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A.V.Kulikov

DYNAMIC CONSERVATION OF ANOMALOUS CURRENT IN GAUGE THEORIES

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Abstract

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The symmetry of classical Lagrangian of gauge fields is shown to lead in quantum theory to certain limitations for the fields interacting with gauge ones. Due to this property, additional terms appear in the effective action in the theories with anomalous currents and its gauge invariance is ensured.

Аннотация

Куликов А.В. Динамическое сохранение аномальных токов в калибровочных теориях: Препринт ИФВЭ 86-83. – Серпухов, 1986. – 7 с., библиогр.: 6 назв.

Показано, что симметрия классического лагранжиана калибровочных полей приводит в квантовой теории к определенным ограничениям для полей, взаимодействующих с калибровочными. В теориях с аномальными токами вследствие этого свойства в эффективном действии возникают дополнительные члены и обеспечивается его калибровочная инвариантность.

Canonical quantizations of gauge theories as systems with constraints[1,2] lead to unsatisfactory results in the models with chiral fermions. The theory becomes inconsistent due to the fact that the initial classical action

$$S_o(A,\psi)=\int dx\{-\frac{1}{4}(F^a_{\mu\nu})^2+\bar{\psi}(i\hat{\partial}+\hat{A}^a t^a\frac{1+\gamma^5}{2})\psi\} \qquad (1)$$

ceases to be invariant in the quantum case. Namely, the exclusion of fermions from (1), i.e. functional integration over fermions, results in the following effective action

$$S_{eff}=\int dx\{-\frac{1}{4}(F^a_{\mu\nu})^2\}+W[A], \qquad (2)$$

whose second term is noninvariant with respect to transformations from the gauge group

$$W[A^g]\neq W[A], \qquad A^g_\mu=gA_\mu g^{-1}+ig\partial_\mu g^{-1}. \qquad (3)$$

In formulae (1)-(3), $A_\mu=A^a_\mu t^a$ and t^a are the generators of a gauge group.

Property (3) leads to the vacuum expectation value of invariant functionals $F[A^g]=F[A]$ being dependent both on the group element and on the choice of the gauge, if the traditional approach is used:

$$<F[A^g]> = \frac{1}{N}\int d\mu_{FP}(A)\exp\{iS_{eff}(A)\}F[A^g] =$$

$$=\frac{1}{N}\int d\mu_{FP}(A^{g^{-1}})\exp\{iS_{eff}(A^{g^{-1}})\}F[A]\neq<F[A]>. \qquad (4)$$

Here $d\mu_{FP}=dA_\mu C(A)\Delta(A)$ is the Faddeev-Popov[2] measure, $C(A)$ is the gauge, and $\Delta(A)$ is a determinant.

The arising contradiction (3) may mean one thing only: the initial problem has been ill-posed. i.e., an a priori approach to (1) as a gauge theory must have been incorrect. For example, if $W[A]$ contains the term $\sim A^2$ there are no reasons whatsoever to consider the effective theory (2) to be a gauge one, but as it is the theory of a massive vector field its existence may well be justified. Therefore in the general case as well, when the properties of symmetry W[A] are unknown, we have no right to treat the theory as a gauge one. (In this connection it should be noted that the definition of the total group of symmetries is a most overriding and complicated problem already in the classical theory). This suggests that to avoid the contradiction of type (4) one should consider in the quantum theory the standard integral "over all fields"

$$\langle F[A]\rangle = \frac{1}{N}\int dA e^{iS[A]} F[A], \qquad N = \int dA e^{iS[A]} \qquad (5)$$

irrespective of the structure of the initial classical action.

As shown by Faddeev and Popov [2], in a gauge-invariant theory (for invariant S and F) integral (5) contains the group volume in a completely factorized form. The same volume is also contained in the norm, therefore in the gauge-invariant theory formula (5) takes the following traditional form:

$$\langle F[A] = \frac{1}{N'}\int d\mu_{FP} e^{iS[A]} F[A], \qquad N' = \int d\mu_{FP} e^{iS[A]} \qquad (6)$$

For the gauge-noninvariant functionals $F[A^g] \neq F[A]$ in invariant theories vacuum means (5), if they exist are invariant,

$$\langle F[A^g]\rangle = \langle F[A]\rangle \quad ,$$

because of the measure dA_μ being invariant.

We suggest that the vacuum expectation values (5) should be considered for all the theories. This offers an attractive possibility of the universal approach to quantization of any theories irrespective of the structure of the initial classical action. Really, formula (5) is by definition used in noninvariant theories and, as is seen from (6), coincides with the Faddeev-Popov integral [2] in gauge-invariant ones. Besides it being universal, integration "over all the fields" ensures the absence of the troubles of type (4) in (7). The reason of why formula (5) was not used in gauge-invariant theories and those of type (1) is explained by the degeneration of the classical Lagrangian. In the long run, this does not allow one to use the perturbation theory without fixing the gauge. But it turns out that this gauge freedom leads to some subsidiary conditions in the functional integral presenting an opportunity to calculate integral (5) consistently. To clarify the situation arising when integrating degenerate Lagrangians, in Section 2 we shall consider a simple example of a linear system and demostrate the role of classical symmetry as a "generator" of conservation laws in quantum theory, In Section 3, the "group" integration is singled out from an arbitrary functional integral in gauge theories. The limitations appearing in this case are considered and the total effective action of gauge fields is constructed (in the form of a functional integral).

2. Let us consider the Gaussian integral

$$Z[j] = \frac{1}{N}\int d\phi \exp\{\frac{i}{2}(\phi,\ D\phi)+i(j,\phi)$$
$$N = \int d\phi \exp\{\frac{i}{2}(\phi,D\phi)\cdot \qquad (8)$$

Here $\phi\in E$, E is a linear space with the scalar product (.,.), D is an operator in it. We focus our attention on the theories with degenerated Lagrangians. Let

$$\exists K\subset E: \psi\in K \Rightarrow D\psi=0$$

Let us choose the basis in K

$$\forall\psi\in K,\quad \psi=\sum_{k=1}^{M} q_k\psi_k,\quad g_k\in \mathbb{R},\quad M=1,\dots,\infty.$$

Let us put ($\psi\in K$)

$$\phi=\phi'+\psi,\quad (\psi,\ \phi')=0.$$

Then, by the definition/3/ of integral (8), we get:

$$Z[j] = \frac{1}{N}\int d\phi'\prod_{k=1}^{M} dq_k \exp\{\frac{i}{2}(\phi',\ D\phi')+i(j,\ \phi')+$$
$$+\sum_{k=1}^{M} iq_k(j,\ \psi_k)\}=$$
$$=\frac{1}{N'}\int d\phi' \exp\{\frac{i}{2}(\phi',D\phi')+i(j,\phi')\}\frac{\prod_{k=1}^{M}2\pi\delta((j,\ \psi_k))}{V^M}, \qquad (9)$$

where

$$N' = \int d\phi' \exp\{\frac{i}{2}(\phi',\ D\phi')$$

$$V=\int dq = 2\pi\delta(0).$$

Note now that the quadratic part of integral (8) is invariant under the translation group

$$\phi\to\phi+\sum_{k=1}^{M} q_k\psi_k,\quad q_k\in\mathbb{R} \qquad (10)$$

therefore we can apply the Faddeev-Popov trick to integral (8). Let us define the "unity" as follows:

$$1=\Delta(\phi)\int dq C(\phi+\Sigma q_k\psi_k). \tag{11}$$

The gauge C will be chosen in the form

$$C(\phi)=\exp\frac{i}{2}(\phi,\psi_i)\Lambda_{ik}(\psi_k,\phi),\ \det\Lambda_{ik}\neq 0,$$

$$\Delta(\phi)=\sqrt{\det\Lambda_{ik}}. \tag{12}$$

Introducting (11) into (8), varying the order of integration and shifting $\phi\to\phi-\Sigma q_k\psi_k$ we obtain

$$Z[j]=\frac{\sqrt{\det\Lambda_{ik}}}{\sqrt{\det(D-\Lambda)}}\exp\frac{-i}{2}(j,(D+\Lambda)^{-1}j)\frac{\prod_{k=0}^{M}2\pi\delta((j,\psi_k))}{V^M}$$

$$\Lambda=|\psi_i>\Lambda_{ik}<\psi_k|. \tag{13}$$

Comparison of the formulas shows that (13) is integral (9) derived in the gauge C (12). Let us pay attention to the fact that the presence of δ-functions results in the gauge-independence of integral (13) and to the existence of D^{-1} in (9).The derivation of formulas (9),(13) shows that such helpful features of integral (8) manifest themselves as a result of "integration over the group" (10).So, the degeneration of Lagrangian (8) leads not to the divergence, as it is commonly considered,but,on the contrary,to the uniqueness of the integrals and to the existence of inverse operators available in them. The complete analysis of these questions in the operator formalism has been performed in the splendid work of A.I.Oksak[4].

To conclude this Section, it should be noted that we have not defined concretely the properties of the space K, in particular, we have not assumed the basis $\{\psi_k\}$ to be normalizable. Consequently, to determine K it is necessary to find not only the normalized zero modes but also all the solutions $\psi\in E$ to the equation $D\psi=0$. Such solutions could also be (depending upon a specific problem) non-decreasing functions, including polynomials, plane waves, etc.

Here is one more remark to the point. Noninvariant local values fit badly to describe the theories with degenerated Lagrangians[4]. In the given example, this manifests itself in the non-existence of the quantities of type

$$<\phi(x)\phi(y)>\sim i(D')^{-1}(xy)-\frac{\delta''(0)}{\delta(0)}P_o(xy), \tag{14}$$

where $P_o=\sum_{k=1}^{M}|\psi_k><\psi_k|$, $D(D')^{-1}=1-P_o$.

A detailed discussion of this question with the construction of (generalized) states can be found in [4]. Here we note that for practical purposes instead of (14) the qualities of type $(D')^{-1}$ and P_o, or $(D+\Lambda)^{-1}$ (13) are always sufficient.

3. As is shown in the previous Section, the degeneration of Lagrangian in linear systems leads to certain dynamic conditions. There is a clear understanding of this phenomenon in the operator interpretation which is based on the procedure of "dressing" the Fock states [4]. It is highly probable that the study of general degenerate theories should be carried out in the frames of these ideas. But a concrete operator realization of the given representations must be hampered by the nonlinearity of the degenerated (group) space. As differed from the operator approach, in the functional one these difficulties turn out to be inessential. The principle of integration "over all the fields" allows to single out explicitly the group degrees of freedom from the integral and to obtain the dynamic consequences of gauge symmetry. Such a procedure is a complete repetition of the linear case.

Let us consider the integrals (the normalizing factors are descarded)

$$Z= \int d\bar{\psi} d\psi e^{iS_o[\psi]} \tilde{Z}[j] \qquad (15)$$

$$\tilde{Z}[j]= \int dA \exp i\{ S_o[A]+ \int dx j^a_\mu(x) A^a_\mu(x) \}. \qquad (16)$$

The notations used in these formulas correspond to interaction (1).

Let us define

$$1= \Delta(A) \int dg C(A^g) \qquad (17)$$

assuming the gauge C to satisfy the conventional conditions (see, for example [5]). On putting (17) into (16) and shifting $A \to A^{g^{-1}}$ we obtain

$$\tilde{Z}[j]= \int d\mu_{FP} e^{iS_o[A]} \Gamma[j,A]$$
$$\Gamma[j,A] = \int dg \exp i \int dx \{ j^a_\mu(x) (A^{g^{-1}}_\mu (x))^a \}. \qquad (18)$$

The last integral is easily calculated in the Abelian ($t^a \to 1$) theory:

$$\Gamma[j,A] = \delta(\partial_\mu j_\mu(x)) \exp i \int dx \{ j_\mu(x) A_\mu(x) \}. \qquad (19)$$

This formula gives foundation for the use of the term "dynamic conservation of current". So, gauge symmetry

(~ degeneration of the classical action) leads to certain
degeneration of the classical action) leads to certain
constraints on the matter fields which should necessarily be taken into account in all the theories. This is the basic quantum consequence of the classical symmetry of Lagrangian.

Further consequences of formulas (18) can be obtained for the whole integral (15). Let us put (18) into (15) and make the replacement of variables

$$\begin{cases} \psi = U(g)\psi' \\ \bar\psi = \bar\psi'\bar U(g) \end{cases} \tag{20}$$

$$U(g) = \frac{1+\gamma^5}{2}g^{-1} + \frac{1-\gamma^5}{2}, \quad \bar U(g) = \gamma^0 U^+ \gamma^0$$ in such a way that

$j_\mu(x) = g j'_\mu(x) g^{-1}$. The measure $d\bar\psi d\psi$ in (15) is noninvariant under transformation (20):

$$d\bar\psi d\psi = d\bar\psi' d\psi' \exp iW[A,g]. \tag{21}$$

With allowance for (21), the effective action in (15) will have the form

$$S_{eff} = S_o[A] + S_o[\psi'] + \int dx \bar\psi' \hat A^a t^a \frac{1+\gamma^5}{2} \psi' + W[A,g] =$$

$$= S_o(A, \psi') + W[A,g]. \tag{22}$$

Omitting the primes we obtain the final expression for Z in (15):

$$Z = \int d\mu_{FP} d\bar\psi d\psi e^{iS_o(A,\psi)} \int dg^{iW[A,g]}. \tag{23}$$

This formula is the main result of the present work and a basis for further study of the theories with anomalies. Note, that in the invariant theories, W=0, expression (23) also transforms into the traditional integral over the Faddeev-Popov measure. On the contrary, the existence of anomalies in the fermion sector results in an additional contribution into the effective action of a gauge field. Quite important is the fact that this contribution is not just an anomalous interaction of the Wess-Zumino-Witten type but a result of integration of "anomalies" over the gauge group. Here the mere fact of integration is important because the result is a functional with the property

$$\int dg \exp iW[A,g] = \int dg \exp i\{W[A^{h^{-1}}, g] + W[A,h]\} \tag{24}$$

for an arbitrary element h(x) of the gauge group. Consequently, the integral of each term of expansion (24) over h is equal to zero. With account of nontrivial topological properties of W, this inspires us with a hope that functional (24) additionally presents a possibility of a further consistent study of the topological structure of anomalous theories.

In conclusion let us bring for illustrative purposes a number of formulas of the Abelian theory. In this case $g \to e^{i\omega}$, $dg \to d\omega$, and

$$W[A,g] \to W[A-\partial\omega]-W[A].$$

Routine calculations of the variation of the fermion determinant (regularized by ξ-function) lead to the following expression:

$$W[A-\partial\omega] = W_{WZ} + \Delta W = \frac{1}{48\pi^2} \int dx \{ 4(\omega(x) - \frac{1}{\partial^2} \partial_\rho A_\rho) \cdot$$

$$\cdot \epsilon^{\alpha\beta\mu\nu} \partial_\alpha A_\beta \partial_\mu A_\nu + \frac{1}{2}(A_\alpha - \partial_\alpha \omega) \partial^2 (A_\alpha - \partial_\alpha \omega) - \frac{1}{4}[(A_\alpha - \partial_\alpha \omega)^2]^2 \}. \tag{25}$$

The symbol ΔW is used to denote the last two terms in (25). Typically (see, for example /6/) they are not considered in the ground, that the local counterterms compensating the contribution of ΔW can be introduced into the Lagrangian. But the inclusion of $-\Delta W$ into the bare Lagrangian is not very much consistent from the viewpoint of gauge invariance. Therefore we consider ΔW together with W_{WZ}.

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А.В.Куликов
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Редактор А.А.Антипова. Технический редактор Л.П.Тимкина.
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Институт физики высоких энергий, 142284, Серпухов Московской обл.

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