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MONTE CARLO DETERMINATION OF THE SPIN-DEPENDENT POTENTIALS

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SUMMARY

The bound states of heavy quark systems can be calculated by a Hamiltonian formulation, based on an expansion of the interaction into inverse powers of the quark mass. To second order in $1/m$ one obtains[1,2]

$$
\begin{aligned}
H = {} & \frac{\vec{p}^2}{m} + V(r) + \frac{\vec{S}_+ \cdot \vec{L}_+ + \vec{S}_- \cdot \vec{L}_-}{2m^2} \frac{1}{r} \left(\frac{dV}{dr} + 2 \frac{dV_1}{dr} \right) \\
& + \frac{\vec{S}_+ \cdot \vec{L}_- + \vec{S}_- \cdot \vec{L}_+}{m^2} \frac{1}{r} \frac{dV_2}{dr} \\
& + \frac{1}{m^2} \left[\left(\vec{S}_+ \cdot \hat{r} \right) \left(\vec{S}_- \cdot \hat{r} \right) - \frac{1}{3} \vec{S}_+ \cdot \vec{S}_- \right] V_3(r) \\
& + \frac{1}{3m^2} \vec{S}_+ \cdot \vec{S}_- V_4(r) . \qquad (1)
\end{aligned}
$$

The potentials $V_1 - V_4$ account for the spin-orbit and spin-spin coupling between quark and antiquark and are responsible for the fine and hyperfine splittings in heavy quark spectroscopy. They can be expressed in terms of

expectation values of Wilson loop factors with suitable insertions of chromomagnetic or chromoelectric fields, denoted by $\langle \ldots \rangle_W$, as in the following equation

$$\epsilon_{ijk}\hat{r}_k \frac{d\widetilde{V}_1(r,T)}{dr} = \int_0^T dt \int_0^T dt' \, (t'-t) \, \langle g^2 B_i(0,t) \, E_j(0,t') \rangle_W ,$$

$$\epsilon_{ijk}\hat{r}_k \frac{d\widetilde{V}_2(r,T)}{dr} = \int_0^T dt \int_0^T dt' \, (t'-t) \, \langle g^2 B_i(0,t) \, E_j(\vec{r},t') \rangle_W ,$$

$$\left(\hat{r}_i\hat{r}_j - \frac{1}{3}\delta_{ij}\right) \widetilde{V}_3(r,T) + \frac{1}{3}\delta_{ij}\widetilde{V}_4(r,T) = \int_0^T dt \int_0^T dt' \langle g^2 B_i(0,t) \, B_j(\vec{r},t') \rangle_W ,$$

$$V_i(r) = \lim_{T\to\infty} \frac{1}{T} \frac{\widetilde{V}_i(r,T)}{\langle 1 \rangle_W} . \tag{2}$$

We have used a Monte Carlo simulation to evaluate the expectation values in Eq. (2) and, from them, the spin-dependent potentials. The calculation was performed on a $16^3 \times 32$ lattice, in the quenched approximation and with Wilson's action, at $\beta = 6.2$. The code has been described in detail in Ref. 3 and the results for the spin-dependent potentials have been presented in Ref. 4. Independent lattice calculations of spin-dependent potentials have also been published in Refs. 5-8.

Our results are illustrated in the figures. The x symbols represent the original Monte Carlo results, the o symbols represent the results after a correction for short distance lattice artifacts. The correction, based on the perturbative behavior of the potentials and on the actual separation of the plaquettes which enter into the formulae for $V_1 - V_4$, is not meant to be rigorous, but is introduced to estimate the size of possible short-distance lattice distortions. The values of the potentials in physical units are obtained assuming that they renormalize like the quark mass squared and using the ratio between bare and renormalized quark masses (~ 0.5), which one infers from a direct lattice calculation of hadron masses.[9] The renormalized values of the potentials are in reasonable agreement with a relation derived by Gromes.[10] One of the most important results of the Monte Carlo calculation is the clear evidence for a long range, non-perturbative component in V_1 (cfr. also Ref. 8). The calculation has now been repeated for $\beta = 6$. The new results, as well as an application to the calculation of the spin splittings, will be presented shortly.[11]

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REFERENCES

1. E. Eichten and F. Feinberg, Phys. Rev. Lett. 43, 1205 (1979); Phys. Rev. D23, 2724 (1981).
2. M.E. Peskin, lecture in *Proc. 11th SLAC Summer Institute on Particle Physics*, (1983)SLAC Report No. 267.
3. M. Campostrini, K.J.M. Moriarty and C. Rebbi, Comp. Phys. Comm. 42, 174 (1986).
4. M. Campostrini, K.J.M. Moriarty, and C. Rebbi, Phys. Rev. Lett. 57, 44 (1986).
5. M. Campostrini, Nucl. Phys. B256, 717 (1985).
6. C. Michael and P.E.L. Rakow, Nucl. Phys. B256, 640 (1985).
7. P. deForcrand and J.D. Stack, Phys. Rev. Lett. 55, 1254 (1985).
8. C. Michael, Phys. Rev. Lett. 56, 1219 (1985).
9. D. Barkai, K.J.M. Moriarty and C. Rebbi, Phys. Rev. Lett. 156B, 385 (1985).
10. D. Gromes, Zeit. Phys. C26, 401 (1984).
11. M. Campostrini, K.J.M. Moriarty and C. Rebbi, in preparation.

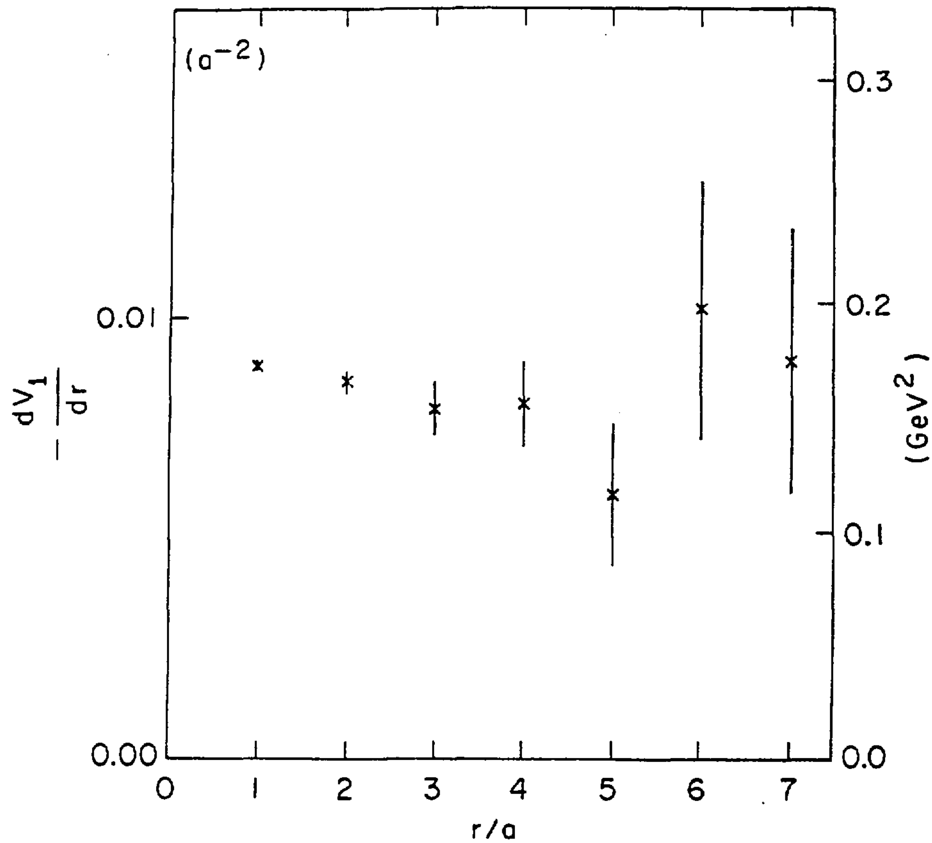

Fig. 1 Monte Carlo results for the spin-dependent potential V_1.

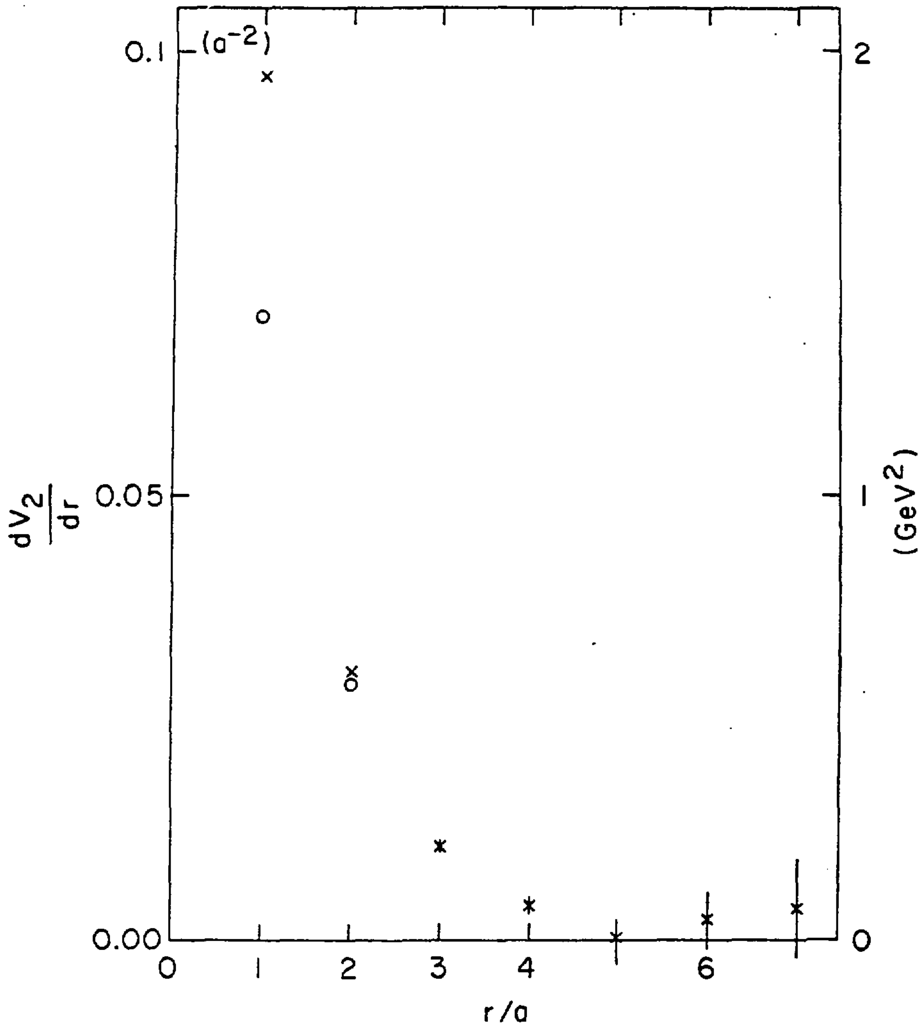

Fig. 2 Monte Carlo results for the spin-dependent potential V_2, before (x) and after (0) the correction for lattice artifacts discussed in the text.

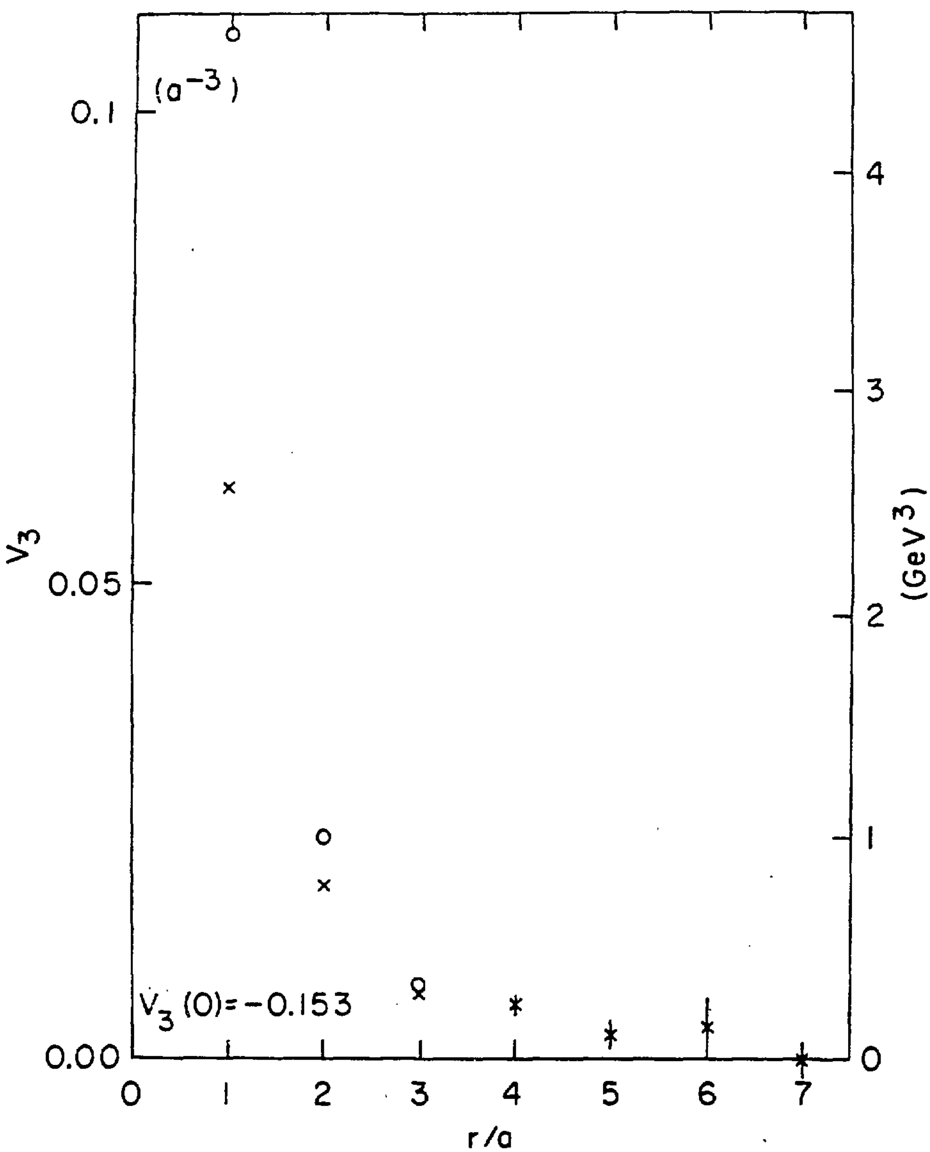

Fig. 3 Same as in Fig. 2, but for the spin-dependent potential V_3.

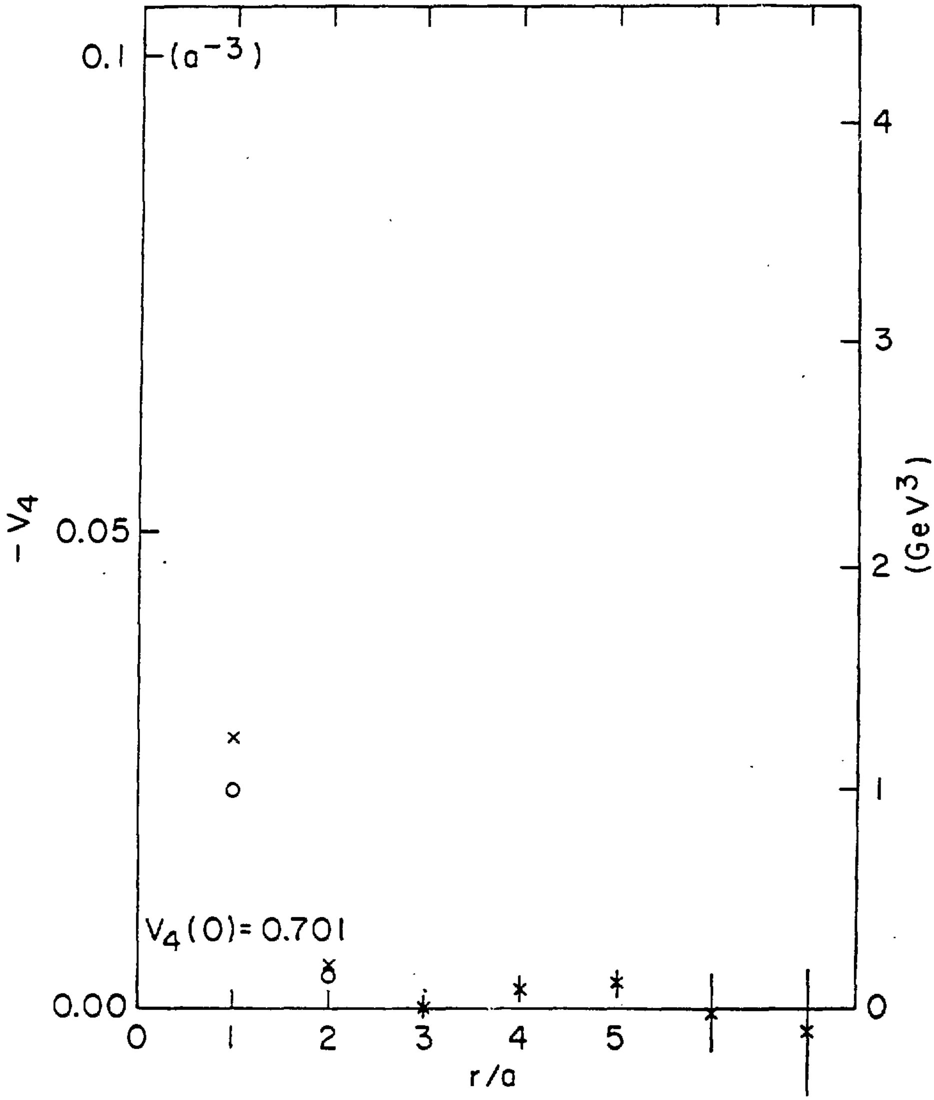

Fig. 4 Same as in Fig. 2, but for the spin-dependent potential V_4.