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THE PHOTON STRUCTURE FUNCTION AT LARGE Q^2

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INTRODUCTION

Quantum chromodynamics is supported by experimental results of many short-distance processes. Among the processes that have been thoroughly investigated, deep-inelastic electron-photon scattering (Fig. 1) is of special interest. This reaction has had a long theoretical history[1-5] before first measurements were reported[6]. The physical interest in the analysis of the photon structure function with regard to QCD is based on the following key points :

i) As a consequence of asymptotic freedom, the structure function rises linearly with $\log Q^2$ in leading order, the slope being predicted by QCD.

ii) In next-to-leading order, the absolute size of all moments of F_2^γ with $N > 2$ is asymptotically fixed by the Λ parameter.

A spurious singularity at $N = 2$ is not expected to spread to larger N values as can be inferred from electron scattering off off-shell photons that is completely calculable perturbatively[7]. Higher orders can only be calculated for yet larger N values since the residual non-perturbative contributions have to fall off faster than the perturbative component.

Applying such a QCD analysis to medium range Q^2 values, supplementary assumptions on the residual non-perturbative part of the structure function are needed. These assumptions are beyond the realm of perturbative QCD calculations, and they must be subject to experimental scrutiny. The $e\gamma$ experiments carried out till now, though limited in statistics, provide in fact a consistent picture.

LEP I[8] and still better LEP II with a total CM e^+e^- energy of approximately 200 GeV offers the unique opportunity to measure the photon structure function over a wide Q^2 range from 10 to 2000 GeV^2 (Fig. 2). The large lever arm in Q^2 can be exploited to study carefully the rise in $\log Q^2$:

i) The linear rise in $\log Q^2$ is a consequence of the running coupling constant. Freezing the coupling constant at an initial value of $Q_0^2 = 5$ GeV^2, for instance, bends $F_2(x,Q^2)$ asymptotically to a scale invariant function that is independent of Q^2[10]. This effect can show up only in the highest Q^2 bins accessible in LEP II.

ii) The slope parameter is reduced in QCD by an amount of O(1) compared with the parton model[4]. This unique prediction of QCD can experimentally be clearly proved at an integrated luminosity of $\int \mathcal{L}\,dt = 500\ pb^{-1}$, unperturbed by higher-twist effects at large Q^2.

STRUCTURE FUNCTIONS

The cross-section for deep inelastic electron scattering off a photon target (Fig. 1), is parametrized by 2 structure functions,

$$\frac{d\sigma}{dxdy} = \frac{4\pi\alpha^2 s(e\gamma)}{Q^4}\left[(1-y)\ F_2(x,Q^2) + y^2 x\ F_1(x,Q^2)\right] \tag{1}$$

F_1 and F_2 are proportional to the cross sections for transversely and longitudinally polarized virtual photons,

$$F_1 = F_T \tag{2a}$$

$$F_2 = 2xF_T + F_L \tag{2b}$$

The Bjorken variable x and y can be expressed in terms of momentum transfer, energies and scattering angle,

$$x = Q^2/2q\cdot p_\gamma = Q^2/(Q^2+W^2) \tag{3a}$$

$$y = q\cdot p_\gamma/k_e\cdot p_\gamma = 1-E'/E.\cos^2\theta/2 \tag{3b}$$

The coefficient y^2x is small under normal experimental conditions, and only F_2 can be measured.

In the <u>quark parton model</u> the structure functions are calculated in a way similar to QED, and quarks are treated as free particles without strong interactions. In this approach, the leading part of F_2 is linear in $\log Q^2$ for light quarks while F_L is asymptotically finite and scale invariant,

$$F_2^{QPM} = \frac{\alpha\langle e^4\rangle}{\pi}\ x\ [x^2+(1-x)^2]\ \log Q^2 + \ldots \tag{4a}$$

$$F_L^{QPM} = \frac{4\alpha\langle e^4\rangle}{\pi}\ x^2(1-x) \tag{4b}$$

Virtual photon exchange dominates at low Q^2 and $\langle e^4\rangle$ is a sum over the fourth power of all quark charges involved,

$$\langle e^4\rangle = 3 \sum_{fl} e_q^4. \tag{5a}$$

For $Q^2 \gtrsim 1000\ \mathrm{GeV}^2$, Z exchange becomes increasingly important so that :

$$\langle e^4\rangle \to 3 \sum_{fl} \frac{1}{4} \sum_{i,j=L,R} e_q^2 \left[e_q - \frac{Q^2}{Q^2+m_Z^2} \; \frac{Z_i(e)Z_j(q)}{\sin^2\theta_W \cos^2\theta_W} \right]^2 \tag{5b}$$

where the electroweak Z quark charges are given in the Glashow-Salam-Weinberg model as

$$Z_L(F) = I_{3L}(F) - e(F)\sin^2\theta_W$$

$$Z_R(F) = \qquad\qquad - e(F)\sin^2\theta_W$$

for left and right-handed couplings (Fig. 3). The different behavior of the γ structure functions in x and Q^2 compared to hadronic targets is a consequence of the unlimited transverse momentum in the pointlike splitting $\gamma \to q\bar{q}$. For heavy quarks the finite mass corrections are given by[9] :

$$F_2^Q = \frac{3\alpha\, e_Q^4}{\pi} \left\{ v_Q\, x \left[4x(1-x)\left(2 - \frac{m_Q^2}{Q^2}\right) - 1 \right] \right.$$
$$\left. + x \left[x^2 + (1-x)^2 + \frac{4m_Q^2}{Q^2} x(1-3x) - \frac{8m_Q^4}{Q^4} x^2 \right] \log \frac{1+v_Q}{1-v_Q} \right\} \tag{6a}$$

$$F_L^Q = \frac{12\alpha\, e_Q^4}{\pi} \left\{ v_Q\, x^2(1-x) - \frac{2m_Q^2}{Q^2}\; x^3 \log \frac{1+v_Q}{1-v_Q} \right\} \tag{6b}$$

where v_Q is the quark velocity in the $Q\bar{Q}$ rest frame and the Bjorken variable is restricted to $x < Q^2/(Q^2 + 4m_Q^2)$. Figure 4 shows the charm contribution for various values of Q^2.

After switching on perturbative gluon radiation, three mechanisms compete with each other in the final build up of the structure function F_2. For rising Q^2 :

i) the number of quarks rises uniformily through the increasing $\gamma \to q\bar{q}$ splitting probability,

ii) at $x > 0.4$, quarks are lost through increasing gluon radiation (accumulating at small x),

iii) gluon radiation is damped due to the logarithmically decreasing coupling constant.

The net effect after solving the Altarelli-Parisi equations asymptotically, is a structure function that keeps rising linearly in $\log Q^2$ but with an O(1) change of the coefficient. (QCD corrections to F_L turn out to be numerically small). A comparison between the Born term prediction of the γ structure function and the O(1) QCD correction is presented in figure 5. Since the structure function can be well determined at $Q^2 > 200\ GeV^2$ in LEP II, this qualitative difference can be established unperturbed by higher-twist effects.

The kinematical increase of gluon bremsstrahlung with Q^2 is just balanced in QCD by the decrease of the running coupling constant, resulting in a uniform rise of F_2 with $\log Q^2$. In a theory with a small but fixed coupling constant α_*, gluon bremsstrahlung moves the increasing number of quarks (due to $\gamma \to q\bar{q}$ splitting for rising Q^2) all down to small x values. For finite $x > 0$ the structure function becomes asymptotically scale invariant at a size (α/α_*). This is illustrated in figure 6 for a toy model in which the coupling constant α_* is frozen at $Q_0^2 = 5\ GeV^2$ with $\Lambda = 200$ MeV. The comparison with the expected behavior of $\int x^2 F_2^\gamma(x,Q^2)$ in QCD proves that LEP II provides a lever arm in Q^2 large enough to observe direct consequences of the running QCD coupling.

Eventhough the high Q^2 data at LEP II cannot shed any new light on the problem of the absolute normalization of the structure function and the QCD Λ parameter, that can be studied equally well at medium Q^2 accessible through PETRA and the (upgraded) PEP, the main points should shortly be summarized for completeness.

In next-to-leading order, needed to fix the absolute scale of the structure function, light-cone expansion plus renormalization group result in the following expansion of the moments of the γ structure function[5] :

$$F_2^N(Q^2) = \sum_{i=\pm NS} A_i^N \left[\alpha_S(Q^2)\right]^{d_N^i} + \frac{1}{\alpha_S(Q^2)} \sum_i \frac{a_N^i}{d_N^i+1} + \sum_i \frac{b_N^i}{d_N^i} + C_N + O(\alpha_S) \qquad (7)$$

The anomalous dimensions d_N^i, the a_N^i, etc. are calculable numbers in perturbation theory whereas the A_i^N are remnants of non-perturbative long-distance QCD. They must also contain contributions that remove the singularities induced when $d_N^i \to 0$ for $N \to 2$,etc. Two approaches have been proposed to solve this problem :

i) Inventing a parametrization of quark and gluon densities to describe the γ structure function at moderate Q^2, allows the calculation of the Q^2 evolution in a way analogous to deep inelastic scattering on hadronic targets[10-11]. In this approach no more singularities are encountered. The slope of the structure function in $\log Q^2$ can well be predicted. The sensitivity on Λ, however, is lost since $\alpha_s^{-1}(Q^2) - \alpha_s^{-1}(Q_0^2) \propto \log Q^2/Q_0^2$ does not depend on Λ in leading order.

ii) In a different approach that does not merely copy well-known patterns in deep-inelastic lepton-nucleon scattering, the singularities in the point-like piece are isolated and canceled against pole terms with properly fixed residue A_N^i [7]. The remnants of the expansion about the poles are summarized in a free parameter λ. Besides this non-perturbative parameter one must expect, on general physical grounds, a VDM type contribution to be present in A_N^i [10]. If the transverse momentum in $\gamma \to q\bar{q}$ is small, the lifetime of the quark-antiquark pair is long, and the overlap with the low-lying resonances ρ, ω, ϕ is large. The x dependence of this component drops as α H (1-x) for $x \to 1$, and since this contribution comes with a coefficient proportionnal to $[\alpha_s(Q^2)]^{d_N}$, it will die out for $Q^2 \to \infty$. Two remarks are mandatory : (a) The structure function is not affected by the regularization procedure beyond $x > 0.2$[7], (b) The strength $H \approx 0.2$ of the hadronic component can be traded for a change of the λ parameter, with moderate impact on Λ [12]. Duality arguments might be invoked to explain this rather natural observation. Adopting the PLUTO values for these two free parameters, the prediction for the structure function at $Q^2 = 200$ GeV^2 is shown in figure 7 for $\Lambda_{\overline{MS}} = 183{+}80/{-}60$ MeV. The errors correspond to an integrated luminosity of 500 pb^{-1} and $100 < Q^2 < 500$ GeV^2. The figure proves that the photon structure function can accurately be measured at large Q^2 in LEP II.

In conclusion, LEP II offers the unique opportunity to measure the photon structure function over a large Q^2 range up to ~ 2000 GeV^2. Two crucial predictions of QCD can be tested in this experiment : the linear rise in log Q^2 as a consequence of asymptotic freedom, and the large renormalization O(1) of the shape of the structure function due to gluon bremsstrahlung, unperturbed by higher-twist effects.

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FIGURE CAPTIONS

1. Deep inelastic electron-photon scattering in e^+e^- colliders.

2. Q^2 distributions for PEP, LEP I and LEP II.

3. Effective quark charges in electron-photon scattering at high energies.

4. Contribution of charmed quarks to the photon structure function for various Q^2 values.

5. O(1) change of the shape of the photon structure function after switching on gluon Bremsstrahlung.

6. Comparison of the Q^2 evolution of the photon structure function in QCD with a theory in which the coupling constant is frozen.

7. Regularized QCD prediction for the photon structure function at $Q^2 = 200$ GeV^2 and sensitivity to the QCD Λ parameter. Errors bars correspond to an integrated luminosity of 500 pb^{-1} and $100 < Q^2 < 500$ GeV^2.

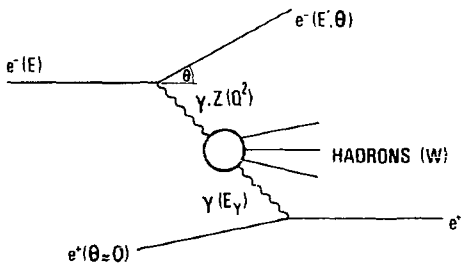

Figure 1

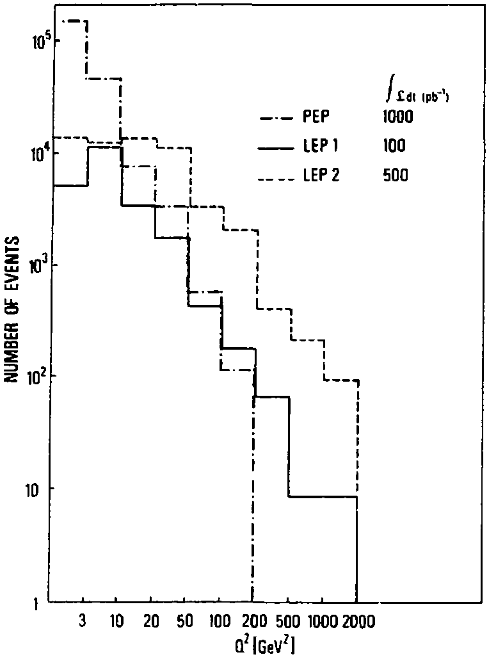

Figure 2

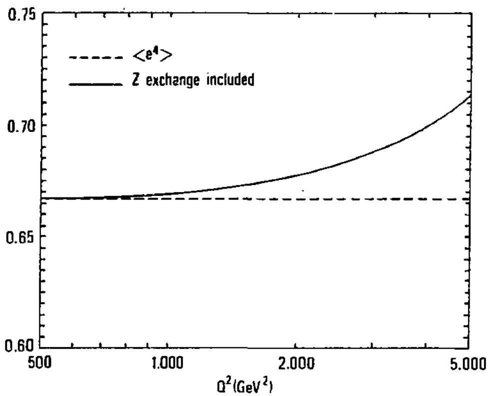

Figure 3

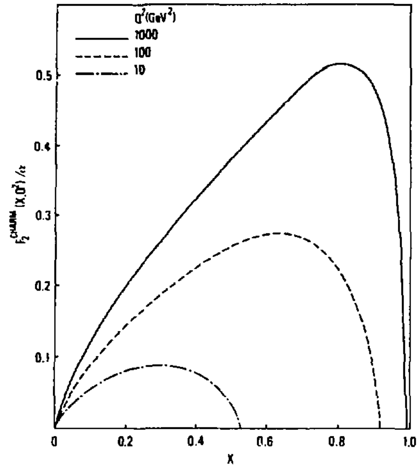

Figure 4

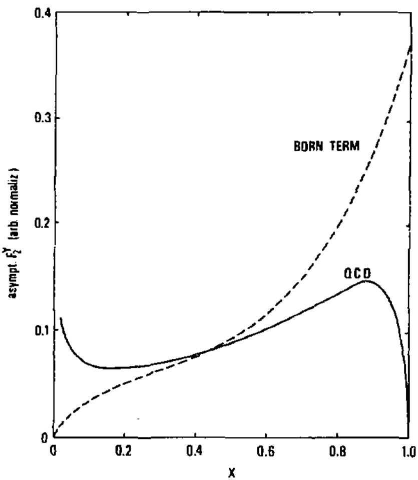

Figure 5

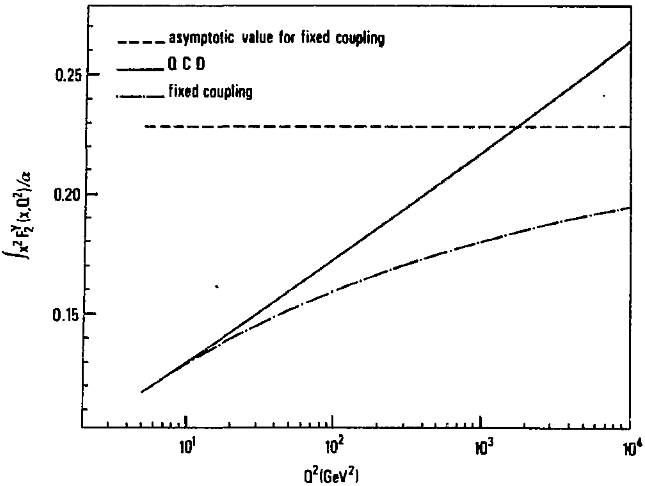

Figure 6

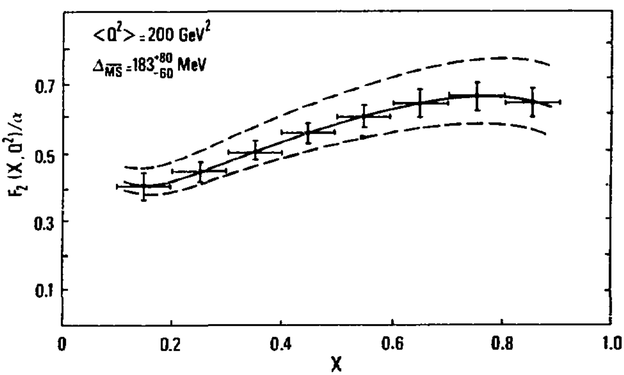

Figure 7