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ABELIAN GAUGE THEORIES WITH TENSOR GAUGE FIELDS

by

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Abstract:

Gauge fields of arbitrary tensor type are introduced. In curved space-time the gravitational field serves as a bridge joining different gauge fields. The theory of second order tensor gauge field is developed on the basis of close analogy to Maxwell electrodynamics. The notion of tensor current is introduced and an experimental test of its detection is proposed. The main result consists in a coupled set of field equations representing a generalization of Maxwell theory in which the Einstein equivalence principle is not satisfied.

In abelian gauge theory in curved space-time the field equations for charged matter fields contain the covariant derivatives

$$D_\mu = \nabla_\mu - i A_\mu(x) \tag{1}$$

where ∇_μ are the usual geometric covariant derivatives and $A_\mu(x)$ are the electromagnetic gauge fields fixed up to gauge transformations

$$A_\mu \to A'_\mu = A_\mu + \nabla_\mu f \tag{2}$$

where f is an arbitrary differentiable function of space-time variables. When higher order derivatives are considered the symmetric products of the first order differential operators (1) are used and the resulting theory is called the minimal coupling theory.

It is easy to see that the minimal coupling theory is not the only possible one. In fact, we may admit a more general procedure in which instead of using symmetrized products of (1) we shall use the most general differential operators which are generally covariant and covariant under arbitrary phase transformations of matter fields. For example, the second order derivatives $\partial_\mu \partial_\nu$ in the presence of gravitation must be replaced by the generally covariant ones

$$\nabla_{\mu\nu} = \frac{1}{2}\left(\nabla_\mu \nabla_\nu + \nabla_\nu \nabla_\mu\right) \tag{3}$$

which in the minimal coupling theory are further replaced by the operators

$$D^{min}_{\mu\nu} = \tfrac{1}{2}\left(D_\mu D_\nu + D_\nu D_\mu\right). \qquad (4)$$

In our more general approach we replace (4) by differential operators

$$D_{\mu\nu} = \nabla_{\mu\nu} - i A_\mu \nabla_\nu - i A_\nu \nabla_\mu - A_\mu A_\nu - i A_{\mu\nu} \qquad (5)$$

where $A_\mu(x)$ are the usual electromagnetic gauge fields with the transformation property given by (2) while the symmetric tensor gauge fields $A_{\mu\nu}(x)$ transform under gauge transformations according to the rule

$$A_{\mu\nu} \to A'_{\mu\nu} = A_{\mu\nu} + \nabla_{\mu\nu} f. \qquad (6)$$

Comparing (5) with (4) we see that the minimal coupling theory arises if

$$A^{min}_{\mu\nu} = \tfrac{1}{2}\left(\nabla_\mu A_\nu + \nabla_\nu A_\mu\right). \qquad (7)$$

In our more general approach the fields $A_{\mu\nu}$ will be treated as independent from the fields A_μ.

In field theories with higher order field equations we may similarly introduce symmetric tensor fields $A_{\mu_1 \dots \mu_n}(x)$ which under gauge transformations behave as follows:

$$A_{\mu_1\dots\mu_n} \to A'_{\mu_1\dots\mu_n} = A_{\mu_1\dots\mu_n} + \sum_{Sym} \nabla_{\mu_1}\dots\nabla_{\mu_n} f \qquad (8)$$

and in this way we get an infinite sequence of gauge fields of arbitrary tensor type. Independently from the field equations for matter fields we may postulate the existence of such tensor gauge fields and develop a complete theory for them. For practical reasons we must, however, restrict the attention to a finite number of gauge fields and the aim of the present paper is to describe the simplest case which goes beyond the minimal coupling theory. It is the case where apart from the vector gauge field $A_\mu(x)$ we have only one tensor gauge field $A_{\mu\nu}(x)$.

The field equations for the tensor gauge fields $A_{\mu\nu}(x)$ we shall find in close analogy to Maxwell electrodynamics. As a first step we shall look for a gauge invariant third order tensor field $F_{\varrho\mu\nu}(x)$ which in our theory will play the same role as the electromagnetic tensor

$$F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu \qquad (9)$$

plays in the Maxwell electrodynamics. The apparent candidate for $F_{\varrho\mu\nu}$ taken in the form

$$F_{\varrho\mu\nu} = \alpha \nabla_\varrho A_{\mu\nu} + \beta \nabla_\mu A_{\nu\varrho} + \gamma \nabla_\nu A_{\varrho\mu} \qquad (10)$$

must, however, be rejected, since it cannot be gauge invariant. It turns out that in curved space-time the correct generalization of (9) is of the form

$$F_{\varrho\mu\nu} = \alpha\left(\nabla_\varrho A_{\mu\nu} - \nabla_\nu A_{\varrho\mu} - R^{\eta}{}_{\mu\varrho\nu} A_\eta\right) +$$

$$+ \beta\left(\nabla_\mu A_{\nu\varrho} - \nabla_\nu A_{\varrho\mu} - R^{\eta}{}_{\varrho\mu\nu} A_\eta\right) \qquad (11)$$

where $R^{\eta}{}_{\mu\varrho\nu}$ is the curvature tensor of the considered space-time. The appearance of the curvature tensor in (11) is caused by the non-commutativity of the covariant derivatives ∇_μ and shows some fundamental difference between the gauge fields A_μ and $A_{\mu\nu}$. In the case of A_μ its relation to the gauge invariant tensor field $F_{\mu\nu}$ given by (9) is not influenced by gravitation, because

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \qquad (12)$$

as the consequence of the symmetry of Christoffel symbols. In the case of $A_{\mu\nu}$ the relation between $A_{\mu\nu}$ and $F_{\varrho\mu\nu}$ explicitly depends on gravitation, because (11) contains the curvature tensor. In addition, the tensor $F_{\varrho\mu\nu}$ depends not only on $A_{\mu\nu}$ but also on A_μ.

A similar situation appears for all higher tensor gauge fields $A_{\mu_1 \ldots \mu_n}$. The gauge invariant combinations formed from these tensor fields always contain different gauge fields and the curvature tensor. Our generalized approach to gauge fields reveals therefore a new role of the gravitational field since it is now the bridge joining various gauge fields. Only for the minimal coupling theory which uses solely the vector

gauge fields A_μ there are no interrelations between different gauge fields and the indicated role of gravitation is invisible.

From (11) it is seen that the tensor $F_{\varrho\mu\nu}$ has the following symmetry property:

$$F_{\varrho\mu\nu} + F_{\mu\nu\varrho} + F_{\nu\varrho\mu} = 0 \qquad (13)$$

as the counterpart of the electrodynamical symmetry

$$F_{\mu\nu} + F_{\nu\mu} = 0 \qquad (14)$$

satisfied by the electromagnetic tensor. In addition, from (11) we see that $F_{\varrho\mu\nu}$ consists of two parts with different additional symmetries. The first part, multiplied by α, satisfies

$$F_{\varrho\mu\nu} = -F_{\nu\mu\varrho}, \qquad (15)$$

while the second part, multiplied by β, satisfies

$$F_{\varrho\mu\nu} = -F_{\varrho\nu\mu}. \qquad (16)$$

The natural requirement of irreducibility (or maximal symmetry) of physical tensors suggests that we should choose in (11) either $\alpha = 0$ or $\beta = 0$. In order to make the correct choice we shall again refer to Maxwell electrodynamics where the tensor $F_{\mu\nu}$ satisfies the Maxwell-Faraday equations

$$\nabla_\varrho F_{\mu\nu} + \nabla_\mu F_{\nu\varrho} + \nabla_\nu F_{\varrho\mu} = 0. \tag{17}$$

Using the antisymmetry of $F_{\mu\nu}$ these equations may be reduced to the equations

$$\partial_\varrho F_{\mu\nu} + \partial_\mu F_{\nu\varrho} + \partial_\nu F_{\varrho\mu} = 0 \tag{18}$$

which show that the Maxwell-Faraday equations are independent of gravitation. We also note that equations (18) are identically satisfied if we use the representation of $F_{\mu\nu}$ in terms of the gauge fields A_μ as given by (12). Taking all this into account we require that in the absence of gravitation the tensor $F_{\varrho\mu\nu}$ should satisfy the equations

$$\partial_\lambda F_{\varrho\mu\nu} + \partial_\varrho F_{\mu\nu\lambda} + \partial_\mu F_{\nu\lambda\varrho} + \partial_\nu F_{\lambda\varrho\mu} = 0 \tag{19}$$

and these equations should be satisfied identically if $F_{\varrho\mu\nu}$ is represented in terms of the gauge fields $A_{\mu\nu}$ as given by (11). A simple calculation then shows that such requirement can be fulfilled only if we choose in (11) $\beta = 0$ and up to normalization we get the gauge invariant tensor $F_{\varrho\mu\nu}$ in the form

$$F_{\varrho\mu\nu} = \nabla_\varrho A_{\mu\nu} - \nabla_\nu A_{\varrho\mu} - R^{\eta}{}_{\mu\varrho\nu} A_\eta , \tag{20}$$

as the counterpart of the electromagnetic tensor $F_{\mu\nu}$. It is clear that in the presence of gravitation we must suitably modify equations (19) in order to make them generally covariant, but we shall not need such modification here.

The field equations for the gauge fields and the matter fields ϕ_i will be obtained from the Lagrangian

$$\mathcal{L} = -\frac{1}{2f^2} F_{\varrho\mu\nu} F^{\varrho\mu\nu} - \frac{1}{4e^2} F_{\mu\nu} F^{\mu\nu} + \mathcal{L}_{matter}(D_{\mu\nu}\phi_i, D_\mu\phi_i, \phi_i), \tag{21}$$

where f and e are appropriate coupling constants. Extremizing the action integral

$$S = \int \mathcal{L} \sqrt{-g}\, d^4x \tag{22}$$

with respect to the variations of A_μ and $A_{\mu\nu}$ we get the following generalized Maxwell equations

$$\nabla_\mu F^{\mu\nu} = -\frac{e^2}{f^2} R^{\nu}{}_{\mu\eta\varrho} F^{\eta\mu\varrho} - e^2 \frac{\delta \mathcal{L}_{matter}}{\delta A_\nu} \tag{23}$$

$$\nabla_\varrho \left(F^{\varrho\mu\nu} + F^{\varrho\nu\mu}\right) = -f^2 \frac{\delta \mathcal{L}_{matter}}{\delta A_{\mu\nu}} . \tag{24}$$

We are now ready to give a physical meaning to the fields $F^{\varrho\mu\nu}$. From (24) we see that matter fields may produce a new type of sources; the tensor currents

$$j^{\mu\nu} = -f^2 \frac{\delta \mathcal{L}_{matter}}{\delta A_{\mu\nu}} , \tag{25}$$

which serve as sources for the fields $F_{\varrho\mu\nu}$. These tensor currents, in addition to the ususal vector currents

$$j^{\mu} = -e^2 \frac{\delta \mathcal{L}_{matter}}{\delta A_{\mu}} , \tag{26}$$

describe new yet unknown electromagnetic characteristics of matter. They lead to the following modification of the Lorentz force

$$F^{\mu}_{Lorentz} = F^{\mu\nu} j_{\nu} + \left(F^{\mu\varrho\nu} + F^{\mu\nu\varrho}\right) j_{\varrho\nu} . \tag{27}$$

It is obvious that if the given matter does not exhibit any trace of tensor current this expression reduces to the usual Lorentz force. A careful and precise investigation of the Lorentz force may therefore serve as a direct experimental test of our theory and as an operational definition of the tensor fields $F_{\varrho\mu\nu}$.

In the absence of tensor currents the field equations (23) admit the solution

$$F_{\varrho\mu\nu} = 0 \tag{28}$$

and the whole theory reduces just to Maxwell electrodynamics. This will be the case if we take the usual gauge invariant matter Lagrangian

$$\mathcal{L}_{matter} = -\left(D_{\mu}\phi\right)^{*}\left(D^{\mu}\phi\right) + m^2 \phi^{*}\phi \tag{29}$$

for a matter described by a single scalar field ϕ.

In the absence of gravitation the field equations (23) always reduce to the Maxwell ones regardless of the presence of tensor currents. But in the presence of gravitation and non-vanishing tensor currents we obtain an essential modification of Maxwell electrodynamics in curved space-time. This is so, for example, with the following Lagrangian of scalar matter

$$\mathcal{L}_{matter} = a\left(D_{\mu\nu}\phi\right)^{*}\left(D^{\mu\nu}\phi\right) - b\left(D_{\mu}\phi\right)^{*}\left(D^{\mu}\phi\right) + c\phi^{*}\phi \tag{30}$$

which leads to the currents

$$\begin{aligned} j^{\mu} = {} & ib\left[\left(D^{\mu}\phi\right)^{*}\phi - \phi^{*}\left(D^{\mu}\phi\right)\right] + \\ & + 2ia\left[\left(D_{\nu}\phi\right)^{*}\left(D^{\mu\nu}\phi\right) - \left(D^{\nu\mu}\phi\right)^{*}\left(D_{\nu}\phi\right)\right], \end{aligned} \tag{31}$$

$$j^{\mu\nu} = ia\left[\phi^{*}\left(D^{\mu\nu}\phi\right) - \left(D^{\mu\nu}\phi\right)^{*}\phi\right] \tag{32}$$

and to the fourth-order wave equation

$$\begin{aligned} & a D_{\mu\nu} D^{\mu\nu}\phi - ia\left(\nabla_{\mu}A_{\nu} + \nabla_{\nu}A_{\mu} - 2A_{\mu\nu}\right)D^{\mu\nu}\phi + \\ & + b D_{\mu} D^{\mu}\phi + c\phi = 0 \end{aligned} \tag{33}$$

In the presence of tensor currents the field equation (23) and (24) represent a theory for which the Einstein equivalence principle is not satisfied. In fact, the field equations (23) remain generally covariant, even if we neglect the term with the curvature tensor, and in this case we get the usual Maxwell equations in curved space-time. However, there is no way to obtain equations (23) from the Maxwell equations in flat space-time through the equivalence principle, because in the conventional formulation of this principle the possibility that the curvature tensor may couple different fields is not taken into account. Our model thus shows that the formulation of the equivalence principle must be modified.

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