INSTITUTE OF THEORETICAL AND EXPERIMENTAL PHYSICS

ITEP -- 99 (1987).

Yu.A. Simonov

QUARK PROPAGATORS AND CORRELATORS IN A CONFINING VACUUM

Preprint №99

Moscow — ATOMINFORM — 1987

УДК 530.145 М-16

QUARK PROPAGATORS AND CORRELATORS IN A CONFINING VACUUM: Preprint ITEP 87-99/

Yu.A.Simonov – M.: ATOMINFORM, 1987 – 29p.

Quark propagators, quark-antiquark Green functions and photon selfenergy operator $\Pi^{(2)}(k)$ are considered in the background (anti)selfdual field. The cases of a homogeneous selfdual field in d=4 and magnetic field in d=2 are studied in detail. Isolated quarks and quark-antiquark pairs are shown to be confined in those cases with the quadratic form of confining potential. In the space filled with domains of the homogeneous field with random directions the confining potential is of linear form, and the colorless $q\bar{q}$ pair [illegible] not confined.

Fig. – , ref. – 24

1. Introduction

The confinement of quarks and gluons may be thought of as an interplay of different mechanisms, where 1) properties of vacuum background fields, 2) quantum fluctuations around this background (gluons) and correlations between quarks and gluons in the vacuum and 3) backward action of quark sources on the vacuum fields all combine to produce the observable picture of vacuum and hadrons.

It is known for some time, however, [1-6] that the picture of strong background vacuum field of (anti)selfdual character can explain most phenomenological and Monte Carlo results, namely: the gluon condensate [3]; the hierarchy of hadronic scales in different J^P channels [2]; OZI rule violation [4]; strong mixing in $0^-,0^+$ channels [2]; appearance of zero quark modes and a possible mechanisms of the chiral symmetry breaking due to zero mode mixing [6]; the fact that both gluon condensate and topological susceptibility correspond to approximately one topological charge per 1 fm^4 [6].

It is tempting therefore to assume that the main properties of the QCD vacuum (such as confinement and chiral symmetry breaking) can be explained by the vacuum background fields only and take into account the effects enumerated above by 2) and 3) in the next approximations. To make our program explicit and in the spirit of work [2] we represent the color vector potential in the vacuum as

$$A_\mu = \bar{A}_\mu + a_\mu \tag{1}$$

where $\bar{A}_\mu = O(1/g)$ is of a quasiclassical origin and is somehow made of classical solutions, whereas $a_\mu = O(g^0)$ is a quantum fluctuation. To be consistent one should assume that the background field $\bar{A}_\mu, \bar{F}_{\mu\nu}$ is large and therefore $g(\bar{F})$ is a small quantity. This way we make our model not very realistic, since in real vacuum $g(\bar{F})$ should be determined dynamically (e.g. from the minimum of the free energy of the vacuum) and it will probably be of the order of unity. Nevertheless keeping $\bar{A}_\mu$, $\bar{F}_{\mu\nu}$ large (resp. $g(\bar{F})$ small) we disentangle the mechanisms 1)-3) and put them in action one after another, having in mind to extrapolate afterwards $g(\bar{F})$ to moderate values of the order of unit.

This scheme was used in [7] to study general features of quark and gluon propagators in vacuum fields. It was found [7] that periodic and "bounded" stochastic vacuum fields (exact meaning see in [7] and Section 4) cannot produce the confinement behaviour of quark and gluon propagators, i.e. a fall-off at large Euclidean distances faster than the exponential one. To ensure this behaviour one must have in vacuum a stochastic ensemble of regions (including arbitrarily large regions) where the field strength is roughly constant. This stochastic regime is different from the ordinary Anderson localization regime [8] (the latter produces only an exponential decay at large distances) and it was called in [7] the superlocalization regime.

The behaviour of quark and gluon Green functions at large distances in the superlocalization regime

$$G(x-y) \sim \exp\left(-\frac{|x-y|^{\alpha}}{\xi^{\alpha}}\right), \alpha > 1 \qquad (2)$$

is similar to the behaviour of an electron Green function in the external homogeneous selfdual field [9]. The case of a homogeneous field and the case of a stochastic ensemble of domains have both distinct and common features. It will be shown in a subsequent publication, that the stochastic ensemble of domains with homogeneous (anti)selfdual fields differs from an infinite region of a homogeneous selfdual field in several respects: a) zero quark modes of the infinite region go over into a kind of (superlocalized) energy band around zero for stochastic ensemble; b) gluons with color direction parallel to the background field propagate freely in the homogeneous abelian field, but are confined in the stochastic ensemble with domains of different color direction; c) there is no average direction in color and space for a stochastic ensemble in contrast to the constant homogeneous field in the whole R^4 space. This makes stochastic vacuum O(4) invariant and color $SU(N_c)$ invariant.

There are also common features. First of all, the homogeneous field propagators [9] behave as in (2) and consequently have no singularities in momentum space at finite p. Using those propagators one can calculate the photon polarization operator $\Pi(k)$ and other gauge invariant correlators.

It appears [10-12] that $\Pi^{(2)}(k)$ in the quasiabelian homogeneous selfdual field also has no singularires at finite k. The potential between the quark and antiquark is quadratic and bound states of q and $\bar{q}$ are confined just as isolated quarks and antiquarks, as will be shown in Section 3. These features are unphysical and it is very important to find background fields ensuring linear confining potential and propagating white states. Here the crucial role is played by a stochastic sequence of fields instead of the everywhere constant field. We present below arguments and estimates that 1) for domains of constant selfdual fields with uncorrelated directions the potential between opposite charges is linear ; 2) a colorless bound state emerges and propagates freely in the vacuum. We try to elucidate all these questions also on simple models. First we consider in Section 2 a quantum mechanical model of two opposite charges in a 2-dimensional plane with a magnetic field perpendicular to it. All main features of 4-dimensional selfdual fields are already seen in this example, whereas the case of a quark and antiquark pair in a homogeneous selfdual field in 4d is considered in Section 3. The influence of stochasticity and derivation of a linear interaction is given in Section 4. In conclusion we list all main features of our models which should correspond to the real vacuum.

2. A quantum mechanical model

2.1. Let us consider an electron and a positron moving in the (x_1, x_2) plane with the magnetic field $\vec{H}$ applied in the 3 direction. We disregerd (the trivial) spin dependences, because they have nothing to do with confinement and also because they generate simple additive terms.

There are different solutions for one particle with mass m moving in the constant magnetic field (depending on which quantum numbers are fixed) with the Hamiltonian:

$$H = \frac{(\vec{p} - e\vec{A}(\vec{r}))^2}{2m} \tag{3}$$

For example for $A_1 = -Bx_2$, $A_2 = 0$ there is a solution with fixed p_1

$$\psi_{np_1} = \left(\frac{eB}{\pi}\right)^{1/4} \frac{e^{ip_1x_1 - (x_2 - x_0)^2 \frac{eB}{2}}}{\sqrt{2^n n!}} H_n\left(\sqrt{eB}\,(x_2 - x_0)\right) \tag{4}$$

where $x_{20} = -p_1/eB$; $E_n = (n + \frac{1}{2})eB/m$ (5)

The one-particle Green function for zero energy satisfies an equation

$$\frac{1}{2m}(\vec{p} - e\vec{A}(\vec{x}))^2 G(\vec{x},\vec{y}) = \delta^{(2)}(\vec{x} - \vec{y}) \tag{6}$$

and has the following form (see e.g. [13]), where we have explicitly separated the gauge-variant factor

$$G(\vec{x},\vec{y}) = \frac{m}{2\pi} \exp\left(i\int_{\vec{y}}^{\vec{x}} A_\mu(z)dz_\mu\right) \cdot \mathcal{K}_0\left(\frac{eB}{4}(\vec{x} - \vec{y})^2\right) \tag{7}$$

and where $\mathcal{K}_0(z)$ is the modified Bessel function. The case of nonzero energy see in Appendix 1. For us it is important, that the asymptotics of G(x,y) at large distances is (for any energy E)

$$G(\vec{x}-\vec{y}) \sim [(\vec{x}-\vec{y})^2]^{-1/2} \cdot \exp\left(-\frac{eB}{4}(\vec{x}-\vec{y})^2\right) \quad (8)$$

which means that any charged particle is confined in the plane (X_1, X_2). It is what one expects in the classical picture, where the electron moves around a circle in the (X_1, X_2) plane and cannot propagate on large distances.

There is no discrepancy between the apparent propagation in x_1 direction in (4) and behaviour (8), since the energy levels (5) do not depend on p_1 and in the spectral representation of the Green function

$$G(\vec{x},\vec{y};E) = \int dp_1 \sum_n \frac{\psi_{np_1}(\vec{x})\,\psi^*_{np_1}(\vec{y})}{E_n - E} \quad (9)$$

one does not encounter any singularities at finite p_1, which means confinement also in the x_1 direction (due to the Gaussian dependence of ψ_{np_1} on p_1 one arrives upon integration over p_1 to the Gaussians in x_1. This type of behaviour is typical for constant fields and we shall meet it in subsequent Sections). Physically speaking, the Green function (7) is the probability amplitude of a particle with coordinates $\vec{y}(y_1,y_2)$ to reach a point $\vec{x}(x_1,x_2)$; therefore at the initial point $\vec{y}$ one should take into account all momenta p_1, p_2, i.e. integrate (4) over p_1 which finally produces confinement also in x_1 due to the destructive interference of ψ_{np_1} (4). This bilocal Green function (7) is of physical interest since the physical correlators are produced by the action of local currents: $\langle 0 | J_1(x)\, J_2(y) | 0 \rangle$.

2.2. Now we turn to the system of a positive and a negative charge in the (x_1, x_2) -plane in the magnetic field with the Hamiltonian:

$$H = \frac{1}{2m}(\vec{p} - e\vec{A}(\vec{r}))^2 + \frac{1}{2m}(\vec{p}\,' + e\vec{A}(\vec{r}\,'))^2 + V(\vec{r} - \vec{r}\,') \quad (10)$$

Just as in the case of the Green function (7) it is convenient also here to separate a gauge-variant factor (this can be done in different ways, see Section 4, here we choose a contour $\vec{r}$, $\vec{r}\,'$ which enables to define a white-state wave function φ)

$$\Psi(\vec{r}, \vec{r}\,') = \exp\left(ie\int_{\vec{r}\,'}^{\vec{r}} A_\mu(z)\, dz_\mu\right) \varphi(\vec{r}, \vec{r}\,')\, e^{i\vec{P}\cdot\frac{\vec{r}+\vec{r}\,'}{2}} \quad (11)$$

where the path ($\vec{r}$, $\vec{r}\,'$) goes along the straight line in the (1,2) plane. Substituting (11) into (10) we obtain an equation for φ ($\vec{r}$,$\vec{r}\,'$) in case of equal masses

$$\tilde{H}\varphi = E\varphi \; ; \qquad \vec{\eta} = \vec{r} - \vec{r}\,' ; \; \vec{R} = \frac{\vec{r}+\vec{r}\,'}{2} , \; \vec{P} = \vec{p} + \vec{p}\,' \quad (12)$$

where

$$\tilde{H} = \frac{1}{4m}(\vec{P} - e(\vec{B}\times\vec{\eta}))^2 - \frac{1}{m}\frac{\partial^2}{\partial\vec{\eta}^2} + V(\vec{\eta}) \quad (13)$$

The case of unequal masses is treated in Appendix 3. The most important feature of (13) is the appearence of a confining potential $(\vec{\eta}\times\vec{B})^2 = \eta^2 B^2$, which keeps two opposite charges together even in the absence of the interaction potential $V(\eta)$. Note also, that $\vec{P}$ commutes with the Hamiltonian $\tilde{H}$.

If $V(\eta)$ is absent, one can always redefine $\vec{\eta} \to \vec{\eta}' = \eta + c(\bar{P} \times \bar{B})$ so that the dependence on P disappears in $\tilde{H}$ and energy eigenvalues do not depend on P. In this case for the four-point Green function $G(\bar{r}, \bar{r}' / \bar{\rho}, \bar{\rho}')$ we can repeat all the arguments concerning the spectral representation of the type (9) and will come to the conclusion that the motion of two particles $+e$ and $-e$ is confined both in coordinates $\bar{\eta} = \bar{r} - \bar{r}'$ and $\vec{R}$. This conclusion is expected since the Green function of two noninteracting particles in the external field is proportional to the product of one-particle Green functions and each of those is confining.

The exact form of the two-body Green function is derived in Appendix 2 in two ways giving the same gauge invariant part. Namely, the Green function satisfies an equation

$$\hat{H} \; G(\bar{r}, \bar{r}' | \bar{\rho}, \bar{\rho}') = \delta(\bar{r} - \bar{\rho})\,\delta(\bar{r}' - \bar{\rho}') \tag{14}$$

where $\hat{H}$ is given in (10). If we put for simplicity $\bar{\rho} = \bar{\rho}' = 0$, one obtains

$$G(\bar{r}, r'; 0, 0) = \exp\left[ie\int_0^{\bar{r}} A_\mu dz_\mu - ie\int_0^{\bar{r}'} A_\mu dz_\mu\right] f(\bar{r}, \bar{r}') \tag{15}$$

and

$$f(\bar{r}, \bar{r}') = \frac{m\,|eB|}{8\pi^2}\;\frac{\exp(-z)}{z} \tag{16}$$

with $$Z = \frac{|eB|}{4}\left(F^2 + F'^2\right) = \frac{|eB|}{4}\left(2R^2 + \frac{1}{2}\eta^2\right)$$

One can see that G is confining both in relative and the center-of-mass coordinates.

If we want our model to resemble the QCD vacuum, we should require that the confining force between the charges $\frac{e^2B^2}{4m}\vec{\eta}^2$ be replaced by a linear potential and a bound state of charges e and −e be propagating freely. An obstacle to the latter property can be seen in (13) − this is the term linear in $\vec{P}$:

$$V_d \equiv -e\frac{\vec{P}}{2m}\left(\vec{B}\times\vec{\eta}\right) \equiv -\vec{E}\vec{d} \qquad ($$

where electric field $\vec{E} = \frac{\vec{P}}{2m}\times\vec{B}$ occurs due to the motion in the magnetic field. It is suggestive that the sto-chastic changes of directions of $\vec{B}$ may kill V_d and change confining force, giving rise to an average Hamiltonian of the type:

$$\langle H\rangle = \frac{1}{4m}\vec{P}^2 + \left\langle\frac{e^2B^2\vec{\eta}^2}{4m}\right\rangle - \frac{1}{m}\frac{\partial^2}{\partial\vec{\eta}^2} \qquad (18)$$

The Hamiltonian (18) has already the correct feature: the bound states can propagate freely due to the fact that $\vec{P}^2$ adds to the energy and no confining force is acting in the center-of-mass coordinate. If in addition $\left\langle\frac{e^2B^2\vec{\eta}^2}{4m}\right\rangle$ goes over into a linear potential, we would have a model embodying main features of a realistic confining vacuum. In Section 4 we will show that this really happens.

3. Quark propagation in the homogeneous selfdual field

In this Section the Schwinger's proper time method [9] is used to derive a quark propagator in a homogeneous selfdual field. We keep notations and units of [9] and consider the Euclidean space; in Schwinger's notations $x^2=x_1^2+x_2^2+x_3^2+x_4^2$, but in contrast to [9] we take x_4 to be real; $\frac{1}{2}\{\gamma_\mu,\gamma_\nu\}=-\delta_{\mu\nu}$, $\gamma_0=-\gamma_4$, $\gamma_0^2=1$. The one-electron Green function

$$G(x,y)= i\langle T(\psi(x)\bar{\psi}(y))\rangle \tag{19}$$

satisfies an equation

$$(\gamma_\mu \Lambda_\mu + m)\, G(x,y)= \delta^{(4)}(x-y) \tag{20}$$

where $\Lambda_\mu = p_\mu - eA_\mu$. In the proper-time formalism the Green-function operator and the effective Lagrangian can be written as

$$\begin{aligned} G &= i(-\hat{\Lambda}+m)\int_0^\infty ds\cdot \exp[-i(m^2-\hat{\Lambda}^2)s]= \\ &= i(-\hat{\Lambda}+m)\int_0^\infty ds\; e^{-im^2s}\, e^{-i\mathcal{H}s} \end{aligned} \tag{21}$$

$$\mathcal{L}^{(1)}= \frac{i}{2}\int_0^\infty \frac{ds}{s} e^{-im^2s}\cdot tr(x|e^{-i\mathcal{H}s}|x) \tag{22}$$

where

$$\mathcal{H} = -\hat{\Lambda}^2 = \Lambda_\mu^2 - \frac{1}{2} e\, \sigma_{\mu\nu}F_{\mu\nu},\quad \sigma_{\mu\nu}=\frac{i}{2}[\gamma_\mu,\gamma_\nu] \tag{23}$$

Exact evaluation of the evolution operator $e^{-i\mathcal{H}s}$ in the homogeneous selfdual field case yields the following final expression [9] for G(x ,y) (with s→-is):

$$G(x,y)=\int_0^\infty ds\, e^{-m^2 s}\Big\{-\gamma_\mu\cdot\tfrac{1}{2}\,[ieB\,\mathrm{cth}(eBs)+eF_{\mu\nu}]\cdot$$
$$\cdot(x-y)_\nu+m\Big\}\frac{i}{4\pi^2 s^2}\,\Phi(x,y)\cdot\frac{2(es)^2B^2}{\mathrm{ch}(2esB)-1}\,\cdot \tag{24}$$
$$\cdot\exp\Big[-\frac{(x-y)^2}{4}eB\cdot\mathrm{cth}(eBs)\Big]\cdot\Gamma(\sigma)\cdot\gamma_+\,,$$

where

$$\Gamma(\sigma)\gamma_+=\Big[\mathrm{ch}(2esB)\cdot\gamma_+ + 1-\gamma_+ + \mathrm{sh}(2esB)\frac{\sigma_{\mu\nu}F_{\mu\nu}\gamma_+}{4B}+ \tag{25}$$
$$+\ \tfrac{1}{2}(e\,\sigma_{\mu\nu}F_{\mu\nu}\,s)(1-\gamma_+)\Big]$$

$$\gamma_5=i\gamma_1\gamma_2\gamma_3\gamma_4\,,\ \gamma_5^2=-1\,,\ \gamma_+=\frac{1+i\gamma_5}{2} \tag{26}$$

$$\Phi(x,y)=\exp\Big(ie\int_y^x A_\mu(z)\,dz_\mu\Big) \tag{27}$$

and

$$B^2=\tfrac{1}{4}F_{\mu\nu}F_{\mu\nu}=\tfrac{1}{2}(\vec{B}^2+\vec{E}^2_{Eucl})$$
$$F_{\mu\nu}=\tilde{F}_{\mu\nu}=\tfrac{1}{2}\,e_{\mu\nu\lambda\sigma}F_{\lambda\sigma}$$

For the homogeneous antiselfdual field one should replace

$\gamma_5 \to -\gamma_5$ in (25).

In the zero field limit we obtain the free electron Green function

$$G_0(x,y) = i\int_0^\infty \frac{ds\, e^{-m^2 s}}{4\pi^2 s^2}\left[-\frac{i}{2s}\gamma_\mu (x-y)_\mu + m\right] e^{-\frac{(x-y)^2}{4s}} \tag{28}$$

The long-range behaviour of the Green function in the presence of field is obtained from (24) by the steepest descent method:

$$G(x,y) \sim \exp\left[-\frac{eB}{4}(x-y)^2\right] \tag{29}$$

with the condition $\underline{eB}(x-y)^2 \gg 1$,. We note thet the asymptotics (29) coincides with the nonrelativistic 2 dimensional case (8). A generalization of the electron Green function to the case of a quark in the external homogeneous quasi-abelian field

$$A_\mu = -f_{\mu\nu} \cdot x_\nu \frac{\tau_3}{2} \;;\; F_{\mu\nu} = f_{\mu\nu}\tau_3 \tag{30}$$

is trivial: one should replace everywhere $F_{\mu\nu}$ according to (30), so that terms linear in $F_{\mu\nu}$ acquire a factor τ_3 , while those containing $F_{\mu\nu}$ to an even power are proportional to a unit matrix in the color space.

The representation (24) obtained for the electron (quark) Green function is not convenient to study the zero-mass limit. Namely, there are zero modes satisfying the equation

$$\gamma_\mu \Lambda_\mu \psi = 0 \tag{31}$$

In the operator G (21) for large S (corresponding to small

$\imath^2$ 0) the term $\hat{\Lambda}$ effectively goes to zero, so that the most diverging at $m \to 0$ contribution behaves as $1/m$. This is exactly the part due to the zero modes in the spectral decomposition

$$G(x,y) = \sum_n \frac{\psi_n(x)\,\psi_n^+(y)}{\lambda_n - im} \tag{32}$$

To study the massless limit of G it is convenient to use another representation [11,14-16]:

$$G(x,y) = -\frac{1}{m}\mathcal{P}_0(x,y) + S(x,y) + m\,\Delta(x,y) + O(m^2) \tag{33}$$

where $\mathcal{P}_0$ is the contribution of zero modes

$$\mathcal{P}_0(x,y) = \sum_n \psi_n^{(0)}(x)\,\psi_n^{+(0)}(y) \tag{34}$$

and for $S(x,x')$ there is a connection to a scalar Green function $\Delta(x,x')$ (we keep definition of γ_μ from [9] as in (26), which differs from [14] $\gamma_5(Schw) = i\gamma_5(Br.)$).

$$S(x,y) = \gamma_\mu D_\mu(x)\,\Delta(x,y)\frac{1-i\gamma_5}{2} + \\ + \Delta(x,y)\,\gamma_\mu \overleftarrow{D}_\mu(y)\,\frac{1+i\gamma_5}{2}\,, \quad D_\mu = \partial_\mu - ieA_\mu . \tag{35}$$

$\Delta(x,y)$ was found in [11,14] for the homogeneous selfdual field (modulo $\varphi(x,y)$ factor)

$$\Delta(x,y) = \frac{1}{4\pi^2(x-y)^2}\, e^{-\frac{eB}{4}(x-y)^2} \tag{36}$$

The zero modes (localized at the point z) have the form

[11] (for the field $A_\mu = -\frac{1}{2} F_{\mu\nu} x_\nu$)

$$\psi_z(x) = \frac{eB}{2\sqrt{2}\pi} \exp\left(\frac{ie}{2} z_\nu x_\mu F_{\mu\nu}\right) \cdot \exp\left(-\frac{eB}{4}(x-z)^2\right) \begin{pmatrix} \varphi_n^+ \\ \pm\varphi_n^+ \end{pmatrix} \tag{37}$$

where φ_n satisfy an equation

$$\vec{\sigma}\vec{n}\,\varphi_n^+ = \frac{\vec{\sigma}\vec{B}}{B}\varphi_n^+ = \varphi_n^+$$

and the minus sign in the bispinor (37) refers to the anti-selfdual field. One can see in (35-37) that both zero modes and non-zero modes have confining behaviour and their contribution to the Green function decreases as a Gaussian.

The Fourier transform of (36) is easily obtained to be:

$$\Delta(p) = \int d^4x\, \Delta(x) e^{ipx} = \frac{1}{p^2}\left(1 - e^{-\frac{p^2}{eB}}\right) \tag{38}$$

From (38) it is seen that $\Delta(p)$ has no singularities in the p- plane except at infinity and specifically at $p=0$ $\Delta(p=0) = \frac{1}{eB}$ is finite. At large Euclidean $p^2 > 0$ $\Delta(p)$ behaves as a usual propagator, $\Delta(p) \sim \frac{1}{p^2}$, whereas in the Minkowskian region, $p^2 < 0$, it explodes.

Now we come to the quark-antiquark Green function. Since the quasi Abelian color dependence enters only in the form $A_\mu \sim \tau_3$ and does not influence any physical results, we shall actually derive the Green function for an electron and a positron, while the final results for the polarization operator is qualitatively the same as for quark-antiquark system.

Neglecting the interaction between electron and positron we have for the Green function operator :

$$(\gamma_\mu \Lambda_\mu + m)(\overline{\gamma}_\nu \overline{\Lambda}_\nu + m)\hat{G} = \hat{1} \tag{39}$$

where Λ_μ is the operator (20) for a particle and $\overline{\Lambda}_\nu$ - for an antiparticle. As in (21) we have

$$\hat{G} = (-\gamma_\mu \Lambda_\mu + m)(-\overline{\gamma}_\nu \overline{\Lambda}_\nu + m)\int_0^\infty ds \cdot \exp(-\mathcal{H}' s) \tag{40}$$

where

$$\begin{aligned} \mathcal{H}' &= [m^2 - \hat{\Lambda}^2][m^2 - \hat{\overline{\Lambda}}^2] = \\ &= (m^2 + \Lambda_\mu^2 - \tfrac{1}{2} e \sigma_{\mu\nu} F_{\mu\nu})(m^2 + \overline{\Lambda}_\mu^2 + \tfrac{1}{2} e \overline{\sigma}_{\mu\nu} F_{\mu\nu}) \end{aligned} \tag{41}$$

To separate the gauge-variant factors we write

$$G(x\,\overline{x}\,/x'\overline{x}') = \exp\left(ie\int_{\overline{x}}^{x} A_\mu dz_\mu\right) \cdot f(x\,\overline{x}\,/x'\overline{x}') \exp\left(-ie\int_{\overline{x}'}^{x'} A_\mu dz_\mu\right) \tag{42}$$

For the gauge-invariant piece, f , we have the same representation as (40), (41) with the replacement $\Lambda_\mu \to \tilde{\Lambda}_\mu$ $\overline{\Lambda}_\mu \to \tilde{\overline{\Lambda}}_\mu$, $\mathcal{H}' \to \tilde{\mathcal{H}}'$, where

$$\Lambda_\mu(x) \cdot \exp\left(ie\int_{\overline{x}}^{x} A_\mu dz_\mu\right) = \exp\left(ie\int_{\overline{x}}^{x} A_\mu dz_\mu\right) \tilde{\Lambda}_\mu \tag{43}$$

$$\tilde{\mathcal{H}}' = \mathcal{H}(\tilde{\Lambda}, \tilde{\overline{\Lambda}}) = (m^2 + \tilde{\Lambda}_\mu^2 - \tfrac{1}{2} e \sigma_{\mu\nu} F_{\mu\nu})(m^2 + \tilde{\overline{\Lambda}}_\mu^2 + \tfrac{1}{2} e \overline{\sigma}_{\mu\nu} F_{\mu\nu}) \tag{44}$$

We introduce the center of mass and the relative coordinates

$$X_\mu = \frac{1}{2}(x_\mu + \bar{x}_\mu)\ ;\quad \eta_\mu = x_\mu - \bar{x}_\mu$$
$$P_\mu = \frac{1}{i}\frac{\partial}{\partial X_\mu}\ ;\quad \Pi_\mu = \frac{1}{i}\frac{\partial}{\partial \eta_\mu} \tag{45}$$

to obtain $\tilde{\Lambda}^2, \tilde{\tilde{\Lambda}}^2$ in the following form:

$$\tilde{\Lambda}_\mu^2 = \left(\frac{1}{2}P_\mu + \frac{e}{2}F_{\mu\nu}\eta_\nu + \Pi_\mu\right)^2 \tag{46}$$

$$\tilde{\tilde{\Lambda}}_\mu^2 = \left(\frac{1}{2}P_\mu + \frac{e}{2}F_{\mu\nu}\eta_\nu - \Pi_\mu\right)^2 \tag{47}$$

For heavy quarks we can keep only m^4 and m^2 terms in (44) which results in (modulo spin-dependent unimportant terms)

$$\tilde{\mathcal{H}}_0 - m^4 = m^2(\tilde{\Lambda}^2_\mu + \tilde{\tilde{\Lambda}}^2_\mu) = 2m^2\left[\left(\frac{P_\mu}{2} + \frac{e}{2}F_{\mu\nu}\eta_\nu\right)^2 + \Pi_\mu^2\right] \tag{48}$$

Note the similarity of (48) and (13). Hence all the results of Section 2 are also valid in the 4d space with "Hamiltonian" $\tilde{\mathcal{H}}_0$, which describes the evolution of the states with the proper time.

Therefore the total momentum P_μ can be absorbed by a redefinition of the relative distance η_ν. As a result $\tilde{\mathcal{H}}_0$ will not depend on P_μ and moreover the Green function $f(\eta, \eta', P)$ does not have any singularities in P^2. Hence the total Green function

$$f(\eta, \eta'; X, X') = \int f(\eta, \eta', P)\, e^{iP(X-X')} \frac{d^4P}{(2\pi)^4} \tag{49}$$

is confining, i.e. the quark-antiquark system does not propagate.

This conclusion holds true not only for nonrelativistic particles (for which case (48) is valid) but also in the general case, since in (46) and (47) P_μ enters in the same manner, as in (48). Hence we conclude that in an infinite homogeneous (anti)selfdual field quarks, antiquarks and white noninteracting combinations thereof do not propagate.

The arguments given above can be illustrated by the photon polarization operator in a constant self-dual electromagnetic field

$$\Pi^{(2)}_{\mu\nu}(Q)=\int e^{iQx}\langle 0|T(j_\mu(x)j_\nu(0))|0\rangle d^4x \qquad (50)$$

For massless fermions it was obtained [11-12] *

$$\Pi_{\mu\nu}(Q)=(Q_\mu Q_\nu - Q^2\delta_{\mu\nu})\Big\{-\frac{1}{12\pi^2}\ln\frac{Q^2}{\mu^2}-\frac{e^2B^2}{3\pi^2Q^4}\times$$
$$\times\Big[1-e^{-\frac{Q^2}{2eB}}-\frac{Q^2}{2eB}e^{-\frac{Q^2}{2eB}}+\frac{Q^4}{4e^2B^2}E_1\Big(\frac{Q^2}{2eB}\Big)\Big]\Big\} \qquad (51)$$

Using the properties of $E_1(x)$ [17] one can show, that $\Pi^{(2)}_{\mu\nu}(Q)$ has no singularities at the finite values of Q; it means that i) quarks do not propagate and therefore their mass-shell singularities do not occur; ii) the $q\bar{q}$ system does not propagate as a whole in the selfdual homogeneous field.

---

* Note a misprint in [11] in the coefficients of two last terms in (51), which are corrected in (51). The author is indebted to A.V.Smilga for a discussion of this point.

The asymptotics of $\Pi^{(2)}_{\mu\nu}(Q)$ at large Q^2 can be obtained from (51)

$$\Pi^{(2)}_{\mu\nu}(Q) = (Q_\mu Q_\nu - Q^2\delta_{\mu\nu})\left(-\frac{e^2B^2}{3\pi^2Q^4} - \frac{1}{12\pi^2}\ln\frac{Q^2}{\mu^2} + O(e^{-Q^2})\right) \quad (52)$$

and coincides with the operator expansion [3,15,16] if one replaces $\langle F_{\mu\nu}F_{\mu\nu}\rangle$ by $4B^2$.

The analytic properties of $\Pi_{\mu\nu}$ in this case should be contrasted with the case of the instanton background field [3,11], where there is no confinement and $\Pi_{\mu\nu}(Q)$ has singularities at $Q^2 = 0$ and free quark-mass shell singularities.

4. <u>Quark propagation in a stochastic field</u>

In this Section we take for simplicity Abelian and quasi-abelian nonhomogeneous fields and consider an equation for the Green function of several nonrelativistic charges of equal mass m and zero energy:

$$\sum_i\left(\frac{1}{i}\frac{\partial}{\partial x^{(i)}_\mu} - e_i A_\mu\right)^2 G(x^{(i)}|x^{(i)\prime}) = 2m\,\delta(x^{(i)} - x^{(i)\prime}) \quad (53)$$

As a first step we separate the gauge-dependent exponent, as in (7), (27), and write an equation for the gauge-invariant factor

$$G = \prod_i \Phi(x^{(i)}, x^{(i)}_0)\, f(x^{(i)}, x^{(i)\prime})\, \Phi^+(x^{(i)\prime}, x_0') \quad (54)$$

where $\Phi(x, x_0) = \exp\left(ie\int_{x_0}^{x} A_\mu(z)\,dz_\mu\right)$

Introducing a gauge-invariant quantity:

$$V_\mu(x,x_0)=e\int_0^1 \alpha\, d\alpha\; F_{\nu\mu}[\alpha(x-x_0)+x_0]\cdot(x-x_0)_\nu \tag{55}$$

we obtain for $f(x\bar{x}, x)$ an equation $(x^{(1)}\equiv x,\ x^{(2)}\equiv \bar{x},\ e_2=-e_1=-e)$
$x^{(1)'}\equiv y;\ x^{(2)'}\equiv \bar{y}$

$$\left[\left(\frac{1}{i}\partial_\mu - V_\mu(x,x_0)\right)^2+\left(\frac{1}{i}\bar{\partial}_\mu+V_\mu(\bar{x},\bar{x}_0)\right)^2\right]f=2m\,\varphi^+(x,x_0)\,\varphi^+(\bar{x},\bar{x}_0)\,\varphi(y,y_0)\,\varphi(\bar{y},\bar{y}_0)\,\delta(x-y)\,\delta(\bar{x}-\bar{y}) \tag{56}$$

which is gauge invariant if $y_0=x_0=y$, $\bar{x}_0=\bar{y}=\bar{y}_0$
and also in the case when $x_0=\bar{x}=\bar{x}_0$, $y_0=\bar{y}=\bar{y}_0$.
The form of the path in the exponentials can be arbitrary [18]; for a quark-antiquark system it is convenient to choose a path from $\bar{x}$ to x along the straight line connecting their positions (the same should be done with the y coordinates). This prescription corresponds exactly to the representation (42).

In this case, introducing as in (13) the relative coordinate and momentum $\eta,\ q=\frac{1}{i}\frac{\partial}{\partial\eta}$ and those of the center-of-mass R, P , l.h.s. of (56) can be rewritten as follows

$$2m\hat{H}=\frac{1}{2}P^2+2q^2+U(x,\bar{x})+w_1+w_2 \tag{57}$$

where

$$U(x,\bar{x})=V_\mu^2(x,\bar{x})+V_\mu^2(\bar{x},x) \tag{58}$$

$$w_1=ie\int_0^1(2\alpha^2-2\alpha+1)\,d\alpha\;\eta_\nu\,\partial_\mu F_{\nu\mu}[\alpha(x-\bar{x})+\bar{x}]$$

$$w_2=-e\,P_\mu\,\eta_\nu\int_0^1 d\alpha\; F_{\nu\mu}[\alpha(x-\bar{x})+\bar{x}]- \tag{59}$$
$$-\,2e\,q_\mu\,\eta_\nu\int d\alpha\,(2\alpha-1)\,F_{\nu\mu}[\alpha(x-\bar{x})+\bar{x}]$$

In the 2-dimentional case and the constant field $F_{12}=B$ the

Hamiltonian (57) goes over into (13), and in the 4-dimansional case it coincides with (48).

We now study the influence of stochastic changes of $F_{\mu\nu}$ from one point to another in the (Euclidean) space-time. In addition to that we must consider a stochastic ensemble of field distributions $F_{\mu\nu}$ weighted with some probability function $\Gamma[F_{\mu\nu}]$ (see e.g. [19-21]). We shall assume here that for the fall-off behaviour of Green functions the Birkhoff-Hinchin theorem is applicable([19], chapter 1), stating that the averaging over ensemble can be replaced by the averaging over space dependence at large distances. To this end one must use a representative (i.e. a "typical") potential with stochastic changes in space-time and consider a long-distance behaviour of Green function for such a potential. A tricky point of a correspondence to the ensemble averaging will be discussed in a future publication. As can be seen in (58), $U(x,\bar{x})$ is a quadratic confining potential for constant fields. Suppose now that our d-dimensional space is divided into domains with the same magnitude of magnetic field B, but with random directions (up and down in the 2-dimensional case). This type of vacuum configurations has been suggested in [22]. Let us fix $\bar{x} = 0$ and take x at the distance L from the origin. One can treat U as the square of a 2x4-dimensional vector $\{V_\mu(x,\bar{x}), V_\mu(\bar{x},x)\} \equiv \vec{W}_\mu$. In case of the quantum mechanical problem of Section 2, $\mu = 3$ is fixed and the change $\Delta\vec{W}_3$ caused by passing the n-th domain is $\Delta\vec{W}_3 = \pm Bl\left\{(1-\frac{n}{N}), \frac{n}{N}\right\}$, where the size of domain is l and the field strength is B, and $lN = L$. The problem of an average dependence $U(L) = \vec{W}_3^{\,2}$ is essentially an one-dimensional random-walk problem with changing size of steps. In the 4-d selfdual field case (when only F_{12} and F_{34} are nonzero) the number of random components in $\vec{W}_\mu$ is two and the problem is respectively a two-dimensional random walk problem.

The average displacement squared $(\Delta\vec{z})^2$ after n steps (each of the length a) is

$$(\Delta\vec{z})^2 = Na^2, \tag{60}$$

for slowly changing length a one can replace the r.h.s. of (60) by an integral $\int_1^N a^2(n)dn$. Using (60) in our situation, we obtain for U(L)

$$U(x,\bar{x}) = \frac{e^2}{2}(B\ell)^2\frac{L}{\ell}\cdot b = \text{const}\,|x-\bar{x}| \tag{61}$$

where the coefficient b depends on the number of dimensions, type of domains etc. In this way, we have obtained the linear confining potential between opposite charges (quark and antiquark). It is essential that this potential occurs already in the zeroth order in the coupling constant $e^2 = g^2$, since we assumed $B \sim \frac{1}{g}$ and large, and not due to the gluon exchanges between charges, which have the order $O(g^2)$. The same linear potential occurs in the equation for one-particle Green function that can be seen in the "proper-time"Hamiltonian" (23). Similary, in the equation for two relativistic particles (46-48) the term $\frac{e^2}{4}(F_{\mu\nu}\eta_\nu)^2$ should be replaced in the stochastic case by (61). Note, that in these cases the confining potential does not depend on quark masses and is flavor independent, in accordance with the quark potential model and Monte Carlo calculations.

Let us now consider the terms w_1, w_2 (59). The first one, w_1, in case of constant field domains [22] reduces to a sum of δ-function-type terms of alternating signs, arising from the walls of domains. Therefore w_1 is bounded for growing $L = |x - \bar{x}|$ and its average value is zero, so we shall discard this term in the future. A similar situation occurs for w_2. It is this term which eventually plays important role in the confinement of bound states in the constant field, as it was discussed at the end of Section 2. For a stochastically changing directions of $F_{\mu\nu}$ the term w_2 oscillates around zero average value and may be neglected in the first approximation. Then we obtain a Hamiltonian

$$\bar{H} = \frac{1}{2} P_\mu^2 + 2 q_\mu^2 + C\eta \tag{62}$$

which explicitly realizes our expectations with respect to $q\bar{q}$ interaction: it confines quark and antiquark, but allows for a free propagation of their center of mass.

The latter property can be seen from the $q\bar{q}$ Green function, corresponding to the Hamiltonian (62)

$$G(R,R';x,\bar{x}) = \sum_n \int \frac{d\mathcal{P}\, \varphi_n(x)\, \varphi_n^+(\bar{x})\, e^{i\tilde{\mathcal{P}}(\vec{R}-\vec{R}')}}{\frac{1}{2}\mathcal{P}^2 + \varepsilon_n} \tag{63}$$

or in the 4d space from (40) if we use (for stochastic domains) an analog of (62) instead of (48) for H ; namely

$$\tilde{\mathcal{H}}_0 = \frac{1}{4} P_\mu^2 + C(\eta_\mu^2)^{1/2} + \pi_\mu^2 \tag{64}$$

The poles of the Green function in $\mathcal{P}_\mu^2$ at $\mathcal{P}_\mu^2 = -\varepsilon_n$ coincide with the poles of $\Pi^{(2)}(P)$ and $\Pi^{(2)}(P)$ has no singularity at infinite $\overline{\rho^2}$ and can be continued to the Minkowskian space.

We can take into account w_2 using perturbation theory for stochastic interaction [24]. Since the average of w_2 is zero, the first nonzero contribution to the Green function is of the second order in w_2 which takes the form of self-energy correction and due to (59) is proportional to ρ^2 . Hence an effective mass of the bound state appears, being different from the sum m_1+m_2.

5. Conclusion

We have elaborated in this paper an idea that vacuum background fields may ensure confinement. As an example we have studied in detail propagators and correlators in the homogeneous selfdual field, whereas 2-dimensional problem of charges in magnetic field served as an illustration. It has been shown that charges (quarks) are indeed confined in this field, but $(q\bar{q})$ bound colorless states are confined either. Splitting the homogeneous field into domains with random field directions makes two important phenomena: 1) a quadratic confining potential of the homogeneous field case turns into a linear confining potential; 2) $q\bar{q}$ colorless bound states are not confined and therefore correlators have correct analytic properties.

The resulting confining potential is universal, i.e. depends only on the vacuum properties such as the field strength B and the domain size l , and does not depend on quark mass and flavor. The quaestion why and whether this type of vacuum is realized in QCD is not discussed.

Appendix 1

Here we calculate the Green function of one particle with nonzero energy in the plane with magnetic field. An equivalent to Eq.(6) reads

$$\left[\frac{1}{2m}\left(\bar{p}-e\bar{A}(x)\right)^2-E\right]G(\bar{x},\bar{y},E)=\delta^{(2)}(\bar{x}-\bar{y}) \qquad (A.1)$$

As in the case of zero energy we look for a solution of the form

$$G(\bar{x},\bar{y},E)=\frac{m}{2\pi}\exp\left(i\int_{\bar{y}}^{\bar{x}}A_\mu(z)dz_\mu\right)f\left(\frac{(\bar{x}-\bar{y})^2 eB}{4}\right) \qquad (A.2)$$

For $f(z)$ from (A.1) we obtain an equation:

$$f''+\frac{1}{z}f'+f\left(\frac{2mE}{eBz}-1\right)=0 \qquad (A.3)$$

with the solution

$$f(z)=\sqrt{\pi}\,e^{-z}\,\Psi\left(\frac{1-c}{2},1,2z\right),\ z=(\bar{x}-\bar{y})^2\frac{eB}{4} \qquad (A.4)$$

where $\Psi(\alpha,\gamma,z)$ is the confluent hypergeometric function of the second kind [17] and $c=\frac{2mE}{eB}$

. In case when $E\to 0$ one can use the relation [17]

$$\Psi\left(\tfrac{1}{2},1,2z\right)=\frac{e^z}{\sqrt{\pi}}K_0(z) \qquad (A.5)$$

and one recovers the Green function (7).

The asymptotics of $G(x,y)$ (A.2) at large $|\bar{x}-\bar{y}|$ obtains using the asymptotics of $\Psi(\alpha,\gamma,z)\sim z^{-\alpha}$, so that we get

$$|G(\bar{x},\bar{y},E)| \sim e^{-\frac{(\bar{x}-\bar{y})^2 eB}{4}} \left|(\bar{x}-\bar{y})\sqrt{\frac{eB}{2}}\right|^{\left(\frac{2mE}{eB}-1\right)} \frac{m}{2\sqrt{\pi}} \qquad (A.6)$$

which qualitatively coincides with the zero-energy asymptotics (8).

Appendix 2

Here we derive the nonrelativistic Green function for an electron and a positron in the external constant field B ;

$$A_1 = 2aX_2, \; A_2 = 0 \;, \quad a = -\frac{eB}{2} \qquad (A.7)$$

then the equation for the Green function is

$$2m\mathcal{H}\, G = \left[\left(\frac{1}{i}\frac{\partial}{\partial X_1} - 2aX_2\right)^2 - \frac{\partial^2}{\partial X_2^2} + \left(\frac{1}{i}\frac{\partial}{\partial \bar{X}_1} + 2a\bar{X}_2\right)^2 - \frac{\partial^2}{\partial \bar{X}_2^2}\right] G =$$
$$= 2m\, \delta(x-x')\, \delta(\bar{x}-\bar{x}') \qquad (A.8)$$

In what follows we put $x' = \bar{x}' = 0$

The substitution

$$G = \exp\left(iaX_1X_2 - ia\bar{X}_1\bar{X}_2\right) F(z) \qquad (A.9)$$

where $z = \frac{|a|}{2}(\vec{x}^2 + \vec{\bar{x}}^2)$, yields an equation for F(z):

$$\frac{d^2}{dz^2}F + \frac{2}{z}\frac{d}{dz}F - F = 0 \qquad (A.10)$$

with the solution

$$F(z) = C\,\frac{e^{-z}}{z} \qquad (A.11)$$

The constant C can be found integrating both sides of (A.8) around $X = \bar{X} = X' = \bar{X}' = 0$; in this way we obtain $C = \frac{m|a|}{4\pi^2}$. The exponent in (A.9) can be rewritten in the form which reveals its gauge dependence:

$$\exp(iaX_1X_2 - ia\bar{X}_1\bar{X}_2) = \exp\left(ie\int_0^{\vec{X}} A_\mu dz_\mu - ie\int_0^{\vec{\bar{X}}} A_\mu dz_\mu\right) \tag{A.12}$$

Note that for $X' \neq 0$ one would obtain the usual gauge-variant factors

$$\exp\left(ie\int_{X'}^{X} A_\mu dz_\mu\right) \cdot \exp\left(-ie\int_{\bar{X}'}^{\bar{X}} A_\mu dz_\mu\right) \tag{A.13}$$

Therefore one realizes that F(z) is a gauge-invariant quantity. The large distance behaviour of G from (A.11) is confining for both particles, including their center of mass supporting the arguments of Section (2), given after the Eqs. (9) and (13).

Another way to obtain the Green function is via the coordinates $\vec{\eta} = X - \bar{X}$, $\vec{R} = \frac{\vec{X}+\vec{\bar{X}}}{2}$. If the Green function has the form

$$G = \exp ie\int_{\bar{X}}^{X} A_\mu dz_\mu \cdot f \tag{A.14}$$

then for f we have an equation $(\bar{X} = 0, \bar{X}' = 0)$

$$\left[\frac{1}{4m}\left(\frac{1}{i}\frac{\partial}{\partial \vec{R}} - e(\vec{B}\times\vec{\eta})\right)^2 + \frac{1}{m}\left(\frac{1}{i}\frac{\partial}{\partial \vec{\eta}}\right)^2\right] f = \delta(\vec{R})\,\delta(\vec{\eta}) \tag{A.15}$$

The solution of (A.15) is readily obtained similarly to (A.9):

$$f = \exp\frac{ie}{2}(\vec{B}\times\vec{R})\vec{\eta} \cdot \frac{eBm}{8\pi^2 z}\exp(-z) \tag{A.16}$$

where z is defined in (A.9). Comparing (A.16) and (A.9) we see that both expressions coincide up to a phase factor, which is different in both cases. This fact is due to the gauge dependence: in (A.7) we have $A_\ell = 0$, whereas in (A.15) $\vec{A} = \frac{1}{2}(\vec{B}\times\vec{\eta})$. We note that the phase factor is gauge variant, while F(z) is the same in both derivations as it should be for gauge invariant quantities.

Appendix 3

Hamiltonian of two charges with unequal masses m_1 and m_2 in the magnetic field B_3

Assuming in (10) unequal masses and introducing relative and center of mass coordinates we obtain

$$\tilde{H} = \frac{1}{2M}\left(\vec{P} - e(\vec{B}\times\vec{\eta})\right)^2 + \frac{e^2(m_1-m_2)^2}{8\mu M^2}(\vec{\eta}\times\vec{B})^2 + \\ + \frac{\vec{q}^{\,2}}{2\mu} + \frac{e(m_1-m_2)}{2m_1m_2}\vec{B}(\vec{\eta}\times\vec{q}) \tag{A.17}$$

where $M = m_1 + m_2$, $\mu = \frac{m_1 m_2}{m_1+m_2}$, $\vec{P} = \frac{1}{i}\frac{\partial}{\partial \vec{R}}$, $\vec{R} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1+m_2}$

$\vec{\eta} = \vec{r}_1 - \vec{r}_2$.

Literature

1 Callan C.G., Dashen R., Gross D.J. - Phys.Rev., 1978, D17, 2717; ibid. 1978, D19, 1826.

2 Novikov V.A., Shifman M.A., Vainshtein A.I. and Zakharov V.I. - Nucl.Phys., 1981, B191, 301.

3 Shifman M., Vainshtein A., Zakharov V. - Nucl.Phys., 1979, B147, 385, 448.

4 Geshkenbein B.V., Ioffe B.L. - Nucl.Phys., 1980, B166, 340.

5 Shuryak E.V. - Nucl.Phys., 1982, B203, 93, 116.

6 Dyakonov D.I., Petrov V.Yu. - Nucl.Phys., 1984, B245, 259; ibid. 1986, B272, 457.

7 Simonov Yu.A. - Vacuum Background Fields in QCD as a Source of Confinement, Preprint ITEP N68, 1987.

8 Anderson P.W. - Phys.Rev.,1958, 109, 1492. Progr.Theor.Phys.Suppl., 1985, N84, Y.Nagaoka ed.

9 Schwinger J. - Phys.Rev., 1951, 82, 664.

10 Mil'shtein A.L., Pinelis Yu.F. - Phys.Lett., 1984, B137, 235.

11 Dubovikov M.S., Smilga A.V. - Nucl.Phys., 1981, B185,109.

12 Finjord J. - Nucl.Phys., 1982, B194, 77.

13 Geiler V.A., Margulis V.A. - Teor.Mat.Fiz., 1987, 70, 192.

14 Brown L.S. et al. - Phys.Rev., 1978, D17, 1583.

15 Novikov V.A., Shifman M.A., Vainshtein A.I., Zakharov V.I. - Nucl.Phys., 1980, B174, 378-396.

16 Novikov V.A., Shifman M.A., Vainshtein A.I.,Zakharov V.I. Fortschr.Phys., 1984, 32, 585.

17 Abramowitz M., Stegun I. - Handbook of Mathematical Func

tions, Natl Bureau of Standards.

18 Mendelstam S. - Ann.Phys. **19**, 1962, 1.

19 Lifshitz I.M., Gredescul S.A., Pastur L.A. - Introduction to the Theory of Disordered Systems, Moscow, Nauka, 1982.

20 Mc Kane A.J., Stone M. - Ann.Phys., 1981, 131, 36.

21 Zittartz J., Langer J.S. - *Phys.Rev.*, 1966, 148, 741.

22 Simonov Yu.A. - Toronic Domains in the QCD Vacuum, Preprint ITEP N14, 1987; Yad.Fiz., 1987, 46, 328.

23 Feller W. - An Introduction to Probability Theory and its Applications, v.I, Chapter 14, John Wiley and Sons, NY.

24 Abrikosov A.A., Gor'kov L.P., Dzyaloshinsky I.E. - Methods of Quantum Field Theory in Statistical Physics, Moscow, GIFML, 1962.

Ю.А.Симонов

Пропагаторы и корреляторы в вакууме с конфайнментом.

Работа поступила в ОНТИ 5.06.87

Подписано к печати I7.06.87 T13499 Формат 60х90 I/I6
Офсетн.печ. Усл.-печ.л.I,75. Уч.-изд.л.I,3. Тираж 290 экз.
Заказ 99 Индекс 3624 Цена I9 коп.

Отпечатано в ИТЭФ, II7259, Москва, Б.Черемушкинская, 25

ИНДЕКС 3624

М.,ПРЕПРИНТ ИТЭФ, 1987, № 99, с.1-29