| - \eta n_e |\mathbf{v}_e| + S_p
\]  

\[
\frac{\partial n^+_i}{\partial t} = \alpha n_e |\mathbf{v}_e| + S_p
\]  

\[
\frac{\partial n^-_i}{\partial t} = \eta n_e |\mathbf{v}_e|
\]  

where \(n_e\): electron density, \(n^+_i\): positive ion density, \(n^-_i\): negative ion density, \(\mathbf{v}_e\): electron fluid velocity, \(\alpha\): electron ionization coefficient, \(\eta\): electron attachment coefficient, and \(S_p\) is a source of electron- ion pairs due to photolonization [52]. The electron fluid velocity is determined from the expression

\[
n_e \mathbf{v}_e = D_e \nabla n_e - \mu_e n_e (E_e + E_a)
\]  

where \(D_e\) is the diffusion tensor for electrons with components \(D_{11}\) and \(D_{T} \), \(\mu_e\) is the electron mobility, and \(E_e\) and \(E_a\) are the space-charge and applied field, respectively. The space-charge field is determined from Poisson's equation as discussed in Section III. An algorithm, based on Flux Corrected Transport techniques [58], was developed to accurately handle the steep gradients and large dynamic range that are encountered when solving Eqs. (5)-(8), coupled to Poisson’s equation. This algorithm has been discussed elsewhere [57]. Using this model, we have simulated the development of breakdown up to the formation of a weakly conducting channel for a number of generic conditions. The results for initial distributions corresponding to classes b) and c) (previous paragraph) are summarized below.

For class b), the characteristics of the early stages of breakdown strongly depend on the amplification factor, \(e^{\text{ad}}\) (which in turn depends on the type of gas, pressure, and applied field), and the magnitude of the secondary cathode processes. These two parameters further define four possible situations depending in their relative magnitude. When the intensity of the secondary emission
processes and the amplification factor are relatively small, the distribution becomes "critical" (see section IIa) far from the cathode and no secondary avalanches are generated from the cathode. The distribution remains finite in radius as it moves towards the anode (see Fig. 4a). After the "critical" stage, anode and cathode directed streamers develop that are also finite in radius (Fig. 4b). The speeds of these streamers are essentially equal (and of the order of $10^8$ cm/sec). This is due to the fact that the condition of the (gas) medium ahead of the streamers is the same. The evolution of the longitudinal electric field on axis, $E_z(z,t)$, for this case is shown in Fig. 5. Note that the field does not go to zero as discussed in the previous section. Moreover, as the streamer fronts move away from each other, the field inside the ionized region increases until it becomes equal to the applied field. When the intensity of the secondary processes is large, secondary avalanches can develop from the cathode that may influence the characteristics of the cathode directed streamer. For example, the speed of the cathode directed streamer may be larger than that of the anode directed streamer.

In both of the above situations, after the streamers reach the electrodes, a weakly ionized channel with charge densities of the order of $10^{14}$ cm$^{-3}$, radius of the order of $10^{-2}$ cm, and total resistance in the 100K ohm to mega ohm range bridges the gap (see Fig. 4). For circuits with series resistance in the ohm range, the voltage across the gap does not collapse at this point. The

![Fig. 4. Evolution of constant electron density contours in r-z space for $E/N = 2.34$ Td.](image-url)
Fig. 5. Evolution of the longitudinal electric field on axis corresponding to the distributions shown on figure.

Further development of breakdown depends on the rate at which the conductivity of the ionized channel increases with time. This rate is governed by the heating of the background gas and by multi-step collision processes.

Depending on the magnitude of the secondary processes at the cathode, a number of weakly ionized (non-overlapping) channels may form either early in the development of the primary channel (for high magnitudes), or later when the cathode directed streamer approaches the cathode (for low magnitudes). The question then is: After the formation of the primary channel, does the conductivity of this channel increase fast enough with time that the voltage across the gap collapses before the formation of additional channels? If it does, breakdown occurs in a single channel; if not, a number of channels develop. In this case, breakdown may occur by either a change in the conductivity of some of the channels or when sufficient number of channels are formed that the total current in the circuit is sufficiently high that the voltage collapses.

A number of models have been proposed to explain the change in conductivity of one channel (see below). This transition may (loosely) be divided into two regions: a) the evolution from a density of $\sim 10^{14}\text{cm}^{-3}$ to $\sim 10^{18}\text{cm}^{-3}$ (for atmospheric pressure discharges), and b) the hydrodynamical region. In the first region, the evolution of the discharge channel is characterized by the
flow of energy from the various internal energy states of the background gas and eventually to gas heating and increased ionization. The details of the energy flow depend on the properties of the background gas. In this region, the composition of the gas changes considerably as the degree of ionization and gas temperature increase. Analysis of this evolution has been carried out by a number of authors (see for example [58-59]). In all of these studies, the discharge has been assumed to be uniform. The consequences of this assumption have not been assessed. As the density of the channel increases to nearly $10^{18}$ cm$^{-3}$, the further development of the channel is largely determined by hydrodynamical considerations. Theories describing the further evolution of the channel in this later period were developed by Drabkina [60] and Braginskii [61]. For further discussion on this subject the reader is referred to these references and to [62-64].

When the amplification factor is large, the initial distribution grows rapidly very near the cathode. If in addition, the magnitude of the secondary processes is small, breakdown develops as described above, except for the possible absence of cathode directed streamers. However, when the magnitude of the secondary processes is large, secondary avalanches that develop from the cathode cause a rapid expansion of the initial distribution in the radial direction (see Fig. 6). At later times, essentially a plane streamer front develops that propagates towards the anode. Subsequently, the evaluation of the distribution is essentially in 1-dimension and is similar to class b above. The distribution for this latter case is shown in Fig. 7. After the streamer reaches the anode, a weakly ionized gas fills the inter-electrode space. As in the other cases, the resistance of this channel is high. Voltage collapse is observed after the impedance of the ionized channel is sufficiently low compared to the external circuit impedance.

V. Concluding Remarks

In the previous sections we have presented: a) a brief review of pulsed breakdown models, b) a kinetic analysis of streamers, and c) a description of the development of breakdown under a number of generic gap conditions. In the discussion of breakdown, we have emphasized the fundamental processes and the role they play in the development of breakdown. The gap conditions then determine how these processes contribute to the breakdown of the gap.

Significant advances have been made in the diagnostics of pulsed discharges which have resulted in more detailed observation of the spatio-temporal development of breakdown for a number of gap conditions [49,50]. Similar advances have been made in understanding the various stages of breakdown and how the fundamental processes (single-particle and collective) contribute to its development. However, a number of obstacles have to be surmounted before more quantita-

![Fig. 6. Density contours at \( t = 1 \) nsec when the amplification factor \( e^{a \tau} \) is large. At later times, the contours are similar to those shown on Figure 7.](image)
Fig. 7. Density contours at t = 0.8 ns for an initial distribution consisting of a disc of electrons in front of the cathode with a density of $5 \times 10^5$ cm$^{-3}$ and a radius of 0.3 cm.

First, there is a significant deficiency of data on the spectral composition of the radiation produced by an electron avalanche, the average lifetime of the radiating states, and the absorption length of the radiation. This data is needed before a more quantitative description of streamer propagation can be accomplished.

Second, the most difficult computational problem, yet to be resolved, is the solution of Poisson’s equation for charge distributions existing during the development of breakdown. Although techniques for solving the Poisson equation are available, (primarily in 2-dimensions) these techniques fail when the photoionization yield is small and the applied field is very large. In these situations the gradients in the charge distributions are too large and localized to be handled by these techniques. Moreover, to investigate such phenomena as streamer bifurcation (branching), and multi-avalanche initiated breakdown, a three dimensional solution of Poisson’s equation is necessary. No computational model to date has made use of Poisson’s equation in 3-dimensional, most assume cylindrical symmetry. Note that techniques are available for solving the equations of evolution (kinetic or fluid) of the various species in 3-dimensional configuration space (see sections III and IV).

A fully kinetic description of the evolution of breakdown (although in principle possible) is very time consuming and at present not feasible with available computers (except for the very early stages as discussed in section III). An alternate, less time consuming, description consists of moment (fluid) equations for each of the particle species (see Section IV). For a number of situations (see section IV), a single moment description (using the equation of continuity for the particle densities) is satisfactory. However, when the photo-ionization yield is low and the applied fields are large (leading large gradients in the particle densities, mean velocities, and mean energies), such description is no longer satisfactory, as previously discussed. For these situations, a proper set of moment equations is yet to be found. This is another of the obstacles to the complete descrip-
tion of breakdown. In essence, it entails the proper termination of the hierarchy of moment equations and a procedure for determining the various coefficients appearing in the various equations.

After the weakly ionized channel has been formed, the quantitative description of its transformation into a highly conducting channel constitutes the last obstacle we shall mention. This consists of an investigation of the energy and mass flow in the discharge. Rather detailed studies of energy flow in discharges have been made using rate equations [54,05] (i.e. zero dimensions). In order to apply these results (or techniques) to pulsed discharges, the finite dimensionality of the channel and the transport of the particle species out of the channel region, must be taken into account. Such analysis have been carried out by restricting the details of the energy flow. However, the tools exist for conducting a sufficiently comprehensive investigation of this transition.

Of the obstacles discussed above, the first appears to be the least likely to be resolved in the near future.

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References


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