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FROISSART TYPE RISE
OF CROSS SECTIONS
AND PREDICTIONS FOR SPECTRA
AND MULTIPlicITIES OF HADRONs
AT FUTURE ACCELERATORS
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Experimental data of ISR and SPS colliders on $\sigma_{\text{tot}}$ and on the diffraction cone slope $\left[ B(\alpha) \right]_{t=0}$ are used for a more precise determination of parameters $\Delta = \alpha_f(\alpha) - 1$ and others of the supercritical Pomeron. With account of all $P^h$ rescatterings it leads to the Froissart type rise of cross sections at high energy

$$\sigma_{\text{tot}} = \left( \frac{8\pi}{C} \right) \left( \frac{\alpha_f + P^2}{(\Delta - 1)} \right) \approx \left( \frac{8\pi}{C} \right) \left( \alpha_f' \Delta \right) x^2,$$

where $\xi = \ln \frac{3}{m^2}$, $\bar{\xi} = \ln \left( \xi + P^2/4t \right) + \text{const}$, $P^2$ is the square of the Pomeron residue radius and $C = 1 + 5^{\frac{1}{4}} \pi \alpha_x \approx 4.5$ for pp interaction. The same formula gives the value of $\left( \frac{8\pi}{C} \right) \left[ B(\alpha) \right]_{t=0}$ after substitution $A = 3^{\frac{1}{2}} A$. These equations determine fairly accurate $\sigma_{\text{tot}}$ and $B_{pp}(\alpha)$ already, at SPS collider energies and yield predictions for their values at energies of future accelerators $\sqrt{s} \sim 1-10^2$ TeV. The quark-gluon string model of Pomeron, describing existing experimental data, leads to predictions for super high energies of spectra of hadrons produced with small $p_t$, in particular, the values of $\left( \frac{dN}{dy} \right)_{y=0}$, hadron average multiplicities $\langle N \rangle = \langle N_x(\xi) \rangle$ and even their distributions over multiplicity $W(N) = S_N/\sigma_{\text{int}}$. The results are presented as curves and tables for energies $\sqrt{s} = 0.9, 2, 4, 10, 20, 40, 10^2, 10^3$ TeV.

Fig. - 6, ref. - 14.
Quark-gluon model of supercritical Pomeron (with $\Delta = \alpha_P(0) - 1 \geq 0.1$) describing experimental values of $d\sigma_{el}/dt$ and $\sigma_{tot}$ reproduces simultaneously the complicated picture of the multiple production of hadrons with small $p_t$ at high energy $\sqrt{s} \gg m_H$ — their energy spectra, multiplicities, the distribution over multiplicities and even over $p_t$.

Pomerons's parameters: $\Delta = \alpha_F(0) - 1$, $\alpha' = \alpha_F'(0)$, the residue $g_F = g_{F1} g_{F2}$ and the residue radius squared $R_F^2$ were determined many years ago — in 1970 by the fit of the experimental values of $d\sigma_{el}/dt$ and $\sigma_{tot}$ and were found to be (for pp, pp interaction) $\Delta = 0.07 \pm 0.02$, $\alpha' = 0.25 \pm 0.03$, $g_F = 3.60 \pm 0.80$, $R_F^2 = 3.40 \pm 0.30$, where the latter three quantities are given in units of $(\text{GeV}/c)^{-2} = 0.389 \text{ mb}$.

Later measurements have given slightly larger values for $\sigma_{tot}$ and $\sigma_{tot}$ (on an average by $\sim 1 \text{ mb}$) at the highest ISR energies $\sqrt{s} = 53$ and 63 GeV than were used in previous fits for Pomeron parameter determination. This small shift of the experimental value of $\sigma_{tot}$ was
found to have a significant effect on Pomeron parameters especially on $\Delta$ and on the value of $\gamma_p^*$ strongly correlated with it (the author is grateful to A.B.Erlykin, who attracted his attention to this sensibility of $\Delta$ to the ISR data on $\sigma_{\text{tot}}^\text{tot}$). Therefore the new fit of the data on $\sigma_{\text{tot}}^\text{tot}$ and on the diffraction cone slope $[B(s)]_{s_0}$ was done with the account of both the corrected ISR $\sigma_{\text{tot}}^\text{tot}$ values and the new (UA4 /12/) SPS-collider data at $\sqrt{s} = 540$ GeV. As a result a much larger $\Delta$ and a bit smaller $\alpha_p^*$, $\gamma_p^*$ and $\rho_p^*$ were obtained:

$$\Delta = 0.12 \pm 0.01, \quad \alpha_p^* = 0.22 \pm 0.02, \quad \gamma_p^* = 2.14 \pm 0.03,$$
$$\rho_p^* = 3.30 \pm 0.02$$

(1)

Below these corrected parameters' values are used and the predictions of the considered model are given for the cross sections, for diffraction cone slopes, for spectra and multiplicities of produced hadrons in the region $\sqrt{s} \sim 1 - 10^2$ TeV.

1) The Froissart type rise of $\sigma_{\text{tot}}^\text{tot}(s)$ and slopes $B(s) = \left[ \frac{\ln (d\sigma/dq^*)}{dq^*_2} \right]_{q_*=0}$ follows from the elastic scattering amplitude $M \propto M_p$ of the form

$$M_p = (i-tg(J\Delta \rho))(x/L) \sum_{n=1}^{\infty} \frac{(-x_0)^n}{n!} \exp(-xq^2/L)$$

(2)

*) The value $\sigma_{\text{tot}}^\text{tot} = 66.5 \pm 2.5$ mb, obtained at $\sqrt{s} = 900$ GeV in the new ran of the SPS-collider, was not taken into account. However, it was found to correspond exactly to the theoretical curve in Fig. 1a.
which comes contributions from exchange of arbitrary number $n = 1, 2, \ldots$ of Pomeron, here

$$
u_0 = \left( \frac{C_0 \mu}{\lambda_0} \right) \exp \nu_0$$

is the rescattering parameter, $\nu = \lambda_0 \mu / \lambda_0$, $\exp(\nu_0) = (C_0 \mu)^\Delta$, $\nu = \nu_0^2 + \nu_0^2$, $\nu_0 = 1 - (\Delta / 2)$.

Equation (1) is written in the quasicontinuum approximation, which does not include the contribution of the so-called "enhanced" Pomeron graphs associated with the interaction between Pomeron; they are not important due to the smallness of the Pomeron-Pomeron vertices (see $\nu^2$). In this approximation the diffraction production of jets of hadrons is taken into account by introduction of the constant factor

$$C = 1 + e^{\text{diff} / \nu_0^2}$$

in the rescattering parameter $\nu_0$, where according to the experimental data $C = 1 + 0.5$ for pp collision case, or $C = 1 + 0.6$ for $\pi N$ case, or $C = 1 + 0.7$ for $K N$ case.

Since

$$\nu_0 = \frac{2 \pi}{\sqrt{2} (M_{\mu^+ \mu^-})} \quad \text{and} \quad B = \Re \left[ \frac{d \ln M^2}{dq^2} \right]_{q^2 = 0}$$

then it follows from (2) at high energy, where $\frac{M}{\sqrt{s}} \ll \Delta$ (i.e., at $\nu_0 \gg 1$):

$$\int e^{\nu_0} (\phi / C)(\alpha^2 + R^2) \Psi(\nu_0) = (8\pi / C)(\alpha^2 + R^2) \left( \frac{s}{2s} \Delta - \nu_0 \right)$$

$$[B(s)]_{\Delta = 0} = \frac{2 (\alpha^2 + R^2)}{\sqrt{2} (M_{\mu^+ \mu^-})} \Psi(\nu_0) / (\phi / C) \approx (\alpha^2 + R^2) \left[ \frac{s}{2s} \Delta - \nu_0 \right]$$

Here

$$\Psi(\nu_0) = \sum_{n=1}^{\infty} \frac{(-\nu_0)^n}{n!} = \nu_0 (1 - e^{-\nu_0}) \frac{d\nu_0}{\nu_0} = \ln(\nu_0) + \mathcal{O}(e^{-\nu_0})$$

is the function determining, according to Eq. (2), the value
of \( M_P \) at \( q_2 = 0 \) and \( e = \exp \frac{\gamma}{c} = 1.781 \) is the Euler constant. The equality \( \ln(v_c) = \frac{3}{2} - l \) was used in Eq. (3) with

\[
l = \ln\left(\frac{(\alpha_0^2 + R^2)}{a_C f_c}\right)
\]

where the logarithmic part of \( l \) rises almost linearly with \( J \) when \( J \) is not too large:

\[
l = \frac{(\delta f - R^2)}{2R^2} - b_0, \quad b_0 = \ln\left(\frac{\alpha C f_c}{2R^2}\right).
\]

It follows also from Eq. (2) that

\[
\tilde{E}(u) = \frac{1}{n!} \sum_{n=0}^{\infty} \frac{(-u)^n}{n!} = \int_{0}^{\infty} \frac{d\ln \left(\mu\right)}{10} \approx \frac{1}{10} \left[ \ln^2 \left(\mu, u\right) + \frac{1}{2} + O(\mu^2) \right],
\]

which reproduces just the lower line of Eq. (3).

At high energies (\( J > \frac{1}{4}\Delta \)) both quantities \( \sigma^{\text{tot}}(s) \) and \( B(s) \) display the Froissart type increase \( \propto \frac{1}{s^2} \):

\[
\frac{\sigma^{\text{tot}}}{s^2} = F_0 + F_1/s + F_2/s^2,
\]

where \( F_0 = \left(\frac{\alpha x}{C}\right) \frac{\alpha_2 \Delta}{g} \) is almost a universal constant and \( F_1 = \left(\frac{\alpha x}{C}\right) \left( R_2^2 - \alpha_2 \Delta \right) \) and \( F_2 = -\left(\frac{\alpha x}{C}\right) R_2^2 \ell \) only weakly (logarithmically) depend on \( J = \ln \frac{1}{m_{\text{th}}^2} \) and, in general, depend through \( R_2^2 \) on the type of colliding particles.

In Fig. 1a and 1b there is plotted the dependence on the energy \( \sqrt{s} \) of \( \sigma^{\text{tot}} \), \( B \) and of

\[
\sigma^2 = \left(\frac{8\pi}{3}\right) \frac{1}{2} \left[ \phi(\mu) - \phi(2\mu)/2 \right] = \left(\frac{4\pi}{3}\right) \left( R_2^2 + \alpha_2 \right) \left( J \Delta - l - \ln 2 \right)
\]

for the values (1) of the parameters of Pomeron (corresponding to \( \min \gamma^2 \approx 0.9 (\text{mb})^2 \) per one experimental point.
In Figs. 1, 2. Besides Pomeron in Fig. 1a, 1b the contribution of Regge poles $\alpha = f, \omega$ is also included (together with all $\alpha P^n$ terms) for correct matching with the low energy region $\sqrt{s} \leq 3.4$ TeV. The account of them gives $M = M_p + M_f + M_\omega$ for $pp$ or $\bar{p}p$ scattering cases, where for $\alpha = f, \omega$

$$M_\alpha = \eta \lambda \rho \sum_{n=1}^{\infty} \frac{(-\lambda \eta)^{n-1}}{n! (n-1)!} \exp \left(-\frac{\lambda \rho q^2}{\rho_0}\right),$$

with $\eta = \varepsilon_0 \lambda_0 / \lambda$, $\lambda_0 = n + (\lambda \rho / 3)^{-1}$, $\varepsilon_0 = \varepsilon_0^f + \varepsilon_0^\omega$, $\varepsilon_0 = (n \lambda / 3)^{\frac{1}{2}}$, $\lambda = \eta \lambda_0$.

While for $\alpha = f$, $\eta f = i - \frac{t_0 \rho^0}{\lambda^2}$, or for $\alpha = \omega$, $\eta \omega = i + \frac{t_0 \rho_0^0}{\lambda^2}$.

The best values of parameters $\lambda$, $\lambda_0$, $\rho_0^0 = g_0 \lambda_0$, $\rho_0^0$ are given in Table 1. Both amplitudes $M_p$, $M_f$ decrease as $1/\sqrt{s}$ at high energy and their contributions are important only at $\sqrt{s} \leq 100$ GeV.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\lambda_{\alpha}$ (GeV/-2)</th>
<th>$\rho_{\alpha}$ (GeV/-2)</th>
<th>$\rho_{0}^0$ (GeV/-2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f$</td>
<td>0.27 ± 0.02</td>
<td>0.70 ± 0.10</td>
<td>7.8 ± 0.5</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.54 ± 0.02</td>
<td>1.00 ± 0.10</td>
<td>4.5 ± 0.25</td>
</tr>
</tbody>
</table>

Values of these parameters also slightly differ from those obtained previously $^{12}$. Even at the largest ISR energy $\sqrt{s} = 63$ GeV the contribution of $f$, $\omega$ poles yields corrections to $\sigma_{pp}^{tot}$ of order of 1 mb (at $\sqrt{s} \sim 10$ GeV it is of order of tens mb), therefore without them it is impossible...
to obtain any exact estimation of Pomeron parameters.

The dependence (4) of $\sigma_{\text{tot}}/s^2$ on $\sqrt{s}$ is represented in Fig. 1b; as it shows this quantity fast approaches the constant Froissart limiting value $F_o = (\pi/\alpha)\sqrt{s}$ as $\sqrt{s}$ increases which is $\sim 0.20$ mb for $C = 1.5$ and for values (1) of Pomeron's parameters.

In general Eq. (4) can be used (with the values of $F_o$, $F_1$, $F_2$ given above) to describe the observed $\sigma_{\text{tot}}$ behaviour in $pp - \bar{p}p$ and in other $\pi\pi$, $\pi\pi$ etc. channels.

At large energies the cross section $\sigma_{\text{tot}}^{pp} = \sigma_{\text{tot}}^{pp}$ rises fast according to Fig. 1a, almost proportionally to $s^2 = (\ln s)^2$ and with the values (1) of the parameters passes close to the cosmic ray values of $\sigma_{\text{tot}}^{pp}$ measured (with large errors) at $\sqrt{s} \sim 10^3$-105 GeV; they are shown in Fig. 1a,b by rectangles and by the dark circle; the open circle represents the value of $\sigma_{\text{tot}}^{pp}$ obtained on the installation "fly eye" also in cosmic rays.

At $\sqrt{s} = 0.54$, 0.90, 2, 4, 10, 20, 40, 102 and 103 TeV the cross section $\sigma_{\text{tot}}^{pp}$ must have the values given in Table 2; their accuracy is of order of 5%. The same is true for the values of $\sigma_{\text{el}}^{pp}$ and $B = [B(\text{el})]$ which are also presented in Table 2.

Table 2.

<table>
<thead>
<tr>
<th>$\sqrt{s}$ TeV</th>
<th>0.54</th>
<th>0.90</th>
<th>2</th>
<th>4</th>
<th>10</th>
<th>20</th>
<th>40</th>
<th>100</th>
<th>103</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{\text{tot}}^{pp}$ mb</td>
<td>61.0</td>
<td>66.5</td>
<td>75.0</td>
<td>85</td>
<td>95.7</td>
<td>105</td>
<td>116</td>
<td>129</td>
<td>169</td>
</tr>
<tr>
<td>$\sigma_{\text{el}}^{pp}$ mb</td>
<td>12.5</td>
<td>13.5</td>
<td>16.8</td>
<td>18.2</td>
<td>22.5</td>
<td>25.4</td>
<td>18.5</td>
<td>33.5</td>
<td>47.2</td>
</tr>
<tr>
<td>$B_{pp}(\text{el})_{++}$</td>
<td>15.7</td>
<td>16.4</td>
<td>17.7</td>
<td>19.0</td>
<td>20.5</td>
<td>22.5</td>
<td>24.1</td>
<td>26.5</td>
<td>34.0</td>
</tr>
</tbody>
</table>
The elastic scattering cross section \( \sigma^{el} \) rises at large energies a bit faster than \( \sigma^{tot} \); it follows from (3) and (5) that the ratio

\[
R = \frac{\sigma^{el}}{\sigma^{tot}} \propto \left( \frac{1}{2C} \right) \left( 1 - \frac{\ln 2}{\xi_{\Delta}} \right)
\]

slightly increases up to its limiting value \( 1/2C = 1/3 \) which it takes at \( \xi_{\Delta} \gg 1 \). Fig. 1a and Table 2 show that \( R \) is yet small for ISR or SPS collider energies, here it is close to \( 1/6 \) or to \( 1/5 \) correspondingly. However, at \( \sqrt{s} \approx 10 \text{TeV} \) \( R \) becomes equal to \( 1/(4.46) \) and at \( \sqrt{s} \approx 100 \text{TeV} \) close to \( 1/(3.8) \).

The lower line of Eq. (3) (see its right hand side) shows that the diffraction cone slope \( B_{pp} \) begins to increase about linearly with increasing of \( \xi = \ln \frac{\sqrt{s}}{M^2} \) as \( \xi_{\Delta} \ll 2 \) does not exceed \( \pi^2 / 6 \) (i.e. up to \( \sqrt{s} \approx 1 \text{TeV} \)) see Fig. 2. In this region the expression in the last brackets in the last line of Eqs. (3) rest close to a constant. Only at much larger energies \( \sqrt{s} \gg 1 \text{TeV} \) the slope \( B_{pp} \) begins to use \( \xi_{\Delta}^2 \).

Simple expressions of the "quasi-sieikonal" approximation in the right-hand parts of Eq. (3)-(5) reproduce the observable values of \( \sigma^{tot}_{pp} \) and \( B_{pp} \) within the accuracy of order of 1% in the region \( \sqrt{s} > 0.1 \text{TeV} \).

However, at super high energies \( \sqrt{s} > 10^5 \) this approximation becomes nonvalid, as here \( C = 1 + \frac{\sigma^{\text{diff}}}{\sigma^{el}} \to 1 \) at \( \xi = \ln \frac{\sqrt{s}}{M^2} \to \infty \). A bit more correct approach leads to the same form (4) for \( \sigma^{tot}_{pp} \) at \( \xi \gg 1 \), but with larger \( R_0 = 2/M^2 \), \( A^2 = 4\xi_{\Delta} \). Moreover, the account of the threshold singularity \( 1/2! \) of the trajectory \( \alpha_F (t) \).
of the form \( n(\ell - 4m_{\pi}^2)^y \), \( y \geq \frac{3}{2} \) and of the interactions between Pomeronem, slightly changes (increases) the value of \( a_0 \), leading to \( a_0 \rightarrow a = \frac{1}{2}m_{\pi} \). The region of the super high energies will be considered separately.

2) Spectra of produced hadrons \( d\sigma^{incl}/dy \) and their average multiplicities are determined in the quark–gluon model of the Pomeron \(^6\) by equations

\[
\frac{d\sigma^{incl}}{dy} = \sum_n \tilde{\sigma}_n(\xi) \phi_n(\xi, y),
\]

\[
\bar{N}(\xi) = \sum_n \left( \frac{\tilde{\sigma}_n}{\tilde{\sigma}_{n+1}} \right) \bar{N}_n(\xi), \quad \bar{N}_n = \int \phi_n(\xi, y) dy,
\]

where \( y = \ln \left( \frac{(p_T^2 + m_1^2)^{1/2}}{\ell} \right) \) is the rapidity of the produced hadron (in c.m. system), \( m_1 = \sqrt{p_T^2 + m_1^2} \) - its transversal mass and \(^7\)

\[
\tilde{\sigma}_n(\xi) = \left( 4\pi/\mathcal{N}_n \right) x_p \left[ 1 - e^{-2\nu_0} \sum_{n=1}^{\infty} \frac{\nu_0}{n!} (x_p)^n \right]
\]

is the cross sections of production of \( n \)-Pomeron hadron showers corresponding to the cut of \( n \) Pomeron in the Pomeron graph for elastic scattering amplitude (at \( n=0 \), \( \tilde{\sigma}_0 = \tilde{\sigma}^{el} + \tilde{\sigma}^{dif} \) is associated with diffraction hadron production; note that \( \tilde{\sigma}^{dif} = \tilde{\sigma}_0 + \sum_{n \geq 1} \tilde{\sigma}_n \), \( \tilde{\sigma}^{el} = \tilde{\sigma}^{el} - \tilde{\sigma}^{dif} \)). Function \( \phi_n(\xi, y) = d\sigma^{incl}/dy \) determine the inclusive spectra of hadrons produced in these \( n \)-Pomeron showers and have the following form \(^6\)

\[
\phi_n(\xi, y) = 2\pi b_0 \cdot F(x_+ \xi) \cdot F(x_- \xi)
\]

where \( b_0 \approx \frac{1}{2} \) is a constant, \( x_\pm = \left( \frac{\sqrt{x_1^2 + x_4^2} \pm x}{2} \right) \), \( x = P_{\ell}/b_0 \), \( x_4 = m_1/p_0 \), \( p_0 = \sqrt{2} \), \( y_0 = \ln (\sqrt{\nu_0^2} + 1/2) \) - is the maximal value of
the rapidity \( y \) of the produced hadron, and

\[
F(x_\pm) = \int_{x_\pm} q(x_\pm) G(x_\pm, p_\pm^2) dx_\pm = (1-x_\pm)^{x-1} \int_{x_\pm} \frac{G(x', y', x^{-x_\pm}, y', 1-x_\pm)}{F(x', y', x^{-x_\pm}, y', 1-x_\pm)}
\]

is the convolution of the quark distribution in the nucleon

\[
q(x_\pm) = \frac{C}{x_\pm^{\alpha_Q}} (1-x_\pm)^{\beta-1}, \quad \beta = 1, \quad \alpha_Q = \alpha_R - 2\alpha_g, \quad \alpha_R = \alpha_W = \alpha_G = 1
\]

and of the fragmentation function \( G(x_\pm, p_\pm^2) \approx (1-x_\pm) x^{-x_\pm} \),

\[
\frac{d\phi}{dx_\pm} = 2\phi_0 p_\pm^2 \approx 0.4, \quad p_\pm^2 = 0.2 \left( \frac{E_0}{c} \right) .
\]

Here, \( \gamma = 2 - 2\alpha_\gamma + x'^2 \), and \( \phi_0 \) is determined by the condition

\[
\int q(x_\pm) dx_\pm = 1.
\]

At the edges of the spectrum i.e. at \( x_+ \to 1 \) (or at \( x_- \to 1 \)) \( F(x_\pm) \) takes the limiting form

\[
F(x_\pm) \approx (1-x_\pm)^{x-1},
\]

which corresponds to the three-

reggeon limit of hadron (\( \Xi \) -meson) spectra. However, the

main contribution to the integrals (6) for \( \vec{N}(\Xi) \) (with

\[ y_\Xi = y_0 - y_{\pm n} \) comes from the central part of spectra

\[ y \approx y_0 \], where the simple form \( F(x_\pm) \approx (1-x_\pm)^{x_\Xi/y_\Xi} \), \( x_\Xi = \sqrt{y_\Xi} \) is valid.

Thus the mean multiplicity \( \langle n \rangle = \frac{1}{N_\Xi} \int q(x_\pm) dx_\pm \) of each of \( n \)-Pomeron showers can be calcuated in the simple analytical form

\[ y_\Xi^2 \]

at high energy.

In general the form of \( q_n(\Xi, y) \) depends on the assumption

about energy distribution among \( n \)-Pomeron jets, in particular,

Eq. (7) corresponds to equal energy partition with \( \sqrt{y_\Xi} \) belonging to each jet. However final results only weakly depend

on the choice of this distributions (see \[ 14-6/ \) and thus we restrict ourselves to the simple form (7). Note that everywhere the mean value \( p_\pm^2 \) is substituted for \( p_\Xi^2 \).

The spectra of charged particles, resulting from (6) and

(7) at values (1) of Pomeron's parameters are shown in Fig. 3; the dashed curves in Fig. 3 indicate the same spectra as
functions of the quasirapidity \( \eta = \ln \tanh \frac{y}{2} \) which is approximately equal to the rapidity \( y \) at \( y > 1 \). Fig. 3 reveals fine correspondence of the theory with the experimental data of Batavia, ISR and SPS-collider and shows that heights of the central parts of spectra \( \frac{dN_c}{d\eta} = \frac{dN_c}{d\eta(y)} \) continue to increase as \( \sim \ln^2 \frac{\eta}{\sqrt{s}} \) at high energies \( \sqrt{s} > 1 - 10^2 \) TeV. The areas below these curves giving the average multiplicities of the produced charged particles \( \bar{N}_c(\eta) = \int_0^{\infty} d\eta \frac{dN_c}{d\eta} d\eta \) also rapidly rise as \( \sim \ln^3 \frac{\eta}{\sqrt{s}} \).

They are shown in Fig. 4. Table 3 represents the values of \( \frac{dN_c}{d\eta(y)} \) and of the average multiplicities \( \bar{N}_c(\eta) \) for SPS-collider and for future accelerator energies.

Table 3.

<table>
<thead>
<tr>
<th>( \sqrt{s} )</th>
<th>0.54</th>
<th>0.9</th>
<th>2</th>
<th>4</th>
<th>10</th>
<th>20</th>
<th>40</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{dN_c}{d\eta(y)} )</td>
<td>3.37</td>
<td>3.6</td>
<td>4.2</td>
<td>4.5</td>
<td>5.2</td>
<td>5.8</td>
<td>6.3</td>
<td>7.3</td>
</tr>
<tr>
<td>( \bar{N}_c(\eta) )</td>
<td>27.6</td>
<td>32.0</td>
<td>40.5</td>
<td>48</td>
<td>59.0</td>
<td>68.5</td>
<td>78</td>
<td>84</td>
</tr>
</tbody>
</table>

3) Multiplicity distributions of charged particles can be obtained in the model considered in Refs. /6-8/ taking into account the almost independent way of production of hadrons in each of 2n quark-gluon chains in Fig. 1 which follows from the observed small value of close correlations of different properties of produced hadrons. It means that distributions over multiplicities \( N \) in each quark-gluon chain, in one Pomeron jet, and also in all n-Pomeron jets are close to the Poisson ones, i.e. to \( P_N(N) = \left( \frac{\bar{N}^N}{N!} \right) e^{-\bar{N}} \).

The resulting distribution over multiplicity \( N = N_{\text{ch}} \) should
are the topological cross sections. These distributions are depicted in Fig. 5 for parameters (1) of Pomeron for high energies $\sqrt{s} = 0.54, 10, 100$ TeV in the KNO variable

$$W(N) = \frac{S_N}{\sigma_{kn}} , \quad \text{where}$$

$$S_N = \sum_{n=0}^{\infty} c_n(s) P_{n}(N) \quad (8)$$

In principle the dips in the distribution in Fig. 5 can be filled by contribution of the "enhanced" Pomerons graphs [14] which include vertices of the interaction of Pomerons. However as SPS collider data show, these contributions are evidently not essential.

Author expresses his graduate to A.B. Kaidalov for a number of discussions and to A.I. Veselov for numerical calculations.
Fig. 1a. $\sqrt{s}$ dependence of $\sigma^{\text{tot}}(s)$ and of $\sigma^{\text{el}}(s)$ for pp and $\bar{p}p$ scattering at the values (1) of the Pomeron parameters ($\Delta = 0.12$) and at the values of the $\rho$, $\omega$ - Regge pole parameters given in Table 1.
The dependence of $\frac{\sigma^{\text{tot}}}{\xi^2}$ on $\sqrt{V}$ given in Eq. (4).
Fig. 2. $\sqrt{s}$ dependence of the diffraction cone slope $\left[\frac{d\sigma}{d\Omega}\right]_t=0$ of the elastic $pp$ and $\bar{p}p$ scattering given by (3) - for the Pomeron contribution. In the region $\sqrt{s} < 0.2$ TeV small contribution of the $f^2$, $\omega$ poles is also taken into account.
Fig. 3. The dependence on the rapidity $y$ of the summary spectra $dN_\mathrm{ch}/dy = \sum_i (dN_i/\sinh(y))$ of the production of all charged particles (there are mainly $K$-mesons) in pp interaction. Different curves correspond to different energies $\sqrt{s}$ of the pp system. The dashed lines show the values of $dN_\mathrm{ch}/dy$, where $\eta = \ln(y/\sqrt{s})$ is the quasirapidity; the corresponding experimental data of ISR and SPS-collider are also shown by the circles and triangles.
Fig. 4. The $\sqrt{s}$ dependence of the total average multiplicity of the charged particles $\langle N_{ch} \rangle$ = $\int_{y_{min}}^{y_{max}}$ $(dN_{ch}/dy)_{pp}$ produced in pp scattering. The dashed curve gives the same quantity obtained without account of the diffractively produced particles (along it $\sum_i (dO_{incl}/d\Omega dy)$, where $\sigma_{incl}$ = $\sigma_{diff}$). Experimental data are shown by points and crosses.
Fig. 5. The distribution \( \psi(\xi) = \langle n_{\text{ch}} \rangle \frac{\text{dn}_{\text{ch}}}{\text{d} \ln \xi} \) of all produced charged particles in the multiplicity in the E90 variable \( \xi = \frac{n_{\text{ch}}}{\langle n_{\text{ch}} \rangle} \). The curves in two groups represent: 0 - E90 distribution, which is close to (8) for all energies \( \sqrt{s} < 50 \text{ GeV} \); 1 - the distribution (8) for SPS-collider energy \( \sqrt{s} = 0.54 \text{ GeV} \) and the corresponding UA4 experimental data. The curves 2, 3, 4 in the upper group correspond to energies 0.9, 2, and 4 TeV, curves 5 and 6 - to the energies 10 and 100 TeV of the future accelerators. They reveal clear maxima and minima strongly violating the E90 scaling.
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