

**JAERI-M**  
**87-110**

**EFFECT OF DISCRETE RF SPECTRUM ON FAST WAVE  
CURRENT DRIVE**  
**— CONCEPTUAL DESIGN STUDY OF FY86 FER —**

August 1987

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編集兼発行 日本原子力研究所  
印刷 日青工業株式会社

Effect of Discrete RF Spectrum on Fast Wave  
Current Drive

- Conceptual Design Study of FY86 FER -

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(Received July 9, 1987)

Effect of discrete RF spectrum has been studied for the fast wave current drive with the ion cyclotron range of frequency. Driven current and power densities decrease in this spectrum than in the continuous spectrum. However, there is a possibility to have the mechanism which allows electrons outside the resonance region to interact with the fast wave, taking into account the electron trapping by discrete RF spectrum. In the case of neglecting the electron trapping effect, driven current and power densities decrease up to 0.6 - 0.8 of those which are obtained for the continuous spectrum for the FER (Fusion Experimental Reactor). However, their driven current and power densities can be almost doubled in their magnitude for the discrete spectrum by taking into account the trapping effect.

Keywords: Current Drive, Fast Wave, RF Spectrum, FER

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離散的放射スペクトルの速波電流駆動への影響  
— 次期大型装置設計 (FY86 FER) —

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(1987年7月9日受理)

イオンサイクロトロン周波数領域で固有に現われるアンテナからの離散的放射スペクトルの速波電流駆動への影響を検討した。上記スペクトルの場合の駆動電流密度・駆動電力密度は連続スペクトルの場合に比べて減少する。しかし電子はプラズマ中の電磁場に捕捉されスペクトルの離散化の程度を緩和して電磁場と相互作用することが考えられ、上記捕捉効果を考慮した電流駆動過程を定式化した。その結果、FERでは、従来の電流駆動モデルで求めた離散的スペクトルの場合の駆動電流密度・駆動電力密度は連続スペクトルの場合の6～8割程度まで減少する。しかし上記捕捉効果を考慮すると、従来の電流駆動モデルで求めた離散的スペクトルの場合の2倍以上まで、駆動電流密度、駆動電力密度が増加する可能性があることがわかった。

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## Contents

1. Introduction .....	1
2. Basic equations .....	1
3. Effect of the discrete RF spectrum on fast wave current drive ....	4
4. Conclusion .....	7
Acknowledgement .....	7
Referenece .....	7

## 目 次

1. 緒 言 .....	1
2. 基礎方程式 .....	1
3. 離散的放射スペクトルの速波電流駆動への影響 .....	4
4. 結 言 .....	7
謝 辞 .....	7
参考文献 .....	7

## 1. Introduction

Fast wave current drive has been studied extensively as the current drive method for the tokamak reactor.<sup>1,2)</sup> In previous theory, uniform RF spectrum was used to obtain the current drive efficiency.<sup>3)</sup> However, the RF spectrum is discrete in plasmas due to a long wavelength of fast waves, compared with a tokamak machine size. Especially, the magnitude of discreteness becomes larger for smaller size of the machine. The driven current will be decreased owing to the discreteness. To do precise evaluations for current drive with such large wavelength, it is necessary to take account of electron trapping by the electromagnetic field in plasmas and of relaxation of the magnitude of discreteness due to the trapping. In the paper, such treatment will be done by using modified orbits of trapped and untrapped electrons for the Fokker-Planck equation.

## 2. Basic equations

Equation of motion for electrons immersed in the electromagnetic fields is written as

$$m \dot{v}_{\parallel} = -eE(z,t) - \mu \frac{\partial B(z,t)}{\partial z}, \quad (1)$$

where  $m$ ,  $e$  and  $v_{\parallel}$  are electron's mass, charge and velocity parallel to the magnetic field and  $\mu$  is the magnetic moment. In this equation, Landau damping and transit time magnetic pumping are considered as the current drive mechanism. To obtain the modified orbits, the electromagnetic fields are, for simplicity, given as traveling waves,

$$E(z,t) = E \sin(k_{\parallel}z - \omega t), \quad (2a)$$

$$B(z,t) = -B \cos(k_{\parallel}z - \omega t), \quad (2b)$$

where  $k_{\parallel}$  and  $\omega$  are wavenumber and angular frequency of fast waves, and the mode-mode coupling is not considered here. Coordinates  $(z, v_{\parallel}, t)$  is Galilei transformed into ones  $(\xi, \eta, \tau)$ , moving with phase velocity

of  $\omega/k_{\parallel}$ . Using the ratio of electromagnetic field energy and kinetic energy of electrons,

$$\kappa^2 = \frac{(2/k_{\parallel}) |eE + k_{\parallel} \mu B|}{m \eta^2 / 2 + (1/k_{\parallel}) |eE + k_{\parallel} \mu B|} , \quad (3)$$

electrons are divided into trapped ( $\kappa^2 > 1$ ) and untrapped ones ( $\kappa^2 < 1$ ). Electron's velocity and position can be obtained from Eq. (1) by using Jacobi's elliptic functions and their Fourier series,<sup>4)</sup>

$$\begin{aligned} \eta(\tau) &= \frac{2\omega_b}{k_{\parallel} \kappa} d_n \left( \frac{\omega_b}{\kappa} \tau + F_0 \right) \\ &= \sum_{n=0}^{\infty} D_n \cos(\omega_n \tau + \phi_n) , \quad \text{for } \kappa^2 < 1 , \end{aligned} \quad (4a)$$

$$\xi(\tau) = D_0 \tau + \sum_{n=1}^{\infty} d_n \sin(\omega_n \tau + \phi_n) , \quad \text{for } \kappa^2 < 1 , \quad (4b)$$

$$\begin{aligned} \eta(\tau) &= \frac{2\omega_b}{k_{\parallel} \kappa} \text{cn}(\omega_b \tau + \kappa F_0) \\ &= \sum_{n=0}^{\infty} C_n \cos(\omega_n' \tau + \phi_n') , \quad \text{for } \kappa^2 > 1 , \end{aligned} \quad (4c)$$

$$\xi(\tau) = \sum_{n=0}^{\infty} c_n \sin(\omega_n' \tau + \phi_n') , \quad \text{for } \kappa^2 > 1 , \quad (4d)$$

where

$$D_0 = \frac{\pi \omega_b}{k_{\parallel} \kappa K_1} , \quad D_n = \frac{4\pi \omega_b}{k_{\parallel} \kappa K_1} \frac{q_1^n}{1 + q_1^{2n}} \quad (n \geq 1) , \quad (5a)$$

$$d_n = \frac{4}{nk_{\parallel}} \frac{q_1^n}{1 + q_1^{2n}} \quad (n \geq 1) , \quad (5b)$$

$$\omega_n = \frac{2\pi n \omega_b}{2K_1 \kappa} , \quad \phi_n = \frac{n\pi}{K_1 \kappa} F_0 , \quad (5c)$$

$$C_n = \frac{4\pi \omega_b}{k_{\parallel} K_2} \frac{q_2^{n+\frac{1}{2}}}{1 + q_2^{2n-1}} , \quad c_n = \frac{8}{(2n+1)k_{\parallel}} \frac{q_2^{n+\frac{1}{2}}}{1 + q_2^{2n-1}} , \quad (5d)$$

$$\omega_n' = \frac{(2n+1)\pi\omega_b}{2K_2}, \quad \phi_n' = \frac{(2n+1)\pi}{2K_2} F_0', \quad (5e)$$

$$\omega_b = \left[ \frac{k_{\parallel}}{m} (eE + k_{\parallel}\mu B) \right]^{1/2}, \quad (5f)$$

$$K_1 = F(\pi/2, \kappa), \quad K_1' = F(\pi/2, \sqrt{1-\kappa^2}), \quad (6a)$$

$$K_2 = F(\pi/2, 1/\kappa), \quad K_2' = F(\pi/2, \sqrt{1-1/\kappa^2}) \quad (6b)$$

$$q_i = \exp(-\pi K_1'/K_1), \quad (i=1,2), \quad (6c)$$

$$F(\phi, \kappa) = \int_0^{\phi} \frac{d\theta}{\sqrt{1-\kappa^2 \sin^2 \theta}}, \quad (\text{elliptic integral of first kind}) \quad (6d)$$

$$F_0 = F(k_{\parallel} \xi_0/2, \kappa), \quad (6e)$$

and  $\xi_0$  is the initial position of electrons.

Figure 1 shows the schematic drawing of electron trapping. In the conventional quasi-linear theory, it is approximated that electrons do straight-line motion and interact with wave of phase velocity  $\omega/k_{\parallel}$ . However, the region of electrons interacting with the wave is spread out as shown in hatched region of Fig. 1, by taking into account the electron trapping. So the electrons can interact with the wave in their velocities spread around the center of the phase velocity. In other words, the magnitude of discreteness of RF spectrum can be mitigated by the velocity spread of the electrons.

According to the quasi-linear theory, diffusion coefficients of the Fokker-Planck equation can be obtained by using the orbits of untrapped electrons ( $\kappa^2 < 1$ ),

$$D = \left(\frac{e}{m}\right)^2 \int_{-\infty}^{\infty} dk_{\parallel} |E(k_{\parallel}, \omega) + (i/e)k_{\parallel}\mu B(k_{\parallel}, \omega)|^2 \\ \times \sum_{n=1}^{\infty} \sum_{\ell=-\infty}^{\infty} \frac{\omega_i J_{\ell}^2(k_{\parallel} d_n)}{\{\omega - \ell \omega_n - k_{\parallel}(v_{\parallel} + D_0)\}^2 + \omega_i^2}, \quad (7)$$

where  $D$  is averaged with the initial position of the untrapped electrons  $\phi_n$  and  $\omega_i$  is the imaginary part of  $\omega$ . The diffusion coefficient for trapped electrons ( $\kappa^2 > 1$ ) can be also obtained as



$$D = \left(\frac{e}{m}\right)^2 \int_{-\infty}^{\infty} dk_{\parallel} |E(k_{\parallel}, \omega) + (i/e)k_{\parallel} \mu B(k_{\parallel}, \omega)|^2$$

$$\times \prod_{n=0}^{\infty} \sum_{\ell=-\infty}^{\infty} \frac{\omega_i J_{\ell}^2(k_{\parallel} c_n)}{(\omega - \ell \omega_n' - k_{\parallel} v_{\parallel})^2 + \omega_i^2} \quad (8)$$

Consider the simple case in the lowest order of  $n$  and  $\ell$ . Resonance conditions for untrapped electrons are  $\omega - (\ell + 1)\omega_1 = k_{\parallel} v_{\parallel}$  for  $n = 1$  and  $\ell = 0, \pm 1$ . Untrapped electrons interact with the waves in the shifted velocities. Similarly, resonance conditions for trapped electrons are  $\omega - \ell \omega_0' = k_{\parallel} v_{\parallel}$  for  $n = 0$  and  $\ell = 0, \pm 1$ , compared with the condition of  $\omega = k_{\parallel} v_{\parallel}$  which is obtained by using the straight-line motion. So it can be seen that chance for the electrons interacting with waves increases by taking into account the electron trapping effect.

### 3. Effect of the discrete RF spectrum on fast wave current drive

We study the effect of the discrete RF spectrum for tokamak reactor such as FER whose parameters are major radius  $R = 4.23$  m, minor radius  $a = 1.14$  m, toroidal field  $B_t = 4.2$  T,  $n = 7 \times 10^{19} \text{ m}^{-3}$ , temperature  $T = 14$  keV, frequency of fast wave  $f = 160$  MHz and its power  $P_d = 12$  MW.<sup>5)</sup> The electromagnetic fields of  $E(z, t) < 10$  V/m and  $B(z, t) \leq 10^{-4}$  T are calculated by the wave equation.

Effect of discrete RF spectrum on the fast wave current drive is simulated qualitatively by using two-dimensional Fokker-Planck code.<sup>6)</sup> Discrete RF spectrum used here is shown in Fig. 2. We take following typical parameters. Spectrum width is chosen as  $w_1 = 3$  and  $w_2 = 5$ , where  $w$  is a normalized velocity by the electron thermal velocity  $v_t$ , and  $w_1$  and  $w_2$  are lower and upper limit of RF spectrum. The spectrum width between the teeth of comb is selected to be  $w_3 = 0.2$ , taking into account the mesh size of the model. Diffusion coefficient of  $D = 10^{-3}$  is adopted under the above magnitudes of the electromagnetic fields.

Figure 3 shows the electron velocity distribution function during RF current drive. Calculation is done in the case that diffusion coefficient exists in  $3 \leq w \leq 5$  and is continuous. The electron velocity distribution function increases in the region of  $3 \leq w \leq 5$  compared with

the region of  $w < 0$ . This increment is varied by the discreteness of RF spectrum. Figure 4 shows the effect of discrete RF spectrum on the electron velocity distribution function. It shows three typical cases, (a) continuous spectrum  $\Delta w = 0$ , (b)  $\Delta w = 2$  and (c)  $\Delta w = \infty$ . For the case (a), electrons are accelerated continuously in the region of  $3 \leq w \leq 5$  and the electron velocity distribution function becomes larger than one for other cases. Since electrons are accelerated by the RF spectrum near  $w = 3$  but are not accelerated by the one between  $3 < w < 5$  for the case (b), the electron velocity distribution function is decreased though its decrement is smaller than the one of the Maxwellian. Electrons are again accelerated by the RF spectrum near  $w = 5$  but the electron velocity distribution function becomes smaller than the one of case (a) since the number of electrons is smaller near  $w = 5$ . Since electrons are accelerated only by the RF spectrum near  $w = 3$  for the case (c), the electron velocity distribution function decreases. The difference of the electron velocity distribution function between cases (b) and (c) is smaller than the one between cases (a) and (b) since the number of electrons have little collision near  $w = 5$ . Figure 5 shows the current drive efficiency  $\eta = \hat{j}_{RF} / \hat{P}_d$  on the width  $\Delta w$ , where the normalized driven current and normalized dissipated power are written as,

$$\hat{j}_{RF} = (en/e n v_t) \int_{-\infty}^{\infty} v_{\parallel} f dV , \quad (9a)$$

$$\hat{P}_d = (m n v v_t^2)^{-1} \int_{-\infty}^{\infty} (m v_{\parallel}^2 / 2) \frac{\partial}{\partial v_{\parallel}} D \frac{\partial}{\partial v_{\parallel}} f dV , \quad (9b)$$

where  $n$  and  $v$  are plasma density and collision frequency.<sup>3)</sup> As  $\Delta w$  increases,  $\hat{j}_{RF}$  and  $\hat{P}_d$  decreases but the current drive efficiency keeps constant. The reason will be described below.

The number of electrons accelerated by the RF spectrum with the teeth of comb around  $w = 3$  is decreased by the collision in the range of  $w \geq 3$ . However the decrement of the number is suppressed if these electrons are accelerated again by next teeth of comb of the RF spectrum. For a smaller  $\Delta w$ ,  $\hat{j}_{RF}$  and  $\hat{P}_d$  have sharp decrease. This dependency is made more clear in smaller velocities such as  $2 \leq w \leq 4$ , where  $v$  becomes larger. For a larger  $\Delta w$ , the number of electrons accelerated by the first teeth of comb of the RF spectrum is decreased by the collision

but the number keeps a certain level, which is larger than that without RF waves, due to  $v \propto w^{-3}$ . Therefore  $\hat{j}_{RF}$  and  $\hat{P}_d$  are saturated in certain levels. The reason why  $\eta$  does not have a large change is that  $\eta$  is the ratio of  $\hat{j}_{RF}$  and  $\hat{P}_d$  and that they have almost same change proportional to the electron velocity distribution function  $f$ .

Using the toroidal mode number of the fast wave  $N$ , normalized phase velocity is given as  $w = \omega R / N v_t$ . The discreteness of the fast wave is derived from the integer  $N$ . The spectrum width is given by  $\Delta w = 0.1 - 0.3$  for the number  $N$  which satisfies  $3 \leq w \leq 5$  in the FER. In the case of neglecting the electron trapping effect,  $\hat{j}_{RF}$  and  $\hat{P}_d$  decrease up to  $0.6 \sim 0.8$  of  $\hat{j}_{RF}$  and  $\hat{P}_d$  which are obtained for the continuous spectrum  $\Delta w = 0$  in Fig. 5.

Next consider the electron trapping effects. For deeply trapped electrons with  $\kappa = 2$ , we get  $k_{\parallel} c_n \ll 1$  and  $J_{\ell}(k_{\parallel} c_n) = 0$  ( $\ell \neq 0$ ) in the electromagnetic fields of the FER plasma. Therefore we do not expect the relaxation of the discrete RF spectrum due to the trapped electrons. Determining  $\kappa$  for the untrapped electrons, we should select  $\eta/v_t = \Delta w$  ( $\equiv \omega R / N^2 v_t$ ) since a single wave affects the electrons within the width  $\Delta w$ , i.e., up to the next wave's phase velocity. In the case we obtain  $q_1 \approx 0$ ,  $k_{\parallel} d_n \approx 0$  and  $J_{\ell}(k_{\parallel} d_n) = 0$  ( $\ell \neq 0$ ). So the resonance condition is reduced to be  $\omega - \omega_1 = k_{\parallel} v_{\parallel}$ . Using the toroidal mode number which satisfies  $3 \leq w \leq 5$  and calculating  $\omega_1$ , effective range of normalized velocity turns to be  $2.9 \leq w \leq 4.7$  due to the Doppler shift of trapping effect. Therefore  $\hat{j}_{RF}$  and  $\hat{P}_d$  can be increased up to about two times larger than the magnitude of them in the spectrum width of  $\Delta w = 0.1 - 0.3$  from Fig. 5.

#### 4. Conclusion

In conclusion, a mechanism taken into account the electron trapping by discrete RF spectrum is proposed in which electrons are allowed outside the resonance region to interact with the fast wave. This effectively spreads out the wave energy over a wider range of velocity space. It causes sustaining more driven current. This effect can bring larger driven current in larger RF power. This mechanism can also be expected for lower hybrid wave current drive. Toroidal effect on ray trajectories may also lead to the broadening of fast wave spectrum. The variation of velocity is obtained from  $v_{\parallel} = \omega R/N$ , such as  $(\Delta v_{\parallel}/v_{\parallel})_N = \epsilon \sin\theta \cdot \Delta\theta$  ( $\epsilon$  is inverse aspect ratio). The variation of velocity is also obtained from particle energy conservation, such as  $(\Delta v_{\parallel}/v_{\parallel})_B = (\mu B/m v_{\parallel}^2) \epsilon \sin\theta \cdot \Delta\theta$ . These ratio may be about unity and hence toroidal effect can not be expected to the broadening of fast wave spectrum. However the effect remains to be fully examined.

#### Acknowledgement

We would like to acknowledge Drs. K. Tomabechi, M. Yoshikawa, S. Tamura and T. Tone for their encouragements. Two of authors (T.O and K.Y) also would like to acknowledge Drs. S. Yamada, N. Ozaki and M. Ohtsuka for their encouragements.

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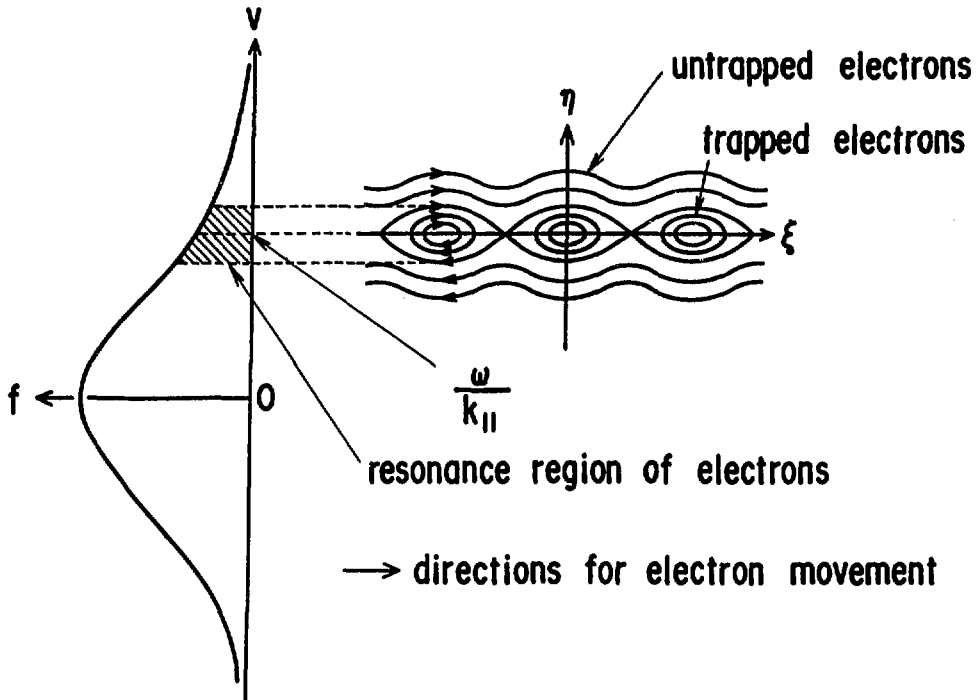


Fig. 1 Schematic drawing of electron trapping

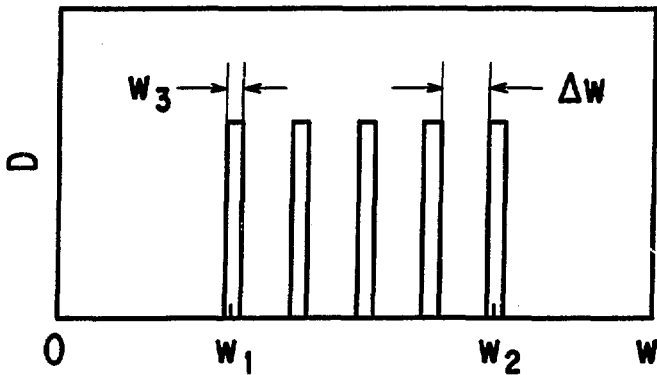


Fig. 2 Modelled discrete RF spectrum

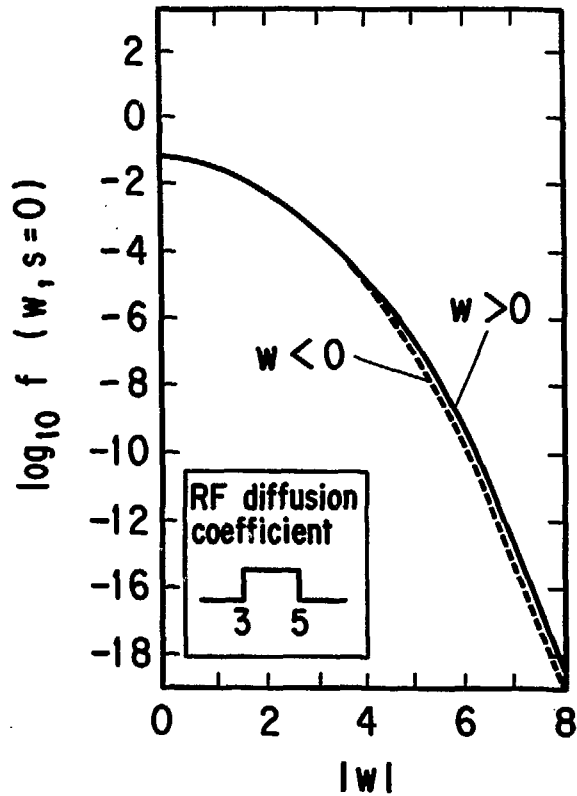


Fig. 3 Electron velocity distribution function during RF current drive

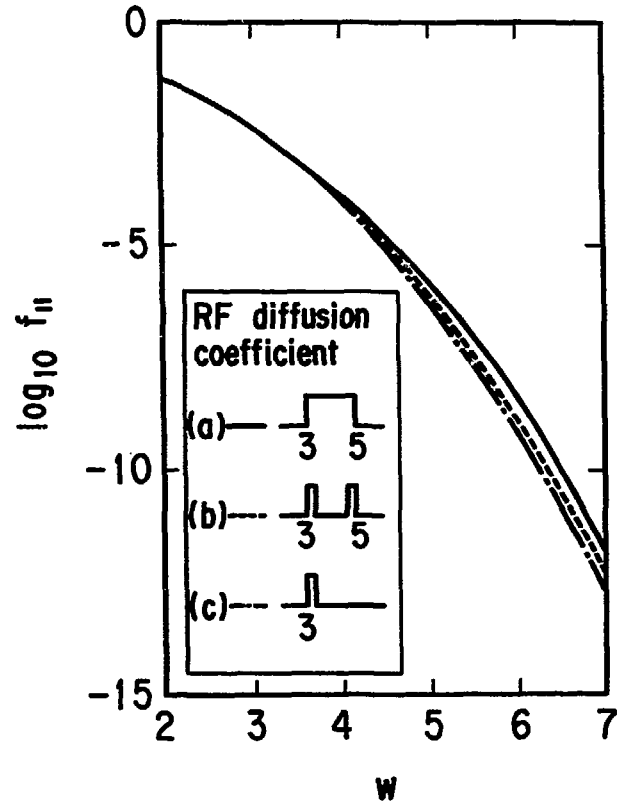


Fig. 4 Effect of discrete RF spectrum on the electron velocity distribution function.

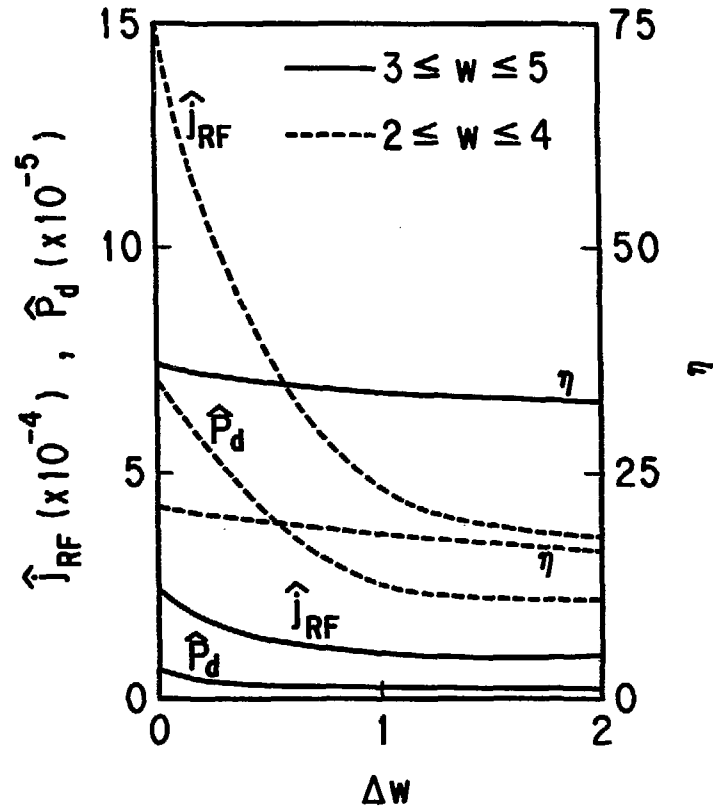


Fig. 5 Dependence of driven current, dissipated power and current drive efficiency on the spectrum width  $\Delta w$