ITEP 86 – 160

INSTITUTE OF THEORETICAL
AND EXPERIMENTAL PHYSICS

B.O.Kerbikov, M.I.Polikarpov,
L.V.Shevchenko*, Al.B.Zamolodchikov

MONTE CARLO STUDY OF THE MULTIQUARK SYSTEMS

Preprint

Moscow – ATOMINFORM – 1986

*Moscow State University

УДК 539.172 Ц-16

MONTE CARLO STUDY OF THE MULTIQUARK SYSTEMS: Preprint ITEP 86-160/

B.O.Kerbikov, M.I.Polikarpov, L.V.Shevchenko, Al.B.Zamolodchikov - M.:ATOMIZDAT,1986

A b s t r a c t

Random walks have been used to calculate the energies of the ground states in systems of N=3, 6, 9, 12 quarks. Multiquark states with $N > 3$ are unstable with respect to the spontaneous dissociation into color singlet hadrons. We have employed the modified Green's function Monte Carlo algorithm which proved to be more simple and much more accurate than the conventional few body methods. In contrast to other techniques, the same equations are used for any number of particles, while the computer time increases only linearly *v.s.* the number of particles.

Fig. - 2, ref. - 26

© Центральный научно-исследовательский институт информации и технико-экономических исследований по атомной науке и технике (ЦНИИатоминформ),1986

1. Introduction

The problem of the existence of multiquark states and their properties is today one of the most important in hadron spectroscopy. This problem is also of vital importance to the derivation of nuclear physics from quantum chromodynamics. The multiquark systems have been investigated in the framework of different approaches: the additive two body potentials, the flux-tube model, the bag model, the lattice QCD, etc. Each of these models has its own merits and drawbacks; the comparison between different models is not our aim in this paper. The important point is that in multiquark spectroscopy one is confronted not only with the difficulties imposed by the complexity of the QCD in the infrared region, but also with the "technical" task of solving the many-body problem. As the number of particles increases and the interaction becomes more complicated the traditional many-body methods (such as variational, Faddeev equations, hyperspherical formalism) inevi-

tably face with untractable difficulties. The accuracy of the calculations, in particular the accuracy in determination of the wave function, becomes eventually uncontrollable, while the computer time increases catastrophically. In addition, for any given number of particles its own rather cumbersome system of equations should be written. In this paper we show that the ground state energy and the wave function of multi-quark system (and in general of any quark or nuclear system) can be easily and very precisely calculated via the Green's function Monte Carlo method. The idea is to use the Euclidean (imaginary time) propagator for the system as a filter producing the ground eigenstate. The advantage of this method, besides its high accuracy, is the uniform type of equations for any number of particles. We propose an improved version of the Green's function Monte Carlo method. We consider the simplest states made of $N=3$, 6, 9, 12 quarks. The application of the same method to more complicated dynamical problems not amenable to analysis through other methods will be the subject of a subsequent paper.

2. Green's Function Monte Carlo (GFMC)

This section introduces the basic ideas of the Quantum Green's Function Monte Carlo method (GFMC). Till now the GFMC was mainly used in statistical and molecular physics and therefore we find it appropriate to present a pedagogical exposition of the method for the readers not acquainted with the original works of Kalos, Ceperley and other authors /1-11/.

The GFMC is based on an old idea, attributed to Fermi /1/, that the imaginaty time Schrödinger equation is equivalent to the diffusion equation with branching (source-sink). However, apart from some simple problems solved in the early 1950's, this approach has received no impetus until Kalos /2/ introduced such a major improvement as importance sampling or guiding function. The first serious success of GFMC was the calculation by Kalos, Levesque, and Verlet /3/ of the ground state energy of 100 hard spheres at densities near melting. Then the method was generalized to treat fermions /4/, and simulations of the properties of the electron gas /4/ and liquid ^{3}He /5/ were performed in good agreement with the experiment. Recently the GFMC yielded the ground state energies of small molecules (H_3, LiH, Li_2, H_2O) with a record accuracy /6/ and the muon-alpha sticking probability /7/. The striking successes of GFMC inspired the field theory specialists to use it in lattice simulation; the corresponding versions of GFMC go by the names of Projector Monte Carlo /8/ and Guided Random Walks /9/. Some papers dealing with applications of the GFMC to nuclear physics have been published /10,11/. Today the most efficient GFMC algorithm seems to be the one described in papers /6/.

Now let us explain the essence of the GFMC following Ref. /6/. Consider a system of N particles with the Hamiltonian H . Let R denote a point in the 3N-dimensional configuration space. Let us analytically continue to purely imaginary time, $\beta = it$ ($\hbar = 1$ in our units), and let E_T represent a constant shift in the zero of energy, whose

introduction proves useful. Then the Schrödinger equation takes the form

$$-\frac{\partial \Psi(R,\beta)}{\partial \beta} = \left[-D\Delta + V(R) - E_T\right]\Psi(R,\beta), \qquad (1)$$

where $D = (2m)^{-1}$, m being the mass of each of the particles (for simplicity we restrict ourselves to particles having the same mass). Let us expand $\Psi(R,\beta)$ in a complete set of eigenfunctions $\psi_i(R)$ of the Hamiltonian H. One finds

$$\Psi(R,\beta) = \sum_i C_i \exp\left[-(E_i - E_T)\beta\right]\psi_i(R).$$

From this expression it follows that at sufficiently long times only the term corresponding to the ground state survives:

$$\Psi(R,\beta) \to C_0 \psi_0(R) \exp\left[-(E_0 - E_T)\beta\right].$$

Hence we may conclude that at large values of β the asymptotic solution is a steady-state solution, provided that E_T is adjusted to the ground state energy E_0.

Let us now introduce the guiding function $\psi_G(R)$. The introduction of this function is also called importance sampling. One tries to choose the guiding function in such a way as to incorporate the known singularities of the potential $V(R)$ (e.g. the singularity of the Coulomb potential at the origin). The product wave function $f(R,\beta) = \Psi(R,\beta)\psi_G(R)$, by virtue of (1), satisfies the equation

$$\frac{\partial f}{\partial \beta} = -\left[E_L(R) - E_T\right] f + D \cdot \Delta f - \vec{\nabla}\left(f \vec{F}_Q\right), \qquad (2)$$

where the quantity $E_L(R)$ is the local energy obtained from the guiding function

$$E_L(R) = \Psi_G^{-1} H \Psi_G , \qquad (3)$$

and the "quantum force" $\vec{F}_Q$ is given by

$$\vec{F}_Q = 2D \Psi_G^{-1} \vec{\nabla} \Psi_G . \qquad (4)$$

According to Eq. (2) the evolution of the distribution $f(R,\beta)$ is a result of three factors: 1) branching with the replication factor $exp\{-\beta\left[E_L(R) - E_T\right]\}$, 2) random diffusion with a mean squared displacement being βD, 3) drift proportional to $\beta \vec{F}_Q$ by the guiding quantum force. The new Eq.(2) has two superiorities over the original Eq. (1). Firstly, the quantum force guides the random walks away from regions where the guiding function Ψ_G is small toward regions where it is large; a reasonable guiding function will therefore substantially reduce the computer time. Secondly, in the original Eq. (1) the replication factor is proportional to $exp\{-\beta\left[V(R) - E_T\right]\}$ and thus it gives rise to large fluctuations of the population in the regions where the potential $V(R)$ has singularities. In contrast to that the branching factor in Eq. (2) contains the local energy E(R) which with a good choice of Ψ_G neeed not become singular when $V(R)$ does.

Now we may write instead of the differential Eq. (2) an equivalent evolution equation

$$f(R,\beta+\tau)=\int dR'\rho_D(R,R';\tau)\,f(R',\beta)\,e^{-(E_L-E_T)\tau}, \qquad (5)$$

where $\rho_D(R,R';\tau)$ is the diffusion density matrix corresponding to diffusion and drift from R to R' :

$$\rho_D(R,R';\tau)=(4\pi D\tau)^{-3N/2}\exp\{-[\vec{R}-\vec{R}'-\tau\vec{F}_Q(R')]^2/4D\tau\}. \qquad (6)$$

For large times by adjusting $E_T = E_0$ one may achieve a steady-state distribution

$$f(R,\beta+\tau)=f(R,\beta). \qquad (7)$$

Equations (5-7) constitute the essence of the Diffusion Monte Carlo method (DMC) /4/. Unfortunately, due to the finite time step it is difficult to estimate the ultimate accuracy of DMC. In the GFMC equations which will be formulated shortly the finite time step error is removed. Also the efficiency of the GFMC is somewhat better than that of DMC.

Let us describe the evolution of the distribution $f(R,\beta)$ using the exact density matrix $\rho=\exp(-\beta H)$ in place of ρ_D . Then Eq. (5) goes into

$$f(R,\beta+\tau)=\int dR'\rho(R,R';\tau)\,e^{E_T\tau}\,\frac{\Psi_G(R)}{\Psi_G(R')}\,f(R',\beta). \qquad (8)$$

The exact density matrix ρ is not known but it can be related via an integral equation to any trial density matrix

and then sampled stochastically using the von Neumann-Ulam random walks. The density matrix ρ obeys the Bloch equation /12/

$$\frac{\partial \rho(R,R';\beta)}{\partial \beta} = -H\rho(R,R';\beta) \tag{9}$$

with the initial condition

$$\rho(R,R';0) = \delta(R-R'). \tag{10}$$

Let us choose an arbitrary density matrix ρ_T (in particular, ρ_T may be the high-temperature density matrix or the kernel for the 3N-dimensional harmonic oscillator). Making use of Eqs. (9-10) it is easy to obtain /12,6/ the following equation for the exact density matrix ρ :

$$\rho(R,R';\beta) = \rho_T(R,R';\beta) + \int_0^\beta d\beta' \int dR'' \rho(R,R'';\beta-\beta') K(R'',R';\beta'), \tag{11}$$

where the kernel K is given by

$$K(R'',R';\beta) = -\left[H(R'') + \frac{\partial}{\partial \beta'}\right] \rho_T(R'',R;\beta'). \tag{12}$$

In case when the trial density matrix ρ_T is an exact one for a certain potential V_T, the kernel K takes the form

$$K(R'',R';\beta) = -\left[V(R'') - V_T(R'')\right] \rho_T(R'',R';\beta'). \tag{12a}$$

Equation (11) is solved iteratively by a stochastic process (see below) and therefore the smaller is the kernel K, the quicker is the convergence. From Eqs. (12) and (9) it follows that the kernel K is small provided that ρ_T is a good approximation to the exact density matrix ρ. Next, it is convenient to go over from Eqs. (8) and (11) to the time independent form of equations. This is done in the same way as the transformation from the time-dependent to the energy-dependent Green's functions

$$e^{-i\hat{H}t} \longrightarrow \frac{1}{\hat{H}-E} .$$

Similar transformation for the density matrix ρ is performed via the Laplace transform

$$\tilde{\rho}(R,R') = \int_0^\infty \frac{d\beta}{\Delta} e^{\beta(E_T - 1/\Delta)} \frac{\Psi_G(R)\rho(R,R';\beta)}{\Psi_G(R')} = \tag{13}$$

$$= \Psi_G(R) \left\langle R \left| \frac{1}{1+\Delta(H-E_T)} \right| R' \right\rangle \Psi_G^{-1}(R') .$$

Making the same operation on the kernel K, we obtain the following final equation for the density matrix

$$\tilde{\rho}(R,R') = \tilde{\rho}_T(R,R') + \Delta \int dR'' \tilde{\rho}(R,R'') \tilde{K}(R'',R') . \tag{14}$$

Now the evolution of the distribution is obtained by applying the operator $\tilde{\rho}(R, R')$ many times to the initial distribution $f_1 = |\Psi_G(R)|^2$:

$$f_{n+1}(R) = \int dR' \tilde{\rho}(R, R') f_n(R'). \qquad (15)$$

Eqs. (14–15) are the basic equations of the GFMC. Let us show that asymptotically the procedure (15) filters the ground state wave function. From (13) and (15) we have

$$f_n = \Psi_G \sum_\alpha |\Psi_\alpha\rangle \frac{1}{[1+\Delta(E_\alpha - E_T)]^{n-1}} \langle \Psi_\alpha | \Psi_G^{-1} f_1 \rangle .$$

If $\langle \Psi_0 | \Psi_G \rangle \neq 0$, then for large n the ground state will dominate

$$f_n \longrightarrow \frac{\langle \Psi_0 | \Psi_G \rangle}{[1+\Delta(E_0 - E_T)]^{n-1}} \Psi_G | \Psi_0 \rangle . \qquad (16)$$

3. Random Walks Algorithm

To obtain the ground state energy and the wave function from the system of equations (14–15) the following algorithm is used /6/.

1. Choose E_T , Ψ_G and ρ_T .

2. An ensemble of points $\{R_i\}$ $(1 < i < P_i)$ is selected according to the distribution $f_1 = |\Psi_G(R)|^2$. This set is referred to as the first generation. The typical population of the first generation is within the interval $10^2 < P_i < 10^4$.

3. A time step is sampled for each point from the distri-

bution $\Delta^{-1} e^{-t/\Delta}$.

4. Deffuse and drift all the coordinates of one of the initial configutations for one time step. The diffusion is performed by means of the diffusion density matrix (6). Thus we arrive at a new configuration $\{R'_i\}$.

5. Then the branching occurs. Immediatelly to the next generation pass the so-called direct point the number of which is equal to

$$m_D(R,R';t) = exp(E_T t) \frac{\Psi_G(R)}{\Psi_G(R')} \frac{\rho_T(R,R';t)}{\rho_D(R,R';t)} . \qquad (17)$$

It is easy to see that direct points correspond to the first term in the right hand side of the integral equation (14). The ρ_D is inserted into the denominator of Eq. (17) in order to compensate for the action of the diffusion density matrix at the previous step. In general, m_D is not an integer. To convert it to an integer we add a random number uniformly distributed in the interval $[0,1]$ and then round off.

6. To sprout another branch of the process we generate the so called intermediate points with multiplicity

$$m_I(R,R';t) = \frac{K(R,R';t)\, m_D(R,R';t)\Delta}{\rho_T(R,R';t)} . \qquad (18)$$

This corresponds to the first iteration of Eq. (14). The quantity m_I is rounded off in the same way as m_D.

7. Then each of the intermediate points is considered the same as the points of the first generation, i.e. it undergoes

all the operations starting from step (3).

8. Each of the intermediate configutations generates both further direct and intermediate configurations. The former are added to the direct configurations already emerged at the step (5), while the latter are again sources for more direct and intermediate configurations.

9. When all initial configurations, their intermediate descendants, the intermediate configurations originated from intermediate fonfigurations, etc. have been exhausted, the sampling of the first generation $f_1(R)$ is finished. This will occur within a finite time interval, provided that the average $m_I < 1$.

10. Obviously, the sequence of operations described above is equivalent to the following iterations of the integral equation (14):

$$\tilde{\rho} = \tilde{\rho}_T + \Delta\cdot\tilde{\rho}_T * \tilde{K} + \Delta\cdot\tilde{\rho}_T * \tilde{K} * \tilde{K} + \ldots \rightarrow$$

$$\rightarrow \tilde{\rho}_T + \Delta\cdot\tilde{\rho} * \tilde{K},$$

where

$$\tilde{\rho} * \tilde{K} \equiv \int dR''\tilde{\rho}(R,R'')\tilde{K}(R'',R').$$

The density matrix obtained in this way acts on the distribution f_1. Therefore, the distribution of points in the second generation is a sample from the function

$$f_2 = \int dR'\tilde{\rho}(R,R')f_1(R').$$

The total population of the second generation is

$$P_2 = \int dR\, f_2(R).$$

This random walks process continues until enough statistics is obtained. The population of the n-th generation is

$$P_n = \int dR\, f_n(R). \tag{19}$$

In our calculations the number of generations n was about 10^2. The scheme of the algorithm is sketched in Fig. 1.

The ground state energy in GFMC may be calculated in two different ways; from the change of the average population and from the average local energy. The first option is based on Eq. (16) which yields

$$E_0 = E_T + \frac{1}{\Delta}\left(\frac{P_{n-1}}{P_n} - 1\right). \tag{20}$$

The second possibility stems from Eq. (3) for the local energy $E_L(R)$. Taking the average of the local energy over the distribution $f_n(R)$, we get

$$E_n = \langle E_L \rangle_n = \langle \Psi_G^{-1} H \Psi_G \rangle_n = \frac{\int dR\, \Psi_G(R) H[f_n(R)/\Psi_G(R)]}{\int dR\, f_n(R)} = \tag{21}$$

$$= \frac{1}{P_n} \sum_{j=1}^{P_n} \Psi_G^{-1}(R_j) H \Psi_G(R_j).$$

In the limit $n \to \infty$ we have $f_n \to \Psi_0 \Psi_G$ and hence $E_n \to E_0$.

The ground state wave function Ψ_0 is obtained from the distribution of the points in the n-th generation. Asymptotically we obtain

$$f_n(R) \to C\,\Psi_0(R)\,\Psi_G(R). \qquad (22)$$

Needless to say that in order to get an absolutely exact value of E_C one should start with infinite number of initial points and proceed on till infinity. The analysis of errors of real calculations is performed according to a scheme which is in common with any Monte Carlo calculation (for details see Ref. /6/). However, two remarks are in order concerning the accuracy of GFMC calculations. Firstly, the local energy estimate usually has a smaller variance than the estimate based on the population growth. However, one should perform both estimates in order to be sure that the program runs properly. Secondly, with the calculation time T increasing the error decreases according to inherent in any Monte Carlo calculation law $\sim T^{-1/2}$.

With growing number of particles the calculation time needed to achieve any given accuracy grows only linearly. Therefore in particle physics and nuclear physics an accuracy surpassing the accuracy of the physical model is easily attainable.

The efficiency of the algorithm may be improved if at each step one sums $m_I + m_D = m$ and then with probability m_D/m considers all these copies as direct points and with probability m_I/m as intermediate points. Also one should throw away some number of initial generations since they carry reminiscenses of the initial distribution.

The outlined method has an essential limitation, namely the kernel K must be positive. If this condition is violated the probability for the point to pass into the next generation may exceed unity or the population may become negative. On the other hand, the kernel K should be small in order for the process to converge quickly. For a small kernel the positivity requirement is especially unconvenient since for some random walks such a kernel may easily change its sign. We propose the following way to circumvent this difficulty. When solving the integral equation (14) we generate the distribution of points with some measure $d\mu(R) = f(R)\,dR$. The nction of the probability density may be introduced provided that $f(R) > 0$. Then the quantity

$$I = \frac{\int \varphi(R) f(R) dR}{\int f(R) dR}$$

may be calculated as the following sum

$$I = \frac{1}{N} \sum_{j=1}^{N} \varphi(R_j),$$

where the points R_j are distributed with the measure $d\mu(R)$. If the function $f(R)$ is free to change sign, or even to become complex, the usual notion of the measure $f(R)dR$ becomes meaningless. It is proposed in this case to genetate points with probability density $d\mu(R) = |f(R)|dR$. Then the quantity I reads

$$I = \frac{\int e^{i\delta(R)} \varphi(R) d\mu}{\int e^{i\delta(R)} d\mu},$$

where $\delta(R)$ is the phase of the function $f(R)$ ($f(R) = e^{i\delta(R)} |f(R)|$). Now we may compute I according to the equation

$$I = \frac{\sum_{j=1}^{N} e^{i\delta_j} \varphi(R_j)}{\sum_{j=1}^{N} e^{i\delta_j}} .$$

To solve the integral equation (14) using the modified GFMC (MGFMC) one should substitute K for $|K|$ in Eqs. (17--18), so that m_D and m_I still have the meaning of probabilities. Each of the points R from any generation is accompanied by its phase $\delta(R)$. This phase remains unchanged if the point passes into the next generation. However if the point becomes an intermediate one, the phase transforms into $\delta \to \delta + \delta_1$, where δ_1 is the phase of the kernel K, $K = exp(i\delta) \cdot |K|$. In our problem the kernel may change sign but remains real, therefore all phases are either 0 or π. Equation (19) for P_n is substituted for the following one

$$P_n = \sum_{j=1}^{N} e^{i\delta(R_j)} , \qquad (23)$$

while Eq. (21) for $\langle E_L \rangle_n$ now reads

$$\langle E_L \rangle_n = \frac{\sum_{j=1}^{N} e^{i\delta(R_j)} \Psi_G^{-1}(R_j) H \Psi_G(R_j)}{\sum_{j=1}^{N} e^{i\delta(R_j)}} . \qquad (24)$$

Necessary to note that in papers /6/ the GFMC is appied to the case when the ground state wave function is antisymmetric, i.e. has some nodes. The MGFMC described above is reminiscent

of the nodal relaxation method proposed in /6/.

4. The masses of the ground states of multiquark systems

In this section the GFMC is used to compute the ground state energies of systems made of N=3, 6, 9, 12 quarks. We shall resort to the simple nonrelativistic constituent quark model. Its validity (and failure) in multiquark systems is discussed e.g. in Refs. /13,14/. Only two-body potentials will be considered since a sound basis for considering many-body forces is still lacking. However in GFMC the many-body forces do not lead to any complications.

The most reasonable choice for the interaction between quarks is the potential due to the exchange of colour octets /15,16/

$$V_{Q_1 Q_2} = \lambda_1 \lambda_2 V_8 (r_{12}), \tag{25}$$

with λ_i being the Gell-Mann color matrices. For $V_8(r)$ we have taken the well-known Cornell potential /17/:

$$V_8(r) = -\frac{3}{16}\left(-\frac{\varkappa}{r} + \frac{r}{a^2} + C\right), \tag{26}$$

where

$$\varkappa = 0.52, \qquad a = 2.34 \text{ GeV}^{-1} \tag{27}$$

The constant C and the mass of the light quark m have been choosen following Ref. /18/:

$$C = -0.975 \text{ GeV}, \quad m = 0.33 \text{ GeV} \tag{28}$$

We note in passing that for this set of parameters the mass of the ground state in the $Q\bar{Q}$ system is equal to $M(Q\bar{Q}) = 0.595$ GeV which has to be compared with the center of gravity of the $\rho - \pi$ system: $M_{c.o.g.}(\rho - \pi) = 0.612$ GeV.

For the number of quarks $N \geqslant 4$ the coupling of the colors of the constituents into a color singlet is not unique, the color degrees of freedom are not factorized and the potential is a matrix in the color space. A proper account of the color mixing for the $Q\bar{Q}Q\bar{Q}$ system is presented e.g. in Ref. /19/. We shall consider only the simplest configurations for which the color mixing is irrelevant /20/. In the simplest symmetric case when $\lambda_i \lambda_j = \langle \lambda_i \lambda_j \rangle$ we have

$$V(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_N) = \sum_{i<j} \lambda_i \lambda_j V_8(r_{ij}) = -\frac{16}{3(N-1)} \sum_{i<j} V_8(r_{ij}). \quad (29)$$

Note that in arriving at the expression (29) we made use of the equation

$$\sum_{i<j} \lambda_i \lambda_j = -\frac{8}{3} N .$$

It is worth reiterating that the potential (29) is written for symmetric states resembling the MIT Bag Model. In this publication we also ignore the spin dependent forces between quarks.

Thus the Hamiltonian for N quarks symmetric configuration reads

$$H_N = \sum_{i=1}^{N} \frac{p_i^2}{2m} - \frac{16}{3(N-1)} \sum_{i<j} V_8(r_{ij}).$$

The next step is to separate the centre-of-mass motion and to

go over to the "Jacobi" coordinates $\vec{x}_i$, i = 1,2,...,N-1, defined according to Ref. /21/

$$\vec{x}_i = \left[i(i+1)\right]^{-1/2}\left[\sum_{j=1}^{i} \vec{r}_j - i\,\vec{r}_{i+1}\right], \quad i = 1,2,\ldots,N-1. \tag{30}$$

The relative coordinates $\vec{r}_{ij}$ are expressed in terms of the Jacobi coordinates according to the following recurrent relation

$$\vec{r}_{ij} = \sqrt{\frac{j}{j-1}}\,\vec{x}_{j-1} - \sqrt{\frac{j-2}{j-1}}\,\vec{x}_{j-2} + \vec{r}_{i,j-1} \ . \tag{31}$$

The following identity will be also needed

$$\sum_{i<j} \vec{r}_{ij}^{\,2} = N \sum_{i=1}^{N-1} \vec{x}_i^{\,2} \ . \tag{32}$$

Now the Hamiltonian may be rewritten in the form

$$H_N = -\frac{1}{2m}\sum_{i=1}^{N-1} \frac{\partial^2}{\partial \vec{x}_i^{\,2}} + \frac{N}{2}C + \frac{1}{N-1}\sum_{i<j} \tilde{V}_8\left[r_{ij}(\vec{x}_k)\right], \tag{33}$$

where

$$\tilde{V}_8(r) = -\alpha/r + r/a^2 \ , \tag{34}$$

and the notation $r_{ij}(\vec{x}_k)$ means that the relative coordinates r_{ij} should be written in terms of the Jacobi coordinates $\vec{x}_k$ according to Eq. (31).

In variational calculations of the $3\bar{Q}$ and $2Q2\bar{Q}$ systems /13/ the harmonic oscillator potential has been used to approximate the potential (34). This invites to take the kernel for the d=3(N-1) dimensional harmonic oscillator as our trial density matrix ρ_T :

$$\rho_T(R,R';\beta)=\left(\frac{m\omega_T}{2\pi\,sh\omega\beta}\right)^{\frac{3}{2}(N-1)} exp\left\{-\frac{N}{2}C_T\beta\right\}\times$$

$$\times exp\left\{-\frac{m\omega_T}{2\,sh\,\omega_T\beta}\left[\left(R^2+R'^2\right)ch\,\omega_T\beta-2\left(\vec{R}\,\vec{R}'\right)\right]\right\}, \qquad (35)$$

where R is a 3(N-1) dimensional vector with components $R=\{x_i^\alpha\}$, i = 1,2,...,N-1; α = 1, 2, 3. The trial parameter C_T is introduced into ρ_T in order to take into account the constant C in the potential (26). The density matrix (35) is an exact solution for the following potential

$$V_T(\vec{r}_1,\vec{r}_2,\dots,\vec{r}_N)=\sum_{i<j}\frac{m}{2N}\omega_T^2(\vec{r}_i-\vec{r}_j)^2+\frac{N}{2}C_T= \qquad (36)$$
$$=\sum_{j=1}^{N-1}\frac{m\omega^2}{2}\vec{x}_j^{\,2}+\frac{N}{2}C_T\,.$$

The N-body problem for the potential V_T admits an explicit solution and the ground state energy is given by

$$E_N^{h.o.}=\frac{N}{2}C_T+\frac{3}{2}\omega_T(N-1)\,. \qquad (37)$$

From Eq. (12) we obtain the kernel K :

$$K(R,R';\beta)=-H'\rho_T(R,R';\beta),$$
$$H'=\frac{1}{N-1}\sum_{i<j}\left(-\frac{\varkappa}{r_{ij}}+\frac{r_{ij}}{a^2}\right)-\sum_{j=1}^{N-1}\frac{m\omega^2}{2}\cdot\vec{x}_j^{\,2}+\frac{N}{2}(C-C_T). \qquad (38)$$

The guiding function in our calculations is also of the oscillator form

$$\Psi_G(R)=\left(\frac{m\omega_G}{\pi}\right)^{\frac{3}{4}(N-1)} exp\left\{-\frac{m\omega_G}{2}\sum_{j=1}^{N-1}\vec{x}_j^{\,2}\right\}. \qquad (39)$$

The frequencies ω_T and ω_G were put equal to each other

$$\omega_T = \omega_G = \omega \,. \tag{40}$$

With this choice of Ψ_G the diffusion and drift density matrix has the form

$$\rho_D(R, R'; \beta) = \left(\frac{m}{2\pi\beta}\right)^{\frac{3}{2}(N-1)} \times \tag{41}$$

$$\times \exp\left\{-\frac{m}{2\beta}\sum_{j=1}^{N-1}\sum_{\alpha=1}^{3}\left[x_j^\alpha - x_j'^\alpha(1-\omega\beta)\right]^2\right\}.$$

Finally, for the local energy (3) we have

$$E_L(R) = \Psi_G^{-1} H_N \Psi_G = \frac{N}{2} C_T + \frac{3}{2}\omega(N-1) + H', \tag{42}$$

where $H' = V - V_T$ (see Eq. (38)).

The numerical calculations have been carried out according to the algorithm of section 3. At the first stage of the calculations the parameters ω and C_T were adjusted in order to make the kernel K small, while the energy E_T was choosen so as to avoid drastic fluctuations of the population from one generation to another. The time step Δ was $\Delta = 0.5\ \mathrm{GeV}^{-1}$.

Table 1 contains the results of our calculations. The energies are given in GeV. The quantity E_P is the estimate of the energy according to the population growth (Eq. (20)), and E_L is the local energy estimate (Eq. (21)). It is seen from the Table that the accuracy of our energy estimates is in excess of the accuracy of the model and therefore the rounded value for the mass $M_N = Nm + E$ is presented. The computer time for the three body problem was about 3 hours of the EC-1060 computer however this time would be only about 45 min if twice as worse accuracy would suffice us. In Fig. 2 we plot the energy $\langle E_L \rangle_n$ v.s. the number of generations for the three quark system.

5. Results and Discussion

The most interesting results are those concerning the systems of 6, 9 and 12 quarks since the three quark system has been already extensively studied within different approaches /13, 18, 22-27/. However it is worth comparing our results with the results of the hyperspherical formalis /18/. Our value of M_3 is about 5 MeV lower than the result of Ref. /18/. The mass M_3 should be compared with the center of gravity of the N - Δ system: $M^{c.o.g}_{N-\Delta} = 1.085$ GeV. A slight difference between our result and $M^{c.o.g}_{N-\Delta}$ is due to the choice of the constant C (we have taken the value of C from Ref. /18/).

The main physical conclusion on multiquark states is that all of them are far above the thresholds for dissociation into color singlet hadrons and therefore they would not manifest themselves as narrow resonances. One should keep in mind however that we have considered only configurations with the simples color structure, namely with the factorized color degrees of freedom and with

$$a_{ij} = \lambda_i \lambda_j = \bar{a} = -16/3(N-1) .$$

Making use of the variational principle it is easy to prove /20/ that the ground state of the symmetric configuration is always above that of any other configuration

$$M_N(a_{ij}) \leq M^S_N(\bar{a}) . \qquad (43)$$

The states with different color structure will be considered in the framework of GFMC in another publication.

One can also prove /20/ that the masses of the ground states satisfy the relation

$$M_2/2 \leq M_3^S/3 \leq \ldots \leq M_N^S/N . \qquad (44)$$

As can be seen from Table 1 our results satisfy this requirement.

The mass spectrum of multiquark states has a property which may be viewed as saturation of quark forces. Namely, our states with a good accuracy lie on a straight trajectory:

$$M_N = Nm + C\frac{N}{3} - \xi . \qquad (45)$$

where

$$C = 0.407 \text{ GeV}, \qquad \xi = 0.304 \text{ GeV} \qquad (46)$$

In conclusion we should emphasize that the MGFMC permits to solve the many body problem with extremely high accuracy and using the same algorithm for any number of particles.

Numerous stimulating and clarifying discussions with Yu.A.Simonov are gratefully acknowledged. The authors are indebted to A.M.Badalyan and A.A.Migdal for valuable remarks. We wish to acknowledge with thanks discussions with V.F.Kerbikova and V.E.Markushin and receipt of import nt information from D.Ceperley and J.Negele. We thank the administrations of the Institute of Theoretical and Experimental Physics and the Institute of Cosmic Studies for their support of our numerical calculations.

Table 1

N	E_p(GeV)	E_L(GeV)	M_L	$M_{L/N}$
3	0.0978 ± 0.0017	0.09778 ± 0.00032	1.088	0.363
6	0.5176 ± 0.0022	0.5163 ± 0.0005	2.496	0.416
9	0.9213 ± 0.0017	0.91936 ± 0.00053	3.890	0.432
12	1.3175 ± 0.0079	1.3185 ± 0.0012	5.428	0.440

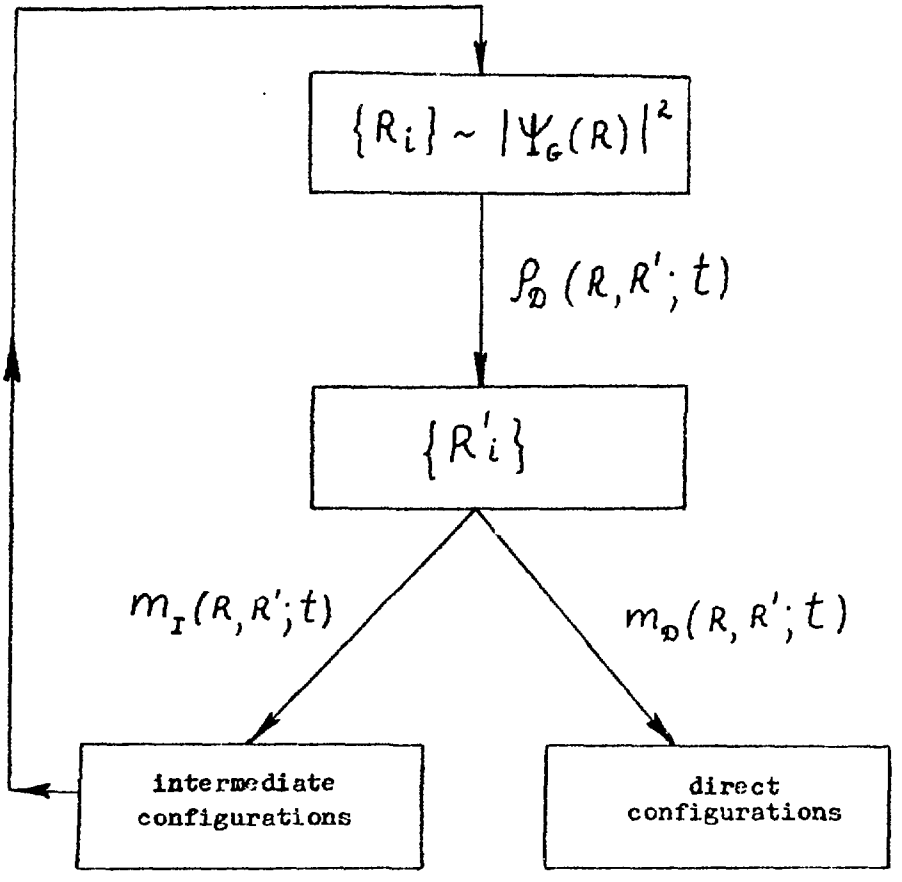

Fig. 1. The scheme of the algorithm.

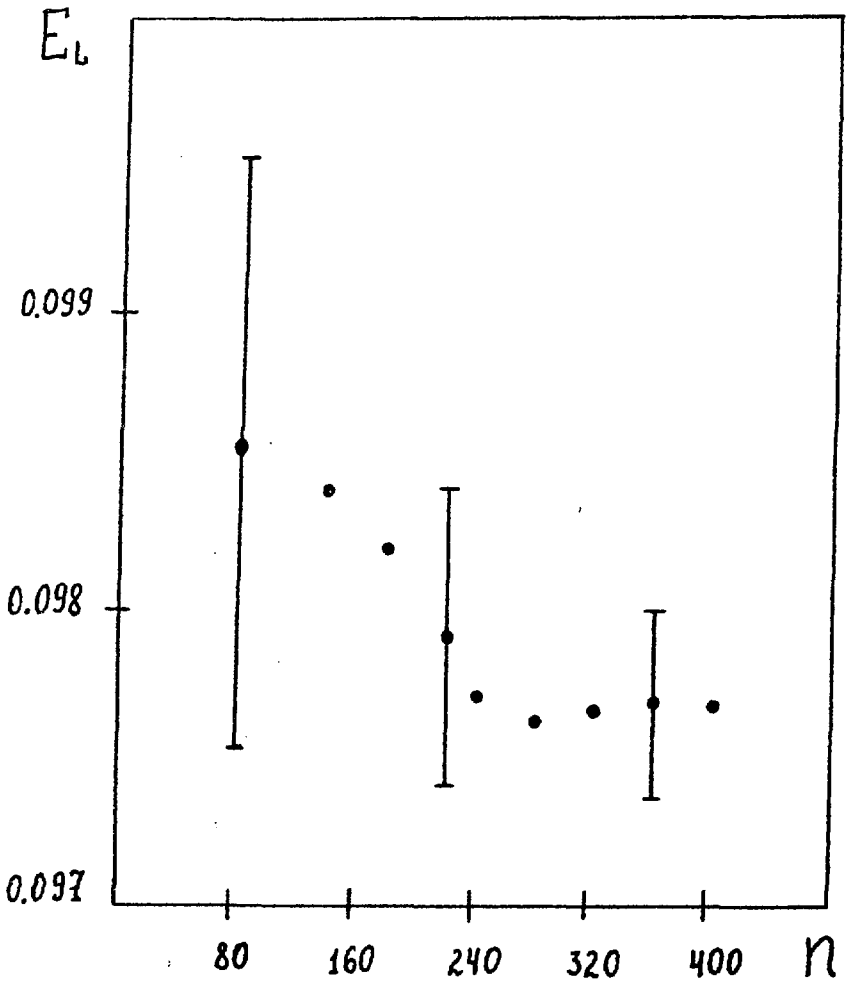

Fig. 2. The energy of the three quark system *V. S.* the number of generations.

References

1. Metropolis N. and Ulam S.–J.Amer.Stat.Assoc., 1949, 44,335
   Kac M. Probability and Related Topics in Physics Science. Interscience, New York, 1959
2. Kalos M.H.–Phys.Rev., 1962, 128, 1791; J.Comput.Phys., 1966, 1, 257; Phys.Rev., 1970, A2, 250
3. Kalos M.H., Levesque D., Verlet L.–Phys.Rev., 1974, A9, 2178
4. Ceperley D., Alder B.J.–Phys.Rev.Lett., 1980, 45, 566
5. Lee M.A., Schmidt K.E., Kalos M.H., Chester G.V.–Phys. Rev.Lett., 1981, 46, 729
6. Ceperley D.M. –J.Comput.Phys., 1983, 51, 404
   Ceperley D.M., Alder B.J.–J.Chem.Phys., 1984, 81, 5833
7. Ceperley D.M., Alder B.J.–Phys.Rev., 1985, A31, 1999
8. Blankenbecler R., Sugar R.L.–Phys.Rev., 1983, D27 , 1304
   Kung D., Blankenbecler R., Sugar R.L. –Phys.Rev., 1985, B32, 3058
9. Chin S.A., Negele J.W., Koonin S.E. Ann.Phys., 1984, 157, 140; Chin S.A., van Roosmalen O.S., Umland E.A., Koonin S.E. –Phys.Rev., 1985, D31, 3201; Barnes T., Daniell G.J., Storey D. Nucl.Phys., 1986, B265, 253
10. Zabolotzky J.G., Kalos M.H. Nucl.Phys., 1981, A356,114; Helmbrecht U., Zabolitzky J.G. Nucl.Phys., 1985, A442,109; Carlson J., Kalos M.H. Phys.Rev., 1985, C32, 2105
11. Serot B.D., Koonin S.E., Negele J.W. Phys.Rev., 1983, C28, 1679; Alhassid Y., Koonin S.E. Ann.Phys., 1984, 155, 108; Horowitz C.J., Moniz E.J., Negele J.W. Phys.Rev., 1985, D31, 1689; Negele J.W. Proceedings of the Interna-

...1 Symposium on Time-Dependent Hartree-Fock, Lecture Notes in Physics 171. K.Geoke and P.G.Reinhardt, eds. Springer-Verlag, NY, 1982

12. Feynman R.P. Statistical Mechanics, Benjamin, New York, 1972

13. Isgur N. Nuclear Physics from the Quark Model with Chromodynamics. Preprint UTPT-85-18, March, 1985

14. Lipkin H.J.-Phys.Lett., 1986, 172, 242

15. Nambu Y. in: Preludes in Theoretical Physics, eds. A.de Shalit, H.Feschbach and L.Van Hove. North Holland, Amsterdam, 1966;
Greenberg O.W., Zwanziger D.-Phys.Rev., 1966, 150, 1177

16. Lipkin H.J.-Phys.Lett., 1973, 45B, 267

17. Eichten E. et al.-Phys.Rev., 1980, D21, 203

18. Badalyan A.M., Kitaroage D.I., Pariisky D.C.-Yad.Fiz., 1986, 44, 1086

19. Badalyan A.M., Ioffe B.L., Smilga A.V. Preprint ITEP-77, 1985

20. Ader J.-P., Richard J.-M., Taxil P.-Phys.Rev., 1982, D25, 2370

21. Calogero F., Simonov Yu.A.-Nuovo Cimento, 1968, 56B, 71

22. Isgur N., Karl G.- Phys.Rev., 1978, D18, 4186; 1979, D19, 2653; 1979, D20, 1191

23. T.De Grand, R.L.Jaffe, K.Johnson, J.Kiskis. Phys.Rev., 1975, D12, 2060; P.J.Mulders, A.T.Aerts, J.J.de Swart -Phys.Rev., 1979, D19, 2635

24. Carlson J., Kogut J.B, Pandharipande V.R.-Phys.Rev., 1983, D28, 2807

25. Richard J.M.-Phys.Lett., 1981, 100B, 515

26. Kvitzinskii A.A., Kuperin Yu.A., Merkuriev S.P., Novozchilov V.Yu. -Yad.Fiz., 1983, 38, 702.

Б.О.Кербиков и др.

Исследование многокварковых систем методом Монте-Карло.

Работа поступила в ОНТИ 11.09.86

Подписано к печати 12.09.86 Т15547 Формат 60x90 1/16
Офсетн.печ. Усл.-печ.л.1,75. Уч.-изд.л.1,3. Тираж 260 экз.
Заказ 160 Индекс 3624 Цена 19 коп.

Отпечатано в ИТЭФ, 117259, Москва, Б.Черемушкинская,25

ИНДЕКС 3624

М., ПРЕПРИНТ ИТЭФ, 1986, № 160, с.1-28