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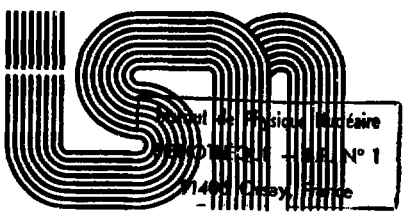

THE MULTIPHONON METHOD AS A DYNAMICAL APPROACH TO OCTUPOLE CORRELATIONS IN DEFORMED NUCLEI

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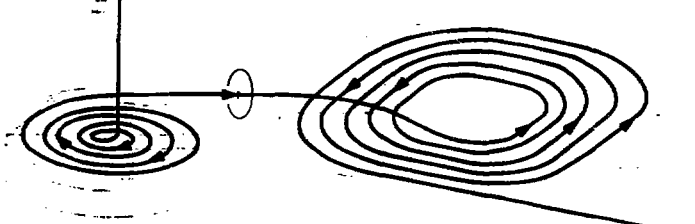

THE MULTIPHONON METHOD AS A DYNAMICAL APPROACH TO OCTUPOLE CORRELATIONS IN DEFORMED NUCLEI

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The octupole correlations in nuclei are studied within the framework of the multiphonon method which is mainly the exact diagonalization of the total Hamiltonian in the space spanned by collective phonons. T is treatment takes properly into account the Pauli principle. It is a microscopic approach based on a reflection symmetry of the potential. The spectroscopic properties of double even and odd-mass nuclei are nicely reproduced. The multiphonon method appears as a dynamical approach to octupole correlations in nuclei which can be compared to other models based on stable octupole deformation.

1. INTRODUCTION

It is well known [1] that strong octupole correlations occur between orbits within a major shell : $2p_{3/2} - 1g_{9/2}$ for the shell 28-50; $2d_{5/2} - 1h_{11/2}$ for the shell 50-82; $2f_{7/2} - 1i_{13/2}$ for the shell 82-126 and $2g_{9/2} - 1j_{15/2}$ for the shell above 126. Typical effects of these correlations (e.g. low lying 1^-, 3^- states, large B(E1) and / or B(E3) values etc...) have been widely observed : in neutron rich Ge-Se around $Z \approx 34$, $N \approx 56$, in neutron rich Ba, around $Z \approx 56$, $N \approx 90$ and

in the actinides around Z = 90 and N = 134. It has been universaly recognized that a deformation of Y_{30} type is needed to understand the low lying level structure of these nuclei. The problem is to know if the octupole degree of freedom leads to a stable deformation or to vibrations. A large amount of work both by experimentalists and theoreticians has been devoted to this problem.

To start our discussion, it seems worthwhile to remind the reader of the different possible forms of the collective potential and of the corresponding schematic pictures of the rotational bands expected in doubly even and odd-mass nuclei. The different possibilities are schematically summarized in Fig. 1. In part a of this figure we show the case of a very high barrier between a reflection asymmetric shape and its mirror image. In such a situation one expects in even-even nuclei, a single band with spin sequence 0^+, 1^-, 2^+, 3^-, 4^+,... In odd-mass nuclei the same situation leads to a rotational band with doubly degenerate states of the same spin but opposite parity.

In part c we give the potential energy curve for a stable octupole deformation where the barrier is lower. In such a case a tunneling between the mirror shapes is possible. In even-even nuclei this gives rise to the existence of two bands of opposite parity. The even spins $I^- = 0^+$, 2^+, 4^+,... are energetically favored compared to the odd spins $I = 1^-$, 3^-, 5^-... In odd-mass nuclei, the lowering of the barrier splits the degenerate states of same spin and opposite parity, and gives rise to *parity doublets* (i.e. couples of two rotational bands with the same spin sequence but with opposite parities).

Finally, in part e , the barrier has entirely disappeared and the potential energy curve corresponds to a

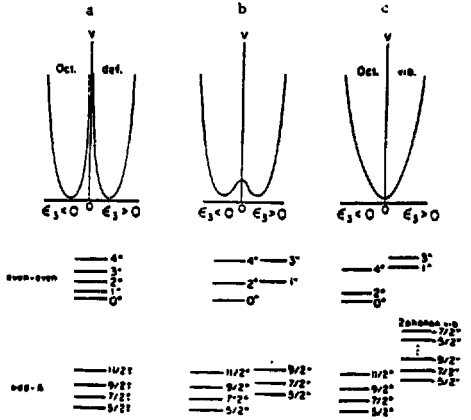

FIGURE 1 Schematic picture of how a rotational band is expected to depend on the potential energy. Part (a) corresponds to a stable octupole deformation; part (b) to a low barrier between a reflection asymmetric shape and its mirror image; part (c) deals with a symmetric shape and octupole K = 0 vibrations.

reflection symmetric deformation ($\varepsilon_3 = 0$). In that case the octupole degree of freedom leads to vibrations. In even-even nuclei one expects, apart from the ground state rotational band with spin sequence $I^- = 0^+, 2^+, 4^+, \ldots$ another low lying rotational band with $I^- = 1^-, 3^-, 5^-, \ldots$ based on an octupole vibrational state with $K = 0$. In odd-A nuclei, each rotational band with band head K^- has a vibrational partner with K^{--}. A first remark becomes evident : at low

energy the expected level schemes in cases b and c are quite similar. As a consequence, it will be difficult, at present, to distinguish between these two situations taking only the mentioned spectroscopic properties into consideration.

In order to relate the predictions of Fig. 1 to experiment, let us choose some illustrative examples in the heavy nuclei (say with $A \geq 218$).

The staggering sequence of levels of positive and negative parity, expected in case a, has been observed in light Ra and Th isotopes which can be reached by (heavy ion, x n) reactions : ^{218}Ra (ref [2]), ^{220}Ra (ref [3-5]), ^{220}Th (ref [6]), ^{222}Th (ref [6-7]) and ^{224}Th (ref [8]). Several remarks may be done. The staggering starts only at spin $I = 4$. The $I=1^-$ and $I=3^-$ members of the band are generally missing since they can usually not be reached in (H.I., x n) reactions. Furthermore in the nuclei, where they are known [4,5,8] their energy does not at all fit into the prediction of an unique band. Finally, one may add that these observations take place in transitional nuclei, with a small β_2 deformation, where the ratio $E(4^+)/E(2^+)$ is not far from the vibrational limit.

Evidence for degenerate states of same spin and opposite parity expected in odd mass nuclei for a stable octupole deformation in case a, is scarce. To my knowledge, only one definite case has been observed [9] in ^{229}Pa.

The rather similar predictions of cases b and c are much easier to relate to experimental results. In even-even nuclei, the displacement of two bands of opposite parity, with even spin energetically favored, is currently observed [10] in heavier Ra and Th. Well separated bands are currently found in heavier actinides.

In odd mass nuclei, several rotational bands with the

same K and different parities have been observed in the same energy range. To illustrate that point let us choose some examples in the odd-mass isotopes of Ac (with odd Z) and Ra (with odd N). In ^{223}Ac a parity doublet with K = 5/2 has been observed [11], in ^{225}Ac two couples of bands with K=3/2$^{\pm}$ and K=5/2$^{\pm}$ have been obtained [12] whereas in ^{227}Ac four bands with K = 1/2^{+}, 3/2^{+}, 1/2^{-} and 3/2^{-} are known [13-16]. Two parity doublets with K = 1/2 and K = 3/2 have been assigned in ^{223}Ra [17-18] and in ^{225}Ra [19], whereas in ^{227}Ra [20] only the couple of bands with K = 3/2 has been confirmed.

Let us now try to sketch what has been done on the theoretical side. The early calculations of Neergard and Vogel [21] treated the low lying negative parity states of even A deformed nuclei as octupole vibrations. The random phase approximation (RPA), in its simplest version, is however unable to reproduce the very low K = 0^{-} band head energy E no matter what the octupole strength may be, and corrections due to the particle-phonon interaction have to be introduced. Since no experimental evidence for two octupole phonon states with K = 0^{+} in the 2E energy area has been found [22-23], it has been gradually recognized that the low frequency octupole K = 0^{-} vibrations known in a wide region of transitional nuclei present anharmonicities which are so strong that one needs a new microscopic approach which is capable of treating such genuine non linear vibrations.

The subsequent effort of the theoreticians has developped in three different directions.

* The largest effort has undoubtedly been done by those who assert that the octupole correlations are so strong that they induce a stable octupole deformation. It is very difficult to give an exhaustive list of all the work done on this aspect, where the Lund-Warsaw collaboration has been

very productive. The breaking of intrinsic reflection symmetry in nuclear ground states has been widely studied. Numerous potential - energy calculations have been done by Möller and Nix[24], Chasman [25], Gyurchovich et al.[26], Leander and coworkers[27-28], Rohozinski and Greiner [29], Nazarewicz et al.[30-31]. In this kind of calculations it has been found that the equilibrium deformation ε_3 is very sensitive to the choice of the single particle potential. The results obtained with a modified oscillator (i.e. with a r^2 term) differ notably from those using a Woods-Saxon or a folded Yukawa potential. A stable octupole deformation is favoured in a rather large number of nuclei around A = 225 only if one introduces a rather flat-bottomed potential, e.g. a folded Yukawa potential fairly constant in the interior region.

* A second possible theoretical approach starts with the assumption that there is no well developed octupole deformation in the ground state. The ε_3 deformation is induced by rotation and shows up only in excited states, at already moderate angular momentum. Such a possibility has been discussed by Nazarewicz et al.[32] and by I.N. Mikhailov et al.[33]. It seems to apply [17] to the transition region, mentioned before, where a staggering sequence of states with opposite parity connected by E1 transitions compete strongly with the stretched E2 transitions.

* The third possible starting assumption for theoreticians is to use a potential with reflection symmetry and to treat the negative parity states as octupole vibrations, exhibiting large anharmonicities. To explain the collective two-phonon states, Soloviev and Shirikova [34] have taken into account the Pauli principle in the two-phonon components of the wave functions within the framework of the quasi particle phonon model. Their model uses, however, a basis

restricted to 0, 1 and 2 phonon states. This approximation appears, *a priori*, too drastic when one looks for the two phonon states. With the use of a reflection symmetric basis, Silvestre-Brac and Piepenbring [35-37] have developed the multiphonon method (MPM). This method is essentially an exact diagonalization of the *total* Hamiltonian in the space spanned by the *collective* phonons. It takes properly into account the Pauli principle. The matrix elements of one and two body operators are calculated with the generalized Wick's theorem and / or recursion formulae [38]. This method is able to describe not only weakly anharmonic vibrations but also the strongly anharmonic motions which prominently appear beyond the RPA critical point. It appears as a dynamical approach to octupole correlations in deformed nuclei and can furnish a microscopic description of finite amplitude collective motion which can be compared to the models based on stable octupole deformation.

In the first part of this series of lectures, we apply the MPM, where the collective space is built only upon the octupole $K = 0^-$ phonon to even-even deformed nuclei. This assumption will be valid for nuclei where the octupole vibrational state $I = 1^-$, $K = 0^-$ is energetically isolated. This will be the case e.g. for light actinides and heavy Barium.

In a second part we extend this version of the MPM to odd-mass deformed nuclei, i.e. we shall restrict ourselves to the case where only one type of phonon plays a role. Special attention will be paid to the arguments usually invoked to assert that some odd-mass actinides have stable octupole deformation.

In a third part we consider two types of phonons $K=0^-$

and $K = 0^+$ as the building blocks for the collective space. This approach will be suited in the even-even nuclei where the 1^- octupole state and the first excited 0^+ state have nearly the same energy, as observed e.g. in some even mass Uranium.

Finally some conclusions are drawn in a last section.

2. APPLICATION OF THE MPM TO EVEN-EVEN DEFORMED NUCLEI WHERE THE OCTUPOLE VIBRATIONAL STATE $K^- = 0^-$ IS ENERGETICALLY ISOLATED.

We introduce an orthonormal basis of quasiparticles a_m fulfilling the usual anticommutation rules

$$\{ a_m, a_n \} = \{ a_m^+, a_n^+ \} = 0$$
$$\{ a_m, a_n^+ \} = \delta_{mn} \qquad (1)$$

The Hamiltonian of the system is expressed in terms of these basic operators

$$H = H_{00} + H_{11} + H_{22} + H_{31} + H_{40} \qquad (2)$$

with

$$H_{00} = c^{te}$$
$$H_{11} = \sum_m \varepsilon_m \, a_m^+ a_m \qquad (3)$$
$$H_{22} = \sum_{pqrs} S_{pqsr} \, a_p^+ a_q^+ a_r a_s$$
$$H_{31} = \sum_{pqrs} R_{pqrs} \, a_p^+ a_q^+ a_r^+ a_s + h.c.$$

$$H_{40} = \sum_{pqrs} P_{pqrs}\, a^{+}_{p} a^{+}_{q} a^{+}_{r} a^{+}_{s} + \text{h.c.}$$

The coefficients E,P,R,S are assumed to be real. In order to condense as much as possible the forthcoming formulae these coefficients must have the same symmetry properties as their corresponding quasiparticle operators e.g.

$$\begin{aligned} P_{pqrs} &= - P_{qprs} = P_{rpqs} = \dots \\ R_{pqrs} &= - R_{qprs} = R_{rpqs} = \dots \\ S_{pqrs} &= - S_{qprs} = - S_{pqsr} = S_{qpsr} \end{aligned} \qquad (4)$$

We suppose that in the two quasiparticle space it is possible to find a collective state described by

$$Q^{+}(K,i) = \frac{1}{2} \sum_{mn} X_{mn}(K,i)\, a^{+}_{m} a^{+}_{n} \qquad (5)$$

where K is the projection of the intrinsic angular momentum on the symmetry axis and where i stands for all other quantum numbers. The X matrix may, for instance, be chosen as the collective solution of the Tamm-Dancoff approximation (TDA). One could also take any other unitary matrix fulfilling the condition

$$X_{mn} = - X_{nm} \qquad (6)$$

Without any loss of generality we assume the X matrix to be real.

Contrary to what is assumed in the quasiboson approximation, or in the interacting boson model (IBM), the entities (5) are no longer considered as bosons since their commutation rules are now

$$\left[\underline{Q}\,(K,i),\ Q^{+}(L,j) \right] =$$

$$- \frac{1}{2}\ \mathrm{Tr}\ (X(K,i)\ X\ (L,j))) + \sum_{mn} \left[X(K,i) X(L,j) \right]_{mn} a_n^{+} a_m \qquad (7)$$

In the first studied case, we restrict ourselves to nuclei where only the octupole $K = 0^{-}$ phonon appears at low energy. We therefore consider only this type of phonon and drop the K and i quantum numbers in (5). It will become necessary to introduce the powers of the matrix X

$$(X^{\lambda})_{pq} = \sum X_{p\, r_1}\ X_{r_1\, r_2} \cdots X_{r_{\lambda-1}\, q} \qquad (8)$$

and the traces

$$\mathrm{Tr}\ (X^{\lambda}) = \sum_{p} (X^{\lambda})_{pp} \qquad (9)$$

We note that matrices X^{λ} with even λ are symmetric while those with λ odd are antisymmetric.

We now introduce the multiphonon states

$$| n > = (n!)^{-1}\ Q^{+\,n}\ | 0 > \qquad (10)$$

These states are not normalized. Their norm $N_n = < n\ n >$ can be calculated[35] with the help of the recursion formula

$$n\ N_n = - \frac{1}{2} \sum_{\lambda=0}^{n-1} N_{n-\lambda-1}\ \mathrm{Tr}\ (X^{2\lambda+2}) \qquad (11)$$

where $N_0 = 1$

Note that the multiphonon states (10) could also have been defined without the factor $(n!)^{-1}$, which is only introduced

in order to simplify relation (11) and the forthcoming formulae.

The matrix elements of the different parts H_{ij} of H can also be evaluated by use of rather simple recursion formulae [36-39] which are hereafter recalled

$$
\begin{aligned}
&\langle n | H_{11} | n \rangle = - \sum_{\ell=0}^{n-1} N_{n-1-\ell} \, \mathcal{E}(\ell+1) \\
&\langle n+2 | H_{40} | n \rangle = 3 \sum_{\ell=0}^{n} N_{n-\ell} \sum_{i,j=0}^{\ell} \mathcal{S}_{i+j,\ell} \, \mathcal{C}(i,j) \\
&\langle n | H_{22} | n \rangle = - \sum_{\ell=0}^{n-1} N_{n-1-\ell} \sum_{i,j=0}^{\ell} \mathcal{S}_{i+j,\ell} \, \mathcal{S}(i,j) \\
&\quad - 2 \sum_{\ell=0}^{n-2} N_{n-2-\ell} \sum_{i,j=0}^{\ell} \mathcal{S}_{i+j,\ell} \, \mathcal{S}'(i,j)
\end{aligned}
\tag{12}
$$

where the dynamical quantities $\mathcal{E}$, $\mathcal{C}$, $\mathcal{S}$ and $\mathcal{S}'$ are

$$
\begin{aligned}
&\mathcal{E}(i) = \mathrm{Tr}\,(E\, X^{2i}) \\
&\mathcal{C}(i,j) = \sum_{pqrs} P_{pqrs} \, (X^{2i+1})_{pq} \, (X^{2j+1})_{rs} \\
&\mathcal{S}(i,j) = \sum_{pqrs} S_{pqsr} \, (X^{2i+1})_{pq} \, (X^{2j+1})_{rs} \\
&\mathcal{S}'(i,j) = \sum_{pqrs} S_{pqsr} \, (X^{2i})_{pr} \, (X^{2j})_{qs}
\end{aligned}
\tag{13}
$$

We note that according to parity conservation the contribution of the H_{31} part, which changes the phonon number by one, vanishes. Furthermore, the matrix of H separates into two parts : one for each parity. The vectors $|2p+1\rangle$ form the basis for the diagonalization of the negative parity states, whereas the states $|2p\rangle$ serve as such for the positive parity ones. To go further, we need to make a choice for H. Since our aim is not to search for a fine agreement with the experimental observations, we simply describe the nuclei as being built of protons and neutrons moving in a deformed Nilsson potential and interacting through a constant monopole pairing force and a charge independent octupole-octupole force. The model Hamiltonian writes then

$$H = H_{sp} + H_p + H_Q \tag{14}$$

where

$$H_{sp} = \sum_{p,m} e_m(p)\, c_m^+(p)\, c_m(p) \tag{15}$$

$$H_p = -\sum_p G_p\, L_p^+\, L_p \tag{16}$$

$$H_Q = -\frac{\chi_3}{2} \sum_{pq} Q_p\, Q_q \tag{17}$$

with

$$L_p = \sum_m c_{-m}(p)\, c_m(p) \tag{18}$$

$$Q_p = \sum_{mn} \langle mp|\, r^3\, Y_{30}\, |np\rangle\, c_m^+(p)\, c_n(p) \tag{19}$$

In these relations p refers to protons or neutrons, whereas m or n labels the single particle states. The canonical Bogoliubov-Valatin transformation is introduced to switch from

particle operators c_m to quasiparticle operators a_m. The explicit form of H_{11}, H_{22} and H_{40} are not reproduced here since they can easily be found in the literature [40].

This formalism has been applied in two regions of nuclei : for even Ra and Th isotopes [39,41] with mass number $222 \leq A \leq 228$ and for ^{146}Ba in ref [42]. Hereafter we present the prominent results of these applications.

In the two cases we use the Nilsson potential with the parameters recommended by Lamm [43]. The calculations are made for a deformation $\varepsilon_2 = 0.15$, which is suited for those nuclei at the edge of a deformed region. The intrinsic matrix elements of the $r^3 Y_{30}$ operator are evaluated according to the prescriptions of Boisson and Piepenbring [44]. The units are chosen so as to express the octupole strength χ_3 in keV. Since the BCS gap parameters Δ_p and Δ_n cannot always be evaluated empirically because the experimental masses are not known, several sets of values have been used for the pairing strength parameters G_p and G_n. The calculations have been made using 30 active levels equitably distributed on each side of the Fermi surface. The BCS equations are solved taking, as usual, the $-Gv^2$ single particle renormalization into account. The X matrix of relation (5) is obtained by solving the secular equation of the Tamm-Dancoff approximation. In fact, the NPM works [35-36] for any choice of an antisymmetric unitary matrix X so that it is not necessary to use the same value χ_3 in the Hamiltonian (17) as for the calculation of the TDA phonon (5). However, if one wants to restrict as much as possible the number of free parameters one may take $\chi_3 = \chi_3^{TDA}$.

For a given set of pairing parameters G_p and G_n, the octupole strength χ_3 is adjusted so as to get, within the

MPM, the lowest 0^- energy $E(0_1^-)$ in the neighborhood of the intrinsic band head energy E_{int}. A rough estimate of the rotational part contained in the observed $E(1^-)$ energy leads to $E(1^-) - E_{int} \approx 10$ keV in the first region and ≈ 20 keV in ^{146}Ba. In the MPM, the eigenstates of the model Hamiltonian are obtained by the diagonalization in a basis containing n_{max} collective octupole phonons. This maximum value n_{max} is chosen so as to get numerical stability for the three lowest eigenvalues. In the numerical calculations reported here $n_{max} = 12$ for the Ra-Th isotopes and $n_{max} = 10$ for ^{146}Ba. For the retained κ_3 value we also search if the random phase approximation (RPA) has any physical solution. The most interesting information consist in the location of the first excited 0^+ state, which we compare to the observed one, whenever this has been confirmed. We note that for the $I^- = K^- = 0^-$ states no rotational energy has to be substracted from the measured value.

In Table I we give the results for four even Ra and two even Th, obtained for the pairing strength G recommended by Nilsson and Prior [45].

$$G_p = \frac{22}{A} \text{ MeV}$$
$$G_n = \frac{16}{A} \text{ MeV} \qquad (20)$$

In each nucleus the selected κ_3 value is so that the secular equation of the RPA has no real collective solution. Furthermore, the TDA energies ω_{TDA} of the first excited 0^- state are quite large compared to the experimental energy $E(0^-)$, demonstrating, once again, the importance of the H_{40} term neglected in the TDA.

The calculated $E(0^+)$ energies of the first excited 0^+ state are systematically larger than $2E(0^-)$ showing the great

TABLE I Calculated intrinsic energies (first line) of the first excited $K^{-} = 0^{-}$ and 0^{+} states for octupole strength parameter χ_3 compared to the measured band head energies (second line) where no rotational effects have been substracted and to the energy ω_{TDA} obtained in the harmonic Tamm-Dancoff approximation. All energies are in keV.

Nucleus	$E(0^{-})$	$E(0^{+})$	$\omega_{TDA}(0^{-})$	χ_3
222	232.	926.	546.	3.925
Ra	242.13	914.0		
224	203.	964.	511.	4.25
Ra	215.98	916.4		
226	246.	997.	528.	4.35
Ra	253.73	824.2		
228	464.	1209.	657.	4.20
Ra	474.14	721.17		
226	221.	939.	523.	3.925
Th	230.4	805.2		
228	318	1017.	566.	3.85
Th	327.9	831.3		

anharmonicity of the octupole vibrations. We note that they are systematically larger than the measured ones. The deviation $E(0^{+})$ calculated $-E(0^{+})$ observed increases with A. It is low for ^{222}Ra and ^{224}Ra but gets quite large for isobars 226 and 228.

These findings suggest that the first excited 0^{+} state in ^{222}Ra and ^{224}Ra could practically be of a pure octupole nature. This is certainly no more the case in the isotopes of Ra and Th with A = 226 and A = 288. In these latter nuclei other $K = 0^{+}$ excited states have been observed [23,46]. The

restriction to a basis (10) with only one building phonon is then no more justified. The coupling to a quadrupole type $K = 0^+$ phonon may play a certain role and a larger basis is needed. This will be the object of our fourth chapter. We have also studied the sensitivity of the results of Table I to variations of the quadrupole deformation ε_2 of the single particle potential and / or of the pairing strength parameters G_p and G_n. The results are qualitatively not altered. The anharmonicities are even enhanced if one introduces stronger pairing like suggested by Neergard and Vogel [21] or Chasman [25]. As a first conclusion, we may say that the MPM, using as a building block the collective octupole $K = 0^-$ phonon, furnishes a microscopic explanation of the existence of a $K = 0^-$ band head at very low energy $E(0^-)$ where the RPA fails in the lightest Ra and Th even isotopes. It explains also why the "two phonon" states were not observed around $2E(0^-)$ in these nuclei.

Our second application concerns ^{146}Ba. This isotope is the most rotational neutron-rich Barium [47-48]. Levels with $I^- = 1^-$ and 3^- have been observed [49] respectively at 739 and 821 keV. The systematics of Pinston [47] suggests a minimum of the 1^- energy in this nucleus. More generally the Barium isotopes with $N \simeq 88$-90 exhibit analogous properties to the heavy nuclei studied before where $Z \simeq 88$-90. In Table II we present the results of the MPM [42]. Several sets of the pairing parameters G_p and G_n have been used. We report the values of the pairing gaps Δ_p and Δ_n, the energy E_{qp} of the lowest two quasiparticle states with $K = 0^-$, the Tamm-Dancoff energy ω_{TDA}, the fitted value χ_3 of the octupole force strength, the energies $E(0_1^-)$ and $E(0_2^-)$ of the two lowest excited states obtained in the MPM. The

TABLE II Calculated intrinsic energies E (0_1^-) and E (0_2^+) of the first excited $K^- = 0^-$ and 0^+ states obtained in the MPM for octupole strength parameter χ_3 and pairing parameters G_p and G_n compared to the result ω_{TDA} of the Tamm-Dancoff approximation and the energy E_{qp} of the lowest two quasiparticles states. All energies are in keV, except the pairing strength parameters for which G/A is in MeV. The ratio R = E (0_2^+) / E (0_1^-) measures the anharmonicities of the octupole vibrations.

G_p	22	23	24	25	26
G_n	15	16	17	18	19
Δ_p	668	752	826	914	1003
Δ_n	431	504	571	655	744
E_{qp}	1019	1167	1303	1472	1649
χ_3	4.75	5.10	5.35	5.5	5.9
ω_{TDA}	901	971	1031	1108	1178
$E(0_1^-)$	724	716	716	727	716
$E(0_2^+)$	1598	1727	1815	1919	1999
R	2.21	2.41	2.53	2.64	2.79

ratio R = E (0_2^+) / E (0_1^-) given in the last line furnishes a measure of the anharmonicities of the octupole vibrations and, in some way, a measure of the importance of the octupole correlations. We would like to emphasize the following points.

The sets of parameters G_p and G_n have been extrapolated from the values suggested for the rare earth nuclei by Nilsson and Prior [45]. They cover a rather large region of values for the corresponding gaps Δ_p and Δ_n including most likely the experimental situation.

In all the cases, the χ_3 value, selected in the MPM, does not lead, in the RPA, to any physical collective solution.

The IDA energies are always much larger than the E (0_1^-) values of the MPM confirming the importance of the parts of the residual interaction neglected in the TDA.

As for the Ra and Th, the calculated E (0_2^+) energies are always larger than 2E (0^-). The exhibited anharmonicities of the octupole vibrations lead to values of R between 2.2 and 2.8. These are significantly smaller than for the light Ra - Th where we found R > 3.

The fact that the octupole correlations appear weaker in ^{146}Ba than in the Ra - Th region may be related to the different opposite parity orbitals involved in each case in the octupole interactions. According to the restricted basis used in this version of the MPM we may urge the reader not to use the results of Table II as a prediction for a precise location of the non yet observed "two phonon octupole vibrational" level in ^{146}Ba.

To measure the strongness of the octupole correlations, in the nuclei we studied, we used the MPM based, *a priori*, on reflection symmetry and we could reproduce the spectroscopic proper-

ties observed at low energy. This result should however not be used, *per se*, as an argument against stable octupole deformation. We simply claim that our description gives similar results to those predicted by models starting with a stable octupole deformation [27,31].

3. EXTENSION OF THIS VERSION OF THE MPM TO THE ODD-MASS DEFORMED NUCLEI

In the preceding chapter, we have seen that the MPM, where only one collective octupole phonon was introduced, was able to explain the spectroscopic properties of some specific even-even nuclei. It seems therefore worthwhile to extend this simple version of the MPM to the odd-mass nuclei of the same region. This new tool will offer the possibility to study the blocking due to the odd particle and its effect on the anharmonicities of the even core. Since it starts with the assumption of a reflection symmetry in the potential it may furnish an alternative approach to the evaluation of the importance of the octupole correlations in odd mass nuclei to be compared to methods starting with a non vanishing stable octupole deformation ($\varepsilon_3 \neq 0$).

It may however be useful to remind the reader some of the changes expected when one switches from even to odd mass nuclei. First, due to the Pauli principle, the odd particle will have some blocking effects. Second, it can also polarize the nuclear core, i.e. change the ε_2 deformation, affect the pairing correlations etc... Third, it is well known that a better knowledge of the single-particle potential is required in the description of odd-mass nuclei than in that of even ones. It is generally admitted that, in the transitional region near Ra and Th, the single proton

states are better determined than the neutron states. We shall therefore illustrate the method with an application to the odd Z nuclei $^{223-225-227}$Ac where some detailed experimental data exist [11-16]. Special attention will be paid to various arguments usually invoked to assert that the odd-mass nuclei of this region have stable octupole deformation.

To extend the MPM with one type of phonon to odd mass nuclei, we introduce the set of states

$$|n;\ p\ > = (n!)^{-1}\ Q^{+n}\ a_p^{+}\ |0\ > \qquad (21)$$

These states are not normalized and in the most general case not orthogonal. Strictly speaking, they do not form a basis. As for the even-even nuclei the problem is to evaluate the overlap matrix elements of states (21) and the matrix elements for one and two body operators between these states. This has been solved in ref. [50] where the following relevant formulas have been derived. For the overlap matrix one has

$$< m;p'|\ n;q\ > = \delta_{mn} \sum_{\ell=0}^{n} N_{n-\ell}(X^{2\ell})_{pq} \qquad (22)$$

where N can be obtained through (11) and where $(X^{0})_{pq} = \delta_{pq}$

For the one-body part H_{11} of the Hamiltonian one gets

$$< m;p\ |H_{11}|n;q\ > = \delta_{mn} \Big\{ \sum_{\ell=0}^{n} N_{n-\ell} \sum_{i,j}^{\ell} \delta_{i+j,\ell}\ (X^{2i}\ E X^{2j})_{pq}$$

$$+ \sum_{\ell=0}^{n-1} N_{n-1-\ell} \sum_{i,j=0}^{\ell} \delta_{i+j,\ell}\ (X^{2i+1}\ E\ X^{2j+1})_{pq} - \tilde{E}(i+1)\ X^{2j}_{pq} \Big\}$$

(23)

where $\mathcal{E}(i)$ is the same as in (13).
The relations for the two body parts of H are more involved

$$\langle m;p|H_{40}|n;q\rangle =$$

$$\delta_{m,n+2} \sum_{i=0}^{n} N_{n-2} \left[\sum \mathcal{P}(i,j)(X^{2k})_{pq} - 4\,\mathcal{P}_{pq}(i,j,k) \right] \tag{24}$$

where $\mathcal{P}(i,j)$ is given in (13) and where

$$\mathcal{P}_{pq}(i,j,k) = \sum_{rscu} P_{rstu} (X^{2i+1})_{rs} (X^{2j+1})_{tq} (X^{2k})_{up} \tag{25}$$

$$\langle m;p|H_{22}|n;q\rangle =$$

$$\delta_{mn} \Big\{ \sum_{i=0}^{n-1} N_{n-i-1} \sum \Big[2\,\mathcal{S}^{(4)}_{pq}(i,j,k) - 4\,\mathcal{S}^{(1)}_{pq}(i,j,k) - \mathcal{S}(i,j)\,(X^{2k})_{pq} \Big]$$

$$+ 2 \sum_{i=0}^{n-2} N_{n-i-2} \sum \Big[2\,\mathcal{S}^{(3)}_{pq}(i,j,k) - \mathcal{S}^{(2)}_{pq}(i,j,k) - \mathcal{S}'(i+1,j+1)(X^{2k})_{pq} \Big] \Big\} \tag{26}$$

where $\sum$ is a summation over i,j,k varying from 0 to ℓ such as

$$i + j + k = \ell$$

The quantities $\mathcal{S}(i,j)$ and $\mathcal{S}'(i,j)$ are those of relation (13). In addition we have introduced four new dynamical quantities

$$\mathcal{Y}^{(1)}_{pq}(i,j,k) = \sum_{rstu} S_{rstu} (X^{2i})_{rp} (X^{2j+2})_{su} (X^{2k})_{tq}$$

$$\mathcal{Y}^{(2)}_{pq}(i,j,k) = \sum_{rstu} S_{rstu} (X^{2i+2})_{rp} (X^{2j+1})_{sq} (X^{2k+1})_{ut}$$

$$\mathcal{Y}^{(3)}_{pq}(i,j,k) = \sum_{rstu} S_{rstu} (X^{2i+2})_{rt} (X^{2j+1})_{sq} (X^{2k+1})_{up}$$

$$\mathcal{Y}^{(4)}_{pq}(i,j,k) = \sum_{rstu} S_{rstu} (X^{2i+1})_{rs} (X^{2j+1})_{up} (X^{2k})_{tq} \tag{27}$$

Contrary to the case of the even-even nuclei, the H_{31} part of the Hamiltonian has now some non vanishing matrix elements when the quasiparticle states labelled by p and q are of different parity

$$\langle m;p | H_{31} | n;q \rangle =$$

$$3 S_{m,n+1} \sum_{i=0}^{n} N_{n-i} \mathcal{R}^{(1)}_{pq}(i,j,k)$$

$$+ \sum_{i=0}^{n-1} N_{n-1-i} \sum_{j} \left[\mathcal{R}^{(2)}_{pq}(i,k,k) - \mathcal{R}^{(3)}_{pq}(i,j,k) - \mathcal{R}(i,j) (X^{2k})_{pq} \right] \tag{28}$$

where

$$\mathcal{R}(i,j) = \sum_{rstu} R_{rstu} (X^{2i+1})_{rs} (X^{2j+2})_{tu}$$

$$\mathcal{R}^{(1)}_{pq}(i,j,k) = \sum_{rstu} R_{rstu} (X^{2i+1})_{rs} (X^{2j})_{tp} (X^{2k})_{uq}$$

$$\mathcal{R}^{(2)}_{pq}(i,j,k) = \sum_{rstu} R_{rstu} (X^{2i+1})_{rq} (X^{2j})_{sp} (X^{2k+2})_{tu}$$

$$\mathcal{R}^{(3)}_{pq}(i,j,k) = \sum_{rstu} R_{rstu} (X^{2i+1})_{rs} (X^{2j+1})_{tq} (X^{2k+1})_{up} \tag{29}$$

Note that the different dynamical quantities given here could be defined in several alternative forms due to the *symmetry properties of the P, R and S coefficients. Further* we remark that the dynamical quantities of the even nuclei appear always as coefficients of an even power matrix element $(X^{2i})_{pq}$. Since the model Hamiltonian is chosen to be an hermitian operator, we have the following symmetry properties :

$$\begin{aligned}
\mathcal{S}(i,j) &= \mathcal{S}(j,i) \\
\mathcal{S}'(i,j) &= \mathcal{S}'(j,i) \\
\mathcal{S}^{(1)}_{pq}(i,j,k) &= \mathcal{S}^{(1)}_{qp}(k,j,i) \\
\mathcal{S}^{(3)}_{pq}(i,j,k) &= \mathcal{S}^{(3)}_{qp}(i,k,j) \\
\mathcal{S}^{(4)}_{pq}(i,j,k) &= -\mathcal{S}^{(2)}_{qp}(k-1,j,i)
\end{aligned} \tag{30}$$

Further simplifications arise according to the conservation of the parity and to the special value K = 0 characterizing the phonon used as the building block. E.g. on one hand the elements $(X^{2i})_{pq}$, $\mathcal{R}_{pq}$ and $\mathcal{S}_{pq}$ vanish if the states p and q have projections Ω_p and Ω_q of the angular momentum of opposite signs; on the other hand, the elements $(X^{2i+1})_{pq}$ and $\mathcal{R}_{pq}$ vanish when $\Omega_p - \Omega_q > 0$.

If, in the truncation of the basis (21), we can restrict ourselves to only one doubly degenerate state for each gi-

ven K^{π} and parity the situation gets even simpler. In such a case the states (21) are orthogonal and standard techniques can be used for the diagonalization procedure.

We now illustrate this extension of the MPM to odd mass deformed nuclei with the odd mass Actinium [51]. In the isotopes with mass number 223, 225 and 227 only six rotational bands with different K^{π} quantum numbers have been identified [11-16] at low energy. They have $K^{\pi} = 1/2^{\pm}$, $3/2^{\pm}$ and $5/2^{\pm}$. In a Nilsson type potential with standard parameters [43], for an odd number Z = 89 of protons, six orbitals having these characteristics arise. For positive parity we have the orbitals 1/2+ 660, 3/2+ 651 and 5/2 + 642 which are $i_{13/2}$ members. For negative parity, one finds the states 1/2- 530 of the $f_{7/2}$ sub-shell and the 3/2- 532 and 5/2- 523 from $h_{9/2}$.

Since the knowledge of the single-proton potential is not good enough to search for a fine agreement with the experimental results, we may, in a first approximation, restrict index p in (21) to the six values corresponding to these six quasiproton states when looking for the lower part of energy spectrum. We note that this restriction does not concern the definition (5) of the octupole phonon, where 30 proton and as many neutron states are considered. We are then in the special conditions where the evaluation of the overlap matrix elements reduces to the calculation of norms. For the results we shall present in this chapter, the maximum number of collective phonons introduced in (21) to obtain, for each K^{π}, the numerical stability of the three lowest eigenstates, is $n_{max} = 12$.

As for the even-even neighbouring nuclei treated in section 2 we have used a quadrupole deformation $\varepsilon_2 = 0.15$ Since the odd proton may polarize the core and lead to a greater

deformation we have also made some runs of our codes with $\varepsilon_2 = 0.20$. The sensitivity of the results versus a variation of the pairing strength has been studied. Three sets of values for G_p and G_n recommended respectively by Nisson and Prior eq (20) and ref [45], Neergard and Vogel [21] and Chasman [25] have been introduced.

Special attention has been paid to three observables.

a) The existence of parity doublet bands (PD)

The [illegible] energy splitting of the observed [illegible] has often been considered [9,16,25,52,53,54] as a signature of stable octupole deformation. It is therefore of special interest to see if the PD arise from a single parity mixed Nilsson orbital ($\varepsilon_3 \neq 0$) or from two different intrinsic orbitals obtained with reflection symmetry ($\varepsilon_3 = 0$) according to some peculiarities of the shell structure in the actinides nuclei. If one wants to compare the MPM energies to the observed ones, one needs to evaluate the rotational contribution to the measured values. This latter contains three parts. The first one $\frac{\hbar^2}{2J}\left[I\,(I+1) - K^2\right]$ is easy to get. The second one corresponding to the recoil term $\frac{\hbar^2}{2J}\left[\langle j^2\rangle - \ldots^2\right]$ cannot be calculated and is expected to be negligible. The third one is due to the Coriolis coupling which connects states with K values differing by one unit. This last part can be obtained within the frame of the unified model of Bohr and Mottelson. Whenever possible, we use a best fit procedure [55,56] in the inversion of the eigenvalue problem of H. Taking into account these corrections, we obtain from the observed energies, the expected parity doublet splittings $\Delta E(K) = E(K^+) - E(K^-)$. For $K = 1/2$ a value $\Delta E(1/2) =$ 123 keV is found in ^{227}Ac; for $K = 3/2$ we get $\Delta E(3/2) =$ − 10 kev in ^{225}Ac and + 9 keV in ^{227}Ac; finally for $K = 5/2$

one has $\Delta E(5/2) = 63$ keV in ^{223}Ac and 37 keV in ^{225}Ac.

TABLE III Energy splittings $\Delta E(K) = E(K^+) - E(K^-)$ (in keV) for ^{223}Ac and a deformation $\varepsilon_2 = 0.15$. The calculations are made for different sets of the pairing force in the BCS approximation (first column) and with the MPM (second column).

$\varepsilon_2 = 0.15$	Nilsson-Prior		Neergard		Chasman	
	BCS	MPM	BCS	MPM	BCS	MPM
$\Delta E(1/2)$	- 75	-64	- 75	-56	-72	-41
$\Delta E(3/2)$	-168	-93	-103	15	-44	111
$\Delta E(5/2)$	160	123	139	117	117	114

TABLE IV Same as in Table 3 but for $\varepsilon_2 = 0.20$

$\varepsilon_2 = 0.20$	Nilsson-Prior		Neergard		Chasman	
	BCS	MPM	BCS	MPM	BCS	MPM
$\Delta E(1/2)$	41	89	36	115	31	147
$\Delta E(3/2)$	-196	-79	-157	- 1	-121	73
$\Delta E(5/2)$	22	36	18	34	14	49

In Tables III and IV we give the results for $\Delta E(K)$ obtained for several sets of pairing force parameters and two values of the deformation $\varepsilon_2 = 0.15$ and $\varepsilon_2 = 0.20$, in a pure BCS calculation (first column) and in the MPM approach (second column). We remark that the PD splittings are quite sensitive to the deformation of the Nilsson field and / or to the pairing force. However, we see that, in each case, the MPM values are closer to the empirical values than the BCS values. For $\varepsilon_2 = 0.20$ and strong pairing strength parameters the three calculated PD splitting of the MPM have values in reasonable agreement with the empirical values. We may therefore conclude that by selecting reasonable values of ε_2, G_p and G_n one can reproduce the empirical PD energy splittings. Furthermore, at least for the K = 3/2 doublets one may obtain very narrow splittings. As a consequence it becomes clear that the existence of narrow energy separations is not typical of a stable octupole deformation as often claimed in the literature.

b) The decoupling factors of the K = 1/2 bands

The comparison of the experimental values of the decoupling parameters to the prediction of theoretical models has often been used [9,16,25,53,54] as one of the criteria for stable octupole deformation. A detailed analysis [57] of the results concerning the odd N nucleus ^{225}Ra has however shown that the observed properties of the decoupling factors are necessary but not sufficient to assert the stable octupole deformation. It is therefore of great interest to determine microscopically the decoupling factors within the MPM where one starts with $\varepsilon_3 = 0$.

Experimental information on K = 1/2 in Ac nuclei exist only for the isotope 227. From the energy level scheme one has

to extract the empirical value of the decoupling factors. Several ways may be used. The first one is to assume that the energies of the members of a K = 1/2 band follow the simple rotational law

$$E(I) = e(K) + \frac{\hbar^2}{2J}\{I(I+1)+(-1)^{I+1/2}(I+1/2)\,a\} \qquad (31)$$

To determine a, one needs three levels (i.e. two energy differences). The problem is then to choose these among all observed members of the rotational band.

For the 1/2 - 530 band, 6 levels are known and there are 20 possibilities for the evaluation of the empirical values of a. The obtained values are spread over a rather large region since

$$-2.47 < a < -1.82 \qquad (32)$$

As a consequence, it is unphysical to give empirical values of a with 4 digits as done by some authors [16].

For the 1/2 + 660 band, 5 levels are identified and one has 10 possibilities to calculate a the values of which being so as

$$4.43 < a < 5.60 \qquad (33)$$

To this way using relation (31) we prefer to take the Coriolis coupling between K = 1/2 and K = 3/2 bands into account. To achieve this program we use again the best fit procedure of ref [55,56]. The fitted empirical values of a vary slightly with the number of levels introduced in the fit but to a much smaller extent than in rel (32,33). Within this procedure we get $a(1/2^-) = -2.16$ and $a(1/2^+) = 5.3$ which are quite different from the values $a(1/2^-) = -2.012$ and $a(1/2^+) = 4.556$ given in [16], putting a part of the conclusions of [16] back into the melting pot. In the MPM calculation one gets several K = 1/2 states of opposite parities exhibiting nearly opposite decoupling factors. As a consequence, this

property cannot either be used as an evidence for stable octupole deformation as often claimed [9,16,25,53,54]. Furthermore, the values obtained within the MPM for the decoupling factors of the two lowest $K^{\pi} = 1/2$ states : - 3.32 for $1/2^{-}$ and 6.51 for $1/2^{+}$ ressemble more to the empirical values (32,33) than to the values ± 3.1 obtained by Sheline [16] using a stable octupole deformation. It is also instructive to compare the values of the MPM to those of the Nilsson model - 3.07 and 6.88. For the positive parity state the MPM result is slightly smaller than the Nilsson value and nearer to the empirical one. The opposite is true for the negative parity state. This should not be interpreted as a failure of the MPM. We should remind the reader that we used a rather restricted basis of diagonalization. An enlargment of the K = 1/2 basis to the state 1/2-541, which in the Nilsson model has a large positive decoupling factor, may certainly have some effects on the MPM which would probably go in the right way.

c) E1 transitions with $\Delta K = 0$

On one hand, the enhancement of the E1 transition linking members of the PD bands has been considered [52] as associated with stable octupole deformation.
On the other hand, early results [58] in the rare earth region have shown that the enhancement of the E1 transition with $\Delta K = 0$ can be explained by a small coupling of the odd quasiparticle to the octupole K = 0 vibration of the even core. It seems therefore worthwhile to study these E1 transitions in the framework of the MPM. In Table V we present the absolute values of the E1 matrix elements between the intrinsic states. In the firstcolumn we give the values in a pure Nilsson model; in the second one the corrections

TABLE V . Absolute values of the E1 matrix elements between the intrinsic Nilsson state, obtained without and with the introduction of the pairing.

	Nilsson	Nilsson + BCS	MPM
$\lvert\langle 1/2^+ \lvert E1 \rvert 1/2^- \rangle\rvert$	0.0115	0.000878	0.136
$\lvert\langle 3/2^+ \lvert E1 \rvert 3/2^- \rangle\rvert$	0.00585	0.00134	0.153
$\lvert\langle 5/2^+ \lvert E1 \rvert 5/2^- \rangle\rvert$	0.00468	0.00221	0.296

due to the pairing correlations have been added; finally the last column gives the values obtained in the MPM. The observed enhancement is of two order of magnitude for the matrix elements, i.e. of 10^4 for the transition probability. The observed E1 transition strength provides evidently from the core where $\langle 0^+ \lvert E1 \rvert 1^- \rangle = 0.384$ in the case reported in Table V. We would like to emphasize the following point : contrary to the pure individual Nilsson model, with or without $\varepsilon_3 \neq 0$ and / or BCS correlations, the neutrons contribute to the E1 transition through the core. As a consequence, if one switches from one Ac isotope to another, the proton field is practically the same, whereas the neutron situation is different. We found that such a modification may be enough to change by a factor of 10 the E1 core matrix element from one isotope to another. This may be a possible explanation of the lowering by two orders of magnitude of the B(E1) between the two members of the $K = 3/2$ parity doublet when one goes from ^{225}Ac to ^{277}Ac.

From these properties of the E1 obtained in the multiphonon method we may therefore conclude that the observed E1 en-

hancement between members of a PD may not be used as an evidence for stable octupole deformation.

The analysis of the properties of the three observables, a, b and c has clearly shown that the related criteria often used as an evidence for stable octupole deformation are necessary, but not at all sufficient to assert that $\varepsilon_3 \neq 0$. It seems therefore worthwhile to search for some specific features obtained when starting with the assumption $\varepsilon_3 = 0$. In Fig. 2 we present a typical intrinsic energy level scheme obtained within the MPM for ^{223}Ac. The quadrupole deformation is $\varepsilon_2 = 0.20$ and the pairing force parameters those of Chasman. All intrinsic states up to 1 MeV are shown. To make the figure readable, it has been separated in several columns. That on the r h s shows the situation in the even core, the three others correspond respectively to K = 1/2, 3/2 and 5/2. On the left of the K = 1/2 levels we give the values of the calculated decoupling factors discussed previously in subsection b. The numbers beside the lines linking the levels give the calculated values of

$< f \quad E1(\Delta K = 0) \; i >$ in arbitrary units. Those corresponding to the lowest states have been discussed in subsection c and Table V.

We would like to point on the most striking feature observed in Fig. 2 : we note a systematic appearance at low energy of two parity doublets for each value of K. Between the levels given in Fig. 2 and higher lying states there exists a gap of $\simeq$ 1 MeV for each K. Further, we remark that the second PD of a given K has a different order of the parities compared to the first PD. This appeared to be a rather general rule in all our calculations. The only exceptions we notice appeared in the case where the first PD energy splitting was very small. This striking property of our MPM cal-

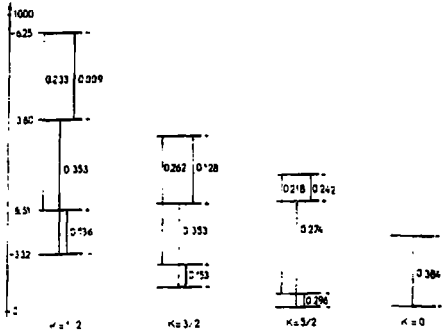

FIGURE 2 Intrinsic energy level scheme obtained within the MFM for ^{223}Ac, $\varepsilon_2 = 0.20$, and the pairing force suggested by Chasman. On the left of the K = 1/2 levels one finds the values of the calculated decoupling factors; beside the lines linking the levels the calculated values of $\langle \psi_f | E1 (\Delta K = 0) | \psi_i \rangle$ in arbitrary units.

culations seems not to be observed in the spectra obtained e.g. by Chasman [25] in his microscopic approach based on a stable octupole deformation. It may therefore furnish an interesting test to choose between the two assumptions $\varepsilon_3 = 0$ or $\varepsilon_3 \neq 0$. Further experimental work in this direction (or further analysis of still available data at higher energy) would be therefore of great interest.

4. APPLICATION OF THE MPM TO EVEN-EVEN DEFORMED NUCLEI WHERE TWO K = 0 COLLECTIVE STATES WITH DIFFERENT PARITY ARE OBSERVED IN THE SAME ENERGY REGION

In section 2, we restricted ourselves to one type of phonons as a building block of the multiphonon basis. This assumption can be considered as justified in the cases where one collective vibrational state appears energetically isolated from all other intrinsic excitations. In the even-even nuclei where several collective states appear in a same energy region such a restricted basis is no more suited and one has to recourse to a more elaborated version of the MPM [38] using either the generalized Wick's theorem or coupled recursion formulae.

In practical situations, one has to calculate the elements of the overlap matrix and of the Hamiltonian. For this kind of problem, the recursion formulation of the problem is more easily handled, at least when only a few types of phonons are involved. Up to now, only cases, where two types of phonons are introduced, have been studied [59-60]. We shall illustrate this version of the MPM [59] for the case where the two types of phonons used as building blocks are

$$Q_1^{\pm} = \frac{1}{2} \sum (X1)_{mn} \; a_m^{\pm} \; a_n^{\pm} \qquad (34)$$

with $K^{\pi} = 0^+$ and

$$Q_2^{\pm} = \frac{1}{2} \sum (X2)_{mn} \; a_m^{\pm} \; a_n^{\pm} \qquad (35)$$

with $K^{\pi} = 0^-$

Such a truncation will be suited in nuclei where two collective bands characterized by $K^{\pi} = 0^+$ and $K^{\pi} = 0^-$ arise at

nearly the same energy. This is the case, e.g. in ^{234}U, which will serve as an illustrative example. As in section 2, we use the TDA solutions as the building phonons. For the $K^\pi = 0^-$ phonon we introduce the collective solution. According to the non conservation of the number of particles, the $K^\pi = 0^+$ TDA solutions contain some spurious contributions. Among the two lowest TDA solutions we retain as the building phonon, the second one, which appears to contain the smallest spurious component and ressembles more to the physical RPA phonon.

With the phonons (34) and (35) we build the multiphonon states

$$|p,q\rangle = (p!\,q!)^{-1}\ (Q_1^-)^p\ (Q_2^-)^q\ |0\rangle \qquad (36)$$

To calculate the overlap matrix elements

$$F(p',q';pq) = \langle p'q'|p\,q\rangle \qquad (37)$$

one needs another matrix

$$A_{mn}(p'q';pq) = \langle p'q'|a_n a_m|p\,q\rangle \qquad (38)$$

An application of the general formulae of [38] to the case of states (36) (or a direct rederivation) leads to the following coupled recursion formulae

$$\begin{aligned} p'\,F(p'q';pq) &= -\tfrac{1}{2}\,\mathrm{Tr}\,(X1\ A(p'-1,q';p\,q)\\ q'\,F(p'q';pq) &= -\tfrac{1}{2}\,\mathrm{Tr}\,(X2\ A(p',q'-1;p\,q)\\ A(p'q';pq) &= X1\ F(p'q';p-1,p) + X2\ F(p'q';p,q-1)\\ &\quad + X1\ A(p-2,q;p'q')X1 + X2\ A(p,q-2;p'q')X2 \end{aligned} \qquad (39)$$

$$+X1\ \ A\ (p-1,q-1;p'q')\ X2 + X2\ A(p-1,q-1;p'q')\ X1$$

where we start with $F\ (00,00) = \langle 0 \mid 0 \rangle = 1$.

We note that :

$$\begin{aligned} A\ (00;\ 10) &= X1 \\ A\ (00;\ 01) &= X2 \end{aligned} \tag{40}$$

If we want the one phonon states to be normalized to 1, we have to choose the antisymmetric matrices so as

$$Tr\ (X1)^2 = Tr\ (X2)^2 = -\ 2 \tag{41}$$

Furthermore, the orthogonality of the two building phonons leads to the condition

$$Tr\ (X1\ X2) = 0 \tag{42}$$

We note that multiphonon states with $p + q > 1$ are neither normalized nor orthogonal.

The computational task is reduced by some selection rules due to the conservation of the number of phonons and the special K values of the building phonons (34) and (35). For instance, the elements (37) vanish if

$$(-1)^q \neq (-1)^{q'}$$

$$\text{or } p' + q' \neq p + q \tag{43}$$

Moreover, there will be two types of A matrices, one connecting states of same parity, and another relating those of opposite parities. Further simplications arise from the time reversal properties of the A matrices

$$A_{m\,\bar{n}}\ (p'q';p\ q) = -\ A_{\bar{m}\,n}\ (p'q';\ p\ q) \tag{44}$$

where $\bar{n}$ is the time reversal state of n.

As shown in [38], all the matrix elements of one body operators can be expressed in function of the quantities F

$$B_{1234}(p'q';pq) = \sum_{abcd} \Big\{ -(X1)_{ab}(X1)_{cd} F'(p'q';p-2,q) - (X1)_{ab}(X2)_{cd} F(p'q';p-1,q-1)$$

$$-(X2)_{ab}(X2)_{cd} F'(p'q';p\,q-2) + (X1)_{ab} A_{cd}(p'q';p-1\,q) + (X2)_{ab} A_{cd}(p'q',p,q-1)$$

$$+(X1)_{1a}(X1)_{2b}(X1)_{3c}(X1)_{4d} B_{abcd}(p-4,q;p'q') + (X2)_{1a}(X2)_{2b}(X2)_{3c}(X2)_{4d} B_{abcd}(p,q-4;p'q')$$

$$+\big[(X1)_{1a}(X2)_{2b}(X2)_{3c}(X2)_{4d} + (X2)_{1a}(X1)_{2b}(X2)_{3c}(X2)_{4d} + (X2)_{1a}(X2)_{2b}(X1)_{3c}(X2)_{4d}$$

$$+(X2)_{1a}(X2)_{2b}(X2)_{3c}(X1)_{4d}\big] B_{abcd}(p-1,q-3;p'q') + \big[(X2)_{1a}(X1)_{2b}(X1)_{3c}(X1)_{4d}$$

$$+(X1)_{1a}(X2)_{2b}(X1)_{3c}(X1)_{4d} + (X1)_{1a}(X1)_{2b}(X2)_{3c}(X1)_{4d} + (X1)_{1a}(X1)_{2b}(X1)_{3c}(X2)_{4d}\big]$$

$$B_{abcd}(p-3,q-1;p'q') + \big[(X1)_{1a}(X1)_{2b}(X2)_{3c}(X2)_{4d} + (X2)_{1a}(X2)_{2b}(X1)_{3c}(X1)_{4d}$$

$$+(X1)_{1a}(X2)_{2b}(X1)_{3c}(X2)_{4d} + (X2)_{1a}(X1)_{2b}(X2)_{3c}(X1)_{4d} + (X1)_{1a}(X2)_{2b}(X2)_{3c}(X1)_{4d}$$

$$+(X2)_{1a}(X1)_{2b}(X1)_{3c}(X2)_{4d}\big]\; B_{abcd}(p-2,q-2;p'q') \Big\} \qquad (47)$$

and A. As an example, for the part H_{11} of H one gets

$$\langle p'q' | H_{11} | p\ q \rangle = - \mathrm{Tr}\ (EX1\ A(p-1,q;p'q')) - \mathrm{Tr}\ (EX2\ A(p,q-1;p'q')) \tag{45}$$

For the two body operators, such as H_{40}, H_{31} and H_{22}, the derivation is much more involved, and one has to introduce a new quantity

$$B_{1234}(p'q';p\ q) = \langle p'q' | a_4\ a_3\ a_2\ a_1 | p\ q \rangle \tag{46}$$

for which the recursion formula (47) holds .

In (47) the summation $\sum_{abcd}$ runs over all permutations (with their sign) of indices 1, 2, 3 and 4 which give different contributions, e.g. :

$$\sum_{abcd} (X)_{ab}(X)_{cd} = X_{12}\ X_{34} - X_{13}\ X_{24} + X_{14}\ X_{23} \tag{48}$$

The H_{40} term is directly given in terms of B. For the H_{31} part, we have

$$\langle p'q' | a_1^+\ a_2^+\ a_3^+\ a_4 | p\ q \rangle = \sum_r (X1)_{4r}\ B_{123r}(p-1\ q;p'q') + \sum_r (X2)_{4r}\ B_{123r}(p,q-1;p'q') \tag{49}$$

Whereas for the H_{22} contribution we may use :

$$\langle p'q' | a_1^+\ a_2^+\ a_4\ a_3 | p\ q \rangle =$$

$$(X1)_{34}\ A_{12}\ (p-1,q;p'q') + (X2)_{34}\ A_{12}\ (p,q-1;p'q')$$

$$+ \sum_{rs} \{ (X1)_{3r} (X1)_{4s} B_{12\,sr}(p-2,q;p'q') + \left[(X1)_{3r} (X2)_{4s} + (X1)_{4s} (X2)_{3r}\right] B_{12\,sr}(p-1,q-1;p'q') + (X2)_{3r} (X2)_{4s} B_{12\,sr}(p,q-2;p'q') \} \qquad (50)$$

Formally the relations (47,49,50)are rather simple. But in practical situations, we are faced with the two usual major difficulties in the numerical task : how to save computer time ? and how to store all the quantities needed in the calculations ?

The main difficulty arises from the matrix element B of rel. (46). A priori, if we consider thirty active levels and, if the calculations are made up to nine phonons, we get for one kind of particles

$$30^4 \times 10^4 = 81 \times 10^8 \text{ elements}$$

Fortunately, the conservation of the number of phonons, of the parity and of K leads to some selection rules for rel. (46) :

$$p' + q' = p + q - 2$$
$$(-1)^{q'} = \bar{\nu}_1 \bar{\nu}_2 \bar{\nu}_3 \bar{\nu}_4 (-1)^q \qquad (51)$$
$$k_1 + k_2 + k_3 + k_4 = 0$$

where k_i and $\bar{\nu}_i$ are the quantum numbers of the quasiparticle state $a_i^+ 0 >$ for the projection of the angular momentum and parity.

Moreover, for a given set of multiphonon states $p,q >$ and $p'q' >$ and for a given set of indices 1 2 3 4 there are 4! = 24 matrix elements B, which differ at most by a phase

factor.
As a consequence, only one of these 24 B is calculated and stored.

Moreover, according to time reversal properties

$$B_{1\ 2\ 3\ 4}\ (p'q';\ p\ q) = B_{\bar{4}\ \bar{3}\ \bar{2}\ \bar{1}}\ (p'q';\ p\ q) \qquad (52)$$

the number of stored B's can further be reduced.
In spite of this, we shall have to restrict ourselves to twenty active levels for each type of particles to be able to handle the problem in reasonable computer time.

In addition to the Hamiltonian (14) of chap. 2, we introduce here a residual quadrupole-quadrupole force

$$H_Q = -\ \frac{\chi_2}{2}\ \sum_{pq}\ Q_p\ Q_p \qquad (53)$$

where

$$Q_p = \sum_{mn} < mp\ |r^2\ Y_{2O}|np > c^{+}_{m}(p)\ c_n(p) \qquad (54)$$

Here too, p refers to the type of particles whereas m or n are relative to the single particle states.

In the illustrative example of ^{234}U, we choose $\epsilon_2 = 0.22$, as proposed by Löbner et al.[61] and the field parameters of Lamm[43]. The BCS gap parameters Δ_p and Δ_n are evaluated empirically from the experimental masses[62]. The values of Δ_p and Δ_n may be obtained by use of different methods[63-65]. We adopt $\Delta_p = 940$ keV and $\Delta_n = 710$ keV from method [65]. The corresponding pairing strength parameters

$$\begin{aligned} G_p &= 156\ \text{keV} \\ G_n &= 129\ \text{keV} \end{aligned} \qquad (55)$$

for 20 active levels are fixed that way. We remain then with only two free adjustable parameters χ_2 and χ_3.

To get their values we fit the calculated energies $E(0_1^-)$ and $E(0_2^+)$ of the MPM to the levels observed [66] respectively at 786 keV and 810 keV. We remind the reader that, for the $K^\pi = 0^+$ state there is no contribution due to the rotation to be substracted, whereas for the $K = 0^-$ a correction of $\simeq$ 10 keV is estimated.

The maximum value of the number of phonons have been limited here to 8. With this choice, the stability of the "one phonon states" 0_1^- and 0_2^+ is obtained within $\simeq$ 3 keV and that of the "two phonon states" 0_3^+, 0_2^-, 0_4^+ within less than 50 keV.

We focuse mainly our attention to the location of the 0_3^+, 0_2^-, 0_4^+ states and their desexcitation through E1 and E2 transitions.

TABLE VI Values (in keV) of the energies ω_{TDA}, ω_{RPA} and E_{MPM} for the fitted values $\chi_2 = 1$ keV and $\chi_3 = 9$ keV.

	0^+	0^-
ω_{TDA}	1224	1159
ω_{RPA}	1424	
E_{MPM}	770	781

In Table VI we give the values of the energies of the building phonons ω_{TDA}, compared to that obtained in the RPA and to the MPM result for the fitted values χ_2 and χ_3.

In Fig. 3 we present the calculated energy spectrum up to 2.5 MeV. On the l.h.s. we give the E1 transitions. The numbers at the right of the arrows linking the initial

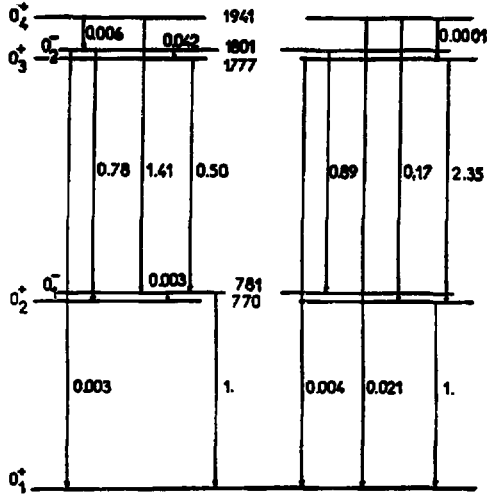

FIGURE 3 Energy spectrum obtained for ^{234}U in the multiphonon method. The intrinsic E1 transitions are given on the left hand side, the intrinsic E2 on the right hand side.

intrinsic states ψ_i to the final intrinsic state ψ_f give the values of $|\langle\psi_f|E1\ |\psi_i\rangle|^2$.
As usual the effective charges are

$$e_p = \frac{N}{A}\ e \text{ and } \qquad e_n = -\ \frac{Z}{A}\ e \tag{56}$$

The units are chosen so that

$$|\langle 0_1^+\ |E1|\ 0_1^-\rangle|^2 = 1$$

On the r.h.s. we give, in a similar way, the information concerning the possible E2 transitions. Here the effective charges are

$$e_p = e(1+\epsilon) \text{ and } e_n = e\,\epsilon \tag{57}$$

The calculations have been done with $\epsilon = 0$ and $\epsilon = 0.1$. The numbers given in Fig. 3 correspond to $\epsilon = 0.1$. The results presented in Fig. 3 appeal the following comments.

The E1 and E2 transitions clearly indicate the nature of the states located between 1770 and 1940 keV :

- The 0_3^+ state is strongly related by E2 to the 0_2^+ and therefore appears to be mainly the two $K = 0^+$ phonon state.

- The 0_2^- state is equally connected by E1 to 0_2^+ and by E2 to 0_1^- and is a good candidate for a ($K = 0^+ + K = 0^-$) phonon state

- The 0_4^+ state decreases mainly to the 0_1^- state by E1 and is, by no doubt, principally of octupole nature.

The anharmonicities of the "two phonon states" obtained in the MPM appear smaller for 0_3^+ since

$$\frac{E(0_3^+)}{2E\,(0_2^+)} = 1.15 \tag{58}$$

and for 0_2^- for which

$$\frac{E(0_2^-)}{E(0_1^-)+E(0_2^+)} = 1.16 \tag{59}$$

than for 0_4^+ since

$$\frac{E(0_4^+)}{2E(0_1^-)} = 1.24 \tag{60}$$

The energy spectrum is evidently dilated.

As a consequence, none of the calculated "two phonon states" can explain the 0_3^+ and 0_2^- states observed [66] respectively at 1044 keV and 1237 keV for which the anharmonicity ratios [58,59] are 0.6 and 0.8 and would lead to a compressed energy spectra.

We observe also that cross-over transitions (with $\Delta N = 2$) or transitions with $\Delta N = 0$ are strongly hindered compared to favored $\Delta N = 1$ transitions, mentioned before.

Furthermore, these conclusions are not changed if one uses $\varepsilon = 0$ instead of $\varepsilon = 0.1$ for the E2 effective charges.

It becomes also clear that the large anharmonicities of the octupole vibrations found in section 2 are not foundamentaly affected by the presence of a second building phonon with $K^\pi = 0^+$.

In Table VI we observe that $\chi_3 > \chi_2$. This should not be considered as an evidence against convergence of the multipole-multipole expansion of the force since most of the quadrupole-quadrupole part has been introduced in the mean field. For 0_2^+ and 0_1^- the energy E_{MPM} is smaller than the values obtained in TDA. For the 0^-, no physical solution is obtained within the RPA. This clearly shows the importance of the parts of H not taken into account in the latter approximations.

5. CONCLUSIONS

The application of the multiphonon method to even-even deformed nuclei has clearly shown that this approach is able to describe not only weakly anharmonic vibrations but also strongly anharmonic motions which prominently appear beyond the RPA critical point.

It is possible, with the MPM, to give a microscopic explanation of the existence of a $K = 0^-$ band head at very low energy $E(0_1^-)$ where the RPA fails. Furthermore, it is now clear why the corresponding "two phonon states" have not been observed at energies near $2\,E(0_1^-)$.

It seems also that the $K = 0^+$ vibration does not couple very strongly to the octupole vibration $K = 0^-$, even if the two one phonon states appear at nearly the same energy.

The application of the extension of the multiphonon method to odd light actinium has shown that the presently known spectroscopic properties of their low lying levels can be explained in a satisfactory way starting with the assumption of a reflection symmetry. Furthermore, it has been demonstrated that the existence of parity doublets, the enhancement of the intra-doublets E1 transitions and the empirical values of the decoupling factors cannot be used as criteria in favour of a stable octupole deformation.

The multiphonon method offers a dynamical approach to octupole correlations in deformed nuclei, which may be compared to the models based on a stable octupole deformation.

A typical property exhibited by the multiphonon method in odd-mass nuclei is the prediction, for each observed parity doublet, of a second parity doublet with, in general, a different order of the parities.

Further experimental work in this direction would therefore be of great interest to help to choose between stable or dynamic octupole deformation.

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