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ABSTRACT . Three body calculations for studying the baryons are performed in a non-relativistic treatment with three quarks interacting via Bhaduri's potential. From the resulting wave functions, it is analyzed under which conditions can a diquark structure occurs Several photos showing quark distributions inside the baryons are presented and discussed in details.

I . INTRODUCTION

Hadron spectroscopy has made rapid and sensible progress since the advent of a theory for strong interactions : the quantum chromodynamics (QCD). Aside from the fundamental approach - the lattice gauge theory - which is still in its infancy for these kinds of problems, there are two main tracks to study hadron properties. The first one gathers the various bag models[1]; they have in common to incorporate covariant formulations, however they are dramatically plagued by a bad description of the center of mass motion. The second one gathers the so-called non-relativistic potential models [2] (NRPM); they are based on QCD inspired potentials and, in principle, allow to treat the center of mass motion in a correct way but lack relativistic invariance. Despite this last drawback, the NRPM encounter a lot of success in meson and baryon spectroscopy. However, they are not restricted to those systems but open the door to more exotic multiquark systems which are under active research presently. Of course their structure in terms of fundamental constituents is morecomplicated and must rely on some approximations. Among these, the cluster theory, which is commonly used in nuclear physics, is specially attractive and was introduced very early. In fact, the notion of diquark - bound state of two quarks - was suggested by Gell-Mann[3] himself in his first paper on quarks. In the last sixties, when the quark model was acquiring a firm existence, the concept of diquark was abundantly used to explain the baryon mass splitting in multiplets based on SU(3)[4], SU(6)[5] or SU(6) x O(3)[6] symmetries. But at that time, the few body problem was not solved very efficiently, the colour degree of freedom was unknown and consequently the dynamics was far to be treated correctly. During the ten following years, the quark model was based on firmer and firmer grounds, the understanding of the strong interactions made considerable progress and curiously models including diquark clusters were almost absent from litterature. However, since around 1977, there exists a renewed interest for cluster models; they were invoked in the description of simple systems like mesons[7] or baryons [8] but also for more complicated structures like dibaryon resonances [9]. The main advantage of cluster models, in a very similar way as in nuclear physics, is to simplify a complicated mathematical problem into a succession of much simpler ones. For example the three body problem for a baryon QQq is reduced to the two body problem of building up a diquark D = (QQ) and further to the two body problem of the interaction D-q between the diquark and the last quark. In the very same way, the four body problem for exotic multiquarks $QQ\bar{q}\bar{q}$ [10] can be reduced to a three body problem $(QQ)-\bar{q}-\bar{q}$ or even to a two body one $(QQ)(\bar{q}\bar{q})$.

Simplicity is the main but not the only argument in favor of models including diquarks. Schmidt and Blankenbecler[11] for instance claimed that the non-scaling properties of the structure functions (in inelastic high energy e^- scattering) of proton and neutron may be due to the interaction of the photon not only with a single quark (this mechanism leads to scaling) but also with a bound state of two quarks. The study of Regge trajectories in baryons with high angular momentum seems to favor a quark-diquark structure inside the baryons [12].

In all these studies done so far the diquarks were considered as entities by themselves whose existence is a fact a priori. In some sense, the system is forced to follow a dynamics which leads to a given structure or, in other words, the system is constrained to move in a specified Hilbert space. Wether or not the true potential favors such a structure is verified a posteriori by analyzing the predictions of the model. Moreover, the diquark being considered as an "elementary" particle, the Pauli principle between the last quark and a quark inside the diquark is generally not taken into account properly.

In this paper we focuse on the baryon internal structure from an other point of view. We start from a quark-quark potential realistic enough to describe correctly the meson properties and let it act freely - that is following the Schrödinger equation - in a baryon. The quarks inside the baryon move as Nature wants and the purpose of this paper is to take photos of the quark distributions, from a probabilistic point of view, once a proper three body calculation is performed.

The signature of a diquark will be the tendency for two quarks to remain close one to each other while the third one orbits at a greater distance.

In the next section we give some indications about the method by which the three body problem is solved and introduce the quantities best suited for making transparent the eventual diquark structure. In the third section, the resulting quark distribution photos are presented and discussed; finally in the last section our conclusions are drawn.

II . BARYON CALCULATIONS

In NRPM, baryons consist of three quarks interacting via a potential. This is of course an approximation since the gluon and meson (or quark-antiquark pairs) degrees of freedom have disappeared. Nevertheless, they are taken into account phenomenologically by renormalizing the quark masses which become the "constituent" masses and by letting free the parameters entering the potential. Generally these parameters are fitted on experimental data (energy levels, magnetic moments or mean square radii).

Let us begin by examining the various degrees of freedom for the problem. Each quark is characterized by a flavor, a colour, a spin and a space position. Concerning the flavor we do not suppose any symmetry for it; in this case flavor and mass are equivalent notions. In all what follows, the term mass must be understood as the constituent mass. In our study we shall consider baryons including u, d, s, c and b flavors with respective masses m_u = 336 Mev, m_d = 339 Mev, m_s = 600 Mev, m_c = 1870 Mev, m_b = 5259 Mev. Each quark comes in three colours : red (r), green(g) yellow (y). Each colour is a weight of the 3 irreductible representation (IR) for the $SU(3)_c$ colour symmetry. Coupling two quarks generates the 6 and $\bar{3}$ (complex IR of 3) representations : $3 \otimes 3 = 6 \oplus \bar{3}$. Coupling the third quark leads to $6 \otimes 3 = 10 \oplus 8$ and $\bar{3} \otimes 3 = 8 \oplus 1$ or

$$3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8 \oplus 1 \qquad (1)$$

One can analyze also these representations in terms of the S_3 group (group for permutations of 3 objects) : it appears that the decuplet 10 is symmetric, the octets 8 are of mixed symmetry and the singlet 1 is antisymmetric. However there is a physical principle which imposes to each observable system to be a colour singlet. Hence, the baryon colour wave function C(1,2,3) must be the antisymmetric 1 irreducible representation. More precisely it is the Slater determinant based on the three colours, namely

$$C = \frac{1}{\sqrt{6}} \left(rgy + gyr + yrg - ryg - ygr - gry\right) \qquad (2)$$

The singlet arises from the coupling of $\bar{3}$ and 3 and consequently in a baryon, two quarks - a diquark - must be in a $\bar{3}$ colour state, which has the same colour content as an antiquark. Similarly each quark (spin 1/2) has two spin components ↑ and ↓ which are the weights of the $2 = \bar{2}$ IR for a SU(2) ≅ O(3) group. Coupling three particles leads to $2 \otimes 2 \otimes 2 = 4 \oplus 2 \oplus 2$ or more conventionally in terms of spin $1/2 \otimes 1/2 \otimes 1/2 = 3/2 \oplus 1/2 \oplus 1/2$. The spin 3/2 wave function $\chi_1^{S=3/2} = |1/2\ (1/2\ \ 1/2)_1\rangle_{3/2}$ is symmetric under particle exchange. On the other hand, there exist two spin 1/2 wave functions which are of mixed symmetry; to distinguish between them we need an additionnal quantum number which is chosen conveniently as the partial coupling for particles 2 and 3. Hence we define

$$\chi_1^{S=1/2} = |1/2\ (1/2\ \ 1/2)_1\rangle_{1/2} \quad \text{symmetric under } 2-3 \text{ exchange}$$
$$\chi_0^{S=1/2} = |1/2\ (1/2\ \ 1/2)_0\rangle_{1/2} \quad \text{antisymmetric under } 2-3 \text{ exchange} \qquad (3)$$

Just as an illustration let us give for each function the component with projection 1/2

$$\chi_1^{S=3/2} \; : \; \frac{1}{\sqrt{3}} (\uparrow\uparrow\downarrow + \uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow)$$

$$\chi_1^{S=1/2} \; : \; \frac{1}{\sqrt{6}} (\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow - 2\uparrow\uparrow\downarrow) \tag{4}$$

$$\chi_D^{S=1/2} \; : \; \frac{1}{\sqrt{2}} (\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow)$$

Lastly each quark i is characterized by its position $\vec{r}_i$ in space. Since the total Hamiltonian is invariant under space translations, the center of mass energy $\frac{\vec{P}^2}{2M}$ $(\vec{P} = \vec{p}_1+\vec{p}_2+\vec{p}_3,\ M=m_1+m_2+m_3)$ decouples from the intrinsic energy and allows the factorization of a plane wave $\exp(i\vec{K}.\vec{R})$ $(\vec{R} = M^{-1}(\sum_i m_i \vec{r}_i)$: center of mass coordinate) in the total wave function. We are left with a Schrödinger equation concerning only intrinsic quantities. In that sense, we say that the center of mass motion is removed properly. It is traditionnal to introduce the relative momenta

$$\begin{aligned}\vec{p} &= \left[2\, m_2\, m_3\, (m_2+m_3)\right]^{-1/2} (m_3\, \vec{p}_2 - m_2\, \vec{p}_3)\\ \vec{q} &= \left[2\, m_1\, (m_2+m_3) M\right]^{-1/2} (m_1\, (\vec{p}_2+\vec{p}_3) - (m_2+m_3)\, \vec{p}_1)\end{aligned} \tag{5}$$

and the relative *Jacobi coordinates*

$$\begin{aligned}\vec{x} &= \left[2\, m_2 m_3/(m_2+m_3)\right]^{1/2} (\vec{r}_2-\vec{r}_3)\\ \vec{y} &= \left[2 m_1 (m_2+m_3)/M\right]^{1/2} \left[(m_2\, \vec{r}_2 + m_3\, \vec{r}_3)/(m_2+m_3) - \vec{r}_1\right]\end{aligned} \tag{6}$$

Let us come to the discussion of the potential to be used in realistic calculations. Starting from QCD, assuming one gluon exchange (perturbative QCD valid in the asymptotic freedom region when the quarks are close one to the other) and performing the Fermi-Breit non relativistic reduction, De Rujula et al [13] have shown that the potential is a sum of two-body terms V_{ij} depending on colour *through the scalar* $\vec{\lambda}_i\, \vec{\lambda}_j$ ($\lambda^{(a)}$ *is the (a) Gellmann matrix notation for the SU(3) generators*): the interquark potential contains a central coulombic term, a contact spin-spin force and also spin-orbit and tensor interactions. *For the non perturbative region* (large separation distances) little is known; the only thing one imposes, in a phenomenological way, is the confinement. To day, there are good arguments, based on lattice gauge theory, that the confining term (at last for q-$\bar{q}$ interactions) is linear. For systems more complicated than q-$\bar{q}$, the confining part is probably no longer a sum of two body forces. These ideas serve as a guide for the form of the resulting potential, but the values of the various parameters are determined by fitting experimental properties. In all our calculations we adopt the potential suggested by Bhaduri et al [4] which gives very good results in meson and baryon spectroscopy. It looks like

$$V = -\frac{16}{3} \sum_{i<j} \vec{\lambda}_i\, \vec{\lambda}_j \left[-\frac{\kappa}{r_{ij}} + \frac{r_{ij}}{a^2} - D + \frac{\hbar^2 c}{m_i\, m_j}\, \frac{\exp(-r_{ij}/r_o)}{r_o^2\, r_{ij}}\, \vec{\sigma}_i\, \vec{\sigma}_j \right] \tag{7}$$

with κ = 102.67 MeV.fm, $a = 0.0326(\mathrm{MeV}^{-1}\mathrm{fm})^{1/2}$, D = 913.5 MeV, r_o = 1/2 2 fm. Some remarks are in order

i) The spin-orbit and tensor forces are absent; this means that orbital and spin angular momenta are good quantum numbers separately; in fact several studies have shown that these types of interaction play a minor role in the baryons.

ii) The spin-spin term is not a pure contact $\delta(\vec{r}_{ij})$ one; this is to avoid a collapse of the system when the pair i-j is in a $\ell = 0$ angular momentum. The smeared short range term is necessary when the spin-spin force is not treated perturbatively.

iii) Additive potentials with a colour factor $\vec{\lambda}_i \vec{\lambda}_j$, like (7), give rise to long range Van der Waals forces between two colour singlets [15]. These forces are not seen experimentally and this argument is the most embarassing drawback against those types of potentials. However, in our present case, baryons do not result from the interaction between two singlets and hence Van der Waals forces are absent.

iv) The colour wave function C is an eigenstate of $\vec{\lambda}_i \vec{\lambda}_j$ with eigenvalue $-\frac{8}{3}$. Hence, from the dynamically point of view one can forget about colour if we replace $-\frac{16}{3}\vec{\lambda}_i \vec{\lambda}_j$ in (7) by the factor 1/2. However, the colour degree of freedom has another effect. If some particles in the baryon are identical, the total wave function must be antisymmetric under their mutual exchange; since the colour function is fully antisymmetric, the remaining part of the wave function (spin-space) must be symmetric. Once these two restrictions are taken into account, the colour degree of freedom can be forgotten. This will be done in the following.

Thus, we are left with the Schrödinger equation

$$\left[\frac{1}{2}(\vec{p}^{\,2} + \vec{q}^{\,2}) + \sum_{i<j} V_{ij}(\vec{x},\vec{y})\right] |(123)\rangle = E\ |(123)\rangle \qquad (8)$$

There exist several methods to solve the three body problem. The Faddeev formalism is certainly the most powerful and rigorous. However, for the purpose of the present paper, an expansion of the wave function in terms of an harmonic oscillator (H.O.) basis is more than enough [16]. The size parameter b of the H.O. functions is a free parameter which is determined in the "best" way; it will be chosen as the length unit. The H.O. functions are written with the Moshinsky's conventions namely:

$$\phi_{n\ell m}(\vec{r}) = \frac{U_{n\ell}(r)}{r}\, Y_{\ell m}(\hat{r}) \quad \text{with}$$

$$U_{n\ell}(r) = \left[\frac{2(n!)}{\Gamma(n+\ell+3/2)}\right]^{1/2} r^{\ell+1}\, e^{-\frac{r^2}{2}}\, L_n^{\ell+1/2}(r^2) \quad \text{and} \qquad (9)$$

$$L_n^{\ell+1/2}(r^2) = \sum_{s=0}^{n} (-1)^s \frac{\Gamma(n+\ell+3/2)}{\Gamma(s+\ell+3/2)} \frac{r^{2s}}{(n-s)!\,s!}$$ are the usual Laguerre polynomials

In terms of these functions the total wave function is expanded as

$$\psi^{LS}(123,\vec{x},\vec{y}) = \sum_{\sigma,n,\ell,\nu,\lambda} C^{LS}_{\sigma n\ell\nu\lambda} \quad \chi^{S}_{\sigma}(123) \left[\phi_{n\ell}(\vec{x})\phi_{\nu\lambda}(\vec{y})\right]_L \qquad (10)$$

The coefficients C^{LS} are determined by diagonalizing the Hamiltonian (8) in the basis (10). The method for calculating the matrix elements has been presented elsewhere[16] and is not needed here. If two particles are identical, we choose them to be particles 2 and 3 (since, by convention, they are privileged by our coupling scheme) and the Pauli principle imposes some restrictions in the basis states (10) (for instance $\sigma+\ell$ must be odd for $S = 1/2$ states). If the three particles are identical the resulting eigenstates are either symmetric or of mixed symmetry.

In order to study diquark structures, or the tendency for two quarks to stay close one from each other, it will be convenient to introduce correlation functions. In a first step we define the simple correlation functions $\rho_\sigma(x)$ and $\rho_\sigma(y)$. They represent the probability density that, for particles 2 and 3 coupled to spin σ, the operators $\vec{x}_i$ and $\vec{y}_i$ defined by (6) take the values x and y . In other words, $\rho_\sigma(x)$ is the probability to find particles 2 and 3 coupled to spin σ and at a distance x from each other, while $\rho_\sigma(y)$ is the probability to find particle 1 at a distance y from the center of mass of 2-3. With help of (10) it is easy to calculate them

$$\rho_\sigma(x) = \sum_{n,\ell,\nu,\lambda,n'} C_{\sigma n\ell\nu\lambda} C_{\sigma n'\ell\nu\lambda} U_{n\ell}(x) U_{n'\ell}(x)$$

$$\rho_\sigma(y) = \sum_{n,\ell,\nu,\lambda,\nu'} C_{\sigma n\ell\nu\lambda} C_{\sigma n\ell\nu'\lambda} U_{\nu\lambda}(y) U_{\nu'\lambda}(y) \qquad (11)$$

They are normalized to unity

$$\sum_\sigma \int_0^\infty \rho_\sigma(x)\, dx = 1 = \sum_\sigma \int_0^\infty \rho_\sigma(y)\, dy \qquad (12)$$

These quantities already give some information on the "geometry" of the quarks inside the baryon. In fact, let us define by R and D the values of x and y for which $\rho_\sigma(x)$ and $\rho_\sigma(y)$ reach their maximum. The situation in which particles 2 and 3 are separated by R and particle 1 at a distance D from the center of mass of 2-3 occurs very frequently in the baryon. Suppose that R << D; this means that geometries like those in Fig (1a) are very probable. But this is precisely the signature for a diquark (2-3). However this is not the end of the story. Suppose now that $R \gtrsim D$. One cannot conclude anything. There may exist situations like those of Fig (1b) where no diquark appears; but there is still possibilities like those of Fig (1c) where diquarks (1-2) or (1-3) manifest clearly. In that case the functions $\rho_\sigma(x)$ and $\rho_\sigma(y)$ are of no use to discriminate between them. This is because it lacks an "angle", the angle θ between the direction of $\vec{x}$ and $\vec{y}$. Thus in a second step, we define the double correlation functions $\rho_\sigma(x,y,\theta)$ which represent, once particles 2 and 3 are coupled to spin σ, the probability density to find particles 2 and 3 separated by a distance x and particle 1 at a distance y from the center of mass of 2-3 and with an angle θ between directions $\vec{x}$ and $\vec{y}$. The calculation is cumbersome but straightforward

$$\rho_J(x,y,\theta) = \sum_{\substack{n'\ell'\nu'\lambda' \\ n\ell\nu\lambda}} C_{Jn'\ell'\nu'\lambda'} \; C_{Jn\ell\nu\lambda} \left[(2\ell+1)(2\ell'+1)\,(2\lambda+1)(2\lambda'+1)\right]^{1/2} \frac{\sin\theta}{2y} \; U_{n'\ell'}(x)\, U_{n\ell}(x)\, U_{\nu'\lambda'}(y)\, U_{\nu\lambda}(y) \tag{13}$$

$$\sum_{\Lambda} (-1)^{L+\Lambda}(2\Lambda+1)\, P_\Lambda(\cos\theta) \begin{pmatrix} \ell' & \ell & \Lambda \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \lambda' & \lambda & \Lambda \\ 0 & 0 & 0 \end{pmatrix} \begin{Bmatrix} \ell' & \lambda' & L \\ \lambda & \ell & \Lambda \end{Bmatrix}$$

It is normalized by

$$\int \rho_J(x,y,\theta)\; y\, dy\, d\theta = \rho_J(x) \tag{14}$$

For a given x, $\rho_J(x,y,\theta)$ gives the photo in polar coordinates of the quark 1 distribution.

In the same way, one can adopt the alternative point of view to see the photo for a given y. In this case it is better to use the derived function

$$\rho'_J(x,y,\theta) = \frac{y}{x}\, \rho_J(x,y,\theta) \tag{15}$$

normalized to

$$\int \rho'_J(x,y,\theta)\; x\, dx\, d\theta = \rho_J(y) \tag{16}$$

For a given y, $\rho'_J(x,y,\theta)$ give the photo in polar coordinates of quarks 2 and 3 distributions.

These ideas will be clarified and illustrated in the next section.

III. DIQUARK STRUCTURES ?

In this chapter, the notions developed previously are applied in actual situations. However the problem is not simple and diquark formation depends on a number of factors:

i) Asymmetry of the system. Two particles with masses much greater than the third one acquire a weaker kinetic energy and tend to stay close one to each other.

ii) Angular momentum effect. If the baryon has a high total angular momentum, each pair of quarks may carry a non negligible angular momentum and the quarks themselves are submitted to an important centrifugal force which prevents them to come close together.

iii) Spin effect. The quark-quark potential contains a spin-spin force (see eq.(7)) which favours a coupling to spin 0 for a diquark. Moreover, since it is of short range, it also tends to bring the two quarks nearer.

iv) Pauli principle. If two quarks are identical there are restrictions in their mutual spin and space motions; it may happen that a situation energetically favoured is strictly forbidden by the Pauli exclusion principle. In particular, this principle does not allow two identical particles to stay at the same place (pure diquark).

v) Quantum mechanics. Quark trajectories inside a baryon make no sense in quantum mechanics; the geometry of quark repartition is not frozen and even the concept of diquark is diffuse. This is why we introduced quantities based on a probabilistic point of view. The photos presented in this paper only reflect maximum probabilities for certain conditions. Indeed quantum mechanics is present in all the above developed arguments i) – iv) (for example the asymmetry argument is based on Heisenberg uncertainty relations implicitely).

Having all this in mind, we studied various different systems. Precisely, we considered twelve baryons with different flavor contents, namely duu, suu, cuu, buu, uss, ucc, ubb, uds, udc, udb, usc, usb. We restricted to states with total spin S = 1/2 but analyzed two possibilities for orbital mome-tum : L = 0 and L = 8. Only the lowest corresponding state was studied. The size H.O. parameter b was determined in order to minimize the energy for each state and the three body problem was solved with a basis up to 8 excitation quanta (it includes 70 states for L = 0 and 78 states for L = 8 with the possibility to take half this number if the Pauli principle is effective). Also, in order to look at the same system differently, we select various quarks as particles 2 and 3; for example the baryon $q_1\ q_2\ q_3$ may be coupled like $q_1(q_2q_3)$ or $q_2(q_1q_3)$ or $q_3(q_1q_2)$. Indeed it appears that structures which are not very transparent in one channel become much evidenc in another one. For each system we plot $\rho_s(x)$, $\rho_s(y)$ and $\rho_s(x,y,\theta)$, $\rho'_s\ (x,y,\theta)$ for some values of x and y. It is out the scope of this paper to present this abundant material. We only focuse on the points which shed some light on our conclusions.

Let us begin with a detailed discussion of the proton structure (duu, S = 1/2) in both L = 0 and L = 8 states. Let us focuse first on L = 0 in the channel d(uu). Since the wave function is essentially restricted to 0 quantum excitation $\left[\phi_{00}(\vec{x})\ \phi_{00}(\vec{y})\right]_0$, the Pauli principle imposes that the uu pair to be in a state with spin $s = S_{23} = 1$. In Fig. 2 we plot the functions $\rho_1(x)$ (dotted line), $\rho_0(x)$ (dot-dashed line) and $\rho(x) = \rho_0 + \rho_1$ (full line) in the upper part (for most physical clarity it is the true distance $R_{23} = \vec{r}_2 - \vec{r}_3$ which is indicated in abscissa); in the lower part the corresponding $\rho_1(y), \rho_0(y)$, $\rho(y)$ are shown again with the physical distance $R_{1-(23)}$ between particle 1 and the center of mass of the pair (23)). The respective maxima of $\rho(x)$ and $\rho(y)$ are obtained for R = 0.66 fm and D = 0.50 fm and we are thus in the doubtful case. Moreover the fluctuations around these values are rather large since the half-width are of order 0.8 fm and 0.6 fm respectively, the half values for ρ being obtained for R_1 = 0.32 fm and R_2 = 1.14 fm and for D_1 = 0.24 fm and D_2 = 0.88 fm. In order to have a better idea of what happens let us look at Fig. 3. Few comments are necessary to understand it. It represents the quark distribution densities ρ_1 and ρ'_1 by projection on a plane. The cross at the center stands for the center of mass of the pair (23) and the scale measured from the bottom left corner always denotes 1 fm. The closed lines represent the various equiprobabilities 0.1, 0.2, ... 0.9, 1 with respect to the maximum of $\rho\ (x,y,\theta)$. The slices between two lines are filled with various grey from white (below 0.1) to black (between 0.9 and 1.0). In the left part of the figure we plot the density $\rho_1\ (x,y,\theta)$ for 3 values of x (R_1,R,R_2). The particles 2 and 3 (here uu) are represented by black circles (particle 2 always in the bottom and particle 3 in the top) and the spin configuration indicated by arrows (here $s = S_{23} = 1$). The various grey shades give the

the probability distribution for particle 1 (here d); there is a symmetry with respect to axis 2-3 : thus the two spots represent the <u>same</u> particle 1. The right part of the figure deals with the density $\rho'_1(x,y,\theta)$ for 3 values of y (D_1, D, D_2) . The particle 1 (here d) is represented by a black circle, the particle 2(u) by the spot on the right and particle 3(u) by the spot on the left in order that the center of mass be at the position indicated by the cross. The integration of these probabilities <u>on the plane</u> (caution ! the integration variables are x and y and not R_{23} and $R_{1-(23)}$) gives the values of $\rho_1(R_1)$, $\rho_1(R)$, $\rho_1(R_2)$ and $\rho_1(D_1)$, $\rho_1(D)$, $\rho_1(D_2)$ respectively. All these views show that we never have a diquark structure (except may be a diquark uu for the bottom right photo but with so weak probability). To confirm this point we have also studied the channel u(ud) for which R = 0.56 fm and D = 0.54 fm: the densities ρ and ρ' are quite similar to those presented in figure 3. The conclusion is that the proton (and the neutron of course) in its ground state is very isotropic, each quark revolving around the other with no angular momentum and at a distance around 0.5 fm, the pair uu being always in a state S = 1 (Pauli principle), while the pairs ud are $\frac{3}{4}$ of time in S = 0 and $\frac{1}{4}$ of time in S = 1 (properties of Racah coefficients).

Let us now come to the more interesting case of the proton in a L = 8 state. Here the centrifugal forces must have some effects. In the channel d(uu) the simple correlation functions $\rho(x)$ and $\rho(y)$ do not give many information since R = 2.44 fm, D = 1.36 fm (the system has a bigger spatial extension as it should be). To see what happens let us look at Fig. 4 which is analogous to Fig. 3 in that case. Now, no doubt is permissible. The system shows a manifest structure with diquark *ud*. Moreover the Pauli principle clearly appears as a "four spot" photo indicating that the d quark does not discriminate between particle 2 (u quark) and particle 3 (other u quark) for forming a diquark. To confirm this conclusion, we study also the channel u(ud). The simple correlation functions, plotted in Fig. 5, are more interesting. At once, it appears that the wave function is not so simple since, depending on the distance, the particles 1 and 3 can couple either to spin S = 0 or 1. However, most of their time, they are coupled to S = 0. One sees that R = 0.92 fm and D = 2.28 fm and, thus, this is the signature for (ud) diquark (case 1 of Fig. 5). *This feeling is reinforced by looking at Figure 6, analogous to Figs. 3 and 4.* Again the ud diquark structure is transparent. In that case, the proton appears like a (ud) diquark with the third particle u orbiting with the full angular momentum L = 8 relative to the diquark and complicated by the exchange for the role of the u quarks due to the Pauli principle. The diquark structure is mainly due to the Coulomb part of the potential which favors a pair to be in a L = 0 momentum. In fact keeping only the linear tail washes up seriously the diquark structure. However, one can raise the question : why is it not the d quark which orbits with L = 8 around an (uu) diquark ? Indeed, from asymmetry considerations, this later case ought to be favored (since its reduced mass is slightly greater than the former one). However spin effects and Pauli principle overcome this handicap. For the uu to form a diquark they must be in a

$\ell = 0$ state and Pauli principle imposes that they must be coupled to $\sigma = 1$, but the spin-spin force is then repulsive and this configuration is penalized in energy. On the other hand, a ud diquark although not favored by its moment of inertia, can be in a $\ell = 0$ state and coupled to $\sigma = 0$ (Pauli does not act for a ud system) and then the spin-spin force is attractive and is able to overcome the asymmetry handicap. We checked this point by removing artificially the spin-spin force in the potential and it appears that the configuration with a uu diquark is now lower in energy.

The study of proton structure has been explored in detail and will serve as a prototype for other systems which are now presented more succinctly. For the following photos, concerning other systems, we restrict ourselves to the most probable situation ($\rho(x,\theta,\theta)$ for $x = R$, $\rho'(x,y,\theta)$ for $y = D$) but for two coupling channels q(QQ) and Q(qQ).

Let us see what happens when one mass is increased; for that we have considered suu, cuu and buu systems. As prototype for Qqq we present the suu case (for clarity in the photos) but the conclusions are quite similar for the others. The $L = 0$ state and $L = 8$ state are shown respectively in Figs. 7 and 8. For $L = 0$, no diquark appears. Each pair qq or Qq can be in a $\ell = 0$ state and thus, since centrifual forces are absent, each particle can stay rather close to the others. Asymmetry tends to favor a (qq) diquark but Pauli principle and spin effects penalize it. The balance between all these effects is null concerning a diquark formation. For $L = 8$ the situation is completely different. Centrifugal forces must act some way or other. Two extreme positions – already discussed for the proton – can arise : (I) formation of a (qq) diquark with Q orbiting with $\ell = 8$, or (II) formation of a (Qq) diquark with q orbiting with $\ell = 8$. Asymmetry favors case (I) but Pauli principle and spin effects favor case (II) and thus the relative importance between these two effects finally decide. As already seen previously, for the proton duu, the second possibility wins. Looking at Figure 8, it appears clearly that it is the first one (diquark uu) which is realized for suu and this is even more pronounced for cuu and buu. The explanation for switching from case II to case I is not mysterious and, in fact, is two fold :

i) the bigger is the ratio m_Q/m_q ($m_d/m_u \simeq 1.0$, $m_s/m_u \simeq 1.8$), the stronger the asymmetry argument prevails

ii) the spin-spin force is proportionnal to $(m_i m_j)^{-1}$ (see eq. 7) and thus the bigger the ratio m_Q/m_q the weaker are spin effects.

Now let us study the case when two masses are increased. The most interesting photos to be presented for prototypes qQQ are ubb for $L = 0$ (Fig. 9) and uss for $L = 8$ (Fig. 10). Let us discuss the $L = 0$ case first. A (QQ) diquark structure emerges. As already stated, centrifugal forces are absent, each particle can come close to each other and the Coulomb attraction becomes effective; kinetic energy fights against and it is much weaker for the QQ pair than for a qQ pair: a QQ diquark is largely favored from that point of view. On the other hand, Pauli principle (QQ must be in a $\sigma = 1$ state) and spin forces (repulsive for $\sigma = 1$, attractive for $\sigma = 0$)

counterbalance this situation. However, the masses are so big that the spin interaction is very tiny. What we gain in energy between a Qq pair and a QQ pair is quasi nothing and the kinetic energy argument is largely dominant. The situation dramatically changes when centrifugal forces play a role; the diquark structure is completely washed up. In analyzing Fig. 10, one sees that the q quark tends to come in the middle of the Q quarks which are rather delocalized. The argument is always the same : for a given L, it is energetically favorable to put the heaviest masses with that angular momentum in order to minimize the centrifugal term $L(L+1)/mR^2$. In that sense, the formation of a QQ diquark (q is in a λ = 8 state relative to ℓ = 0 Q Q pair) is strongly penalized (contrary to the L = 0 case); the formation of a qQ diquark (Q is in a λ = 8 state relative to ℓ = 0 qQ pair) is better preferable. But it is not the optimal case because q can be some of its time in high angular momentum relatively to the QQ pair. The best situation is to put the QQ pair in ℓ = 8 with q orbiting with λ = 0. Pauli principle and spin effects tend to destroy this situation (because the spin interaction is repulsive in S = 1 state), however its effect is largely attenuated because the spin interaction is short range and in the first case the qQ pair has some extension while in the second case (qQ diquark) the big mass of Q weakens spin effects strongly. The resulting motion is thus that of the two heavy quarks revolving with all the angular momentum and the light one staying most of its time in between with zero angular momentum relative to the heavy pair. We have studied a number of other systems but general trends are already contained in the above discussion. In particular, completely disymmetric systems like usc or usb show features intermediate between qqq, qqQ, qQQ depending on the mass ratios but it does not appear new phenomena. A detailed comparison between uus and uds sheds some light on the role of Pauli principle (since $m_d \approx m_u$ these two systems only differ by Pauli correlations). The structure of these two baryons (Σ and Λ) is very similar except that the uds is a more compact object. This last point is a simple consequence of the short range character for the spin-spin interaction which is more effective for the ud pair (in a S = 0 state) than for the uu pair (in a S = 1 state) because of Pauli experiment.

III . CONCLUSIONS

In this paper we have performed three body calculations with a harmonic oscillator basis for systems of three quarks interacting via the Bhaduri's potential. With the help of the resulting wave functions, we have searched for a diquark structure inside the baryons. The problem is tackled from a probabilistic point of view and a number of quark density photos are presented and analyzed. Emphasis is put on the various effects leading to a diquark structure. Loosely speaking, diquark formation occurs in two schematic situations:

i) In qQQ baryons with low angular momentum, a diquark QQ is formed (in a ℓ = 0 state). The origin results mainly from the disymmetry in the masses but quantum mechanics is implicitely present through the Heisenberg uncertainty relations.

ii) In qqQ baryons with high angular momentum. In that case, the centrifugal force is the main ingredient. However the type of diquark results from a sublte balance between the asymmetry which favors a qq diquark, and the spin-spin interaction allied to Pauli principle which favors a qQ diquark. For $m_Q/m_q >> 1$ a qq diquark will emerge, whereas for $m_Q/m_q \approx 1$ a qQ diquark occurs. The arguments explaining diquark formation are "classical" in essence, however quantum mechanics cannot be ignored specially the Pauli principle which forbids some classical situations more favored energetically. We stressed again that diquarks have been studied from a probabilistic point of view and not imposed dynamically a priori. The interplay between these two approaches needs further investigations

ACKNOWLEDGMENTS

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FIGURE CAPTIONS

Figure 1 . Various geometries for quark distributions

a) $R \ll D$; b) $R \approx D$, $\theta \approx \frac{\pi}{2}$; c) $R \approx D$ $\theta \approx 0$ or π

Figure 2 . Single correlation functions $\rho_\sigma(x)$ (upper part) and $\rho_\sigma(y)$ (lower part) for the d(uu) baryon in L = 0 state. Particles 2 and 3 are coupled either to spin $\sigma = 0$ (dot-dashed line), to spin $\sigma = 1$ (dotted line) or no matter the spin (full line).

Figure 3 . *Quark distributions for the d(uu) baryon in L = 0 state. The cross denotes the posi-*tion of the center of mass of particles 2 and 3. The scale measured from the bottom left corner is 1 fm. Black circles denote the positions of the quarks which are fixed while the various grey shades represent the different probability ranges for other quarks. In the left part the distribution $\rho_{\sigma=1}(x,y,\theta)$ is plot for 3 values of x (R_1 = 0.32 fm, R = 0.66 fm, R_2 = 1.14 fm). In the right part we show $\rho_{\sigma=1}(x,y,\theta)$ for 3 values of y (D_1 = 0.24 fm, D = 0.50 fm, D_2 = 0.88 fm). For more details see the text.

Figure 4 Same as figure 3 for the L = 8 state of d(uu) baryon

Figure 5 Same as figure 2 for the channel u(ud) in L = 8

Figure 6 Same as figure 3 for the L = 8 state in the channel u(ud)

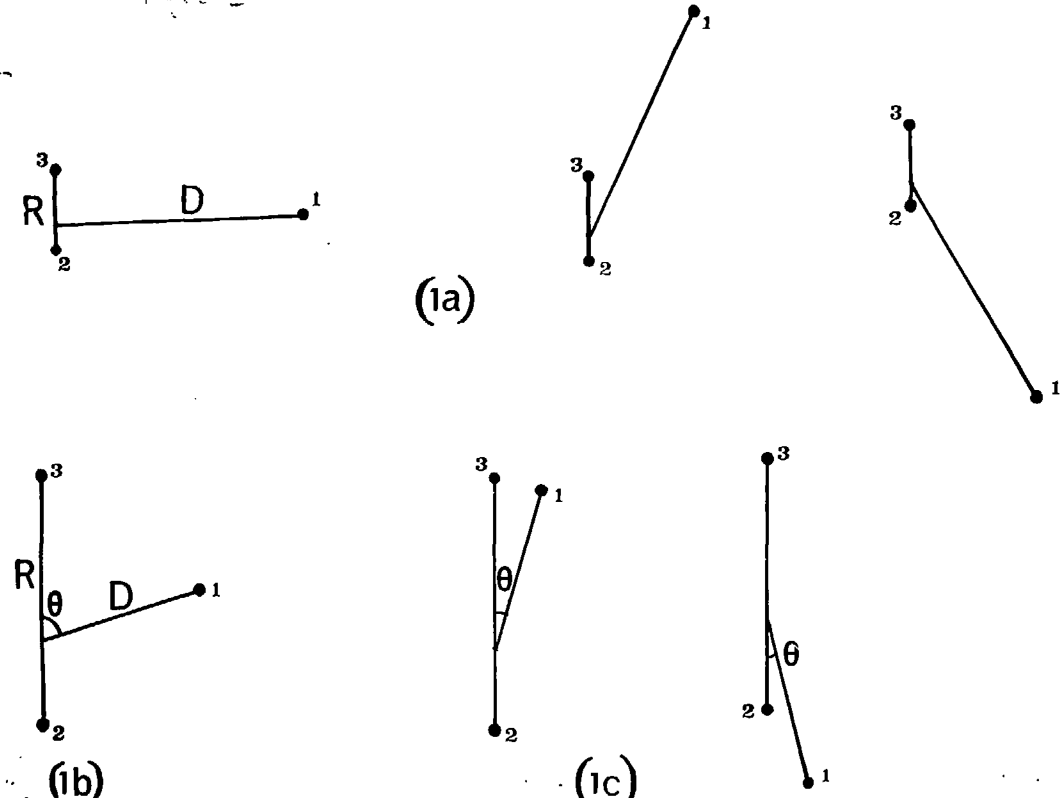
Figure 7 Same as figure 3 but for only one value of x and y (R and D) and for two different channels of the same system. Here L = 0 of the suu baryon

Figure 8 Same as figure 7 for the L = 8 state of suu baryon

Figure 9 Same as figure 7 for the L = 0 state of ubb baryon

Figure 10 Same as figure 7 for L = 8 state of uss baryon.

Figure 1



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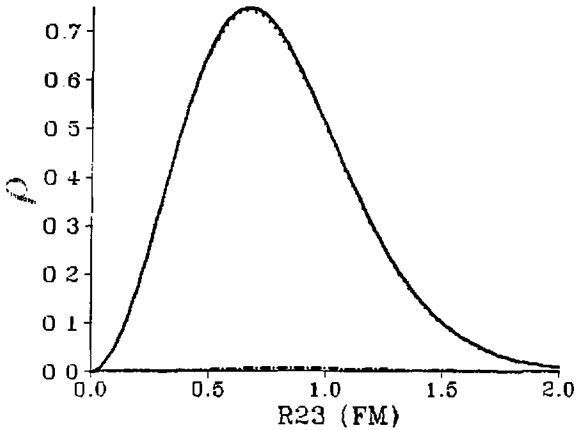
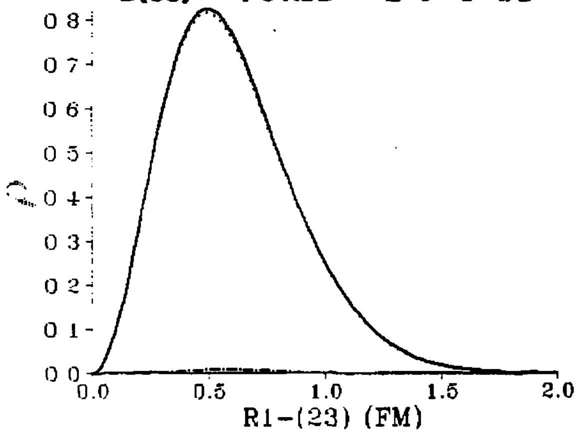
D(UU) POT:BD L=0 S=1/2



D(UU) POT:BD L=0 S=1/2



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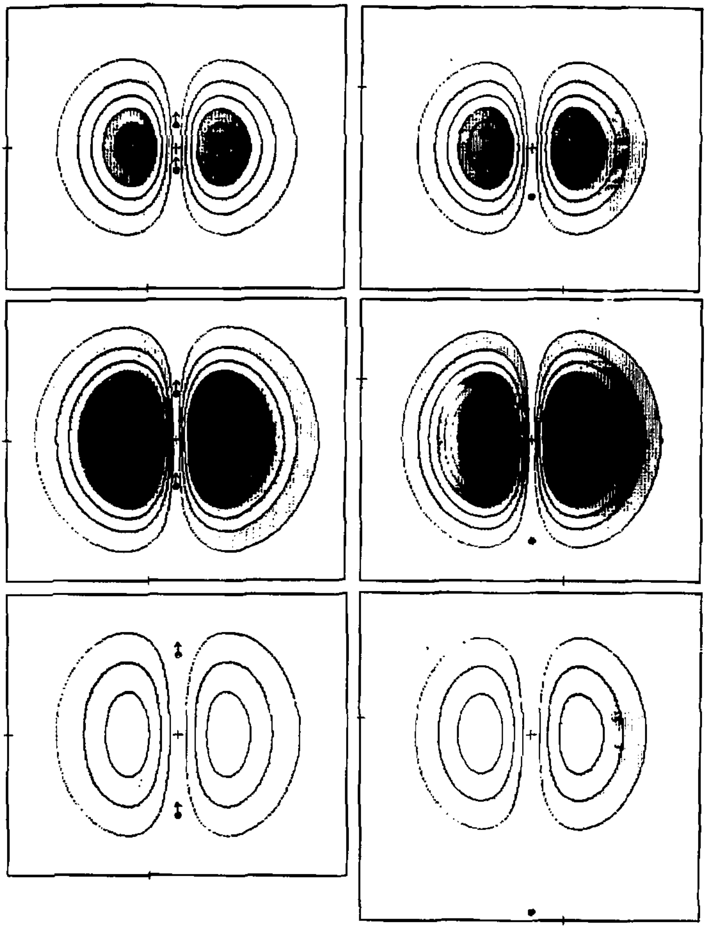
POT : BD   L = 0   S = 1/2



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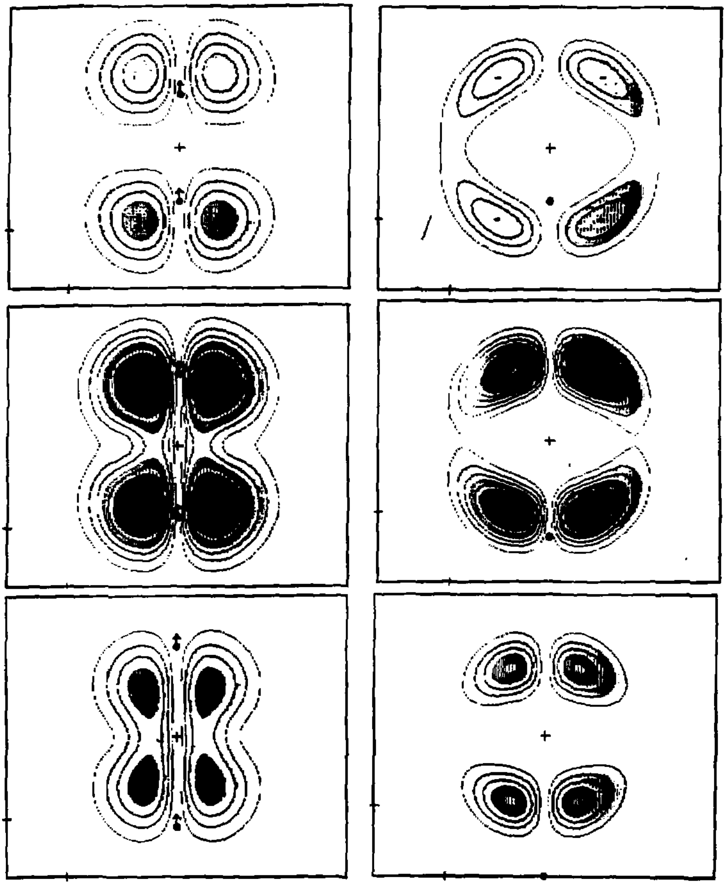
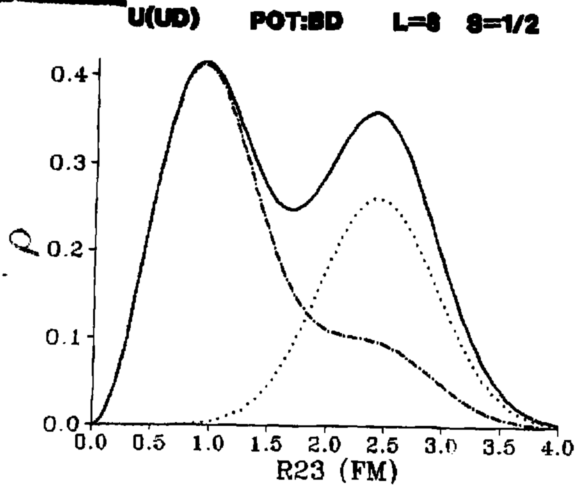
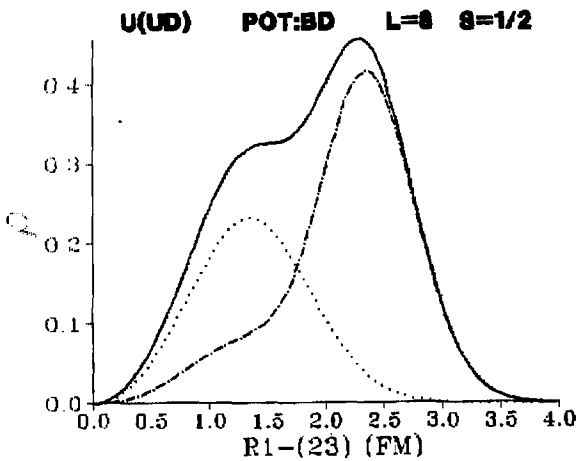
[redacted]

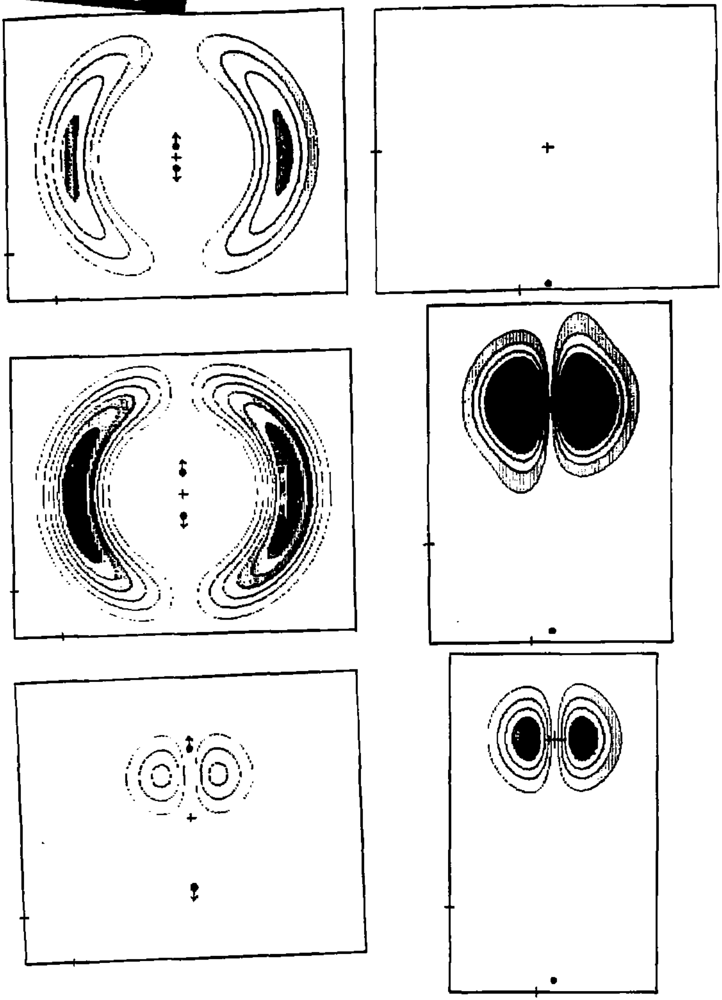
D(UU) POT: BD $L=8$ $S=1/2$ $S_{23}=1$

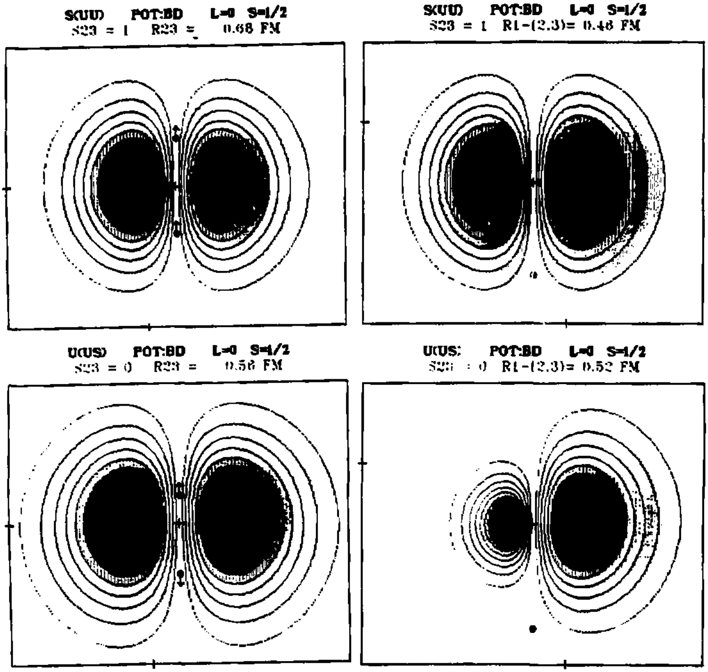



U(UD) POT:BD L=8 S=1/2
ρ
0.0
0.1
0.2
0.3
0.4
0.0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0
R23 (FM)


U(UD) POT:BD L=8 S=1/2
ρ
0.0
0.1
0.2
0.3
0.4
0.0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0
R1-(23) (FM)

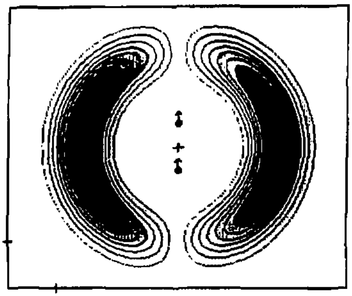
[redacted] T : BD   L = 8   S = 1/2 $S_{23} = 0$



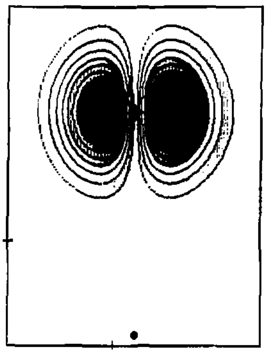

S(UU) POT:BD L=0 S=1/2
S23 = 1 R23 = 0.68 FM
S(UU) POT:BD L=0 S=1/2
S23 = 1 R1-(2,3)= 0.46 FM
U(US) POT:BD L=0 S=1/2
S23 = 0 R23 = 0.56 FM
U(US) POT:BD L=0 S=1/2
S23 = 0 R1-(2,3)= 0.52 FM

[redacted]

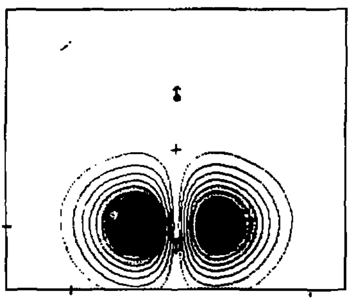
S(UU) POT:BD L=0 S=1/2
S23 = 1 R23 = 1.00 FM



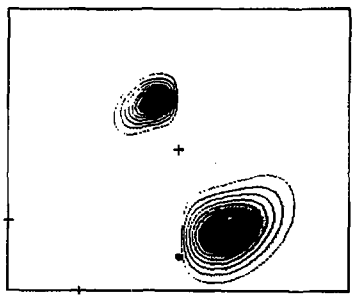
S(UU) POT:BD L=0 S=1/2
S23 = 1 R1-(2,3)= 2.12 FM



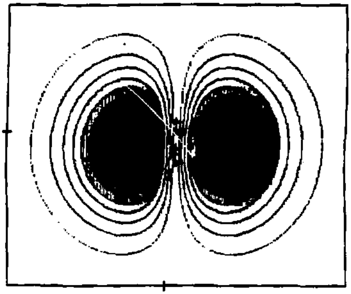
U(US) POT:BD L=0 S=1/2
S23 = 0 R23 = 2.24 FM



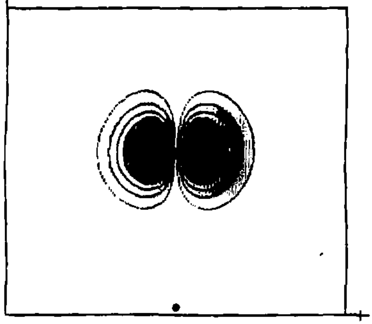
U(US) POT:BD L=0 S=1/2
S23 = 0 R1-(2,3)= 1.52 FM



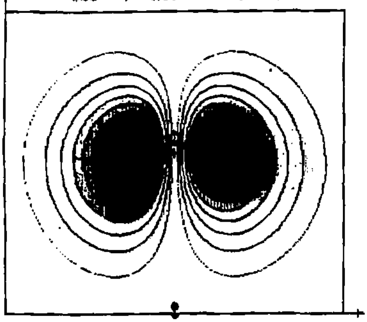
U(BB) POT:BD L=0 S=1/2
S23 = 1 R23 = 0.20 FM



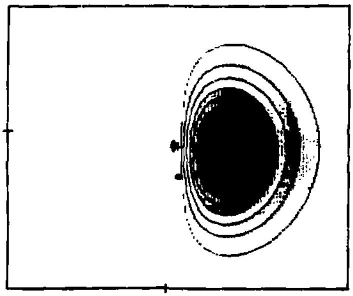
U(BB) POT:BD L=0 S=1/2
S23 = 1 R1-(2,3)= 0.45 FM



B(UB) POT:BD L=0 S=1/2
S23 = 0 R23 = 0.48 FM



B(UB) POT:BD L=0 S=1/2
S23 0 R1-(2,3)= 0.20 FM



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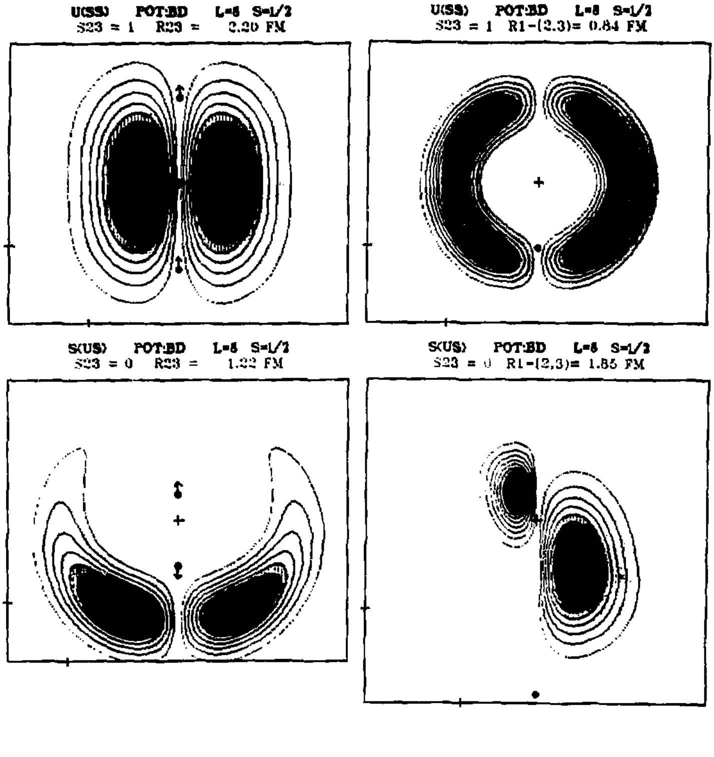
U(SS) POT:BD L=6 S=1/2
S23 = 1 R23 = 2.20 FM
U(SS) POT:BD L=6 S=1/2
S23 = 1 R1-(2,3)= 0.84 FM
S(US) POT:BD L=6 S=1/2
S23 = 0 R23 = 1.22 FM
S(US) POT:BD L=6 S=1/2
S23 = 0 R1-(2,3)= 1.85 FM