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POLARIZATION TRANSFER FROM POLARIZED NUCLEAR SPIN TO μ^- SPIN IN MUONIC ATOM

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Abstract:

A theoretical study of polarization transfer from an initially-polarized nuclear spin to a μ^- spin in a muonic atom is given. The switching of the hyperfine interaction at excited muonic states as well as at the ground 1s state is taken into account. The upper state of hyperfine doublet at the muonic 1s state is considered to proceed down to the lower state. It is found that as the hyperfine interaction becomes effective at higher excited muonic orbitals, a less extent of polarization is transferred from the nuclear spin to the μ^- spin. The theoretical values obtained are compared with the recent experiment of μ^- repolarization in a polarized ^{209}Bi target.

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1. Introduction

Polarization phenomena in nuclear muon capture provide a useful tool for studying the weak interaction, especially for determining the induced pseudoscalar and other hadronic form factors in the nuclear weak current. These phenomena include an asymmetric distribution of neutrons produced by muon capture, that of γ-rays in radiative muon capture, hyperfine effects on nuclear muon-capture rates and so on [1]. All these effects are directly proportional to the magnitude of residual polarization of muons at the 1s state of a muonic atom. However, since negative muons lose about $\frac{5}{6}$ of their initial polarization during atomic capture and cascade process down to the 1s state [2-4], attempts to measure these polarization phenomena have faced large difficulties. In addition, for muonic atoms of nuclei with a non-zero nuclear spin, muons suffer additional depolarization (about $\frac{1}{3}$ in the case of a large nuclear spin) owing to the hyperfine interaction between a nuclear and a muonic magnetic moment [5,6].

For the purpose of recovering the residual muon polarization in muonic atoms, Nagamine and Yamazaki [7] proposed an artificial polarization of the muons by making use of a polarized nuclear target of ^{209}Bi in ferromagnetic compound BiMn ; this method called "repolarization" is based on the polarization transfer from the nuclear spin to the μ^- spin through the strong hyperfine interaction between them. The first experiment was successfully carried out recently at SIN [8].

The repolarization mechanism using a polarized nuclear target was already discussed theoretically by Nagamine and Yamazaki [9,10] in the statistical tensor approach, and by Hintermann and Mukhopadhyay [11] in the density matrix method. Its principle is almost similar to that of the Overhauser effect : when a muonic orbital (J) is coupled with a nuclear spin (I), the hyperfine states ($\mathbf{F} = \mathbf{I} + \mathbf{J}$) become energy eigenstates, and the nuclear polarization brings about the polarization of hyperfine states (F), eventually yielding the muon polarization.

In addition, for a high-Z muonic atom with a large nuclear spin, a fast M1 transition between the hyperfine doublet at the muonic 1s state ($F = I \pm \frac{1}{2}$) is known to be of great importance for the repolarization mechanism [10]. This transition called the hyperfine conversion occurs mostly by emitting conversion electrons, and its transition rate is much faster than a muon life time for high-Z muonic atoms [12]. The enhancement of the repolarization effect due to the hyperfine conversion is understood as follows. At the muonic 1s states, muon spins with $F^{+} = I + \frac{1}{2}$ and $F^{-} = I - \frac{1}{2}$ are parallel and antiparallel to the nuclear spin (I) respectively, and the hyperfine conversion takes place mostly by flipping the muon spin in the case of a large nuclear spin such as in the ^{209}Bi nucleus ($I = \frac{9}{2}$). As a result, most muon spins are directed toward the same direction.

Thus far, theoretical estimations on the repolarization have been made only based on the assumption that the hyperfine coupling is switched on immediately at the instance when muons reach the muonic 1s state [7,9–11]. Obviously, there exist no reasonable explanations for this assumption. In general, the hyperfine interaction becomes effective when the hyperfine splitting is larger than the natural width of the atomic level. This condition might be met even at excited states of the muonic atom. In fact, experimental studies of the negative muon spin rotation, which measured the residual muon polarization at the 1s state of muonic atoms with non-zero nuclear spins, revealed some evidences that the hyperfine coupling is switched on at excited muonic states. Therefore, we intend to extend the repolarization studies to the cases in which the hyperfine interaction becomes effective at excited muonic states as well as the muonic 1s state. In the present calculations, after switching of the hyperfine interaction at muonic excited states, the muons are considered to cascade down to the 1s state by allowed E1 transitions through the hyperfine states of circular orbits. The effect of the hyperfine conversion at the muonic 1s state is also taken into account.

Besides various practical applications to researches of polarization phenomena in muon captures, the repolarization studies will give us information on the role of hyperfine interaction in muonic atomic cascade. Because, as discussed later, the extent of the repolarization is strongly dependent on the muonic state from which the hyperfine coupling is switched on. Therefore, compared with the experimental results, this present work allows us to study the mechanism of the hyperfine depolarization more precisely than the previous studies done by negative-muon spin rotation [13].

In the following section we present a brief formalism of the statistical tensors which are used to describe the orientation of spin states. The details about the present calculation will be shown in Section 3. The result will be discussed in section 4 and the comparison with the experimental results of the ^{209}Bi muonic atom will be shown in Section 5.

2. Statistical Tensor Formalism

Let us deal with the orientation of spin states by the statistical tensors $B_k(J)$ defined as [9,14,15];

$$B_k(J) = \sqrt{2J+1}\sum_m (-)^{J-m} P(m) < JJm - m \mid k0 > \tag{1}$$

where $< JJm - m \mid k0 >$ is a Clebsh-Gordan coefficient, J is a spin of the state of interest, and m and $P(m)$ are the substate of J and its population, respectively. The subscript k refers to a rank of the tensor. The statistical tensors with rank of $k = 0,1,2$ are related to the population, the polarization (P_J) and the alignment (A_J), respectively. They are expressed explicitly in the following:

$$B_0(J) = \sum_m P(m) \tag{2}$$

$$\begin{aligned} B_1(J) &= \sqrt{\frac{3}{J(J+1)}}\sum_m mP(m) \\ &= B_0(J)\sqrt{\frac{3J}{J+1}}P_J \end{aligned} \tag{3}$$

$$B_2(J) = \sqrt{\frac{1}{(2J-1)J(2J+1)(2J+3)}}\sum_m (3m^2 - J(J+1))P(m)$$

$$= B_0(J)\sqrt{\frac{9J^3}{(2J-1)(2J+1)(2J+3)}}A_J \quad (4)$$

When the muonic states of spin-orbit splitting ($J = L\pm\frac{1}{2}$) with the orbital angular-momentum of L are coupled with the nuclear spin (I), namely the hyperfine coupling is switched on, the resultant statistical tensors of the hyperfine states (F) are expressed by those of nuclear and muonic states as follows,[16]

$$B_k^f(F) = \sum_{k_1,k_2} \frac{(2F+1)^{\frac{1}{2}}(2k+1)^{\frac{1}{2}}}{(2J+1)^{\frac{1}{2}}(2I+1)^{\frac{1}{2}}} \begin{Bmatrix} I & J & F \\ I & J & F \\ k_1 & k_2 & k \end{Bmatrix} B_{k_1}^i(I)B_{k_2}^j(J) \quad (5)$$

where {} is a 9j-symbol and $B_k^i(I)$, $B_k^j(J)$ and $B_k^f(F)$ are the statistical tensors of the rank k for the spin states of nucleus, muons and hyperfine states, respectively. From now on, the superscripts of i, j and f will be used to identify the corresponding states in the same meanings. The polarization of the hyperfine state $B_1^f(F)$ can be formed by some combinations of statistical tensors of rank k_1 and k_2. These combinations include not only both the nuclear polarization ($B_1^i(I)B_0^j(J)$) and the initial muon polarization ($B_0^i(I)B_1^j(J)$), but also the coupling of the nuclear alignment with the initial muon polarization ($B_2^i(I)B_1^j(J)$). It should be noted that the combination of $B_1^i(I)B_1^j(J)$ does not contribute to the polarization of the hyperfine state ($B_1^f(F)$), but to the population ($B_0^f(F)$) and the alignment ($B_2^f(F)$) as clearly seen through the property of the 9j-symbol. This characteristic of $B_1^i(I)B_1^j(J)$ was already discussed in relation to the selective population of the F states to determine the magnetic hyperfine splitting by measuring muonic x-ray intensities from 2p to 1s state of ^{209}Bi atoms [9].

Successive propagation of the orientation of $B_k^f(F)$ through atomic cascades is evaluated by

$$B_k^f(F'L'J') = \sum U_k(LL'JJ'FF'I1)B_k^f(FLJ) \quad (6)$$

for allowed E1 transitions from the state of (L, J, F) to $(L'J'F')$. Here, U_k is the extended

U-coefficient defined by

$$U_k(LL'JJ'FF'I1) = (2L+1)(2J'+1)W(LL'JJ';1\frac{1}{2})^2 \times$$
$$(2J+1)(2F'+1)W(JJ'FF';1I)^2 \times u_k(F1F') \qquad (7)$$

where u_k is an ordinary U-coefficient [15]. Here, the energy dependence of the transition is ignored so that only the geometrical factors are taken into account. The selection rule of E1 transition allows only the transitions from J to $J' = J-1$, J and, at the same time, from F to $F' = F-1$, F, $F+1$. The factors of $(2L+1)(2J'+1)W(LL'JJ';1\ \frac{1}{2})^2$ and $(2J+1)(2F'+1)W(JJ'FF';1\ I)^2$ in eq.(7) correspond to the branching ratios of E1 transitions between the spin-orbit splitting states and between those of the hyperfine states, respectively. This is a natural extension of eq.(22) in ref.9).

The hyperfine conversion is expressed by using the ordinary U-coefficient as the transition from F_1 to F_2 through unobserved radiation of angular momentum of L, which is presented by

$$B_k^f(F_2) = B_0^f(F_2)u_k(F_1LF_2)B_k^f(F_1) \quad \text{and,} \qquad (8)$$

$$u_k(F_1LF_2) = (-)^{k+L-F_1-F_2}\sqrt{(2F_1+1)(2F_2+1)}W(F_1F_1F_2F_2;kL) \qquad (9)$$

where $W(F_1F_1F_2F_2;kL)$ is a Racah coefficient. Eq.(8) can be derived from the special case of eq.(5) in which the statistical tensor of rank k couples with that of rank 0 such as $B_0(L)B_k^f(F_1)$ to produce $B_k^f(F_2)$. It is worthwhile to note that the U-coefficient with rank 1 is regarded as the projection operator of the spin vectors, and it is expressed by

$$u_1(F_1LF_2) = \frac{F_2(F_2+1)+F_1(F_1+1)-L(L+1)}{2\sqrt{(F_2(F_2+1)F_1(F_1+1)}} \simeq \cos\theta \qquad (10)$$

where θ is an angle between $\mathbf{F}_1$ and $\mathbf{F}_2$. This is a classical projection of the spin F_1 on the total angular momentum ($\mathbf{F}_2$=$\mathbf{F}_1$+$\mathbf{L}$) with unobserved direction of spin $\mathbf{L}$. Therefore, the ordinary U-coefficient in eq.(10) can also be used to evaluate the residual muon polarization by projecting the polarization of F states to the initial muon spin direction.

3. Atomic Cascade Calculation

For simplicity, let us divide the history on the formation of muonic atom and the cascade transition of μ^- into four stages: The first stage is that of the capture and the formation of a muonic atom. In the second stage, the muon cascades down rapidly by emitting Auger electrons, so that the natural width of muonic levels is greater than the fine structure splitting. In the third stage, the atomic cascade of muons occurs through radiative transitions and the natural width becomes smaller than the fine structure splitting but greater than the hyperfine splitting. In the fourth stage, the natural width becomes comparable to the hyperfine splitting. Each stage will be described in detail in the following.

In the first stage, the atomic capture process produces substantial depolarization which was discussed by Mann and Rose[2] based on the assumption that the direction of muon momentum was completely randomized. As described in ref.9), the muon polarization $B_1^i(J)$ in the fine-structure states of $J = L \pm \frac{1}{2}$ are given by

$$\begin{aligned} B_1^i(J) &= B_0^i(J) u_1(\tfrac{1}{2} L J) B_1^i(\tfrac{1}{2}) \\ &= + \frac{(2J+1)}{2(2L+1)} \sqrt{\frac{J+1}{3J}} P_\mu^0 \quad \text{for} \quad J = L + \frac{1}{2} \\ &= - \frac{(2J+1)}{2(2L+1)} \sqrt{\frac{J}{3(J+1)}} P_\mu^0 \quad \text{for} \quad J = L - \frac{1}{2} \end{aligned} \tag{11}$$

where P_μ^0 is an initial muon polarization before the atomic capture. In the second stage, no depolarization occures. In the third stage, the depolarization takes place as a result of the spin-orbit interaction[2-4]. This cascade depolarization is expressed by

$$B_1^i(J', L') = \sum (2L+1)(2J'+1) W(LL'JJ'; 1\tfrac{1}{2})^2 u_1(J1J') B_1^i(J, L) \tag{12}$$

for the E1 transition from the state of (J, L) to (J', L'), where J' is allowed to be either $J-1$ or J by the selection rule of E1 transition. The orbital angular momentum L at which the cascade depolarization becomes effective is usually difficult to be determined precisely. In the present calculation, it is assumed to be 14, where the muons enter into its own orbit passing through the electronic K-shell.

In the fourth stage, the hyperfine interaction becomes effective. The polarization of the hyperfine states $B_1^f(F)$ is calculated in terms of eq.(5). As for the muon polarization $B_1^i(J)$, the results of the atomic cascades by using eqs.(11) and (12) are used. The atomic cascades through the hyperfine states is calculated by using the extended U-coefficient defined in eq.(7). In the atomic cascade process, the following assumptions are adopted. (1) All the cascade processes are subject to allowed E1 transitions. (2) Nuclear level mixing is not effective. (3) The effect of energy splitting of the spin-orbit and the hyperfine structure is neglected for the E1 transition probability. (4) The difference of the radial wave functions of spin-orbit doublet states is not taken into account. (5) Only the cascades through circular orbits are considered. Schematic energy levels and possible transitions in ^{209}Bi muonic atom ($I = \frac{9}{2}$) are presented in Fig.1.

The hyperfine conversion from the higher state of hyperfine doublet $F^>$ to the lower one $F^<$ at the muonic 1s state is evaluated by

$$B_1^f(F^<) = u_1(F^>, 1, F^<) B_1^f(F^>) \tag{13}$$

In the case of a positive nuclear magnetic moment, $F^+ = I + \frac{1}{2}$ is the higher hyperfine state, but on the contrary, for a negative magnetic moment, it is the lower one. Here, we assume that the hyperfine conversion takes place very rapidly compared with negative-muon mean life time. It is verified for high-Z muonic atoms, although in low-Z muonic atoms it is not the case. In this work, the time dependence of the residual muon polarization due to a conversion rate comparable to the muon life time will not be discussed and it has been examined elsewhere

[12,17]. Finally, in order to obtain the residual muon polarization P_μ at the muonic 1s state, the polarization of the hyperfine state $B_1^f(F^<)$ is projected onto the initial muon-spin direction by using the ordinary U-coefficient.

$$P_\mu = B_1^j(\frac{1}{2}) = u_1(F^<, I, \frac{1}{2}) B_1^f(F^<) \qquad (14)$$

4. Result of Calculations

The muon polarization P_μ resulting from the repolarization mechanism is composed of three components: the contribution from the nuclear polarization $(B_1^i(I)B_0^j(J))$, that from the initial muon polarization $(B_0^i(I)B_1^j(J))$ and that from the coupling between the nuclear alignment and the initial muon polarization $(B_2^i(I)B_1^j(J))$. It should be noted that the contribution of the initial muon polarization is nothing but the residual polarization at the 1s state of the muonic atom of a nucleus with a non-zero unpolarized nuclear spin. These three contributions can be expressed as

$$P_\mu = C_I P_I + C_\mu P_\mu^0 + C_A A_I P_\mu^0 \qquad (15)$$

where P_μ^0, P_I and A_I are an initial muon polarization before muon capture, a nuclear polarization and a nuclear alignment, respectively. The coefficients C_I, C_μ and C_A refer to the magnitudes of each contribution mentioned above.

4.1 ^{209}Bi MUONIC ATOM

The results calculated for ^{209}Bi muonic atom ($I = \frac{9}{2}$, $\mu_N = 4.07$) are shown in Fig.2. where each coefficient is plotted against the muonic excited states at which the hyperfine coupling is switched on. These calculations include the hyperfine conversion at the muonic 1s state from the higher state (F=5) to the lower one (F=4). As shown in Fig.2, the contribution of the nuclear polarization (C_I) is the largest among them, and its direction is opposite to that of the nuclear polarization owing to the hyperfine conversion from F=5 to F=4. It also turns out that as the hyperfine interaction becomes effective at the higher muonic state, C_I decreases drastically, while C_μ increases slightly, approaching to a constant value. On the other hand, C_A is negligibly small compared with the other contributions. The comparison with the experimental result will be described in Section 5.

4.2 GENERAL CASES

We extend these calculations to the case of a muonic atom with an arbitrary nuclear spin I from $\frac{1}{2}$ to 5. In these calculations, the muonic state from which the hyperfine interaction becomes effective ranges from the principle quantum number n of 1 to 14. In addition, the cases in the presence and absence of the hyperfine conversion are separately considered. In the case of no hyperfine conversion, we average the polarizations of each state of the hyperfine doublet by taking account of their statistical populations. In the following, each contribution will be examined separately. The results are shown in Fig.3.

4.2.1 *Contribution of C_I*

It turns out that the hyperfine conversion plays a very important role for C_I as long as the hyperfine interaction becomes effective at a relatively lower state. And, as the state for the hyperfine coupling to be switched on moves up to a higher level, C_I after the hyperfine conversion drops down sharpely ; eventually its magnitude is almost the same as that for the case of the absence of the hyperfine conversion. This behavior indicates the following facts;

(1) When the hyperfine interaction takes place at lower muonic states, the polarizations of each of the hyperfine doublet ($F^{\pm} = I \pm \frac{1}{2}$) are found to be approximately close to *each other in magnitude but opposite in sign.* As a result, the hyperfine conversion can enhance the muon polarization by flipping the muon spin of the upper hyperfine state,

while in the absence of the hyperfine conversion these polarizations may be cancelled so that the residual polarization becomes small.

(2) The switching of the hyperfine interaction at a higher muonic state reduces the polarization at the $F^- = I - \frac{1}{2}$ down to almost zero, while that of $F^+ = I + \frac{1}{2}$ does not decrease much. Accordingly, the hyperfine conversion does not make any difference for the contribution of C_I in magnitude. These are illustrated schematically in Fig.4.

4.2.2 *Contributions of C_μ and C_A*

The contribution from the muon polarization C_μ is smaller than C_I. In addition, in contrast to C_I, the hyperfine conversion reduces C_μ when the hyperfine interaction becomes effective at lower muonic states. As the muonic state for the hyperfine interaction to become effective move up to a higher level, C_μ after the hyperfine conversion increases and its absolute value approaches that without the hyperfine conversion. This result can be understood from the analogy to C_I. The contributions of C_μ at each of the hyperfine doublet have the same in magnitude and sign, as long as the hyperfine interaction becomes effective at lower muonic states. When the hyperfine coupling is switched on at higher muonic states, C_μ at $F^- = I - \frac{1}{2}$ decreases greatly. As described before, C_μ is completely the same as the residual polarization in an ordinary muonic atom with unpolarized non-zero nuclear spin. Therefore, this result can be directly compared with that by Bukhvostov[6], and found to be almost consistent. It should be noted that since C_I depends more strongly on the muonic state at which the hyperfine interaction becomes effective than C_μ does, the repolarization study is more useful than the negative muon spin rotation to obtain the information about the hyperfine interaction in muonic atoms.

The contribution from the coupling of the nuclear alignment with the muon polarization C_A is found to be small. Furthermore, because of a small nuclear alignment, C_A can be ignored in all the practical case.

5. Comparison with the experimental results

The first measurement of the repolarization of ^{209}Bi muonic atom was performed at SIN by the Tokyo-Zürich-Lausanne-Louvain collaboration. The result of this experiment was already published [8]. A polarized ^{209}Bi target was obtained with use of ferromagnetic intermetallic compound of BiMn under a very low temperature reached by a dilution refrigerator. In this compound, a strong internal field of about 1 MG was obtained at a site of Bi nucleus by Mn atomic moments which was oriented by an external field of about 6 kG [18,19]. Since the ^{209}Bi nuclear target was polarized perpendicular to the direction of muon beam in this experiment, only the nuclear polarization contributed to the repolarization effect. This experiment reported the polarization-transfer coefficient C_I of -1.07 ± 0.35.

By comparing this experimental value of C_I with the present calculation, the hyperfine interaction was found to become effective at extremely low excited muonic states such as either 1s or 2p state, as shown in Fig.5. This result shows a striking contrast to the case of low-Z muonic atoms where the hyperfine coupling is known to be switched on at relatively higher muonic excited states studied by negative muon spin rotations [13]. This might be explained qualitatively by considering the Z-dependence of the hyperfine splitting and that of the natural width for muonic levels. The hyperfine splitting is proportional to Z^3 due to the overlapping of muonic wave function with the nucleus [5]. On the other hand, the natural width is determined by the strength of radiative E1 transition, whose rate is proportional to Z^4 [3]. Therefore, for high-Z muonic atoms, the hyperfine splitting is smaller than the natural width, so that the hyperfine interaction is switched on at extremely low muonic states. This situation is quite different from the case for the spin-orbit splitting. Because, the spin-orbit splitting is proportional to Z^4, so that the muonic state for the spin-orbit interaction to become effective is expected to have less Z-dependence than that for the hyperfine interaction. Therefore, it is very interesting to consider the Z-dependence of the hyperfine depolarization compared with spin-orbit depolarization. For the sake of detailed discussions, further measurements will be required, but it can be stressed that the extent of repolarization is more sensitive to the hyperfine interaction than the residual polarization in a muonic atom of unpolarized nuclei measured by the negative-muon spin rotation method.

6. Conclusions

In conclusion, we summarize the results of the present studies as follows: (1) For the repolarization mechanism, there are three potential sources: the nuclear polarization, the initial muon polarization and the coupling of the nuclear alignment with the initial muon polarization. Among them, the contribution from the nuclear polarization is the largest. (2) As long as the hyperfine interaction becomes effective at lower excited muonic states, the hyperfine conversion at the muonic 1s state significantly enhances the repolarization effect contributed from the nuclear polarization (C_I), but decreases that from the muon polarization (C_μ). This can be understood by the fact that the μ^- polarizations at F^+ and F^- transferred from the nuclear polarization have the same magnitude with the opposite sign, but those from the muon polarization have the same magnitude with the same sign. (3) Occurrence of the hyperfine interaction at higher excited muonic states and the subsequent atomic cascade reduce drastically the repolarization effect. In particular, when the hyperfine interaction becomes effective at extremely high muonic states, the hyperfine conversion does not increase the repolarization effect any more ; ' resultant effect with the hyperfine conversion is almost the same as that without the hyperfine conversion. This implies that the switching of the hyperfine interaction at higher muonic excited states especially reduces the μ^- polarization of $F^- = I - \frac{1}{2}$ more than that of $F^+ = I + \frac{1}{2}$.

The recent experiment of the repolarization in ^{209}Bi muonic atom suggests that the hyperfine interaction becomes effective at very low excited states in high-Z muonic atoms in contrast to the case of low-Z muonic atoms. This result can be explained qualitatively by considering the Z-dependence of the hyperfine splitting and that of the natural width of the muonic level. The extent of repolarization depends more sharply on the muonic state from which the hyperfine interaction becomes switched on than the residual μ^- polarization in a muonic atom of an unpolarized nucleus with non-zero nuclear spin. Thus, the measurement of the repolarization will give us more information on the role of the hyperfine interaction in atomic cascade process.

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Figure Captions

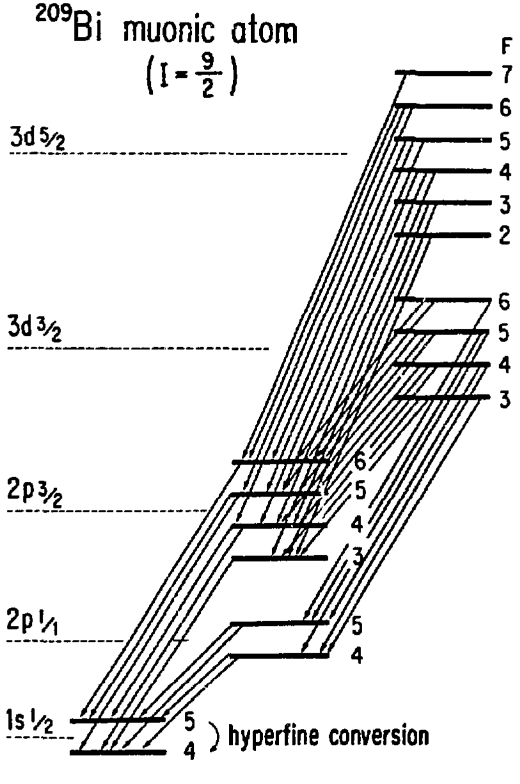
Fig.1 Schematic energy levels of a ^{209}Bi muonic atom. These energy levels are not scaled. The left side of the figure shows the levels unperturbed by the hyperfine interaction. The remainder of the figure shows the splitting due to magnetic dipole hyperfine interaction, although the quadrupole hyperfine interaction is not taken into account for simplicity. Allowed E1 transitions are indicated by arrows. The numbers next to the levels denote the total angular momentum of the corresponding levels.

Fig.2 The polarization-transfer coefficients, C_I, C_μ and C_A, defined in eq.(15) for the ^{209}Bi muonic atom ($I = \frac{9}{2}$, $\mu_N = 4.07$). The hyperfine conversion from the higher hyperfine state ($F = 5$) to the lower state ($F = 4$) are taken into account in this calculation.

Fig.3 The polarization-transfer coefficients C_I, C_μ and C_A are given for the muonic atoms with the nuclear spins I from $\frac{1}{2}$ to 5 as a function of the muonic state from which the hyperfine coupling is switched on. The solid line and dash lines refer to the case of the presence and absence of the hyperfine conversion, respectively. (a) and (b) present the case of the positive nuclear magnetic moment and negative one, respectively.

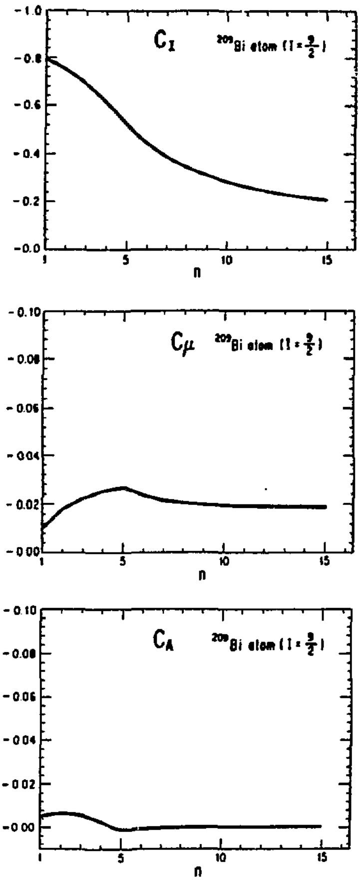
Fig.4 Schematic figures to understand qualitatively the role of the hyperfine conversion to the contribution of C_I. The white and dark arrows refer to the nuclear and muonic polarizations, respectively. (a) is the case when the hyperfine interaction becomes effective at lower excited muonic states, showing that the hyperfine conversion enhances the repolarization effect significantly. (b) is the case when the hyperfine interaction becomes effective at higher excited muonic states and in this case the hyperfine conversion does not increase the residual muon polarization. The detail discussion can be referred to the text. This figure shows the case for positive nuclear magnetic moment, but for negative nuclear magnetic moment, the situation is the same except the sign of the polarization.

Fig.5 The comparison with the experimental result of ^{209}Bi muonic atom recently done at SIN is shown. The result is $C_J = -1.07 \pm 0.35$. This indicates that the hyperfine interaction is switched on at extremetly low excited states in ^{209}Bi muonic atom like either 1s or 2p state.



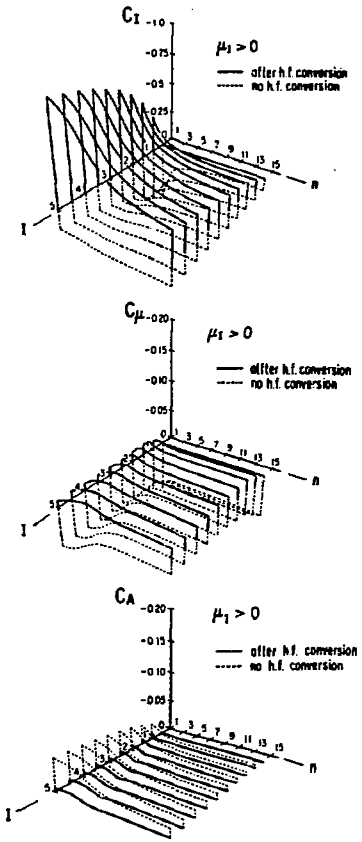
^{209}Bi muonic atom
(I = 9/2)
F
7
6
5
4
3
2
3d 5/2
6
5
4
3
3d 3/2
6
5
4
3
2p 3/2
5
4
2p 1/2
5
4
1s 1/2
hyperfine conversion

Fig. 1



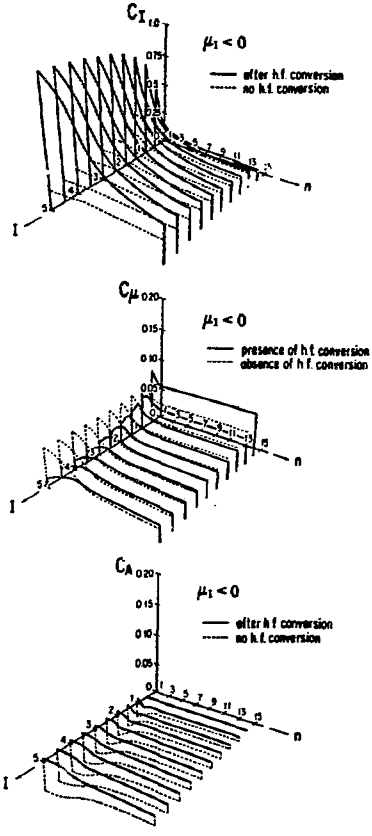
-1.0
-0.8
-0.6
-0.4
-0.2
-0.0
C_I
^{209}Bi atom ($I = \frac{9}{2}$)
1
5
10
15
n
-0.10
-0.08
-0.06
-0.04
-0.02
-0.00
C_μ
^{209}Bi atom ($I = \frac{9}{2}$)
1
5
10
15
n
-0.10
-0.08
-0.06
-0.04
-0.02
-0.00
C_A
^{209}Bi atom ($I = \frac{9}{2}$)
1
5
10
15
n

Fig. 2



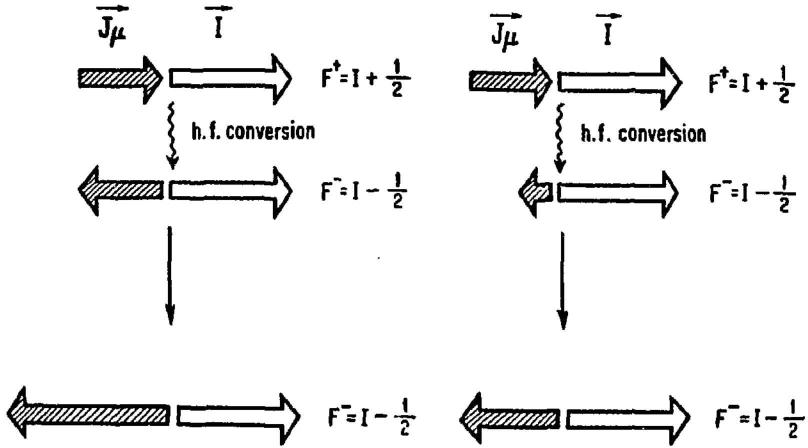
C_I
-1.0
-0.75
-0.5
-0.25
0
μ1 > 0
after h.f. conversion
no h.f. conversion
1 3 5 7 9 11 13 15
n
I
Cμ
-0.20
-0.15
-0.10
-0.05
0
μ1 > 0
after h.f. conversion
no h.f. conversion
1 3 5 7 9 11 13 15
n
I
C_A
-0.20
-0.15
-0.10
-0.05
0
μ1 > 0
after h.f. conversion
no h.f. conversion
1 3 5 7 9 11 13 15
n
I

Fig. 3(a)



C_I
1.0
0.75
0.5
0.25
0
$\mu_1 < 0$
after h.f. conversion
no h.f. conversion
n
I
C_μ
0.20
0.15
0.10
0.05
0
$\mu_1 < 0$
presence of h.f. conversion
absence of h.f. conversion
n
I
C_A
0.20
0.15
0.10
0.05
0
$\mu_1 < 0$
after h.f. conversion
no h.f. conversion
n
I

Fig. 3(b)



(a) from low excited state (b) from high excited state

Fig. 4

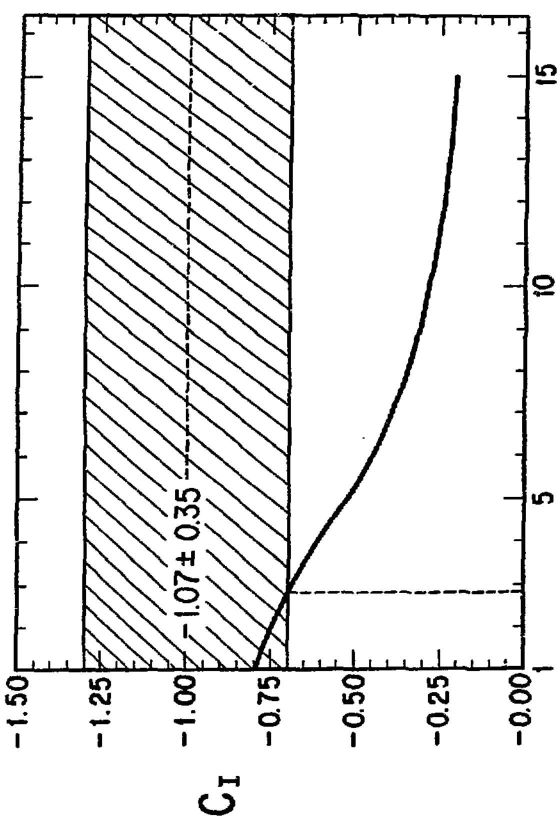


Fig. 5