DECISION ANALYSIS
MULTICRITERIA ANALYSIS

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1. INTRODUCTION

The purpose of the ALARA (As Low As Reasonably Achievable) procedure is to compare various protection options in order to determine which is the best compromise between cost of protection and residual risk (see /1/) When there are several protection options or their introduction affects several dimensions, finding the compromise is not always easy and it is no longer possible merely to use "engineering judgement". In addition, one should be able to justify the choices made when they concern health, whatever the problems posed. In such cases, the use of quantitative decision-aiding techniques is valuable and it is hardly surprising that their use is recommended by the ICRP (see /2/), and has been adopted in various countries such as the United Kingdom (see /3/).

The ALARA procedure covers a wide range of decisions from the simplest to the most complex one. For the simplest one the engineering judgement is generally enough and the use of a decision aiding technique is therefore not necessary. For some decisions the comparison of the available protection option may be performed from two or a few criteria (or attributes) (protection cost, collective dose, ...) and the use of rather simple decision aiding techniques, like the Cost Effectiveness Analysis or the Cost Benefit Analysis, is quite enough. These two basic decision aiding techniques are presented in /4/. For the more complex decisions, involving numerous criteria or for decisions involving large uncertainties or qualitative judgement the use of these techniques, even the extended cost benefit analysis (see /4/), is not recommanded and appropriate techniques like multi-attribute decision aiding techniques are more relevant. There is a lot of such particular techniques and it is not possible to present all of them. Therefore only two broad categories of multi-attribute decision aiding techniques will be presented here : decision analysis and the outranking analysis.

2. DECISION ANALYSIS

2.1. Particularity of this technique

Although the term Decision Analysis is sometimes used to describe any technique to assist in the analysis of decisions, it is generally understood as the title given to a specific set of ideas and analytical methods, developed over the past 25 years, for application to complex decision problems involving uncertainties and multiple conflicting objectives . This type of approach has evolved from several disciplines, including psychology, engineering and management science /5, 6, 7/, and may be considered as the most broadly applicable technique for decision making under uncertainty, based on the mathematical foundations of multi-attribute utility theory.

The essence of Decision Analysis is to construct a scoring scheme (or multi-attribute utility function) for alternative protection options facing a decision maker, with the property that if the score (or utility) is the same for two options, the decision maker is indifferent between them. If, however, the score for option A exceeds that of option B, then by definition A is preferred to B.

This technique is the most general method of aggregation. It is a non-monetary generalization of the cost-benefit analysis.

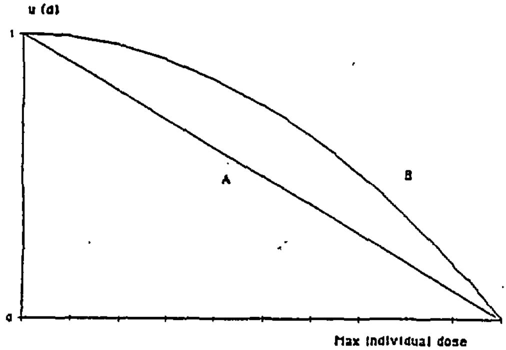
In cost benefit analysis the various options are aggregated (cost of protection, radiological detriment) by using a monetary transformation (e.g. expressing the radiological detriment in monetary terms through the man Sievert value). This transformation rate is fixed and applies in the same way to all possible detriment values. Thus, for example, for cost benefit analysis, the value of alpha being fixed, the cost of the man-Sievert alpha is constant whether applied to a collective dose of one micro or one million man-Sievert. This single value concept is reconsidered during the extension of cost benefit analysis proposed by the NRPB (see /2/) which differentiated alpha according to the per-caput dose.

With this technique the cost of a one man-Sievert collective dose depends on the per caput dose and there is no more a direct linearity between the collective dose and the cost of the detriment associated with this collective dose. This example illustrate the interest of a non linearity between the performance of the options (collective dose) and the assessment of the relative interest, or utility of these performance. That is one of the basic interest of Decision Analysis.

2.2. <u>How does it work</u> ?

Having formally specified the relevant factors (e.g. protection costs and collective dose) and corresponding attributes (e.g. $ and man-Sv) to be included in the optimisation study, and having quantified the consequences of each protection option in terms of these attributes, it is necessary to incorporate preferences and judgements about such consequences into the analysis of options (see /1/). This is established for each attribute j through a single attribute utility function u_j traducing the relative desirability of the possible consequences x_j for the attribute j. Generally to the best outcome or consequence (lower cost, collective dose) is associated a single attribute utility Uj of 1 and to the worst consequence single attribute utility of 0.

The major interest of this technique is that these single utility functions are not necessarly linear. This allows the introduction of different attitudes as it is illustrated by the following figure.



Two different single utility functions A and B are presented here, as an example, for the attribute **maximum individual dose.** The first one, A, is linear and traduces a classical continious monotomme decreasing appreciation of the consequences. On the contrary the second one, B, is non linear and decreases faster near the worst consequence (the highest maximum individual dose). This second function traduces the fact that the decision maker focusses his attention on the high individual doses and is then more sensitive to any variation in these range of consequences. This type of attitude corresponds to the extension of the cost benefit analysis to the β individual dose factor which allows an higher cost of the man-Sievert to doses linked to high individual doses.

The flexibility of these single utility functions allows also the introduction of factors which are not easy to quantify particularly in monetary terms as it is required in a Cost Benefit Analysis. As an example, if for one criteria one can only express the performances of the various protection options in a qualitative way (very good, good...., bad, very bad) it is possible to translate these evaluation as follows :

EVALUATION	SINGLE ATTRIBUTE UTILITY FUNCTION
Very good	1
Good	0.8
Good enough	0.6
Bad enough	0.4
Bad	0.2
Very bad	0

At the end of this first step, each option is characterized by various scores established from the different single attribute utility functions.

From these scores uj expressing the various utilities of the n consequences ($x_{1,i}$; $x_{2,i}$; ... ; $x_{j,i}$;..... $x_{n,i}$) associated to each protection option i, a multi-attribute utility function $\mathbf{U}_i$ must be elaborated. This global function provides a figure of merit of each option i (called "total utility"). When the attributes are all preferentially independant /8/ the multi attribute utility function can be expressed in an additive form :

$$\mathbf{U}_i = \mathbf{U}(x_{1,i} ; ... ; x_{j,i} ; x_{n,i}) = \sum_{j=1}^{n} k_j \, U_j \, (x_{j,i})$$

where the kj's are scaling constants reflecting the relative importance, or weight, assigned to each attribute j. (Generally $\Sigma k_j = 1$).

The more the "total utility" $\mathbf{U}_i$ is, the more interesting is the option. The analytical solution is the option which maximizes these total utility $\mathbf{U}$.

One can notice that the cost benefit analysis is a particular form of additive multi attribute utility function where all the single utility functions are linear (equal to the consequence $U_j(x_{j,i}) = x_{j,i}$ and the k_j are the monetary values of each unit of consequences (as the cost of man Sievert).

2.3. How to implement

In order to implement such technique, the two major steps are : the assessment of the single attribute utility functions $\mathbf{U}_j(x_{j,i})$ and the scaling constants k_j.

2.3.1. Assessment of the single attribute utility functions $\mathbf{U}_j(x_{j,i})$

The relative preferences of the decision maker for each criteria are described by a single attribute utility function. Conventionally a utility of 0 is assigned to the worst consequences (x_*) and a utility of 1 to the best one (x^*) and a utility in the range of [0,1] for the other consequences.

The assessment of this single attribute utility function can be performed by many techniques and there are also many variants of each of them /see 9,10/.

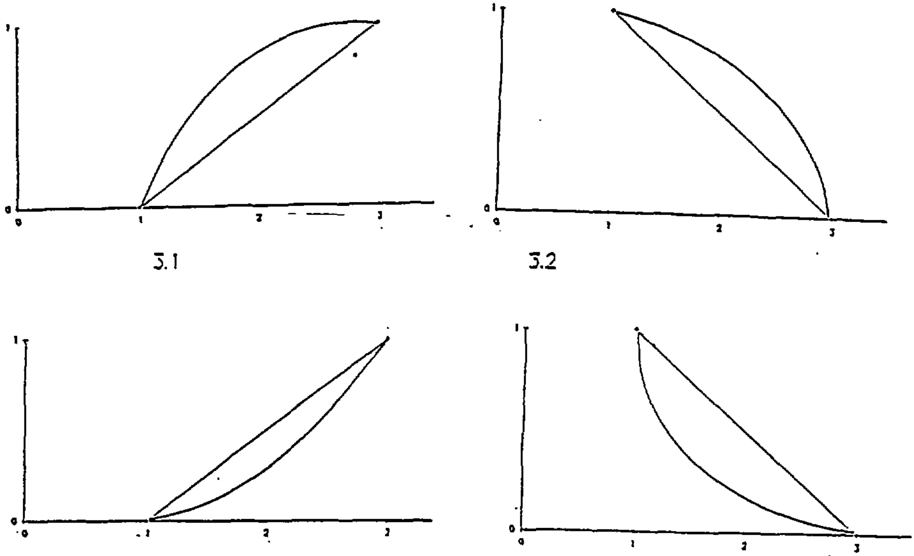
For some attributes, the single attribute utility function is linear. For these criteria the equation of the function is assessed easily from the two points ($x^{\bullet}$; $u(x^{\bullet}) = 0$) and (x^{*} ; $u(x^{*}) = 1$).

For the other attributes the simplest way is based on the hypothesis that the function is monotonic and that the knowledge of a third point (the two first being [$x^{\bullet}$, $u(x^{\bullet}) = 0$] ; [x^{*} ; $u(x^{*}) = 1$] is enough to characterize the function. One can then use different type of function joining these three points : piece wise linear, exponential, logarithmic ; generally the exponential form is chosen.

Depending on the position of the third point and therefore of the shape of the function a generic expression of the function is used, the assessment of the various parameters being performed from the coordinates of the three points :

Shape of function	Figure	Expression of the function	Parameters
CONCAVE ; increasing	3-1	$a - b\,e^{-cx}$	a>o b>o c>o
decreasing	3-2		a>o b>o c<o
CONVEX ; increasing	3-3	$a + be^{cx}$	a>o b>o c>o
decreasing	3-4		a>o b>o c<o

CONCAVE : third point supra linear
CONVEX : third point infra linear



3.1 3.2 3.3 3.4

Sometimes the coordinates of the third point can be assessed directly by the decision maker which is able to give, as an example, the point $(x, u(x) = 1/2)$.

If it is not possible to assess directly the third point, this assessment is often obtained through various questions. Each question is based on a simple choice between 2 different situations. As an example the following choice based on the two extrems points and the medium point $(x^{\circ} + x^{*}/2)$ can be proposed.

Do you prefer the situation A or B ?

A : to be sure to have the consequence $(x^{\circ} + x^{*})/2$

B : probability of 0,5 to have the consequence x° and probability of 0,5 to have the consequence x^{*}.

.If the two situations are judged equivalent, the two utilities u(A) and u(B) are equivalent : $u(x^{\circ} + x^{*})/2 = u(x^{\circ})/2 + u(x^{*})/2$ and the single attribute utility function is linear ($u(x) = a+b.cx$ with $a>o$ $b>o$ $c>o$ for increasing functions $c<0$ for decreasing ones).

.If the situation A is prefered then $u(A)>u(B)$ or $u[(x^{\circ}+x^{*})/2] > u(x^{\circ})/2 + u(x^{*})/2$ and the utility function is concave (so called "risk adverse" attitude).

.If the situation B is prefered then $u(A)<u(B)$ or $u[(x^{\circ}+x^{*})/2] < u(x^{\circ})/2 + u(x^{*})/2$ and the utility function is convex (so called "risk prone" attitude).

In the two last cases, other similar questions allow progressively the assessment of the coordinate of the third point $(x^{\circ}+x^{*}/2 \, ; u[(x^{\circ}+x^{*})/2)]$.

From the three coordinates of these points, the three parameters a,b and c can then be assessed and then all the values of the single attribute utility function.

For some attributes the shape of the function are often choosen as follows :

ATTRIBUTE	SHAPE	
Protection cost	Decreasing	Linear
Collective dose	Decreasing	Linear or concave
Individual dose	Decreasing	Concave

In a few particular cases the single attribute utility function is notmonotonic and three points are not enough for assessing the expression of the function. The assessment of the function is then more complex.

2.3.2. Assessment of the scaling constants k_j

When the single attribute utility functions are assessed the evaluation of the scaling constants k_j is performed. Generally the scaling constants are first ranked from the most important attribute to the less one.

When this ranking procedure is obtained some trade offs, similar to the choices proposed for the assessment of the single attribute utility function, are proposed to the decision maker.

Suppose that you start from the worst situation for all the attributes.

Do you prefer then the situation A or B ?

A : To be sure to get the best performance only for the attribute m (in a middle position of the ranking).

B : To have probability of 0.5 to get the best performance only for the most important attribute k^* and probability of 0.5 to get the best performance only for the less important attribute $k^\bullet$.

If A is prefered then $km > 1/2\,(k^* + k^\bullet)$
If B is prefered then $km < 1/2\,(k^* + k^\bullet)$
If A and B are judged equivalent then $km = k^\bullet + k^*/2$

Similar questions are then asked for the assessment of all the k_j.

3. MULTICRITERIA OUTRANKING TECHNIQUES

3.1. Particularity of these techniques

The decision analysis, as well as the cost benefit or cost effectiveness analysis are "aggregative" techniques. These techniques combine for all options all the attributes representing the relevant factors influencing a decision into a single figure of merit (cost effectiveness ratio, total cost, total utility...). This figure of merit represents, by definition all the advantages and drawbacks associated with each option. This assumption is relevant if some conditions are satisfied.

First, it is necessary for all the attribute considered to be commensurable · such that the global valuation which is finally assigned to each protection option in monetary terms or utility, adequately expresses the contribution of the consequences on each of the individual factors involved. Secondly, it must be accepted that a poor performance on one attribute can be fully compensated by better performances on other attributes and that such trade-offs are acceptable over the full range of consequences arising from all protection options under consideration.

These two basic conditions may pose some difficulties for some specific decisions. Sometimes the evaluation of the various options for the different attributes can only be performed in a qualitative manner. Even if these qualitative assumptions can be translated in single attribute utilities it will not be always possible to aggregate these rough evaluation and a direct comparison of the options may be more simple or realistic. Alternatively where protection options are disparate, it may be judged that the option associated with say, minimum cost and maximum detriment is not really comparable to that leading to maximum cost and minimum detriment. In such circumstances, the use of a multicriteria outranking technique will prove more appropriate than an aggregative technique.

Instead of expressing the global performance of each option in term of a single figure of merit, the multicriteria outranking technique initially compares each option i to every other option j, in order to evaluate whether option i outranks (or is preferred to) option j. This pairwise comparison is generally based on two indicators :

- The first one establishes if there is a sufficient majority among the attributes (taking into account there relative importance) to consider that option i is not worse than j.

- The second one establishes if no attribute, in disagreement with the majority preceeding result, in too a great superiority of j with respect to i.

Option i outranks then j if these two indicators are satisfied and therefore i is better or equal to j for a great part of the relevant factors and for the other one the difference between i and j is not too important.

From these different pairwise comparison, a specific algorithm is then applied in order to select one or a few options.

Some of these techniques are relatively simple (see /11/), some other one are based on more sophisticated theory, like the fuzzy set theory (see /12/) which allows the introduction of subjective uncertainties in the analytical process. The application of these techniques stay however rather difficult and must generally be undertaken by professionals in the field.

Generally these type of techniques are usefull when the number of attribute is high enough or when some particularities of the decision are present and are not compatible with aggregative technique. These techniques are generally very flexible but the result of the procedure is not always easy to interpret. On the contrary aggregative techniques provides always a "best" option even if this result is not realistic.

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