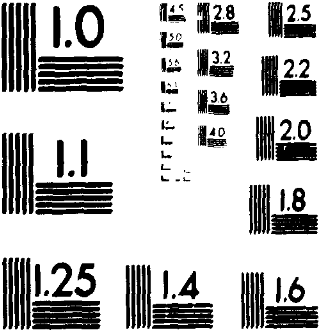


MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS
STANDARD REFERENCE MATERIAL 1010a
(ANSI and ISO TEST CHART No. 2)

ISSN/0393-6287

RT/FUS/86/13

E. BITTONI, M. HAEGI

THE CALCULATION OF THE TRITIUM BURNUP IN TOKAMAKS

COMITATO NAZIONALE PER LA RICERCA E PER LO SVILUPPO DELL'ENERGIA NUCLEARE E DELLE ENERGIE ALTERNATIVE

ISSN/0393-6287

COMITATO NAZIONALE PER LA RICERCA E PER LO SVILUPPO
DELL'ENERGIA NUCLEARE E DELLE ENERGIE ALTERNATIVE

ASSOCIAZIONE EURATOM-ENEA SULLA FUSIONE

THE CALCULATION OF THE TRITIUM BURNUP IN TOKAMAKS

E. Bittoni
ENEA - Dipartimento Tecnologie Intersettoriali di Base, Centro ricerche energia «Ezio Clementel», Bologna

M. Haegi
ENEA - Dipartimento Fusione, Centro ricerche energia Frascati

RT/FUS/86/13

Testo pervenuto nel gennaio 1987

This report has been prepared by: Servizio Studi e Documentazione - ENEA, Centro Ricerche Energia Frascati, C.P. 65 - 00044 Frascati, Rome, Italy.

This Office will be glad to send further copies of this report on request.

3/4

Summary In a deuterium plasma tokamak, the contained fusion-produced tritons are supposed to be decelerated down to thermalization according to classical Coulomb scattering. A fraction of these fast tritons undergoes the DT fusion reaction producing 14.1 MeV neutrons. It is thus possible to get information on the confinement of these fast tritons by comparing the measured and the calculated ratio of the 14.1 MeV to the 2.45 MeV neutron flux. This report describes the calculation of this flux ratio by means of a numerical Monte Carlo-like code.

Riassunto Nel plasma di deuterio di un tokamak, si suppone che la frazione contenuta dei tritoni prodotti sia decelerata fino alla termalizzazione secondo processi di collisioni Coulombiane. Una frazione di questi tritoni rapidi reagisce con i deuteroni del plasma producendo quindi neutroni di 14,1 MeV. Diventa possibile così ottenere informazioni sul confinamento di questi tritoni, paragonando il rapporto calcolato e misurato dei flussi di neutroni di 14,1 MeV con quello dei neutroni di 2,5 MeV. In questa relazione, descriviamo il calcolo di questo rapporto di flusso ottenuto con un codice numerico tipo Monte Carlo.

1. INTRODUCTION

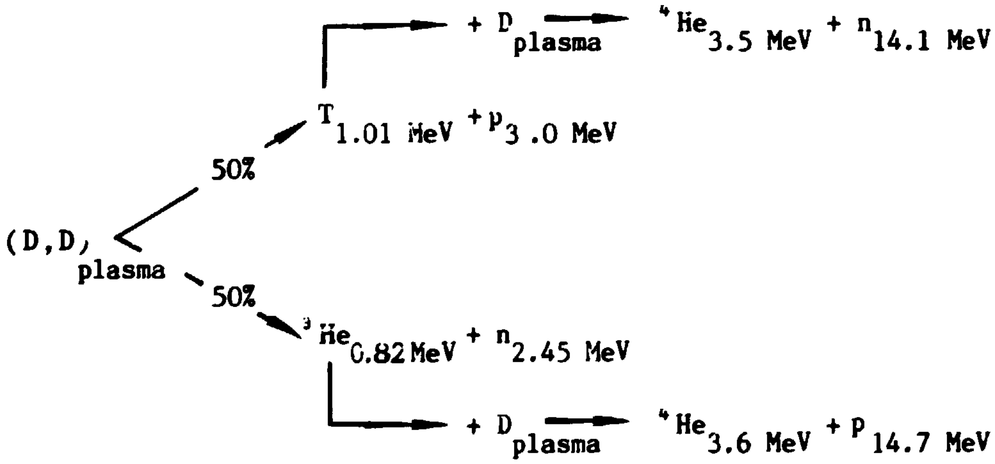
Alpha-particle physics can be treated at three levels:

1) The single α-particle ignores the others and evolves in a field of particles which is unperturbed.
2) The α-particles interact between themselves as well as with the plasma, the latter being macroscopically unperturbed.
3) The α-particles interact strongly with the plasma, possibly producing a measurable heating.

In this paper we will limit ourselves to the first case, studying the confinement of fusion-produced tritons during the slowing down in a tokamak deuterium plasma.

In order to compare the calculations with the expermental measurements, we have to evaluate the ratio of the 14 MeV neutron flux to the 2.5 MeV neutron flux.

Indeed, the nuclear reactions that take place in a deuterium plasma are:



It can easily be shown that at least 99% of the DT reactions take place between a fast triton during its deceleration and a plasma deteuron. The contribution of the thermalized tritons can be neglected because the confinement time and the slowing down time are comparable and the DT cross section of the thermalized tritons is much smaller than that of fast tritons. The contribution to DT reactions due to the bombardment of deuterons adsorbed on the wall by fast tritons escaping from the plasma can also be ignored.

It is thus possible to obtain information on the confinement of the fast triton by comparing the measured and the calculated DT fusion reaction probability [1-5].

Let us call $(N_{14}/N_{2.5})_{experimental}$ the ratio of 14 MeV to 2.5 MeV neutron fluxes as deduced from the experimental measurements and $(N_{14}/N_{2.5})_{classical\ slowdown}$ the corresponding value calculated assuming a classical Coulomb scattering slowdown.

Let:

$$\Gamma \equiv \frac{(N_{14}/N_{2.5})_{experimental}}{(N_{14}/N_{2.5})_{classical\ slowdown}} ,$$

thus for $\Gamma = 1$ the slowdown is classical

for $\Gamma < 1$ there are two possibilities:

a) the slowdown is classical, but the triton gets lost before thermalizaiton because the orbit is perturbed by the magnetic ripple, MHD activity, turbulence, etc.

b) the slowdown is faster than classical.

for $\Gamma > 1$ errors or an exotic scenario.

2. CALCULATION OF THE TRITON BURNUP

In order to calculate the triton burnup in a deuterium plasma, one can resolve the Fokker-Plank equation with the conditions that the tritons are generated at an energy of 1.01 MeV at a rate $R(\bar{r},\bar{v},t)$, which corresponds to a distribution function $f_D(\bar{r},\bar{v},t)$ for the deuterons, and that they are lost when thermalized or when the velocity enters the loss cone. In order to get the distribution function $f_T(\bar{r},\bar{v},t)$ of the tritons, the main difficulty in resolving the Fokker-Plank equations is due to the fact that the fast tritons have orbits with large radial excursions; thus the diffusion and drag coefficients have to be calculated numerically.

Therefore, in this paper, we will not use distribution function calculations, but a Monte Carlo-like guiding center simulation.

We assume that the deuterium plasma is in equilibrium and has the given parameters: $n_e(r)$, $T_e(r)$, $n_D(r)$, $T_D(r)$ and Z_{eff}. In addition to the total current I_p we also assume the shape of the plasma current $J(r)$.

The triton production rate per unit of azimuthal angle is given by

$$\frac{dN_T}{dt} = \frac{1}{4}\, n_D^2(r) < \sigma_{DD} v > R_0\, r\, dr\, d\theta$$

The total production rate of 14.1 MeV neutrons due to the triton burnup is

$$\frac{dN_{14}}{dt} = 2\pi R_0 \int_0^{a-r_\ell} r\,dr \int_0^{2\pi} d\theta \int_{\cos\chi_1}^{\cos\chi_2} \frac{1}{4} \langle \sigma_{DD} v \rangle n_D^2 p(\chi) d(\chi) \int \sigma_{DT}(\ell) n_D(\ell) d\ell$$

total length of the trajectory of the triton born in (r,θ,χ)

where $p(\chi)d\chi$ is the probability that the triton has its initial pitch angle between χ and $\chi+d\chi$, and $\cos\chi_1$, $\cos\chi_2$ are the limiting values of the loss cone for the tritons generated at (r,θ). We assume that the tritons generated at a distance of less than a Larmor radius from the edge of the plasma are immediately lost. We also assume that a triton gets lost once its guiding center radial coordinate becomes $r \geq a-r_L$.

The total production rate of 2.5 MeV neutrons is given by

$$\frac{dN_{2.5}}{dt} = 2\pi R_0 \int_0^a r\,dr \int_0^{2\pi} d\theta \frac{1}{4} n_D^2(r) \langle \sigma_{DD} v \rangle$$

The slowing down of the tritons produced at (r,θ) is simulated by the slowing down of a number N_χ of macroparticles of weight $\frac{dN_T}{dt} / N_\chi$, each one having a probability $p(\chi)$ of being created with an initial pitch angle $\cos\chi = v_\parallel / v$ within the interval -1 to +1. For every point (r,θ), the position of creation of the N_χ macroparticles is randomly generated on a circle of center (r,θ) and radius equal to the Larmor radius. This mechanism, which has a relatively small effect in toroidal devices with a high magnetic field, becomes more important in low field devices because it tends to substantially broaden the triton source function (Fig.1).

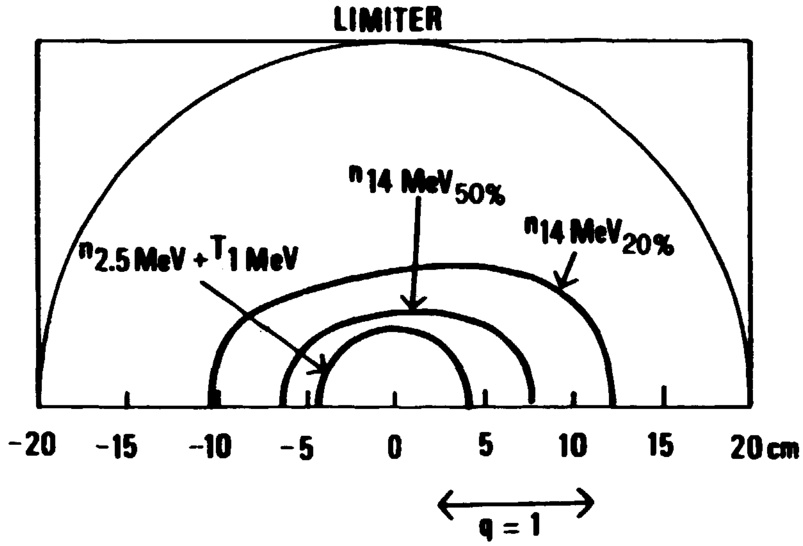


Fig.1 The calculated triton and 2.5 MeV neutron source function $S_{2.5}(r)$ at 50% the maximum value. The calculated 14 MeV neutron source function $S_{14}(r,\theta)$ at 50% and 20% the maximum value.

The probability density $p(\chi)$ is a function of the triton distribution given by the DD reaction. In this paper an isotropic distribution is assumed:

$$p(\chi)d\chi = \Delta(\cos\chi) = 2/N_{\chi}$$

N_{χ} should be sufficiently large to permit a statistical simulation.

The calculation of the burnup integral

$$\int \sigma_{DT}(\ell)n_D(\ell)d\ell$$

total length of the
trajectory of the
tritons born in (r,θ,χ)

is carried out according to the following physical assumptions:

- The slowdown of the tritons is classical and is due only to the Coulombian interaction with the electrons and ions of the plasma.
- As the DT cross section in the energy range of 1010--30 keV is about 10^3 higher than the cross section at thermalization, and as the slowdown time and the confinement time of thermalized tritons are similar, the 14.1 MeV neutron is produced essentially during the triton slowdown. We thus limit our calculation to the above energy interval.
- The slowing down is due to the drag on the electrons and ions of the plasma. At high energies, the drag is essentially due to the electrons; at critical energy, the contributions of the electrons and ions are equal.
- The effect of pitch angle scattering on the confinement has not been taken into account. Pitch angle scattering, which is very small at high energies, becomes important at lower energies, when the DT cross section has already substantially decreased. We assume therefore that tritons which are confined on their first orbit remain so confined during their whole slowdown within the above energy interval.
- The effect on the confinement of tritons due to the magnetic ripple is also disregarded. The ripple has two effects on the triton trajectory: on one hand, the particle may be directly trapped in the local mirrors generated by the ripple; on the other hand, the

modulation of the magnetic field produces small stochastic deviations with respect to an axisymmetric trajectory which results in a radial diffusion affecting the fast triton confinement. Banana orbits are particularly sensitive to both these effects. To illustrate these calculations and their confrontation with experimental results, we will use the FT device. For burnup experiments, the FT has been operated at a plasma current of less than 600 kA. At such currents, practically all banana orbits get immediately lost; hence, we will ignore the effect of the ripple on these trajectories. The remaining particles have essentially [6] circulating orbits for which $v_\parallel \neq 0$ is always satisfied. As these orbits are much less sensitive to the magnetic ripple, we shall ignore this effect for the circulating orbits too.

2.1 Single Orbit Approximation (SOA)

The calculation of the burnup integral is performed assuming that the triton always follows the same orbit during its slowing down, neglecting the displacement or the contraction of the orbit during its deceleration ($\equiv$ SOA). With this assumption, a triton born at the point (r,θ) with a given initial pitch angle X has a well-defined orbit on which nonlocal mean values such as $\langle n_D \rangle$ and $\langle T_e^{3/2}/n_e \rangle$ can be calculated by defining

$$\langle A \rangle \equiv \int_\ell A d\ell/\ell$$

where ℓ is the total length of the poloidal orbit corresponding to the motion of the guiding center of the Larmor orbit.

Using only the trajectory of the guiding center for the calculation of these mean values, we tend to compensate the overevaluation of the burnup that the SOA implies, neglecting the contraction of the orbit during slowing down.

With this hypothesis:

$$\int_{\substack{\text{real length of}\\ \text{the trajectory}}} \sigma_{DT}(\ell) n_D(\ell) d\ell \simeq \langle n_D \rangle \int_{\substack{\text{real length of}\\ \text{the trajectory}}} \sigma_{DT}(\ell) d\ell$$

2.2 Classical slowing down

For classical slowing down of a 1.01 MeV triton, by Coulombian interaction with the electrons and ions of the plasma, down to the critical energy, the mean energy loss by unit of time is given by [7]

$$\frac{dw}{dt} = \alpha w^{-1/2} + \beta w$$

with

$$\alpha = - 1.81 \cdot 10^{-7} \ln\Lambda \, A^{1/2} Z^2 \sum_j \frac{n_j Z_j^2}{A_j}$$

$$\beta = - 3.18 \cdot 10^{-3} \ln\Lambda \, \frac{Z^2}{A} \, \frac{n_e}{T_e^{3/2}}$$

where we have omitted the contribution of the electric field, parallel to the triton velocity. A and Z are respectively the atomic weight and atomic charge of the triton,

A_j and Z_j have the same meaning for impurity plasma ions. Using the above-defined mean value over the orbit for n_D and $n_e/T_e^{3/2}$, it is possible to integrate analytically:

$$w(t) = \left[\alpha w_0^{3/2} e^{-3/2 \beta t} + \frac{\alpha}{\beta}(e^{-3/2 \beta t} - 1)\right]^{2/3} \qquad w \text{ in eV}$$

$$v(t) = 1.39 \cdot 10^6 \frac{1}{A^{1/2}} w(t)^{1/2} \qquad v \text{ in cm/s}$$

where w_0 is the initial energy of the tritons.

The slowing down time τ_{sd} from w_0 to w_{cr} is

$$\tau_{sd} \simeq \frac{2}{3} \frac{1}{\beta} \ln \left[\frac{\frac{\alpha}{\beta} + w_0^{3/2}}{\frac{\alpha}{\beta} + w_{crit}^{3/2}} \right]$$

and therefore, for the single macroparticle we have

$$\int_{\substack{\text{real length of} \\ \text{the trajectory}}} \sigma_{DT}(\ell)d\ell \simeq \int_0^{\tau_{sd}} \sigma_{DT}(E_{cm}(t))v(t)dt$$

where $E_{cm}(t)$ is the energy in the center of mass of the system triton-deuteron.

$$E_{cm}(t) = \frac{A_D}{A_D + A} w(t)$$

2.3 Guiding center equations

As shown previously, the mean values $<n_e>$ and $<\frac{n_e}{T_e^{3/2}}>$ are calculated by means of the guiding center equations.

Assuming a toroidal coordinate system (r,θ,φ); due to the symmetry in φ, the toroidal component of the angular momentum is a constant of motion

$$p_\varphi = mR^2\dot{\varphi} - \frac{Ze}{c} RA\varphi = mRv_\varphi - \frac{Ze}{c} RA_\varphi = \text{const.}$$

A_φ is the toroidal component of the vector potential

$$RA_\varphi = \int_0^r RB_\theta dr \qquad R = R_0 + r\cos\theta$$

where R_0 is the radius of the magnetic axis. The formulation of the guiding center orbit equation is done for two cases:

a) The model of magnetic field is

$$\bar{B} = \frac{R_0}{R} (0, B_{\varphi_0}, B_\theta^0(r))$$

where B_{φ_0} is the toroidal field for $R=R_0$, and $B_\theta^0(r)$ is the poloidal field generated by a cylindrical current distribution having the density

$$J(r) \sim J_0(1-(r/a)^\alpha)^\beta$$

α and β being real.

b) The poloidal flux function

$$\psi = RA_\varphi$$

is given directly by the solution of the MHD equilibrium equations.

Let us first treat case a):

Assume

$$v_\varphi = v_\parallel \frac{B_\varphi}{B} = \frac{v_\parallel}{(1+B_\theta^2/B_\varphi^2)^{1/2}} = \frac{v_\parallel}{b(r)} \qquad \text{for } B_\theta/B_\varphi << 1;$$

using the conservation of

1) the energy

$$E = \frac{1}{2} mv_\parallel^2 + \mu B$$

2) the magnetic momentum

$$\mu = \frac{1}{2} mv_\perp^2/B$$

we get the equation of the guiding center orbit in a dimensionless form

$$\cos\theta = \frac{A}{x}\left\{\left[B \mp \sqrt{B^2 + (c + \frac{f(x)}{p\cdot A})^2}\right] - 1\right\}$$

where

$x = r/a$

$A = R_0/a$ (aspect ratio)

$$B = \frac{1-\cos\chi_0}{2}(1 - x_0/A)$$

$$C = (1 + x_0/A)\cos\chi_0 - \frac{f(x_0)}{p\cdot A}$$

$$f(x) = \frac{1}{c_{\alpha\beta}}\int_0^x \frac{2}{x'}\left(\int_0^{x'} t(1-t^\alpha)^\beta dt\right)dx'$$

$x, x' \leq 1$

Values with the index o correspond to the initial coordinates and pitch angle.

$$c_{\alpha\beta} = \int_0^1 t(1-t^{\alpha})^{\beta} dt$$

is an integration constant.

The parameter p which appears in the orbit equation is given by

$$p = 2\frac{\rho_\theta}{A}\cdot\frac{1}{a} = \frac{\sqrt{2E\cdot m}}{Ze} \quad \frac{c}{0.1 I_p A}$$

where I_p is in amperes, E in ergs, ρ_θ is the Larmor radius in the poloidal magnetic field, and p gives a measure of the maximum excursion, across the magnetic surfaces, executed by the particle.

b) As we can directly use the poloidal flux function

$\psi(R,Z)$

the problem of the guiding center orbit equation can be treated in the following way:

The conservation of the toroidal angular momentum can be written in the first approximation as an expansion in B_θ/B_φ:

$$p_\varphi \simeq mRv_\parallel - \frac{Ze\psi}{c} \qquad \text{where } v_\parallel = v\cdot\cos\chi$$

Using the conservation of the magnetic moment and of the energy, the particle that leaves the point (R_i, Z_i) with a given value of the pitch angle intersects the general level curve of the ψ at the point $(\bar{R},\bar{Z})$ where

$$\bar{R} = R_0\left(\frac{\mu B_{\varphi_0}}{2E} + \sqrt{\left(\frac{\mu B_{\varphi_0}}{2E}\right)^2 + \frac{(p_\varphi + \frac{Ze}{c}\bar{\psi})}{2mER_0^2}}\right)$$

The value of $\bar{Z}$ is then determined by an inverse interpolation of the $\psi(\bar{R},\bar{Z})$.

Repeating this step for a sufficiently large number of values of $\bar{\Psi}$, the trajectory of the particle in the (R,Z) plane can be constructed point by point.

3. DATA FROM THE EXPERIMENT

As we have seen in the previous sections, the following plasma parameters and profiles are necessary to perform the burnup calculations:

$T_e(r), T_i(r), n_e(r), n_i(r), J(r), Z_{eff}$

The most sensitive parameters are the electron temperature profile $T_e(r)$ which essentially controls the slowing down and the plasma current density profile $J(r)$ which controls the confinement of the fast triton.

If the burnup calculations are done to check the experimentally measured values, in order to have a fair comparison, we find it essential to put shot-by-shot the most exact plasma parameters into the theoretical classical slow down calculations. It is then possible to discriminate if the shot-to-shot variation of the burn-up corresponds to fluctuations of the confinement or fluctuations of the plasma parameters.

On FT particular care has been taken to enter the most accurate and reliable experimental plasma parameters into the calculations. The electron temperature, for example, has been measured by Thomson scattering, soft X-ray emission, electron cyclotron emission ECE, and neoclassic resistivity. The best shot-to-shot correlation was obtained by taking the absolute value from the neoclassic resistivity and the radial profile from ECE or Thomson scattering. The uncertainty of the input plasma parameters on the cross sections used, and the approximations of the code provide a $(N_{14}/N_{2.5})_{classical}$ slowdown prevision affected by an error of ±30%.

The input data description of the numericl code is given in the Appendix.

4. COMPARISON OF SLOWDOWN CALCULATIONS WITH EXPERIMENTAL RESULTS FROM THE FT

The experimental procedure, based on the activation of ^{115}In and ^{63}Cu to detect respectively the 2.5 MeV and the 14 MeV neutrons has been described in [3,4,5]. For the calculation of the neutron flux emitted from the plasma, a 3D MCNP code has been used to take account of the 60-ton structure around the plasma and of the azimuthal neutron flux variations due to the nonaxisymmetry of the structure. The output of the procedure is

$$(N_{14}/N_{2.5})_{experimental}$$

affected by a standard error of ±30%.

4.1 Results

For the experiments described in [4,5], the calculated value of Γ, which is the ratio of the experimental to the theoretical classical Coulomb slowdown, is shown in Fig. 2 as a function of the total plasma current I_p for a toroidal magnetic field of 80 kG.

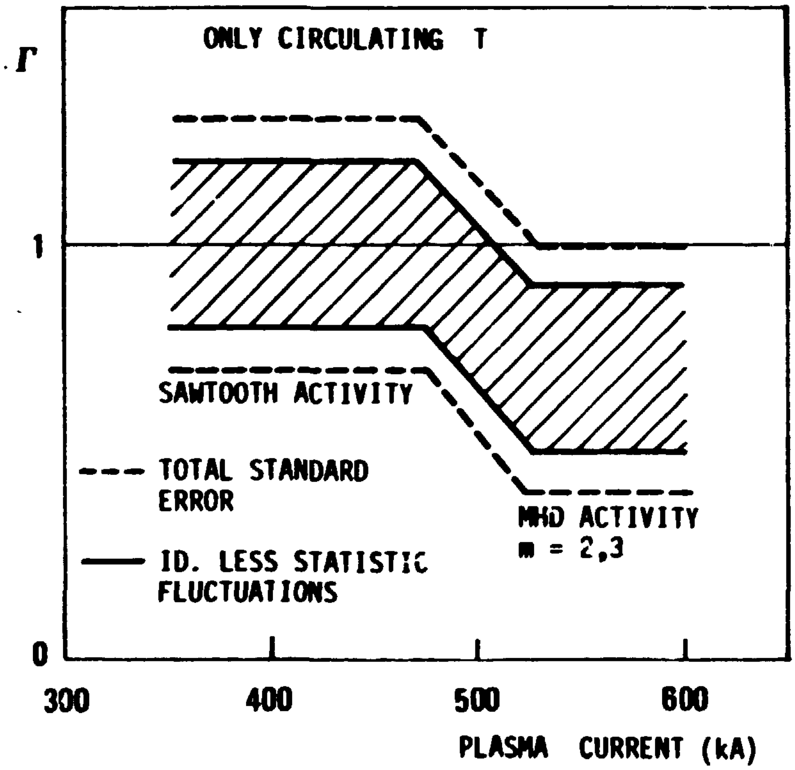


Fig.2 The deviation factor from the classical slowdown

$$\Gamma = \frac{(N_{14}/N_{2.5})_{experimental}}{(N_{14}/N_{2.5})_{classical\ slowdown}}$$

for FT as a function of plasma current at 80 kG.

We see that below a certain current, Γ is substantially near to 1, and that for higher currents (for which MHD activity is present), the value of Γ drops to about 0.7.

If this result is interpreted (case a) of §1) as a loss of the fast tritons during the slowdown, we can say (Fig.3) that in the absence of MHD modes the triton is decelerated up to 0-200 keV, while in the presence of these modes they are statistically decelerated up to only 100-300 keV [5].

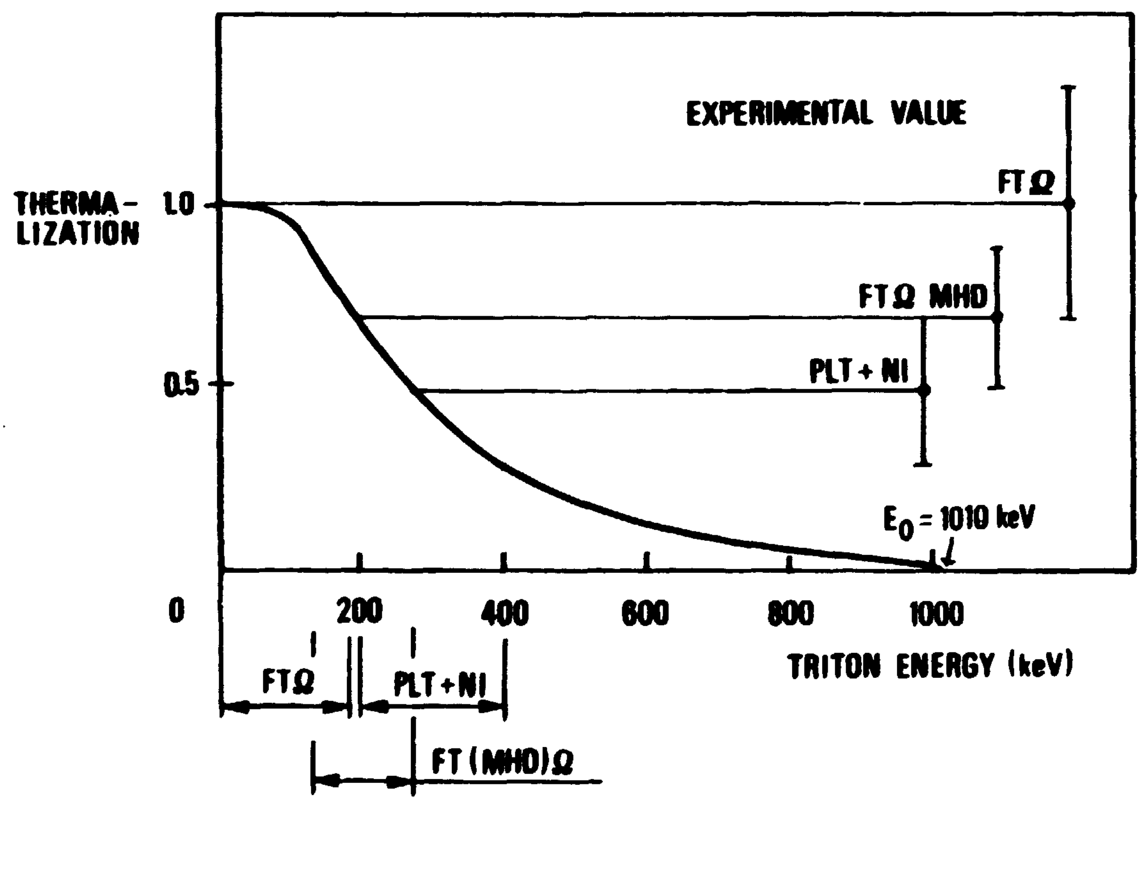


Fig.3 Cumulative DT fusion probability of a fast triton while slowing down.

APPENDIX

Input data description

The input data are given in free format. The first string contains one integer (ISCR in the code) which indicates the number of the plasma discharge considered.

The second string contains four integers, respectively NR, NTH, NCHI, NORB, where NR and NTH define the spatial grid and NCHI the number of birth directions in $\cos\chi$ for the simulation of the tritium source (typically we use NR=NTH=NCHI=20).

NORB is the number of points for which the particle orbit is followed (typically NORB=100).

The third string contains five floating parameters indicating respectively

BZER	toroidal field	(in Gauss) on the magnetic axis
RTORUS	major radius	(in cm)
RMINOR	minor radius	(in cm)
CURR	plasma current	(in Amperes)
COUL	mean Coulomb logarithm	

The fourth string contains two parameters defining the electron density which is supposed in the form

$$n_e(r) = n_{eo}\,(1 - (r/R_{minor})^2)^\alpha$$

DNZ = n_{eo} (in cm^{-3})

EXPN = α

The fifth string contains parameters relative to fusion products:

AMAS	mass number	
ZCHÀR	charge number	
EZERO	birth energy	(in eV)
EFIN	thermalization energy	(in eV)

The sixth string contains the parameters relative to the function defining the plasma ion temperature

$$T_i(r) = (T_{io}-T_{ie})\,(1-(A(r/R_{minor})^{E_1})^{E_2})+T_{ie}$$

TDZ	$= T_{io}$	(in eV)
TDA	$= T_{ie}$	(edge temperature in eV)
ATT	$= A$	
BTT	$= E_1$	
EXPT	$= E_2$	

The seventh string contains the parameters for the definition of the electron temperature

$$T_e(r) = T_{eo}(1-(r/R_{minor})^{E_3})^{E_4})$$

TIZ	$= T_{eo}$	(in eV)
BTTE	$= E_3$	
EXPTE	$= E_4$	

The eighth string contains two parameters describing the shape of the plasma current density

$$J(r) = J_o(1-(r/R_{minor})^{\alpha})^{\beta}$$

We suppose $J(r) \propto T_e(r)^{3/2}$, so

EXPJ1 $= \alpha = E_3$

EXPJ2 $= \beta = \frac{3}{2} \times E_4$

The four following strings contain respectively

1) ZCP, AMASP charge and mass number of plasma ions
2) ZEFF, ZIMP, AIMP mean Zeffective, charge and mass number of impurity species (only one)
3) NT (integer) number of step formats in the integral on the slowing down time (typically 100)
4) RES plasma eletric resistance.

The CPU time for each simulation is typically 20" on a CRAY 1 computer.

REFERENCES

[1] W. Heidbrink, R. Chrien, J. Strachan, Nucl. Fusion, 23 (1983) p.917.

[2] J. Strachen et al., Proceedings of Workshop on the Basic Processes in Toroidal Fusion Devices, Varenna, (1985), ed. Monotypia Franchi.

[3] E. Bittoni, A. Fubini, M. Haegi, E. Pedretti, M. Pillon, M. Vanucci, 12th Eur. Conf. on Controlled Fusion and Plasma Physics, Budapest (1985) p.211, ed. L. Pócs, A. Montvai.

[4] E. Bittoni, M. Haegi, A. Fubini, JET-IR (86) 04, p.191, JET, Abingdon, England.

[5] E. Bittoni, M. Haegi, Symposium on "The Role of Alpha Particles in Magnetically Confined Fusion Plasmas", Göteborg, Sweden (1986). Proceedings to be published in Physics Scripta.

[6] E. Bittoni, L. Feliciani, CNEN Report RT/FI(78)13, Centro Calcolo, Bologna, Italy (1978).

[7] T. Stix, Plasma Phys. 14 (1972), p.367.

Edito dall'ENEA, Direzione Centrale Relazioni.
Viale Regina Margherita 125, Roma.
Finito di stampare nel marzo 1987
Fotoripr. e Stampa La Casa della Stampa
Via Empolitana 120/C - Tivoli (Roma)

Questo fascicolo è stato stampato su carta riciclata