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FERMIONS AND NON-ABELIAN VORTEX

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ABSTRACT

We discuss here some aspects of the system fermion-non Abelian vortex. We show that this system presents properties analogous to the fermion-non-Abelian magnetic monopole one. But, differently from the fermion-monopole case, our system does not present fermion condensate $\langle \bar{\psi}\psi \rangle_V = 0$.

I - INTRODUCTION

We present in this work some aspects of a physical system composed by fermions and non Abelian vortices in $d = 2 + 1$ dimensional space. We show that this system presents properties analogous to the fermion-magnetic monopole system recently studied by V. Rubakov and C. Callan(1) in $d = 3 + 1$ dimensional space.

The method used here to study this system, fermion-vortex , is similar to that of reference (1), but, some modification due to boundary conditions, are introduced.

As we know vortices are solutions with non trivial topology for a system defined in $d = 2$ dimensional space composed by gauge and Higgs fields which present breaking of gauge symmetry by the Higgs mechanism. For a non Abelian theory, which is the case of our interest, the vortex arises when the gauge symmetry breaking is maximum. By maximum we understand that the vacuum of the theory is invariant only by the unit matrix in the adjoint representation (2).

The linear mass density of the vortex, given by $\mu = \alpha^{-1}M^2$, is 10^{-4}g/cm or 10^{22}g/cm for eletroweak and grand unification scales, which are of order 10^{2}Gev and 10^{15}Gev respectively.

In sect II of this work we present the Lagrangian density, $L_V(x)$, for the gauge-Higgs system with a symmetry group SU(2) and the Hamiltonian that describes the fermion-vortex interaction, and some properties are also shown. In sect III, we quantize the system by functional integral in the vortex sector. If we restrict ourselves to quantum fluctuation of the vortex (background) field with radial symmetry, the dynamics of $J = 0$ component of the fermionic wave function(*) becomes effectively two-dimensional, as in the magnetic monopole problem. In sect IV, we obtain the full effective action of the system fermion-vortex in this radial approximation, and the quantum electric field of the vortex. Boundary conditions for the relevant bosonic field are also discussed there and an exact solution obtained. (In the $d = 3 + 1$ dimension the condensate $<\bar{\psi}\psi>$, in the monopole field, which is associated with chiral symmetry breaking (driven by the anomaly) is of order $1/r^3$, where r is the distance from the monopole's center). In our model, we will see in sect V that the mass term (condensate) vanishes. This is due to the fact we are in an odd dimensional space where the anomalies are absent. In the appendices A and C we present a relevant demonstration for our problem, and some properties of the special function used in this work to obtain propagators. In appendix B we present a charged vortex configuration with finite energy.

II - THE SYSTEM

The vortex is a solution with non trivial topology, of the system composed by gauge and Higgs fields defined in

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(*) In the non Abelian vortex field the total angular momentum J of the fermion is integer, like in the non Abelian magnetic monopole case.

a d = 2 dimensional space whose symmetry gauge group G is spontaneouly broken, by Higgsmechanism, in the H sub-group of G. These topological stable solution make a mapping from the quottent group G/H to the circle at $\rho = \infty$ in the ordinary space, and are labelled by $\pi_1(G/H)$.

We consider the Lagrangian density below with a gauge group SU(2)

$$L_V(x) = -\frac{1}{4}(F_{\mu\nu}{}^a)^2 + \frac{1}{2}[(D_\mu\phi^a)^2] + \frac{1}{2}[(D_\mu\chi^a)^2] + V(\phi^a,\chi^a) \quad , \qquad (2\text{-}1a)$$

where

$$F_{\mu\nu}{}^a = \partial_\mu A_\nu{}^a - \partial_\nu A_\mu{}^a - e\varepsilon^{abc} A_\mu{}^b A_\nu{}^c \quad , \qquad (2.1b)$$

$$(D_\mu\phi^a) = \partial_\mu\phi^a - e\varepsilon^{abc} A_\mu{}^b\phi^c \quad , \qquad (2.1c)$$

and

$$V(\phi^a,\chi^a) = \frac{1}{2}\left[\mu_1^2(\phi^a)^2 + \mu_2^2(\chi^a)^2\right] - \frac{1}{4}\left[\lambda_1(\phi^a)^4 + \lambda_2(\chi^a)^4\right]$$

$$- \frac{1}{2}\beta(\phi^a\chi^a)^2 - \frac{1}{2}\gamma(\phi^a)^2(\chi^a)^2 . \qquad (2.1d)$$

Note that we have two triplets of Higgs fields ϕ^a and χ^a (a=1,2,3). The spatial coordinates are $\rho = \sqrt{x_1^2 + x_2^2}$ and θ ($x_1 = \rho\cos\theta$ and $x_2 = \rho\sin\theta$).

In order to get the maximum symmetry breaking the parameter of $V(\phi^a, \chi^a)$ must satisfy specific conditions[2], and the vortex solution A_μ^a is labelled by $\pi_1(SU(2)/Z_2) = \pi_1(SO(3)) = Z_2$ which has no vanishing element. In this model we have vortex or anti-vortex besides the vacuum solution, which are characterized by a conserved topological number: the magnetic flux .

The vortex solution is obtained by the "ansatz"

$$\phi^{cl}(\vec{\rho}) = f(\rho) \begin{pmatrix} \cos\theta \\ \sin\theta \\ 0 \end{pmatrix} , \qquad \chi^{cl}(\vec{\rho}) = g(\rho) \begin{pmatrix} -\sin\theta \\ \cos\theta \\ 0 \end{pmatrix} , \qquad (2\text{-}2a)$$

$$\vec{A}_a^{cl}(\vec{\rho}) = -\hat{\theta}\, \frac{(1 - H(\rho))}{e\rho}\, \delta_{a,3} = -\hat{\theta}\, \frac{A(\rho)}{e\rho}\, \delta_{a,3} , \qquad (2\text{-}2b)$$

$$A_a^{ocl}(\vec{\rho}) = 0 .$$

The functions $f(\rho)$, $g(\rho)$ and $H(\rho)$ must obey the following boundary conditions:

$$f(\infty) = \left(\frac{\mu_1^2 \lambda_2 - \mu_2^2 \gamma}{\lambda_1 \lambda_2 - \gamma^2} \right)^{1/2} , \qquad (2\text{-}3a)$$

(2-3a)

$$g(\infty) = \left(\frac{\mu_2^2 \lambda_1 - \mu_1^2 \gamma}{\lambda_1 \lambda_2 - \gamma^2} \right)^{1/2} .$$

$$f(\rho) \xrightarrow[\rho \to 0]{} 0 \quad , \qquad g(\rho) \xrightarrow[\rho \to 0]{} \rho \qquad (2\text{-}3b)$$

and

$$H(\infty) = 0 \quad \text{and} \quad H(0) = 1.$$

Explicitly, $H(\rho)$, as $\rho \to 0$ is given by $H(\rho) \approx 1 + \rho^2$, and as $\rho \to \infty$ by $H(\rho) \approx e^{-a\rho}$, when "a" is a parameter of order M, the scale where the symmetry is broken.

The interaction of massless fermions with the vortex field is obtained by additing to (2-1a) the Lagrangian density below.

$$L_F = \bar{\psi}\,(i\not\partial - eA)\psi \quad , \qquad (2.4)$$

whose equation of motion is:

$$\vec{\alpha} \cdot (\vec{p} - eA)\psi = i\partial_t \psi . \qquad (2.5)$$

For two spatial dimensions, the "Dirac" $\vec{\alpha}$ matrices are: $\vec{\alpha} = (-\sigma^2, \sigma^1)$, where σ^i are the Pauli matrices. The fermionic spinor has two components only.

The β matrix, which would be present if there were mass term in (2.4), is taken to be σ^3. Since $\beta = \sigma^3$ anticommutes with H, it makesa relation between the eigen modes of H with positive and negative energy, i.e. $\psi_E = \sigma^3 \psi_{-E}$. The zero-energy modes, ψ_o, are eigen modes of σ^3 also. $\psi_o = \begin{pmatrix} u \\ 0 \end{pmatrix}$ or $\psi_o = \begin{pmatrix} 0 \\ v \end{pmatrix}$.

In the presence of mass terms this symmetry is broken.

III - THE QUANTIZATION OF THE SYSTEM

To study the dynamics of the fermionic field in the vortex sector, whose configuration (2.2b) represents the background field, we use the functional integral technique as to obtain the fermionic Green function's from the functional generator below.

$$Z\,[\bar{\zeta},\zeta] = \int [dA]\,[d\phi]\,[d\bar{\chi}]\,[d\chi]\,[d\bar{\psi}]\,[d\psi]\exp[iS^{(q)} + \text{gauge} +$$

$$\text{gauge-fixing terms} + \text{ghost terms} + i\int d^3x(\bar{\chi}\psi + \bar{\psi}\chi)] \quad (3.1)$$

In our problem we consider

$$A_\mu = A_\mu^{cl} + A_\mu{}^q \quad ,$$

$$\phi = \phi^{cl} \text{and } \chi = \chi^{cl}.$$

The Higgs field will be handled classically. We will show in appendix A that for a convenient choice for $(A_a^\mu)^q$ given by

$$A_a^{o\,q} = \frac{A(\rho,\ t)}{e}\ \delta_{a,3}. \quad (3.2a)$$

$$A^{i}_{a}{}^{q} = \frac{A(\rho,t)}{e}\,\hat{x}^{i}\delta_{a,3} \;, \qquad (3.2b)$$

the classical Higgs sector is left invariant in (2.1a). We claim that $A(\rho = 0,t) = 0$ in order to avoid singularities at the origin. We can observe that $A_{\mu}{}^{q}$ presents a radial symmetry and that it is parallel, in isospace, to $A_{\mu}{}^{cl}$. With this choice, the system presents a great simplification. The term $S^{(q)}$ of (3.1) is the action correspondent to the quantum terms only.

In the presence of the quantum gauge field the Dirac equation is expressed by

$$H\psi = [\vec{\alpha}.(\vec{p} - eA) + eA_{0}]\psi = i\partial_{t}\psi \;. \qquad (3.3)$$

The total angular momentum vector of the fermion $J = L_z + \frac{1}{2}\sigma_3 + \frac{1}{2}\tau_3 = -i\partial_\theta + \frac{1}{2}\sigma_z + \frac{1}{2}\tau_3$, that consists of an orbital plus a spin and isospin parts, commutes with H and has integer eigenvalues: $0, \pm 1, \pm 2 \ldots$.

To solve the Dirac equation (3.3) we will use for the matrices $\vec{\sigma}$ (spin) and $\vec{\tau}$ (isospin) the representation

$$\sigma_i = I_{(2)} \otimes \sigma_i \;,$$

$$\tau_i = \tau_i \otimes I_{(2)} \,,$$

where $I_{(2)}$ is a 2 x 2 unit matrix. In this representation we have

$$J = -i\,\mathrm{diag}\,(1,1,1,1)\partial_\theta + \mathrm{diag}\,(1,0,0,-1), \tag{3.4a}$$

and the spinor ψ has now four components.

$$\psi = \mathrm{col}(\psi_1^+,\psi_1^-,\psi_2^+,\psi_2^-) \; . \tag{3.4b}$$

The indices 1,2 refer to the charge and +, - to the spin.

Due to $[H,J] = 0$, we can expand the spinor ψ, of H, in the basis of eigenfunctions of J:

$$\psi^j(\vec{\rho},t) = \frac{e^{ij\theta}}{\sqrt{2\pi\rho}} \begin{pmatrix} \psi_{1,j}^+(\rho,t)\; e^{-i\theta} \\ \psi_{1,j}^-(\rho,t) \\ \psi_{2,j}^+(\rho,t) \\ \psi_{2,j}^-(\rho,t)\; e^{i\theta} \end{pmatrix} \tag{3.5}$$

where J can have some integer value..

Now the fermionic action will be:

$$S_F = \int dt\; d^2\vec{\rho}\; \bar{\psi}[i\partial\!\!\!/ - eA\!\!\!/]\psi$$

$$= \sum_j |C_j|^2 \iint d\rho dt (\bar{\eta}_1^{\,j}(\rho,t)[i\tau_2(\partial_\rho - i\frac{A(\rho,t)}{2}) - i(\partial_t + i\frac{A(\rho,t)}{2}) +$$

$$\tau_1 \frac{(J - H(\rho)/2)}{\rho}]\eta_1^{\,j}(\rho,t) + \bar{\eta}_2^{\,j}(\rho,t)[i\tau_2(\partial_\rho + i\frac{A(\rho,t)}{2}) - i(\partial_t - i\frac{A(\rho,t)}{2}) +$$

$$\tau_1 \frac{(J + H(\rho)/2}{\rho}]\eta_2^{\,j}(\rho,t)) \qquad (3.6a)$$

where the two component spinor $\eta_i^{\,j}(\rho,t)$ is defined by

$$\eta_i^{\,j}(\rho,t) = \begin{pmatrix} \psi_{i,j}^{+}(\rho,t) \\ \\ \psi_{i,j}^{-}(\rho,t) \end{pmatrix} , \qquad (3.6b)$$

and $\bar{\eta}(\rho,t) = \eta^{+}(\rho,t)\bar{\gamma}_0$, where $\bar{\gamma}_0 = \tau^3$.

Eq.(3.6a) correspondsto the action for two flavors of two dimensional fermion living on the half line interacting with an Abelian vector potential A_μ. Note also that $\eta_1^{\,j}(\rho,t)$ and $\eta_2^{\,j}(\rho,t)$ interact with A_μ with opposite chargesand that they do not interact among themselves.

We also can obtain the Dirac equation for each spinor :

$$[\tau_3 (\partial_\rho - i\frac{A(\rho,t)}{2}) + i\tau_1(\partial_t + i\frac{A(\rho,t)}{2}) - \frac{(J - H(\rho)/2)}{\rho}]\eta_1^{\,j}(\rho,t) = 0 \qquad (3.7a)$$

$$[\tau_3 (\partial_\rho + i\frac{A(\rho,t)}{2}) + i\tau_1(\partial_t - i\frac{A(\rho,t)}{2}) - \frac{(J + H(\rho)/2)}{\rho}]\eta_2^{\,j}(\rho,t) = 0 \quad . (3.7\ b)$$

If we define two news spinors by: $\eta_1{}^j = \tilde{\eta}_1^j \exp[\int^\rho \frac{J - H(\rho)/2}{\rho'} d\rho']$, and $\eta_2{}^j = \tilde{\eta}_2{}^j \exp[\int^\rho \frac{J + H(\rho')/2}{\rho'} d\rho']$, we obtain equations equivalent to (3.7a,b).

$$[\tau_3(\partial_t + i \frac{A(\rho,t)}{2}) + i\tau_1(\partial_\rho - i \frac{A(\rho,t)}{2}) + i \tau^+ \frac{(J - H(\rho)/2)}{\rho}]\tilde{\eta}_1{}^j(\rho,t) = 0 ,$$
(3.7a')

$$[\tau_3(\partial_t - i \frac{A(\rho,t)}{2}) + i\tau_1(\partial_\rho + i \frac{A(\rho,t)}{2}) + i \tau^+ \frac{(J + H(\rho)/2)}{\rho}] \tilde{\eta}_2{}^j(\rho,t) = 0,$$
(3.7b')

where $\tau^+ = \tau^1 + i\tau^2$.

Equations (3.7a',b') present singularities at the origin. (This is analogous to what occurs in the monopole problem). The components $\tilde{\eta}_1{}^{j=0}$ and $\tilde{\eta}_2{}^{j=0}$ can be made singularity-free if we recall that the function $H(\rho)$ is different from zero only at $\rho \lesssim \rho_v$ M^{-1}, where ρ_v is the vortex radius, and that for fermions with energy $E \ll M$ the details of fermion wave function in this region are irrelevant. Then, we can consider $\rho_v \to 0$. But, $H(0) = 1$. In order to have a non-singular solution at $\rho = 0$ one should impose the boundary condition $\tau^+\tilde{\eta}(\rho,t) \underset{\rho\to 0}{\to} 0$. (The other fermionic components are kept away from the vortex core by a centrifugal potential, so, in the $\rho_v \to 0$ limit, we can neglect the term $H(\rho)$ in (3.7a',b')[(3)]). Then, in this limit, the equations for the components $\tilde{\eta}_1^{j=0}$ and $\tilde{\eta}_2^{j=0}$ are:

$$[\tau_3(\partial_t \pm i \frac{A(\rho,t)}{2}) + i\tau_1(\partial_\rho \mp i \frac{A(\rho,t)}{2})]\tilde{\eta}^{j=0}_{1,2}(\rho,t) \qquad (3.8a)$$

with

$$\tau^{+} \eta^{j=0}_{1,2}(\rho=0,t) = 0 \ , \tag{3.6b}$$

that results in a two-dimensional model whose space-temporal coordinates and Dirac matrices are: $x^0 = t$, $x^1 = \rho > 0$, $\bar{\gamma}^0 = \tau^3$ and $\bar{\gamma}^1 = i\tau^1$. The matrices satisfy the anti-commuting relation $\{\bar{\gamma}^\mu, \bar{\gamma}^\nu\} = 2\bar{g}^{\mu\nu}$, where $\bar{g}^{\mu\nu}$ is the two-dimensional Minkowski metric.

If we restrict ourselves to the j=0 wave component, we can obtain, exactly , some results which are analogous to the monopole problem studied by C. Callan[1].

IV - EFFECTIVE ACTION

In this section we would like to obtain the effective action for this model, which consists of a fermionic part plus a quantic fluctuation of the gauge potential. The total fermionic action is:

$$i\, S_F^1 = \ln\left[\int [d\psi][d\bar{\psi}]\, e^{iS_F}\right] = \ln\det\left[i\gamma^\mu(i\partial_\mu - eA_\mu)\right] \ . \tag{4.1}$$

We know that $\delta \ln \det[i\gamma^\mu(i\partial_\mu - eA_\mu)] = ie\int d^3x\, \delta A^\mu \langle J_\mu\rangle_A$. If the current $\langle J_\mu\rangle_A$ is linear in A_μ, we can write

$$S_F^1 = \frac{e}{2}\int d^3x\; A^\mu(x)\; \langle J_\mu(x)\rangle_A \ . \tag{4.2}$$

The charge and radial eletric current densities are(*)

$$J_0^a(x) = \bar{\psi}\,\frac{\tau^a}{2}\,\gamma_0\psi = \frac{1}{4\pi\rho}\,(\bar{\eta}_1\,\tilde{\gamma}_0\eta_1 - \bar{\eta}_2\,\tilde{\gamma}_0\eta_2\,) + \sum_j \text{ terms with } j\neq 0 +$$

$$\sum_{j,j'} \text{ terms with } j\neq j' \; (\sim e^{i\theta(j-j')}) \quad ,$$

$$J_\rho^a(x) = \bar{\psi}\,\frac{\tau^3}{2}\,\vec{\gamma}.\hat{x}\psi = \frac{1}{4\pi\rho}\,(\bar{\eta}_1\,\tilde{\gamma}_1\eta_1 - \bar{\eta}_2\,\tilde{\gamma}_1\eta_2\,) + \sum_j \text{ terms with } j\neq 0 +$$

$$\sum_{j,j'} \text{ terms with } j\neq j \; (\sim e^{i\theta(j-j')}) \quad .$$

At this moment we would like to call attention for similarity between this model and that studied by V. Rubakov and C. Callan. Rubakov shows explicitely that the dominant contribution to the fermionic action is given by the S fermionic component. Then, due to their argument, in order to calculate S_F^1 in (4.2) we approximate for the charge and radial eletric curent density by

$$J_0^a(x) = \frac{1}{4\pi\rho}\,[\bar{\eta}_1\,\tilde{\gamma}_0\eta_1 - \bar{\eta}_2\,\tilde{\gamma}_0\eta_2\,] \quad . \qquad (4.3a)$$

$$J_1^a(x) = \frac{1}{4\pi\rho}\,[\bar{\eta}_1\,\tilde{\gamma}_1\eta_1 - \bar{\eta}_2\,\tilde{\gamma}_1\eta_2\,] \quad . \qquad (4.3b)$$

(*) Hence forward we will use for two dimensional spinor $\tilde{\eta}_i^{j=0}$ the more simplified notation: $\eta_i(\rho,t)$.

But, to obtain $S_F^{\prime}$, we must have some relation between the fermionic field and the Abelian potential in (3.8a,b). We then use to solve that equation Schwinger's trick:

$$\eta_i^{j=0}(\rho,t) = \exp[i[\tilde{\gamma}_5 a_i(\rho,t) + b_i(\rho,t)]]\eta_i^{(0)}(\rho,t) \quad , \qquad (4.4)$$

where $\tilde{\gamma}_5 = \tau_2$. In order that η_i satisfy the chosen boundary condition $\tau^{+}\eta_i(\rho=,0,t) = 0$, we need to impose for the scalar field the condition:

$$a_i(\rho,t) \xrightarrow[\rho\to 0]{} 0.$$

The spinor $\eta_i^{(0)}(\rho,t)$ satisfy the free Dirac equation and the boundary condition $\tau^{+}\eta_i^{(0)}(\rho=0,t) = 0$.

It is easy to show that $\eta_{1,2}^{j=0}(\rho,t)$ satisfy the Dirac equation (3.2a) if

$$\dot{a}_{1,2}(\rho,t) - b'_{1,2}(\rho,t) = \mp \frac{A(\rho,t)}{2} \quad , \qquad (4.5a)$$

$$\dot{b}_{1,2}(\rho,t) - a'_{1,2}(\rho,t) = \mp \frac{A(\rho,t)}{2} \quad , \qquad (4.5b)$$

or

$$\Box a_{1,2}(\rho,t) = \Box b_{1,2}(\rho,t) = \mp \frac{1}{2}(\dot{A}(\rho,t) + A'(\rho,t)) \quad , \quad (4.5c)$$

where $\Box = \partial^2{}_t - \partial^2{}_\rho$.

From the boundary condition $a(0,t) = A(0,t) = 0$, we have $b'(0,t) = 0$.

The propagator of the field η_i (ρ,t), which satisfies the same boundary condition as η_i itself is:

$$G_i(\rho t,\rho' t') = \exp[i[\tau_2 a_i(\rho t) + b_i(\rho t)]]G_0(\rho t,\rho' t')\exp[i[\tau_2 a_i(\rho t) - b_i(\rho t)]], \tag{4.6a}$$

such that

$$G_0(\rho t,\rho' t') = -i[\tau_1\partial_t + i\tau_2\partial_\rho][S(\rho-\rho',t-t') + S(\rho+\rho',t-t')\tau_3], \tag{4.6b}$$

where $S(\rho,t) = \frac{1}{4\pi}\ln(t^2 - \rho^2)$ is the two-dimensional scalar free field propagator.

Returning to the calculus of S_F^1, in the (4.3a,b) approximation, we have:

$$S_F^1 = \frac{e}{2}\int d^2x\, A^\mu\langle J_\mu\rangle = \frac{1}{4}\int dt d\rho A(\rho,t)[\langle\bar\eta_1\tilde\gamma_0\eta_1\rangle_A - \langle\bar\eta_2\tilde\gamma_0\eta_2\rangle_{-A} + \langle\bar\eta_1\tilde\gamma_1\eta_1\rangle_A -$$

$$- \langle\bar\eta_2\tilde\gamma_1\eta_2\rangle_{-A}] .$$

Using the point-separation procedure

$$\langle\bar\eta\tilde\gamma_\mu\eta\rangle = \frac{1}{2}\lim_{\varepsilon\to 0}[F_\mu(x,x+\varepsilon,A) + F_\mu(x,x-\varepsilon,A)] ,$$

where
$$F_\mu(x_1,x_2,A) = \exp\left[i\int_{x_1}^{x_2} A_\mu dx\right] \mathrm{tr}\left[\gamma_\mu G(x_1,x_2)\right],$$

we obtain

$$<\bar{\eta}_1 \gamma_1 \eta_1> = -<\bar{\eta}_2 \gamma_1 \eta_2> = -\frac{\dot{a}(\rho,t)}{\pi},$$

$$<\bar{\eta}_1 \gamma_0 \eta_1> = -<\bar{\eta}_2 \gamma_0 \eta_2> = -\frac{b'(\rho,t)}{\pi},$$

where we call $a_1(\rho t) = -a_2(\rho t) = a(\rho,t)$, and $b_1(\rho,t) = -b_2(\rho,t) = b'(\rho,t)$.

In this problem the time-varying gauge function $A(\rho,t)$ can be interpreted as an Abelian potential in the effective two-dimensional problem. So we can use the result above to calculate the fermionic action

$$S'_F = -\frac{1}{2\pi}\iint dt\,d\rho\, A(\rho,t)\left[\dot{a}(\rho,t) + a'(\rho,t)\right] \qquad (4.7)$$

Integrating by parts,

$$S'_F(a) = \frac{1}{2\pi}\iint dt\,d\rho\, a(\rho,t)\left[\dot{A}(\rho,t) + A'(\rho,t)\right]$$

$$= -\frac{1}{\pi}\iint dt\,d\rho\; a(\rho,t)\,\Box\, a(\rho,t) \qquad (4.8)$$

From equations (3.5a,b) we can see that $\dot{A} = a'$, $A' = b'$, then, by (4.7), we can express S'_F as a functional of the b field:

$$S'_F(b) = -\frac{1}{\pi}\iint dt\,d\rho\; b(\rho,t)\,\Box\, b(\rho,t). \qquad (4.8')$$

The action of the quantum gauge field A_μ^q is given by,

$$S_{YM} = \frac{1}{4}\iint dt\, d^2r\; 2(E_i^a)^2 = \frac{1}{2e^2}\int dt\, d^2\rho \left[\dot{A}(\rho t) + A'(\rho,t)\right]^2 ,$$

$$S_{YM}(a) = \frac{4\pi}{e^2}\iint dt\, d\rho\; \rho(\Box a(\rho t))^2 . \qquad (4.9)$$

Again, we can also express S_{YM} by

$$S_{YM}(b) = \frac{4\pi}{e^2}\iint dt\, d\rho\; \rho(\Box b(\rho,t))^2 . \qquad (4.9')$$

The full effective action is the sum of $S_F^{\prime}$ and S_{YM}.

$$S_{eff}(a) = \frac{4\pi}{e^2}\iint dt\, d\rho\; \rho(\Box a(\rho t))^2 - \frac{1}{\pi}\iint dt\, d\rho\; a(\rho t)\,\Box\, a(\rho t),$$
$$(4.10)$$
$$S_{eff}(b) = \frac{4\pi}{e^2}\iint dt\, d\rho\; \rho(\Box b(\rho,t))^2 - \frac{1}{\pi}\iint dt\, d\rho\; b(\rho t)\,\Box\, b(\rho t).$$

We see that the effective action above is quadratic, then, it is possible to find the low-lying exitations of the vortex massless fermion system. Due to the fact that these full effective actions can be expressed as functionals of "a" or "b" field, we can obtain the differential equation for these fields by varying S_{eff} with respect to itself following the variational principle. For $S_{eff}(a)$ we impose that $\delta a = 0$ in the extremes, and for $S_{eff}(b)$ that $\delta b' = 0$. In both the cases qe obtain the equations:

$$\frac{4\pi}{e^2} \Box (\rho \Box a(\rho,t)) - \Box a(\rho t) = 0 . \qquad (4.11)$$

with $a(\rho = 0,t) = 0$, and

$$\frac{4\pi}{e^2} \Box (\rho \Box b(\rho t)) - \Box b(\rho t) = 0 , \qquad (4.11')$$

with $\partial\rho \, b(\rho = 0,t) = 0$.

From the above equations we can obtain the solutions for $a(\rho,t)$ and $b(\rho,t)$, and consequently, the expression for the radial electric field $E^a_\rho(\rho,t) = E^a_i(\rho,t)\,\hat{x}^i = -2\,\frac{\Box a(\rho t)\delta_{a,1}}{e}$ $(= -2\,\frac{\Box b(\rho,t)\delta_a}{e}$
Before solving the full equations, let us obtain the solution when we neglect $\frac{1}{\rho}$ in (4.11). In this case the equation to be solved is:

$$\Box (\rho \Box a(\rho t)) = 0 .$$

In this specific case the problem reduce to obtain a vortex configuration with a "electric" field $E^a_\rho(\rho,t)$. Of course, in this theory where the unbroken subgroup is Z_2, all the vector mesons are massive and E^a_ρ is not an electric field in the sense of that Callan's paper. In Appendix B we return to this problem.

Considering now the full effective action (4.11) we note that a solution of

$$\left(\Box - \frac{e^2}{4\pi^2\rho}\right) a(\rho t) = 0 \quad ,$$

with $a(\rho = 0, t) = 0$,

is also its solution. We can look for solutions for $a(\rho,t)$ of the form $a(\rho,t) = e^{i\lambda t} g(\lambda\rho)$, when $g(y)$ satisfies the differential equation below:

$$\frac{d^2 g(y)}{dy^2} + \frac{e^2}{4\pi^2\lambda}\frac{g(y)}{y} + g(y) = 0 \ ,$$

whose solution is:

$$g(y) = \exp(-iy)\ {}_1F_1\left(1 + i\frac{e^2}{8\pi^2\lambda};\ 2;\ 2iy\right) \ .$$

where ${}_1F_1(a;b;z)$ is confluent hypergeometric function. Some of its properties and integral representation are given in the appendix C.

Now we can write, respectively, the solution for both fields $a(\rho t)$ and $b(\rho t)$, which satisfy different boundary conditions at the origin:

$$a(\rho t) = e^{i\lambda(t-\rho)} \lambda\rho \; {}_1F_1(1 + i\frac{e^2}{8\pi^2\lambda}; 2; 2i\lambda\rho) \;,$$

and

$$b(\rho,t) = e^{i\lambda(t-\rho)} \lambda\rho \; {}_1F_1(1 + i\frac{e^2}{8\pi^2\lambda}; 2; 2i\lambda\rho) + i\, e^{i\lambda(t-\rho)}.$$

The electric field is now given by

$$E^a_\rho(\rho,t) = -\frac{e\lambda}{2\pi^2} e^{i\lambda(t-\rho)} \; {}_1F_1(1 + i\frac{e^2}{8\pi^2\lambda}; 2; 2i\lambda\rho)\,\delta_{a,3}$$

In the small coupling constant limit, $e^2 \to 0$, we can obtain

$$E^a_\rho(\rho t) = -\frac{e}{2\pi^2\rho} e^{i\lambda t}\left(\frac{\lambda\rho\pi}{2}\right)^{1/2} J_{1/2}(\lambda\rho)\,\delta_{a,3}$$

$$= -\frac{e}{2\pi^2\rho} e^{i\lambda t} \operatorname{sen}\lambda\rho\,\delta_{a,3} \;.$$

We can see from the expression for the electric field that it is oscillatory in time and hence its average value vanishes. This type of phenomenon also occurs in the fermion-monopole problem(4).

V - THE MASS TERM

In the massless fermion-non Abelian magnetic monopole system, it was shown that the condensate $\langle \bar{\psi}\psi \rangle$ is non vanishing. This fact is related with chiral symmetry breaking. In this section we show that this condensate does not appear in our system. This happens because we are working in an odd dimensional space when chirality does not make sense.

Consider now the term below:

$$\bar{\psi}\psi = \frac{1}{2\pi\rho}\left[\bar{\eta}_1(\rho t)\,\eta_1(\rho t) + \bar{\eta}_2(\rho t)\,\eta_2(\rho t)\right] + \text{other comp.}$$

To calculate the expectation value $\langle \bar{\psi}\psi \rangle_V$ in the vortex field, we will consider only the contribution of the first term(5). So, we have

$$\bar{\psi}\psi = D_1(\rho t) + D_2(\rho t)\ .$$

where $D_i(\rho t) = \frac{1}{2\pi\rho}\,\bar{\eta}_i(\rho t)\,\eta_i(\rho t)$. We wish to obtain $D_i(\rho t)$. The two-point function is then

$$\langle D_2^{*}(x_1)D_1(x_2)\rangle = \frac{1}{4\pi^2\rho_1\rho_2}\langle \bar{\eta}^{(0)}(x_1)\, e^{2i\tau_2 a(x_1)}\eta^{(0)}(x_1)\, \bar{\eta}^{(0)}(x_2) e^{2i\tau_2(x\ )}\eta^{(0)}(x_2)\rangle$$

$$= \frac{1}{8\pi^2\rho_1\rho_2}\{[e^{-2\langle a(x_1)a(x_1)\rangle}e^{-2\langle a(x_2)a(x_2)\rangle}[e^{-4\langle a(x_1)a(x_2)\rangle} +$$

$$e^{4\langle a(x_1)a(x_2)\rangle}]]\mathrm{Tr}[G_0(x_1,x_2)G_0^T(x_1,x_2)] - [e^{-2\langle a(x_1)a(x_1)\rangle}e^{-2\langle a(x_2)a(x_2)\rangle}$$

$$[e^{-4\langle a(x_1)a(x_2)\rangle} - e^{4\langle a(x_1)a(x_2)\rangle}]] [\mathrm{Tr}G_0^2(x_1,x_2) - [\mathrm{Tr}G_0(x_1,x_2)]^2]\} ,$$

where $G_0(x_1,x_2)$ is given by (4.6b).

From $S_{eff}(a)$, (4.10), we can obtain the propagator $\langle a(x_1)a(x_2)\rangle$.

$$\langle a(x_1)a(x_2)\rangle = K(x_1,x_2) = \int [da]\ a(x_1)a(x_2)e^{iS_{eff}(a)},$$

which obeys the boundary condition $K(\rho = 0,t,\ \rho'\ t') = 0$, and must satisfy the differential equation

$$\Box[\frac{4\pi^2}{e^2}\rho\,[\Box - \frac{e^2}{4\pi^2\rho}]]\ K(\rho t,\rho' t') = \frac{\pi}{2}\ (\rho-\rho')\ (t-t')\ .$$

the expression for $K(x_1,x_2)$ is given by

$$K(x_1,x_2) = -\frac{\pi}{2}\,[D^{(o)}(x_1,x_2) - D^{(e_2)}(x_1,x_2)]\ .$$

where

$$\Box D^{(o)}(x_1,x_2) = \delta^{(2)}(x_1 - x_2)$$

with $\qquad D^{(o)}(\rho = 0,t;\ \rho',t') = 0 \qquad$ and

$$(\Box - \frac{e^2}{4\pi^2\rho})\ D^{(e^2)}(x_1,x_2) = \delta^2(x_1 - x_2)$$

with $D^{(e^2)}(\rho = 0, t; \rho', t') = 0.$

The solution for $D^{(0)}(\rho t, \rho', t')$ is:

$$D^{(0)}(\rho t, \rho', t') = S(\rho - \rho', t - t') - S(\rho + \rho', t - t'),$$

We can obtain the solution for $D^{(e^2)}(\rho t, \rho', t')$ from the solution of the homogeneous differential equation associated with the propagator which satisfies the same bounday condition.

We can write $D^{(e^2)}(x_1, x_2)$ by making an expansion in e^2:

$$D^{(e^2)}(x_1, x_2) = \bar{D}^{(0)}(x_1, x_2) + \bar{D}^{(e^2)}(x_1, x_2) + O(e^4) \ldots$$

With a little work we can show that

$$\bar{D}^{(0)}(\rho t, \rho' t') = -\frac{1}{2} Q_0\left[1 + \frac{(\rho-\rho')^2 - (t-t')^2}{2\rho\rho'}\right] = \frac{1}{4\pi} \ln\left[\frac{(t-t')^2 - (\rho-\rho')^2}{(t-t')^2 - (\rho+\rho')^2}\right].$$

($Q_m(x)$ is a Legendre function). So, the propagator $K(\rho t, \rho' t')$ is given by

$$K(\rho t, \rho' t') = \frac{\pi}{2} \bar{D}^{(e^2)}(\rho t, \rho' t') + O(e^4) ,$$

which is of order e^2. (In the monopole problem, the propagator used to obtain the condensate $\langle \bar{\psi} \psi \rangle$ was $\langle b(x_1)\, b(x_2) \rangle =$ $K'(x_1, x_2)$ which satisfies another bounday condition at the origin, implying that the dominant term in $K'(x_1,x_2)$ is of order 1). The explicit form for $K(\rho t,\rho' t')$, in two specific limits, are:

$$K(\rho t,\rho t) = \frac{\alpha\pi\rho}{12} + O(e^4),$$

and

$$K(\rho t,\rho' 0) \underset{t\to\infty}{\approx} \frac{\alpha\rho\rho'(\rho + \rho')}{4\pi^2 t^3} + O(e^4),$$

where $\alpha = e^2/4\pi$.

For $t - t' = t \to \infty$ the terms $Tr[G_0(x_1,x_2)G_0^+(x_1,x_2)]$, $T_r G_0^2(x_1,x_2)$ and $[TrG_0(x_1,x_2)]^2$ behave like $1/t^2$. Then

$$< D_1^+(\rho t)\ D_1(\rho' 0) > \underset{t\to\infty}{\longrightarrow} 0 \ .$$

Using the cluster theorem for two-point function we conclude that

$$< D_1(\rho t) > = 0$$

and consequently the condensate $<\bar{\psi}\psi>_V$ vanishes in the vortex field.

VI - DISCUSSION AND CONCLUSIONS

We saw that the fermionic-vortex system presents some properties analogous to the fermion-monopole one. For example: The dynamics of interaction of specific wave fermionic component is given by the Schwiger's model. For this component the full effective action is a quadratic functional of the fields, so, we can obtain exactly the propagator associated with this field. (The fermionic propagator for the spinor $\eta_1^{j=0}(\rho,t)$ is also possible to be obtained). Differently from the monopole problem, in our case the propagator $< a(x_1)a(x_2) >$ does not present a short distance divergence[6].

Physical results which depend on the dimension of the space, like the value of condensate $< \bar{\psi}\psi >$, are different for these two models fermion-monopole and fermion-vortex.

About the vortex field we would like to make some pertinent commentaries: The Dirac equation in the presence of the vortex background field, (2.2b,c), does not present bound-state solutions[2]. (Generaly, for a vortex background field which has a magnetic flux $\phi = 1/4\pi \int d^2\vec{\rho}\, \epsilon^{ij} F_{ij} = 1/2\pi \oint dx^i A_i = N/e$, the Dirac equation presents $|N| - 1$ zero-energy bound states [7]).

Unlike with magnetic monopoles, an interaction with a vortex field does not force quantization of the fermionic charge. In our treatment we set (arbitrarily) the Higgs' charge $q = -e$. For any other choice of the Higgs' charge we would have obtained different results. For example: for $q = n\,e$ with $|n|>1$ there is no component of fermionic wave function for which the system would present a two-dimensional behavior. It is also interesting to mention that the number of bound states would be different for different relations between q and e[8].

APPENDIX A

In this appendix we show that the choice (3.2a,b) for $A_\mu{}^q$ leaves the classical Higgs sector invariant.

A-1 ϕ^a field.

The covariant derivative for the Higgs field is given by

$$D_\mu \phi^a = \partial_\mu \phi^a - e\, \varepsilon^{abc} A_\mu{}^b \phi^c \ . \tag{A-1}$$

The classical ϕ^a field has the form

$$\phi(\vec{\rho}) = f(\rho) \begin{pmatrix} \cos\theta \\ \sin\phi \\ 0 \end{pmatrix} = f(\rho) \begin{pmatrix} \hat{x}_1 \\ \hat{x}_2 \\ 0 \end{pmatrix} \ . \tag{A-2}$$

For the background gauge field (2.2b,c) we obtain

$$(D_0 \phi^a)^{cl} = 0 \ ,$$

$$(D_i \phi^a)^{cl} = f'(\rho)\, \hat{x}_i \hat{x}_a + \frac{f(\rho)}{\rho} (1 + A(\rho))\, \varepsilon^{ij} \hat{x}_j \varepsilon^{ab} \hat{x}_b \ . \tag{A-3}$$

where the index a(isospin) and i(spatial) assume value 1 and 2, and $f'(\rho) = \dfrac{df(\rho)}{d\rho}$.

Considering now the quantum gauge field (3.2a,b) we have:

$$(D_o\phi^a) = A(\rho,t)f(\rho)\ \epsilon^{ab}\hat{x}_b\ ,$$

$$(D_i\phi^a) = (D_i\phi^a)^{cl} + A(\rho,t)\ f(\rho)\ \hat{x}_i\ \epsilon^{ab}\hat{x}_b\ . \tag{A-4}$$

We can see that in spite of the covariant derivative of the Higgs field getting modified by the A_μ^q, the kinetic term in the Lagrangian density of this field will preserve its form:

$$(D_\mu\phi^a)^2 = (D_o\phi^a)^2 - (D_i\phi^a)^2 = [A(\rho,t)f(\rho)]^2 - [(D_i\phi^a)^{cl}]^2 - [A(\rho,t)f(\rho)]^2 -$$

$$- 2(D_i\phi^a)^{cl}\ A(\rho,t)f(\rho)\ \hat{x}_i\ \epsilon^{ab}\ \hat{x}_b = -\ [(D_i\phi^a)^{cl}]^2. \tag{A-5}$$

A-2 χ^a field

For the triplet χ^a given in (2.2a)

$$\chi(\vec{\rho}) = g(\rho)\begin{pmatrix}-\sin\theta\\ \cos\theta\\ 0\end{pmatrix} = g(\rho)\begin{pmatrix}-\hat{x}_2\\ \hat{x}_1\\ 0\end{pmatrix}\ . \tag{A-6}$$

its classical covariant derivative is :

$$(D_o\chi^a)^{cl} = 0\ ,$$

$$(D_i\chi^a)^{cl} = \frac{g(\rho)}{\rho}\,(1 + A(\rho))\ \hat{x}_a\ \epsilon^{ij}\ \hat{x}_j - g'(\rho)\ \hat{x}_i\ \epsilon^{ab}\ \hat{x}_b\ . \tag{A-7}$$

Now, considering the quantic gauge potential A_μ^q, we have:

$$(D_0\chi^a) = A(\rho,t)\; g(\rho)\; \hat{x}^a ,$$

$$(D_i\chi^a) = (D_i\chi^a)^{cl} + A(\rho,t)\; g(\rho)\; \hat{x}_i x_a . \qquad \text{(A-8)}$$

Again, we can see that the kinetic term in (2.1a) for this field is not modified:

$$(D_\mu\chi^a)^2 = (D_0\chi^a)^2 - (D_i\chi^a)^2 = [A(\rho,t)g(\rho)]^2 - [(D_i\chi^a)^{cl}]^2 - [A(\rho,t)g(\rho)]^2 -$$

$$- 2(D_i\chi^a)^{cl}\; A(\rho,t)\; g(\rho)\; \hat{x}_i\hat{x}_a = -\;[(D_i\chi^a)^{cl}]^2 . \qquad \text{(A-9)}$$

APPENDIX B

Here we treat the problem of the vortex configuration with an "electric" massive field $E^a_i(\rho,t)$.

For the gauge configuration

$$A_\mu = A^{cl}_\mu + A^q_\mu .$$

where A^{cl}_μ is given by (2.2b) and A^q_μ by (3.2a-b), we have

$$F^a_{ij} = (F^a_{ij})^{cl} \qquad \text{(B-1)}$$

and

$$E^a_\rho(\rho,t) = \frac{(\Delta'(\rho,t) + \Delta'(\rho,t))}{e}\,\delta_{a,3} = -2\,\frac{\Box a(\rho,t)}{e}\,\delta_{a,3} \qquad \text{(B-2)}$$

The field $a(\rho,t)$ must satisfy the following equation

$$\Box(\rho\,\Box\, a(\rho,t)) = 0 .$$

For a solution with a nonzero charge Q^a given by $Q^3 = \int d^2\vec{\rho}$ div $\vec{E}^3 = \oint_{C_\infty} d\vec{\rho}\cdot\vec{E}^3 - \oint_{C_\epsilon} d\vec{\rho}\cdot\vec{E}^3$ where C_∞ and C_ϵ denote a circle of radius $\rho = \infty$ and $\rho = \epsilon$ ($\epsilon \to 0$) respectively, remembering that the massive electric field $\vec{E}$ drops off exponentially at large ρ, we must have $E^3_\rho \sim \frac{1}{\rho}$ as $\rho \to 0$. In this region, for $\Delta(\rho,t)$, a static solution that vanishes at $\rho = \rho_V$ is $\Delta(\rho) \sim \ln(\rho/\rho_V)$, and for $a(\rho,t)$ is $a(\rho) \sim \rho(\ln\rho - 1) - \rho_V(\ln\rho_V - 1)$.

A discussion about charged abelian vortex can be found in the paper of Julia-Zee [9]. There it is shown that a charged vortex configuration presents infinite energy. However, to the Abelian Higgs model if we add a topological mass term, we can obtain a charged Abelian vortex configuration with finite energy. This system is given by

$$L_V = -\frac{1}{4}(F_{\mu\nu})^2 + \frac{1}{2}|(\partial_\mu - ieA_\mu)\phi|^2 - \frac{1}{4}\lambda(|\phi|^2 - m^2/\lambda)^2 + \frac{\mu}{4}\varepsilon^{\mu\nu\lambda} F_{\mu\nu}A_\lambda . \qquad \text{(B-3)}$$

If we use the "ausatz"

$$A_0 = g(\rho)$$

$$A_i = \varepsilon_{ij} \hat{x}_j A(g)$$

and

$$\phi(\rho) = f(\rho)e^{in\theta}$$

the equations of motion are:

$$\frac{1}{\rho}\frac{d}{\rho}\left(\rho \frac{dg(\rho)}{d\rho}\right) + \frac{\mu}{\rho}\frac{d}{d}(\rho A(\rho)) + e^2 g(\rho)\, f^2(\rho) = 0 \ , \qquad \text{(B-4a)}$$

$$\frac{d}{d\rho}\left(\frac{1}{\rho}\frac{d}{d\rho}(\rho A(\rho))\right) - \mu\, g'(\rho) - e f^2(\rho)\left(e A(\rho) - \frac{n}{\rho}\right) = 0 \ , \qquad \text{(B-4b)}$$

and

$$-e^2 g(\rho)\, f(\rho) - \frac{1}{\rho}\frac{d}{d\rho}\left(\rho \frac{df(\rho)}{d\rho}\right) + f(\rho)\left(e A(\rho) - \frac{n}{\rho}\right)^2 + \lambda f(\rho)(f^2(\rho) - m^2/\lambda) = 0 \qquad \text{(B-4c)}$$

From the equation above, we can see that for $\mu \neq 0$, we must have $g(\rho) \neq 0$. In order to obtain a finite energy solution, we must impose the following boundary conditions:

(i) For $\rho \to 0$,

$$f(\rho) \sim f_0 \rho^{|n|} ,$$

$$A(\rho) \sim A_0 \rho^{2|n|+1} ,$$

$$g(\rho) \sim 0 .$$

(ii) For $\rho \to \infty$,

$$f(\rho) \sim \frac{m}{\sqrt{\lambda}} ,$$

$$A(\rho) \sim \frac{n}{e\rho}$$

$$g(\rho) \sim 0 .$$

From the equation of motion for the Lagrangian density (B-3) we have

$$\partial^\beta F_{\beta\alpha} + \mu\varepsilon_{\alpha\mu\nu}(\partial^\mu A^\nu) = j_\alpha = -\frac{ie}{2}\left[\phi^*(\partial_\alpha\phi) - \phi\,(\partial_\alpha\phi^*)\right] - e^2 A_\alpha|\phi|^2 .$$

Its temporal component is:

$$\operatorname{div}\vec{E} - \mu B = -e^2 g(\rho)\, f(\rho)^2 . \qquad \text{(B-5)}$$

The electric charge of this vortex configuration is,

$$Q = - e^2 \int d^2\vec{\rho}\ g(\rho)\ f^2(\rho), \tag{B-6}$$

which is nonzero for $\mu \neq 0$. Integrating (B-5) over all space, and remembering that the massive field drops off at large distances, we have:

$$\int d^2\vec{\rho}\ B(\rho) = \frac{e^2}{\mu} \int d^2\vec{\rho}\ g(\rho)\ f^2(\rho) = \frac{Q}{\mu} .$$

But, $\int d^2\vec{\rho}\ B(\rho) = \frac{2\pi n}{e}$, therefore,

$$\frac{Q}{\mu} = \frac{2\pi n}{e} ,$$

and, thus, we obtain a quantization relation for the ration Q/μ .

APPENDIX C

In this appendix we summarize some relevant properties about the confluent hypergeometric function.

The confluent hypergeometric function ${}_1F_1(a;b;z)$, also represented by $M(a;b;z)$, has the expansion[10]

$$M(a;b;z) = 1 + \frac{a}{b}\frac{z}{1} + \frac{(a)_2}{(b)_2}\frac{z}{2!} + \ldots + \frac{(a)_n}{(b)_n}\frac{z^n}{n!} + \ldots \quad (C\text{-}1)$$

where

$$(a)_n = a(a+1)\ldots(a+n-1),\ (a)_0 = 1 .$$

For $b \neq -n$ and $a \neq -m$, $M(a;b;z)$ is a convergent seriesfor all values of a, b and z,

When $|z| \to \infty$

$$M(a;b;z) = \frac{\Gamma(b)}{\Gamma(a)}\, e^{z}\, z^{a-b}[1 + O(|z|^{-1})] . \qquad (C\text{-}2)$$

The hypergeometric $M(a;b;z)$ has several integral representations. One of them is:

$$\frac{\Gamma(b-a)\Gamma(a)}{\Gamma(b)}\, M(a;b;z) = z^{1-b}\, e^{z/2} \int_{-1}^{1} dt\, e^{-zt/2}\, (1+t)^{b-a-1}(1-t)^{a-1} . \quad (C\text{-}3)$$

In our problem, the confluent hypergeometric function

$$M(1 + \frac{ik}{2\lambda};2;2i\lambda\rho) ,$$

where $k = e^2/4\pi^2$, has the expansion below:

$$M(1 + \frac{ik}{2\lambda};2;2i\lambda\rho) = 1 + \frac{(2i\lambda - k)}{2}\frac{\rho}{1!} + \frac{(2i\lambda - k)(4i\lambda - k)}{2x3}\frac{\rho^2}{2!} + \ldots \quad \text{(C-4)}$$

We see that this function is not singular in the $\lambda \to 0$ limit.

$$M(1 + ik/2\lambda;2;2i\lambda\rho) \xrightarrow[\lambda \to 0]{} \frac{1}{\sqrt{-2k\rho}} I_1(2\sqrt{-2k\rho}) . \quad \text{(C-5)}$$

An integral representation of this function is:

$$M(1 + ik/2\lambda;2;2i\lambda\rho) = \frac{\lambda e^{i\lambda\rho}}{\pi k} \text{ senh } (\frac{\pi k}{2\lambda})\int_{-1}^{1} dt \ e^{i\lambda\rho t}(\frac{1 + t}{1 - t})^{ik/2\lambda} \quad \text{(c-6)}$$

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