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ABSTRACT

A steady state high beta plasma flow through a channel along the magnetic field increasing downstream can be regarded as a "steady state theta pinch", because if we see the plasma riding on the flow we should observe very similar process taking place in a theta pinch. Anticipating to produce an FRC without using very high voltage technics such as the ones required in a conventional theta pinch, we have studied after the analogy a "steady state reversed field theta pinch" which is brought about by steady head-on collision of counter plasma streams along the channel as ejected from two identical co-axial plasma sources mounted at the both ends of the apparatus. The ideal Poisson and shock adiabatic flow models are employed for the analysis of the steady colliding process. It is demonstrated that an FRC involving large numbers of particles is produced only by the weak shock mode which is achieved in case energetic plasma flow is decelerated almost to be stagnated through Poisson adiabatic process before the streams are collided.

1. INTRODUCTION

FRC (field reversed configuration) as produced by a reversed field theta pinch has begun to receive much attention again recently, because in many places it is demonstrated that plasma having <β> close to unity can be controlled to decay smoothly free from violent $n=2$ rotational mode, which had been the fatal defect in the early experiments pursued during the decade of 1960. Indeed, for example, recent FRX-C experiments at Los Alamos [1] recorded the life times as long as 300 μs using the multipole stabilizing method [2] . In this sense we believe the best strategy at this time to advance FRC toward a fusion reactor is to challenge to the steady state experiment that demonstrates further heating and enhancement of the trapped flux is really possible. It is pointed out that, in a steady state FRC, simpler stabilization technics controlling the plasma spin up [3] may be available, because required beams to supply power, particles and the emf [4] for steady state can also be utilized to render the plasma the torque eliminating the spin up. In order to carry out steady state experiment successfully, we would say the preparation only of high power beams will not be sufficient but we need an improved initial target FRC having higher temperature, smaller density and larger separatrix radius size than the ones obtained by the present reversed field theta pinch technics.

In this paper a method is presented to produce such target plasma, using a slow formation technic based on a high beta steady intensive plasma flow which is ejected from a CAPS

(co-axial plasma source) to pass through a channel formed along magnetic field . We recognize that, if the guiding field pressure $B^2/(2\mu_0)$ is set equal to the average pressure $\langle p \rangle$ of the flow, beta value of the plasma becomes very high so that the plasma channel with thin enough sheath may be produced. Examples of such high beta plasma channels are seen in the early cusp experiments [5,6] .

Here, it is pointed out that if such a high beta stream is induced in steady state in passing through the guiding field increasing downstream we may regard it as a "steady state theta pinch", because if we see the plasma riding on the flow we should observe the very similar process taking place in the compression phase of a conventional theta pinch. We would say the analogy to the usual method to produce FRC has yieled the basic concept of the present one making use of a "steady state reversed field theta pinch". The sequence how the present method operates is schematically shown in Fig.1. It is seen that we utilize the two CAPSs mounted on both ends of the apparatus, and operate them to produce the target FRC through the following four stages in sequence:

the stage 1 - two cusp configurations in front of each CAPS are generated,

the stage 2 - the plasmas are ejected from the CAPSs stretching out the field lines,

the stage 3 - after the collision of the counter streams which accompanies the reconnection of the field lines trapped inside, the plasma is set in steady state keeping the balance between the injection from the CAPSs and flowing out to the loss channel formed on the surface of the plasma along the field lines,

the stage 4 - the plasma from the CAPSs are ceases to be injected so that the reconnection takes place to form an FRC.

As may be seen, the present concept is quite similar to the spheromak production technics using CAPS [7,8] except we utilize the two.

In order to simplify the analysis, we assume to use an ideal CAPS ejecting plasma without carrying any azimuthal field B_θ in spite of the fact that presently available ones involve considerably. We believe such ideal CAPS may be developed in a future because a steady state analysis of CAPS, which is expected to be published elsewhere, implies reduction of B_θ to zero is possible in principle in case some particular construction is adopted. It is also noted that B_θ may be annihilated in the stage 3 around the central plasma colliding region, because the two CAPSs eject plasma carrying B_θ in opposite direction by each other. Actually such annihilation has been observed in the TS-3 spheromak experiment at Tokyo University [9] . Therefore, we may say, even if a small amount of B_θ is included at the muzzle, the ideal analysis will not yield significant errors provided $B_\theta \ll B$ is satisfied.

In this paper a steady head-on collision of high beta counter plasma flows taking place at the stage 3 is analysed by using the ideal Poisson and shock adiabatic flow models so as the required parameters of the CAPS can be found from the given data set of the target FRC. The main body of the analyses is given in subsequent section and in section 3 examples of the results are shown with discussions on the advantages of the present method. In the final section the conclusions of the paper are presented.

2. Steady State Modeling Of Colliding Counter Flows

In order to analyse physical process taking place at the stage 3 in detail, we refer a more specific schematic picture shown in Fig.2 instead of the one in Fig.1. As may be seen an energetic plasma is ejected out continuously from the plasma source through the surface 0 at the muzzle into the axial field channel along which a steady state high beta plasma is conducted. Because of the symmetry of the counter streams, the plasma flows must be completely stagnated at the surface C which is located at the center of the system. We postulate sudden deceleration of the flow takes place between the surface 1 and 2, and at the surface 2 the flow is almost stagnated to form a reservoir filled up with a high beta plasma in the central part. We assume the plasma in the reservoir steadily leaks out along the field lines existing in the thin sheath formed on the surface of the plasma. It is noted we may compose a truly stagnated plasma reservoir if we place a virtual stagnation surface C' close to the surface 2, which eventually claims the plasma sink and source for the channel flows and the escaping peripheral flows, respectively, in the thin volume surrounded by the surface 2 and C^{*}.

In the subsection 2.1 the ideal isentropic Poisson adiabatic process important between the surface 0 and 1 is presented while the shock adiabatic process between the surface 1 and 2 is given in subsection 2.3. The particle loss model from the reservoir through the sheath on periphery is formulated in subsection 2.2. In the final subsection we deduce the required specification to the CAPS from the given parameters set of the target FRC.

2.1 A high beta Poisson adiabatic flow

A high beta plasma flow channel along the axial guiding field has a big difference to an usual gas flow through a solid duct or to a low beta plasma flow along the field lines [10] in the point that it is surrounded by a flexible wall deformable in response to the plasma pressure. For the analysis a schematic drawing in Fig.3 is referred.

We postulate an energetic high beta plasma capable of forming a sharp interface between the field and the plasma is ejected out continuously from the plasma source through the surface 0 at the muzzle into the axial guiding field and is completely stagnated at the surface C. Here, we employ a simple single fluid model of cold electrons assuming plasma temperature T is equal to ion temperature and specific heat ratio γ is 5/3, so that the basic three conservation laws of particle numbers, enthalpy (the Bernoulli's law) and entropy (the Poisson's formula) are presented in the following way if we assume plasma flow velocity v and density n are uniform over the cross-section area A of the channel:

$$I = envA \quad , \tag{1}$$

$$E = \frac{1}{2}\frac{m}{e}v^2 + \frac{5}{2}T, \tag{2}$$

$$\frac{T^{3/2}}{n} = \frac{T_0^{3/2}}{n_0} \quad , \tag{3}$$

where I and E are constants along the flow measured by Ampere and eV, respectively, and suffix 0 is given to the quantities at the surface 0. For completing the closed set of equations we must add the requirement of radial pressure balance. Because plasma is postulated to be in high beta and p is uniform over the cross-section, we easily have

$$\frac{B^2}{2\mu_0} = p = nkT \quad , \tag{4}$$

where B is the field strength of the guiding field. In the first place we wish to know how the parameters specified at the surface 0 change along the flow in case B is a given function of z. Then, we have following set of solutions from eqs.(2),(3) and (4) using dimensionless field strength $b(=B/B_0)$ and representing the quantities at the stagnation surface C by suffix C:

$$\frac{T}{T_0} = b^{4/5} \quad , \tag{5}$$

$$\frac{n}{n_0} = b^{6/5} \quad , \tag{6}$$

$$\frac{v}{v_0} = \frac{1-(b/b_C)^{4/5}}{1-b_C^{-4/5}}, \tag{7}$$

where

$$b_C = (\frac{T_C}{T_0})^{5/4} = (1 + \frac{mv_0^2}{5kT_0}) \quad . \tag{8}$$

Using eqs.(1), (6) and (7) we also have

$$\frac{A}{A_0} = \frac{(1-b_C^{-4/5})^{1/2}}{b_C^{6/5}} \cdot \frac{1}{(b/b_C)^{6/5}[1-(b/b_C)^{4/5}]^{1/2}} \quad . \tag{9}$$

Algebraic inspections of eq.(9) show, if v_0 is greater than the local sound velocity $[(5/3)kT_0/m]^{1/2}$, then there appears a waist in the cross-sectional area A_W of

$$\frac{A_W}{A_0} = \frac{16}{3\sqrt{3}}(\frac{T_0}{T_C})^{3/2}(1-\frac{T_0}{T_C})^{1/2} \tag{10}$$

at the field strength b_W given by

$$b_W = (\frac{3}{4})^{5/4} b_C \quad . \tag{11}$$

Consequently, we can see a high beta plasma ejected out from the plasma source with velocity greater than that of the local sound into the increasing field channel is gradually compressed (eq.(6)), heated up (eq.(5)) and decelerated (eq.(7)) to be completely stagnated at the point where the field strength achieves the value of B_C (eq.(8)). It is also seen in eq.(9) that the cross-section A is once decreased to form a waist but expand again to diverge out at the stagnation surface, which originates rather artificial fact that any plasma sink is not introduced in the present system. We believe these plasma behaviors through the increasing field channel are qualitatively in agreement with the early cusp injection experiments at Osaka University [5] , because it was demonstrated that a high beta plasma blob penetrated through the weaker entrance cusp (B=0.4 T) but was reflected back from the higher exit cusp(B=0.8 T) to be confined in the cusp trap: the reflection of the blob is considered to be equivalent to stagnation in steady state operation.

2.2 Particle loss rate from the reservoir

Because of the continuous plasma injection from the CAPSs through the main channel, the plasma in the reservoir is able to leak out only through a thin sheath on the plasma boundary into the vacuum particle sinks existing at the end of the field lines. Figure 2 suggests the loss process is quite similar to the one taking place in an open field theta pinch [11] suffering the end shortening so that we may regard the thickness δ the order of the ion Larmor radii ρ_i. If we assume the beta value of the sheath is low and the flow through it is described by the Poisson adiabatic process, then the Laval nozzle model may be adequate for the flow rate I to be estimated from the given parameters in the reservoir. We postulate the waist of the nozzle where flow velocity achieves sound speed appears at the surface 1 if $B_1 \geqq B_C$ while if $B_1 < B_C$ it moves to the surface C', where we put suffix 1 and C for the quantities at the surface and the reservoir, respectively. Consequently, it may be possible to infer the cross-section of the waist takes a simple form

$$A_{es} = \alpha_f \ \rho_{ies} \ l_{es} \quad , \tag{12}$$

where suffix es is rendered to the quantities at the waist so that l_{es} denotes the total length of the circumference of the nozzle, ρ_{ies} the ion Larmor radii evaluated by external field B_1 or B_C and α_f empirical adjusting factor of the order 1 to reflect the actual size of the sheath as measured by ρ_{ies} . As a result, the well known solution to the Laval nozzle [12] makes it possible to write down I in the following form in terms of the quantities in the reservoir :

$$I = en_{es}v_{es}A_{es} = \frac{9\sqrt{5}}{32}\left(\frac{e}{\mu_0}\right)^{1/2} \frac{\alpha_f l_{es}}{R_{Mes}}(n_C T_C)^{1/2}, \tag{13}$$

where R_{Mes} is the mirror ratio defined by $R_{Mes}=B_1/B_C \geqq 1$ and $R_{Mes}=1$ if $B_1/B_C<1$, which comes from the fact that the waist moves to the surface C'. If we apply the Bernoulli's law (eq.(2)) we may easily find out the power W through the nozzle:

$$W = \left(\frac{1}{2}n_{es}mv_{es}^2 + \frac{5}{2}p_{es}\right)v_{es}A_{es} = \frac{5}{2}T_C I \quad . \tag{14}$$

2.3 The shock adiabatic plasma deceleration

As schematically shown in Fig.2, we postulate the Poisson adiabatic plasma flow from the source is stagnated through the shock adiabatic process taking place between the surface 1 and 2. Thus, we are able to apply the conservation laws of I and W at the surface 1, and the momentum flux balance in z direction in the volume bounded by the surface 1 and C, from which we have three set of equations [12] :

$$e n_1 v_1 A_1 = I \quad , \tag{15}$$

$$A_1 v_1 \left(\frac{1}{2} m n_1 v_1^2 + \frac{5}{2} p_1\right) = W, \tag{16}$$

$$(p_1 + m n_1 v_1^2) A_1 = p_C A_C \quad . \tag{17}$$

Here, we wish to know the quantities at the surface 1 in terms of parameters in the reservoir and the given value of B_1. Inserting $T_1 = T_C - (m/e) v_1^2/5$ as derived from eqs.(15),(16) and (14) into eq.(17) to eliminate T_1, we have quadratic equation for v_1 and consequently obtain bifurcated solutions in the dimensionless form:

$$\xi_1 = \frac{v_1}{v_{tC}} = \frac{\sqrt{5}}{2} \frac{1}{\nu_{C*}} \left[1 \pm (1 - \nu_{C*})^{1/2}\right] \quad , \tag{18}$$

where $v_{tC} = (kT_C/m)^{1/2}$ is the thermal speed in the reservoir and $\nu_{C*} = N_{C*}/N_C$ where $N_C = n_C A_C$ is the line density and N_{C*} the minimum value of N_C for the solution to exist. On the other hand, we know eq.(13) is available to express N_{C*} in the form

$$N_{C*} = (\frac{9}{8})^2 \frac{m}{\mu_0 e^2} (\frac{\alpha_f}{R_{Mes}})^2 \frac{l_{es}^2}{A_C} \quad . \tag{19}$$

Considering the flow is stagnated in the reservoir, we may say (+) sign in eq.(18) gives the strong shock mode while (-) sign the weak shock mode. Using v_I thus obtained we may also have the solutions to T_I and N_I in the dimensionless forms:

$$\tau_I = \frac{T_I}{T_C} = 1 - \frac{1}{2\nu_{C*}} [1 - \frac{1}{2}\nu_{C*} \pm (1-\nu_{C*})^{1/2}] \quad , \tag{20}$$

$$\nu_I = \frac{N_I}{N_C} = \frac{1}{2} \frac{\nu_{C*}}{1 \pm (1-\nu_{C*})^{1/2}} \quad . \tag{21}$$

The value of A_I as required to deduce n_I from N_I may be evaluated if we remind B_I is a given quantity. It may be convenient to introduce mirror ratio R_M defined by $R_M = B_I/B_C$ instead of using B_I itself. And we we have the dimensionless form of A_I from eqs.(20) and (21):

$$\alpha_I = \frac{A_I}{A_C} = \frac{\nu_I \tau_I}{R_M^2} . \tag{22}$$

It is noted R_M can be smaller than 1 to make $R_{Mes}=1$, but if $R_M>1$ we should put $R_{Mes}=R_M$.

In Fig.4 graphical plots of ξ_I, τ_I and ν_I as a function of ν_{C*} are presented. It may be seen the strong shock mode on the branch indicated by (+) sign does not have solution for ν_{C*} less than 16/25 because T_1 is required to be negative. This indicates the momentum flux (F_1) through the surface 1 becomes greater than the one (F_C) through the surface C unless T_1 is negative. Inexistence of the solution at greater ν_{C*} than unity is also understood by inability to achieve momentum flux balance. We may rewrite I in eq.(13) in terms of N_C and N_{C*} and also LHS of eq.(17) in the form:

$$I = \frac{\sqrt{5}}{4} e \left(\frac{N_{C*} N_C T_C}{m}\right)^{1/2} , \tag{23}$$

$$F_1 = \left(\frac{T_C}{v_1} + \frac{4m}{5e} v_1\right) I . \tag{24}$$

Then, from eqs.(23) and (24) we find F_1 takes a minimum value $F_{1*} = kT_C (N_{C*} N_C)^{1/2}$ at $v_1 = (\sqrt{5}/2) v_{tC}$. On the other hand the RHS of eq.(17) is simply $kT_C N_C$ so that if N_C were less than N_{C*} the momentum flux balance is unable to be satisfied. As may be seen inexistence of steady state solution can always be ascribed to the excess of F_1, or in other words, to the over compression of the stagnated plasma at the center by excess momentum flux through the surface 1. Consequently, it is demonstrated we may have any quantities at the surface 1 using eqs.(18),(19),(20),(21) and (22) only if parameters in the reservoir and R_H are given.

2.4 Deduction of the specifications of the CAPS

The purpose of this section is to find out the required specifications of the CAPS from the given data set of an FRC as produced after the termination of continuous plasma injection in the stage 3. We consider the reconnection of the field lines probably takes place around the surface 1 instantaneously and the plasma begins to fall into the final stage 4, in which equilibrium state the plasma is subject not only to radial pressure balance but also axial tension balance. Generally speaking, the plasma motion may be set forth for the new state from the instant of reconnection. Indeed, it is reported in a theta pinch experiment [13] that axial plasma contraction appears after the reconnection for a larger separatrix radius R_s in case the amount of the trapped flux is large or equilibrium $\langle\beta\rangle$ is low. Here, detailed general analyses of the plasma motion toward the equilibrium are postponed in a future but we consider only the case that the equilibrium of an FRC is obtained insitu, assuming following four postulates are satisfied:

1) length L of the FRC is much larger than the distance between the surface 1 and 2,
2) reconnection takes place rapid enough as to conserve plasma particles and energy involved in the reservoir,
3) sharp boundary low flux model [14] can be applied,
4) conducting external wall is placed at the radius R_{wall} given by Suzuki-Hamada [15] ,

$$R_{wall} = \frac{R_s}{\left[1-\frac{(2\langle\beta\rangle-1)}{R_M}\right]^{1/2}} \quad . \tag{25}$$

In order to initialize the problem we take the following five parameters of the FRC to be the given quantities:
n_C, T_C, R_s, $\langle\beta\rangle$ and R_M .

Let R_{Ce} and R_{Ci} be the external and the internal plasma radius in the reservoir at the stage 3 in Fig.1 then sharp boundary model [14] give rise to the relations, $R_s^2=R_{Ce}^2+R_{Ci}^2$ and $\langle\beta\rangle=(R_{Ce}^2-R_{Ci}^2)/R_s^2$, from which we easily have

$$R_{Ce} = [\frac{1}{2}(1 + \langle\beta\rangle)]^{1/2}R_s, \tag{26}$$

$$R_{Ci} = [\frac{1}{2}(1 - \langle\beta\rangle)]^{1/2}R_s, \tag{27}$$

and

$$A_C = \pi R_s^2\langle\beta\rangle \quad . \tag{28}$$

If we claim the trapped flux φ_i into the inner boundary of the plasma is conserved along the channel, we are able to obtain

$$\varphi_i = \frac{\pi}{2}(1 - \langle\beta\rangle)R_s^2B_C \quad , \tag{29}$$

where $B_C=(2\mu_0 n_C kT_C)^{1/2}$ holds by pressure balance.

Because in the preceding section we demonstrated all the quantities at the surface 1 may be found if we give the ones in the reservoir and the mirror ratio R_M, the problem posed in this section becomes equivalent to the one to estimate the specifications of the CAPS from the given parameters at the surface 1. In subsection 2.1, we have solved the problem to find out the downstream quantities under given parameters of the source, so that present problem is just the inverse one. For evaluating the source quantities, we assumed the plasma temperature T_0 is a given parameter, because T_0 is an intrinsic quantity of the source as determined by some complicated heat balance process. Owing to rapid cooling of electrons through thermal conduction along the field lines, we infer $T_0(\sim T_{0i}+T_{0e})$ may be limited within several ten electron volts as suggested by the CTX experiment at Los Alamos [7] . As a result, we are able to find from eqs.(6), (21), (20) and (22) that the source density n_0 is obtained by

$$n_0 = (\frac{T_0}{T_C})^{3/2}\frac{R_H^2}{\tau_1^{5/2}}n_C \quad . \tag{30}$$

Because eqs.(2), (15) and (16) show E is conserved along the flow regardless of the shock, we easily find the kinetic energy E_{k0} of an ion at the source muzzle must be

$$E_{k0} = \frac{5}{2}T_C(1 - \frac{T_0}{T_C}), \tag{31}$$

so that the source velocity should be

$$v_0 = \sqrt{5}\,v_{tC}(1 - \frac{T_0}{T_C})^{1/2} \quad . \tag{32}$$

Here, we wish to know the radius $R_0(=(R_{0e}+R_{0i})/2)$ and the aperture $d_0(=R_{0e}-R_{0i})$ of the CAPS. For simplicity we assume $2R_0 \gg d_0$ in order to have approximation $R_{0i}=R_0$. Then, the flux conservation (eq.(29)) and eqs.(3), (20), (21) and (22) give rise to

$$\frac{R_0}{R_s} = \left[\frac{1}{2R_M}(1 - \langle\beta\rangle)\right]^{1/2}(\tau_1\frac{T_C}{T_0})^{5/8} \quad , \tag{33}$$

and the scaling relation between A_0 and A_C yields

$$\frac{d_0}{R_s} = \frac{1}{\sqrt{10}}\frac{\langle\beta\rangle}{(1-\langle\beta\rangle)^{1/2}}(\frac{T_C}{T_0})^{7/8}\frac{\tau_1^{5/8}\nu_1\xi_1}{(1-T_0/T_C)^{1/2}}\frac{1}{R_M^{3/2}} \quad . \tag{34}$$

Because ξ_1, τ_1 and ν_1 bifurcate, further deductions on the strong shock mode and also the weak shock mode are required for the solutions.

2.4-a The strong shock mode

The branch of the solution taking (+) sign in eqs.(18), (20) and (21) describes the case that an energetic plasma flow is suddenly stagnated somewhere between the surface 1 and 2, which eventually indicates a sharp increase in T and N must always accompanied to this mode. Figure 4 indicates the largest jump appears at $\nu_{C*}=16/25$, we examine the behavior near by using a new variable ε_ν satisfying $\nu_{C*}=(16/25)(1+\varepsilon_\nu)\leq 1$ instead of ν_{C*} itself. In case ε_ν thus defined is an arbitrary given small parameter much less than unity, the definition of ν_{C*} gives rise to

$$N_C = \frac{25}{16}(1 - \varepsilon_\nu)N_{C*} \quad , \tag{35}$$

so that the results in the preceding subsection can be rewritten in terms of ε_ν in the following way:

$$\tau_1 = \frac{5}{3}\varepsilon_\nu \quad , \tag{36}$$

$$\nu_1 = \frac{1}{5}(1 + \frac{4}{3}\varepsilon_\nu) \quad , \tag{37}$$

$$\xi_1 = \sqrt{5}(1 - \frac{5}{6}\varepsilon_\nu) \ , \tag{38}$$

$$\alpha_1 = \frac{1}{3}\frac{\varepsilon_\nu}{R_H^2} \quad . \tag{39}$$

Here, evaluation of N_{C*} becomes necessary for N_C owing to eq.(35). Because of the large jump in T and N appearing at $\varepsilon_\nu \ll 1$, eq.(39) claims the choice of $R_H \sim \varepsilon_\nu^{1/2} < 1$ for A_1 not to be shrunk too much. Then, we may put $R_{Hes}=1$ and $l_{es}=2\pi(R_{Ci}+R_{Ce})$ to find N_{C*} by eq.(19). After manipulations of eq.(19) using eqs.(26), (27) and (28), the final form of N_{C*} is turned out to be

$$N_{C*} = 8.2\times10^{17}\ \alpha_f^2\mu_A \frac{[1 + (1-\langle\beta\rangle^2)^{1/2}]}{\langle\beta\rangle} \quad , \tag{40}$$

where μ_A denotes atomic mass number. Therefore, if $\alpha_f=1$ is assumed for a deuterium FRC eqs.(40) and (28) yield

$$n_C R_s^2 = 2.6\times10^{18}(1 - \varepsilon_\nu)\frac{[1+(1-\langle\beta\rangle^2)^{1/2}]}{\langle\beta\rangle^2} \ , \tag{41}$$

which shows the maximum achievable value of n_C will be limited to 1.2×10^{20} m^{-3} even for a small size FRC of R_s=0.1 m and $\langle\beta\rangle$=0.95. Now, using N_{C*}, τ_1, ν_1 and ξ_1 as evaluated above, eqs.(30), (33) and (34) turn into following final forms giving the answer to the problem:

$$\frac{n_0}{n_C} = (\frac{3}{5})^{5/2}(\frac{T_0}{T_C})^{3/2}\frac{R_H^2}{\varepsilon_\nu^{5/2}} \quad , \tag{42}$$

$$\frac{R_0}{R_s} = 0.97\ (\frac{1-\langle\beta\rangle}{R_H})^{1/2}(\frac{T_C}{T_0}\varepsilon_\nu)^{5/8} \quad , \tag{43}$$

$$\frac{d_0}{R_s} = 0.29\ \frac{\langle\beta\rangle}{(1-\langle\beta\rangle)^{1/2}}(\frac{T_C}{T_0})^{7/8}\frac{\varepsilon_\nu^{15/8}}{R_H^{3/2}} . \tag{44}$$

Consequently, we may say the strong shock mode is not very convenient for actual application because not only very small value of $n_C R_s^2$ is allowed by eq.(41) but also n_0, R_0 and d_0 as given by eqs.(42), (43) and (44), respectively, show quite sensitive dependence on the choice of ε_ν. We fear for such sensitive dependence on ε_ν to indicate reproducible operation is difficult to be achieved because a small change in a source parameter might give a big influence.

2.4-b The weak shock mode

If we take (−) sign in eqs.(18), (20) and (21), the solution moves to the branch of the weak shock mode, which at the surface (1) gives v almost stagnated, and makes T and N approximately equal to T_C and N_C, respectively. The weak shock mode is realized in case the energetic plasma flow from the CAPS is decelerated and heated up by the increasing axial magnetic field before the counter flows are collided.

We are interested in producing FRC having much greater N_C than N_{C*} , so that we may regard ν_{C*} a small parameter much less than unity. If we neglect smaller term than the order of ν_{C*} , eqs.(18), (20) and (21) give rise to

$$\xi_1 = \frac{\sqrt{5}}{4}\nu_{C*}^{1/2} \quad , \tag{45}$$

$$\tau_1 = 1 - \frac{1}{16}\nu_{C*} \quad , \tag{46}$$

and also

$$\nu_1 = 1 - \frac{1}{4}\nu_{C*} \quad . \tag{47}$$

Evaluation of N_{C*} is pursued only for the case $B_1 > B_C$ i.e. $R_{Hes} = R_H \geqq 1$, because necessary I and W are expected to be smaller at greater R_H in addition to the advantage that the reconnection of the field lines to the stage 3 takes place more rapidly. Then, since $l_{es} = 2\pi(R_{1e} + R_{1i})$ holds in this case, the conservation of φ_i and eq.(19) give rise to

$$N_{C*} = 8.2\times 10^{17}\mu_A \frac{\alpha_1^2}{R_H^4}\{1 + [2R_H(1-\langle\beta\rangle)]^{1/2}\} \quad , \tag{48}$$

where assuming $1-\langle\beta\rangle \sim \nu_{C*}$ we neglected the terms of the order smaller than $\nu_{C*}^{1/2}$. Here, we may easily check the postulate

$\nu_{C*} \ll 1$, because even if we take out FRC having relatively small deuterium particle numbers such that $n_C = 1 \times 10^{20}$ m^{-3} and $R_s = 0.2$ m, eq.(48) yields $\nu_{C*} = 1.3 \times 10^{-3}$ for $R_M = 2$ and $<\beta> = 0.95$.

After similar treatment to the strong shock mode using eqs.(45), (46), (47) and (48), we have derived following equations to deduce required source parameters:

$$\frac{n_0}{n_C} = \left(\frac{T_0}{T_C}\right)^{3/2} R_M^2, \tag{49}$$

$$\frac{R_0}{R_s} = \left(\frac{T_0}{T_C}\right)^{5/8} \left[\frac{1}{2R_M}(1-<\beta>)\right]^{1/2}, \tag{50}$$

$$d_0 = 9.1 \times 10^7 \frac{\alpha_f \mu_A^{1/2}}{n_C^{1/2} R_M^{7/2}} \left(\frac{T_C}{T_0}\right)^{7/8} \left\{\frac{1+[2R_M(1-<\beta>)]^{1/2}}{1-<\beta>}\right\}^{1/2}, \tag{51}$$

$$I = 1.4 \times 10^{-6} \alpha_f (n_C T_C)^{1/2} \frac{R_s}{R_M^2} \{1+[2R_M(1-<\beta>)]^{1/2}\}^{1/2}. \tag{52}$$

It is noted eqs.(14) and (31) may be utilized for W and E_{k0}, respectively.

3 Typical Examples Of The Results And Discussions

Using the weak shock model which is suitable to produce FRC involving large numbers of particles, we evaluated major parameters of the CAPS as required to produce the FRC of given specifications. In Table 1, the results are presented of deuterium plasmas with $\alpha_f=1$ and $T_0=30$ eV for three different specifications: spec.1 reflects the theta pinch FRC of the largest R_s ever obtained [13] , spec.2 and 3, which have higher temperature but lower density, are designed as target FRCs for steady state experiment.

If the total length L of FRC is given, particle replacement time $\tau_p^{(3)}$ at the stage 3 may be obtained by

$$\tau_p^{(3)} = \frac{kn_C\pi R_s^2 L\langle\beta\rangle}{2I} \quad . \tag{53}$$

And evaluated results of $\tau_p^{(3)}$ are also presented in Table 1. Because $\tau_p^{(3)}$ is considered to give the minimum steady operation time $t_{op}^{(3)}$, so we may read $t_{op}^{(3)}$ less than 1 ms may be enough for every three specification in Table 1. In the case of an FRC produced by a theta pinch FRC, $t_{op}^{(3)}$ corresponds to formation time t_f at early transient phase, of which typical value in the FRX-C/LSM [13] may be approximately 5 μs. Therefore, since it is reported in ref. [13] that the plasma having internal energy of 17 kJ is produced, the theta pinch circuit delivers power of 3.4 GW on the average during 5 μs to the plasma. On the other hand, spec.1 in Table 1 shows required power level is about 130 MW, which is much smaller than the theta pinch case. Smaller power is compensated by longer operation time allowing the deposit of plasma particles and energy, and brings about the advantage to eliminate the necessity of very high voltage technics. Considering the result of CTX experiments [7] , we believe required power level and pulse length for spec.1 does not seem to have very serious problem in electrode erosion. As for spec.2 and 3, we may refer more recent CTX experiments using larger CAPS in its bore designed as a long pulsed helicity source [16] . Therefore, if we succeed in constructing an ideal long pulsed CAPS capable of ejecting an energetic plasma flow with several keV range carrying small amount trapped B_θ field, we may well anticipate that a high temperature target FRC having any size of R_s may be produced free from troublesome high voltage technics usually coming into a conventional theta pinch.

4 Conclusions

Steady state theta pinch concept based on energetic plasma flow through a high beta magnetic channel has been revealed to offer a simple tool in analysing slow FRC formation. It is demonstrated that weak shocked encounter of two identical high beta plasma channel is quite effective in producing FRC involving large numbers of particles. The favourable weak shocked relaxation mode is achieved only in case energetic plasma flows from the two CAPSs are mainly decelerated and heated up by the mirror field at the end of FRC through the Poisson adiabatic process.

We point out lastly the most important item which must be prepared before the present slow formation technics actually working is to develop an energetic CAPS having the pulse length of the order of 1 ms and ejecting a plasma involving small amount of B_θ .

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FIGURE CAPTIONS

Fig.1. A series of conceptual pictures to show how FRC is produced by colliding plasma counter flows as ejected from two CAPSs (co-axial plasma source). At stage 3 a steady state is postulated to be established.

Fig.2. A simplified specific picture at the stage 3 illustrating the plasma life in steady state: The energetic plasma produced in the source is ejected from the surface 0 at the muzzle into the high beta guiding plasma channel to be decelerated and heated up by the Poisson adiabatic process. The plasma is relaxed to stagnate in the reservoir by the shock adiabatic process taking place between the surface 1 and 2. The plasma in the reservoir escapes out via the peripheral sheath to the particle sinks at the end of the field lines. It is noted the central surface C where plasma truly stagnates due to symmetry can be moved to C' close to the surface 2 to form a virtual central surface for the completely stagnated reservoir.

Fig.3. A schematic drawing to show the Poisson adiabatic high beta channel flow as ejected from the source into a increasing solenoidal field . For the flow be high beta, the sheath thickness δ is required to be smaller than the size of the plasma.

Fig.4. Dimensionless values of T_1, ν_1 and v_1 as a function of $\nu_{C*}(=N_{C*}/N_C)$. The attached (+) and (−) sign on the curves correspond to the branches of the strong and

weak shock mode, respectively.

TABLE CAPTIONS

Table 1. Specifications of the CAPS as deduced from the given data set of the FRC.

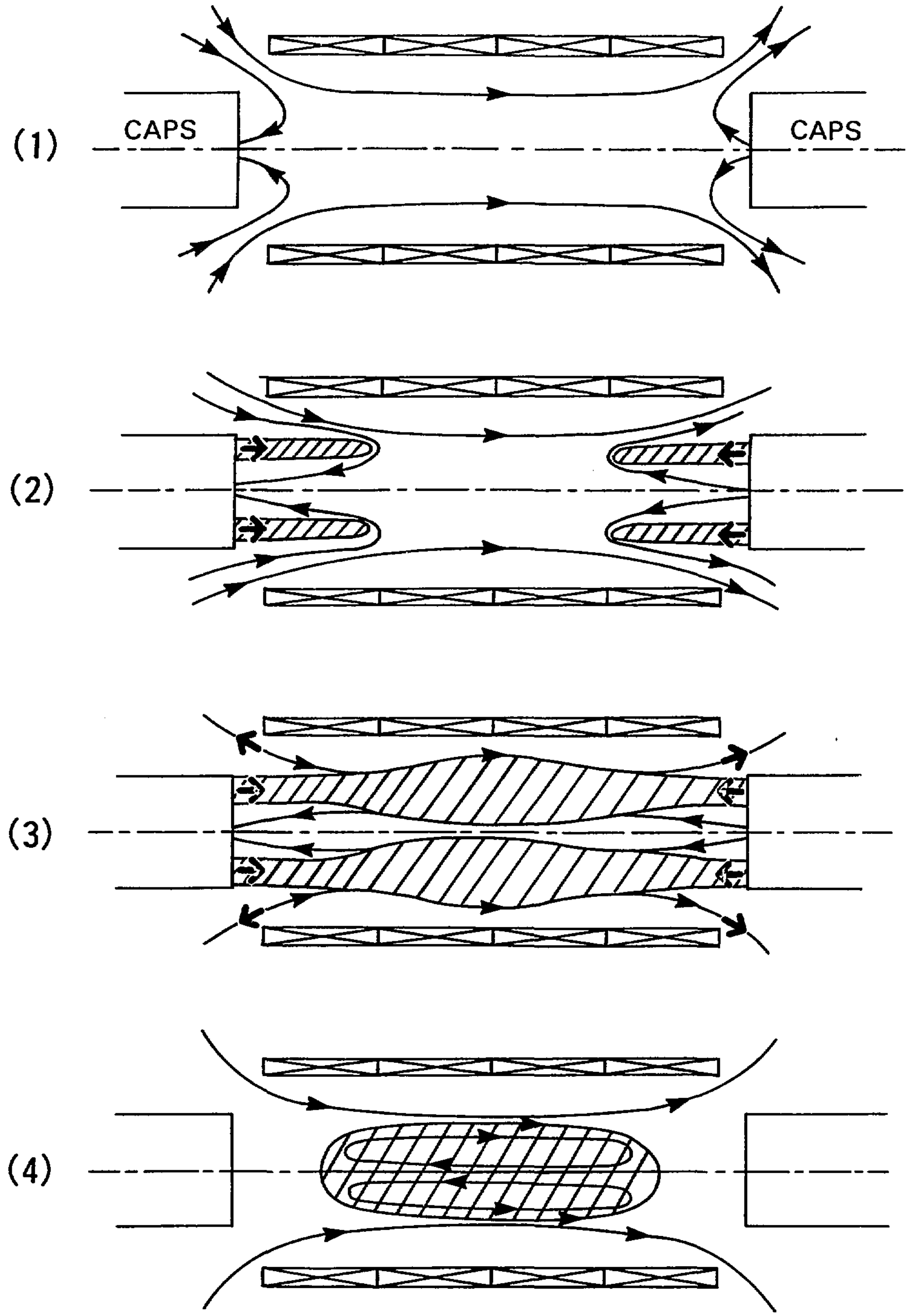


Fig. 1

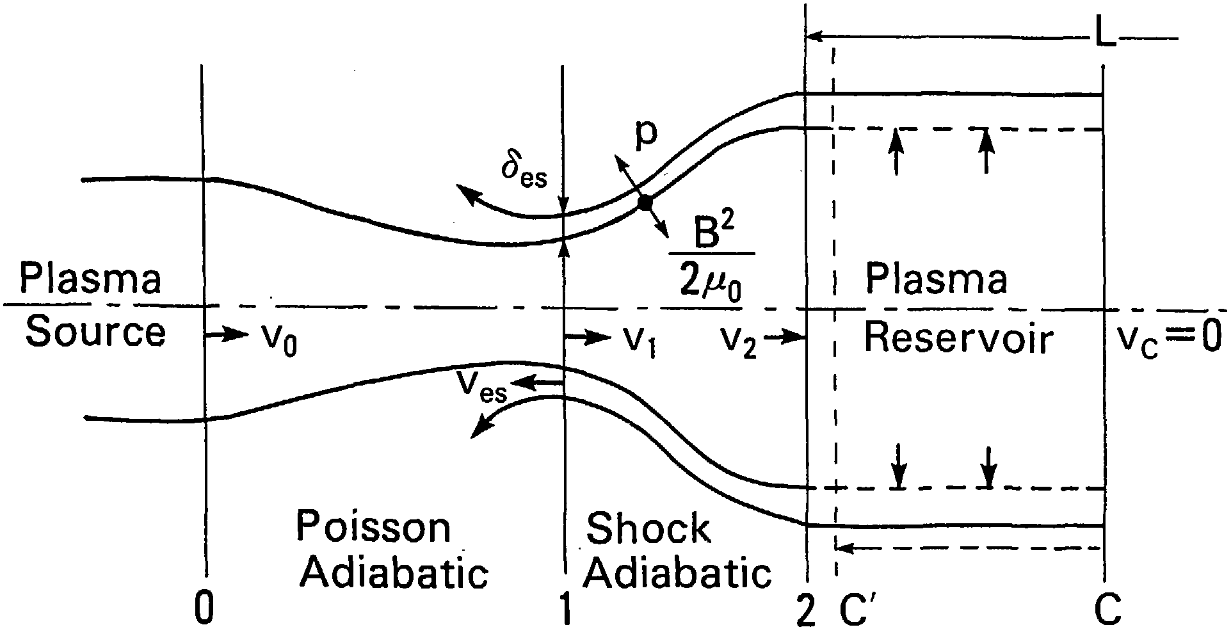

L
p
δes
B²/2μ0
Plasma Source
v0
v1
v2
Plasma Reservoir
vC=0
ves
Poisson Adiabatic
Shock Adiabatic
0
1
2
C′
C

Fig. 2

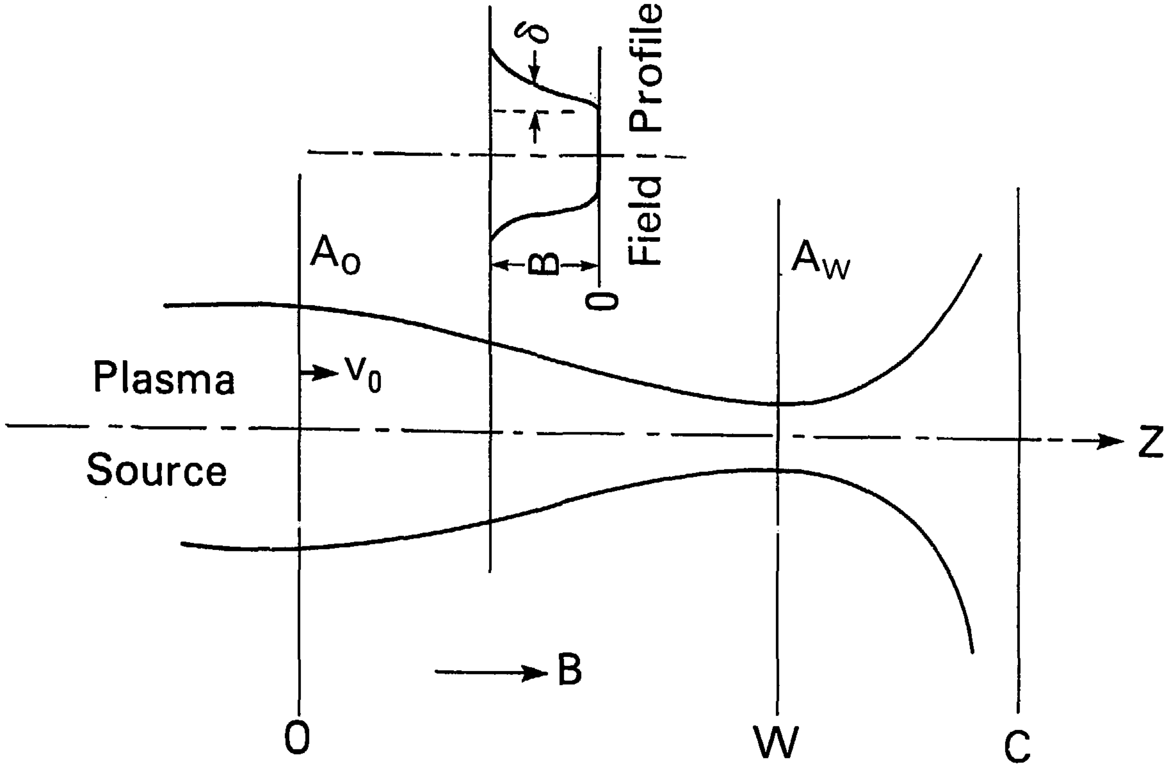
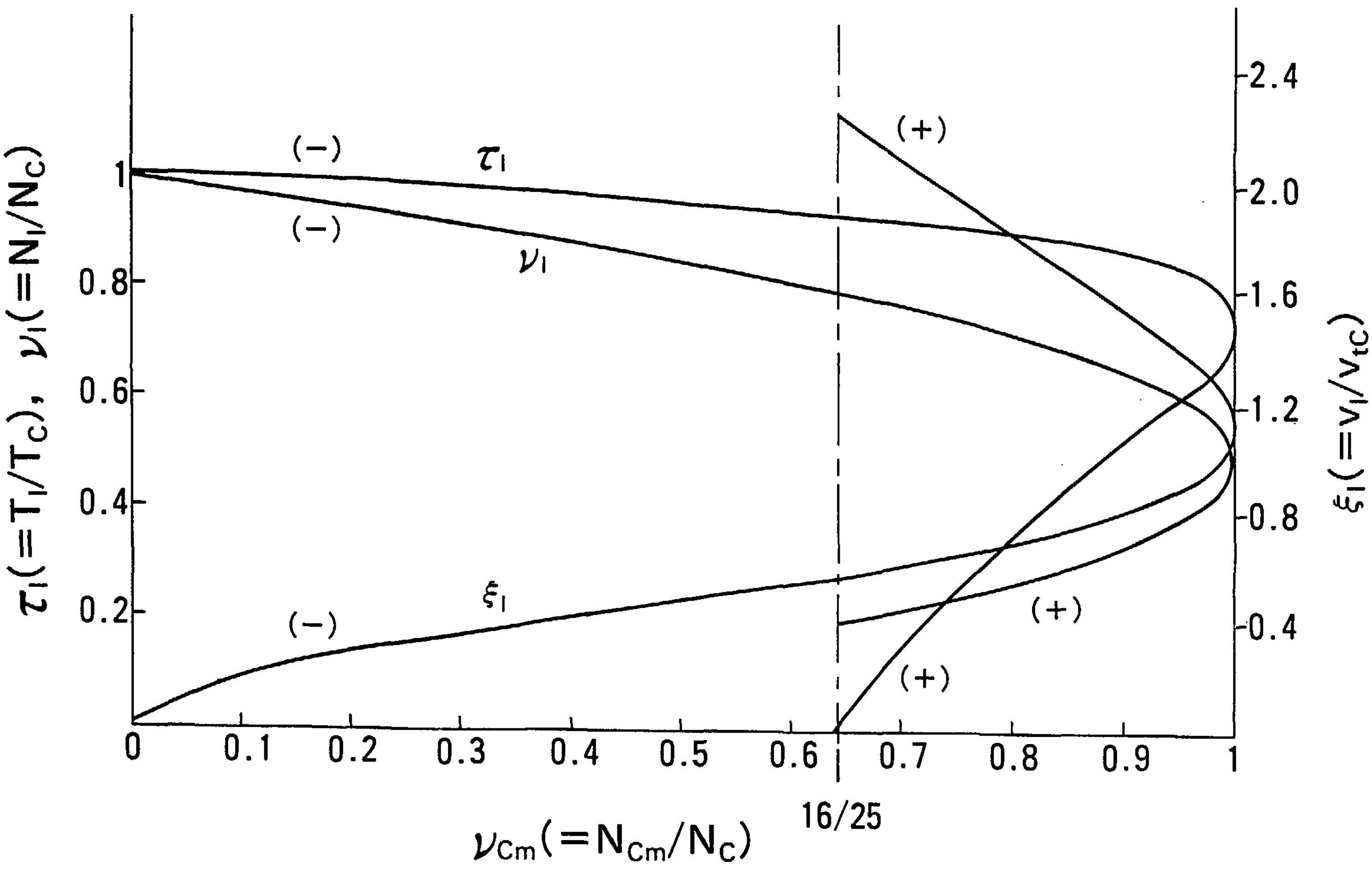


Fig. 3



(−)
τ_l
(−)
ν_l
ξ_l
(−)
(+)
(+)
(+)
1
0.8
0.6
0.4
0.2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
16/25
-2.4
-2.0
-1.6
-1.2
-0.8
-0.4
$\tau_l(=T_l/T_C)$, $\nu_l(=N_l/N_C)$
$\nu_{Cm}(=N_{Cm}/N_C)$
$\xi_l(=v_l/v_{tC})$

Fig.4

TABLE 1

FRC spec.	1	2	3
n_C ($10^{20}m^{-3}$)	20.	1.	1.
T_C (keV)	0.2	1.2	2.0
R_S (m)	0.25	0.3	0.5
L (m)	1.4	5.	5.
$<\beta>$	0.95	0.95	0.95
R_H	1.4	3.	3.
B_C (T)	0.40	0.22	0.28
$\tau_p^{(3)}$ (ms)	0.32	0.53	0.18
CAPS spec.			
R_0 (m)	0.11	0.27	0.63
d_0 (cm)	2.4	3.8	6.0
n_0 ($10^{18}m^{-3}$)	230	3.6	1.7
E_{k0} (keV)	0.43	2.9	4.9
I (kA)	133.	20.3	43.6
W (MW)	66.8	60.8	218.

N.B. : estimations are done for a deuterium plasma flow with the assumptions that α_f=1 and T_0=30 eV.