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DEVELOPMENT OF A COMPUTER CODE 'CRACK' FOR ELASTIC AND ELASTOPLASTIC FRACTURE MECHANICS ANALYSIS OF 2–D STRUCTURES BY FINITE ELEMENT TECHNIQUE

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SINGULARITY

ABSTRACT

The fracture mechanics analysis of nuclear components is required to ensure prevention of sudden failure due to dynamic loadings. The linear elastic analysis near to a crack tip shows presence of stress singularity at the crack tip. The simulation of this singularity in numerical methods enhance convergence capability. In finite element technique this can be achieved by placing mid nodes of 8 noded or 6 noded isoparametric elements, at one fourth distance from crack tip. Present report details this characteristic of finite element, implementation of this element in a code 'CRACK', implementation of J-integral to compute stress intensity factor and solution of number of cases for elastic and elastoplastic fracture mechanics analysis.

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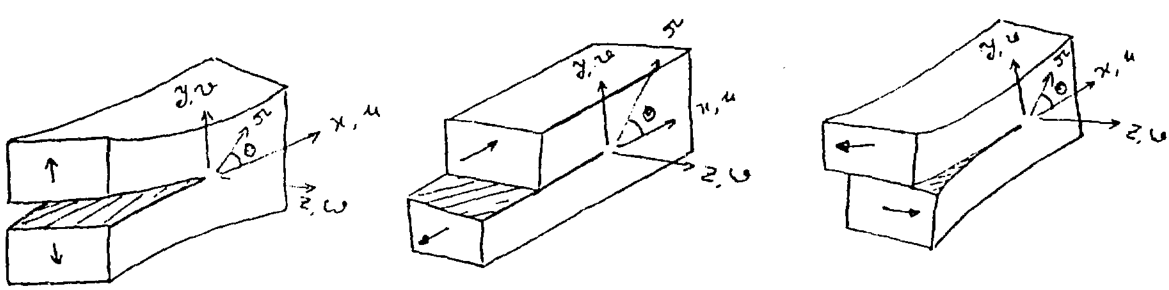
1. **Introduction :-** Cracks may be formed in a structure or a component due to fabrication and/or installation error or it may be due to mechanical and thermal cyclic loadings during inservice operation. Presence of a crack acts like a stress riser and may lead to failure of the component at loads much below its design capability. For nuclear components analysis of components with existing cracks is necessary to judge their safety against sudden failures. This necessitates the development of necessary capability to analyse the structures with a crack.

Present report details the development of a computer code "CRACK" for elastic and elastoplastic fracture mechanics analysis of a 2-D structures under the presence of a crack by finite element technique. The stress intensity factor is calculated by J integral for different paths surrounding the crack tip. The code is modified to analyse orthotropic materials also. Number of cases have been solved to check the correctness and accuracy of this code.

2. Theoretical Requirement of Singularity Element



Mode 1 (opening) Mode 2 (Sliding) Mode 3 (tearing)

Upper figures illustrate the three deformation modes that can exist near a crack. Theory provides expressions for linear elastic stresses $\{\sigma\}$ and displacements $\{\delta\}$ in the immediate vicinity of a crack tip. These expressions are independent of overall geometry and have the form.—

$$\{\sigma\} = \frac{1}{\sqrt{r}} \left(K_I \{\sigma_1\} + K_{II} \{\sigma_2\} + K_{III} \{\sigma_3\} \right)$$

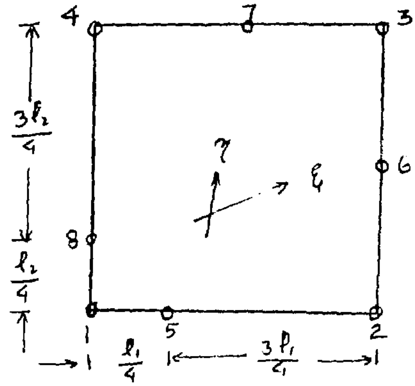
$$\{\delta\} = \sqrt{r} \left(K_I \{\delta_1\} + \text{[illegible]} \} + K_{III} \{\delta_3\} \right)$$

Here r is the distance from the crack tip. Vectors $\{\sigma_i\}$ are functions of θ only, and vectors $\{\delta_i\}$ are functions of θ and material properties. Factors K_I, K_{II} and K_{III} are stress intensity factors for the respective modes, as shown above. Their magnitude depends on the geometry of the specimen and crack and on the distribution and magnitude of loading. Stress intensity factors are tabulated in handbooks for several geometries and loadings. In cases that are not tabulated the factors can be found by the finite element methods.

Irrespective of w[illegible]ther the material is isotropic or orthotropic, there is a stress singularity (as well as strain singularity) of order $1/\sqrt{r}$ at the crack tip. This can be also seen that the various displacements from crack tip (u, v) vary as a function of $\sqrt{r}$. When such a variation of stress and displacement is modelled using the conventional finite [illegible] problems arise. First, a very fine discretization is required near the crack tip. Second, the rate of convergence is slow. Tong and [illegible] [1] first showed that the convergence rate is dominated by the [illegible]. Therefore, the convergence rate can be enhanced using elements th[illegible] approximate the singular strain/stress field at the crack tip properly.

3. Isoparametric Finite Element - Quarter Point Singularity Element

Out of many crack tip elements, the quarter point singularity element is found to be widely accepted. This makes use of isoparametric finite element concept [2]. The singularity in these elements is achieved by placing the mid side node near the crack tip at the quarter point.



Upper figure shows an eight - noded quadrilateral with mid side nodes of two sides at the quarter points. Investigating the role of this placement along the side 1-5-2, we have shape functions along this line as :

$$N_1 = \frac{-\xi}{2}(1-\xi) \quad ; \quad N_2 = \frac{\xi}{2}(1+\xi) \quad ; \quad N_5 = (1-\xi^2)$$

hence using isoparametric concept

$$x = -\frac{\xi}{2}(1-\xi)x_1 + \frac{\xi}{2}(1+\xi)x_2 + (1-\xi^2)x_5$$

choosing $x_1 = 0$; $x_2 = L$; $x_5 = \frac{L}{4}$, we have

$$x = \frac{\xi}{2}(1+\xi)L + (1-\xi^2)L/4$$

or $$\xi = \left(2\sqrt{\frac{x}{L}} - 1\right)$$

and Jacobian term

$$\frac{\partial x}{\partial \xi} = \frac{L}{2}(1+\xi) = \sqrt{\frac{x}{L}}$$

considering only the displacements of points 1, 2 and 5, the displacement u along the line 1-2 is given by

$$u = -\frac{\xi}{2}(1-\xi)u_1 + \frac{\xi}{2}(1+\xi)u_2 + (1-\xi^2)u_5$$

writing in terms of x,

$$u = -\frac{1}{2}\left(-1 + 2\sqrt{\frac{x}{L}}\right)\left(2 - 2\sqrt{\frac{x}{L}}\right) u_1 + \frac{1}{2}\left(-1 + 2\sqrt{\frac{x}{L}}\right)\left(2\sqrt{\frac{x}{L}}\right) u_2 + \left(4\sqrt{\frac{x}{L}} - 4\frac{x}{L}\right) u_5$$

The strain in the x direction is then

$$\varepsilon_x = \frac{\partial u}{\partial x} = J^{-1}\frac{\partial u}{\partial \xi} = \frac{\partial \xi}{\partial x}\cdot\frac{\partial u}{\partial \xi}$$

$$= -\frac{1}{2}\left(\frac{3}{\sqrt{xL}} - \frac{4}{L}\right) u_1 + \frac{1}{2}\left(-\frac{1}{\sqrt{xL}} + \frac{4}{L}\right) u_2 + \left(\frac{2}{\sqrt{xL}} - \frac{4}{L}\right) u_5$$

The strain singularity along the line 1-2 is therefore $1/\sqrt{x}$, which is the required singularity for elastic analysis.

Similarly it can be shown that required singularity is obtained along line 1-8-4, by placing node 8 at quarter point towards node 1. But this does not confirm singularity at node 1 in any direction in the element. It is then proved that this behaviour in a triangular element is observed when the two sides meeting at the crack-tip are equal, i.e. the triangle is isosceles, and the node coinciding with the crack-tip is specified as the node of coalescence to obtain triangular element from a quadrilateral [3].

4. <u>Computation of Stress Intensity Factor by J Integral</u> -

There are various methods to compute stress intensity factor near to a crack by finite element technique. One of the methods is direct evaluation of energy release by involving only a single analysis, propose by Rice. This involves calculation of an integral surrounding the crack tip. This integral, known as the J integral, is path independent and is of the form [4].

$$J = \int_{\Gamma} \left(U - \sigma_x \frac{\partial u}{\partial x} - \tau_{xy} \frac{\partial v}{\partial x} \right) dy + \int_{\Gamma} \left(\tau_{xy} \frac{\partial u}{\partial x} + \sigma_y \frac{\partial v}{\partial x} \right) dx$$

in two-dimensional stress field. Here U is the strain energy density, given in linear elasticity by

$$U = \frac{1}{2} \{\sigma\}^T \{\varepsilon\}$$

and x is the direction of the propagating crack. Once calculated, J can be used to calculate stress intensity factor from the following relationships.

$$K_I = \left(\frac{JE}{1-\nu^2}\right)^{1/2} \text{ for plane strain}$$

$$= (JE)^{1/2} \text{ for plane stress}$$

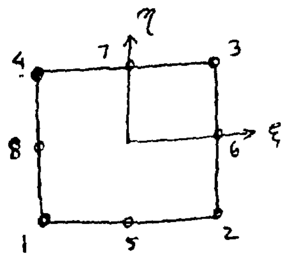
A special advantage of the above integral form is that it is also applicable in non-linear elastic cases and indeed with suitable modifications can be used for elastic-plastic investigation.

According to the theoretical formulation, this integral should be independent of the path Γ. But, because of the fact that the finite element solution is approximate, the integral becomes marginally path-dependent. It is recommended that the maximum value of J integral should be used to evaluate the stress intensity factor.

5. Computer Code "CRACK"

In one existing computer code for plane stress, plane strain and axisymmetric elastoplastic analysis, [5], following modifications are done for crack tip analysis and evaluation of J integral.

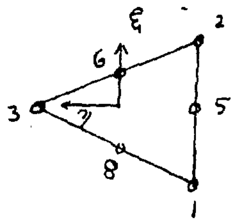
a) The triangular isoparametric element is implemented by collapsing one mid node and two corner nodes of a quadrilateral isoparametric element. The following modifications are done in shape functions for this purpose.



$$N_1 = N_1^* - \frac{1}{2}(N_8 + N_5)$$
$$N_2 = N_2^* - \frac{1}{2}(N_5 + N_6)$$
$$N_3 = N_3^* - \frac{1}{2}(N_7 + N_6)$$
$$N_4 = N_4^* - \frac{1}{2}(N_7 + N_8)$$

Here * represents shape functions of Corner nodes for four noded element.

For this case $N_4 = 0$; $N_7 = 0$



Hence

$$N_1 = N_1^* - \frac{1}{2}(N_8 + N_5)$$
$$N_2 = N_2^* - \frac{1}{2}(N_5 + N_6)$$
$$N_3 = N_3^* - \frac{1}{2}(N_6)$$

b. The integral for J is implemented for the computation along different paths of the discretised structure, identified by gauss points of different elements. Proper modifications are also done to extent the path upto the surface of the crack. Strain energy density U is calculated by incremental theory, so that expression should be applicable also to elasto-plastic case. The values of $\frac{\partial u}{\partial x}$ and $\frac{\partial v}{\partial x}$ are calculated at gauss points from elemental nodol values by the help of shape functions as follows:

$$\frac{\partial u}{\partial x} = \sum_{i=1}^{n} \frac{\partial N_i}{\partial x} u_i \quad ; \quad \frac{\partial v}{\partial x} = \sum_{i=1}^{n} \frac{\partial N_i}{\partial x} v_i$$

c) The program is also modified to take into consideration orthotropic material . This modification is confined to plane stress case only. The [D] matrix for this case is given by:

$$[D] = \frac{E_x}{E_x - \nu_{xy}^2 E_y} \begin{bmatrix} E_x & \nu_{xy} E_y & 0 \\ \nu_{xy} E_y & E_y & 0 \\ 0 & 0 & \frac{(E_x - \nu_{xy}^2 E_y)}{E_x} G_{xy} \end{bmatrix}$$

6. Identifications of main variables –

LNCDS	- element - node array
CORD	- nodal coordinates array
MAT	- element material number array
NBC, NFIX, FIX	- boundary node numbers, fixidity code and specified displacement arrays
DD	- material properties array
SG, TG, WG	- arrays for storing gauss point coordinates and weights for numerical integration
RCL	- concentrated direct load
PR	- normal pressure value
X LOAD, Y LOAD	- if pressure is not normal, its x-y components
NBN	- node no. on which pressure is acting
TEMP	- nodal temperature for thermal loads
R	- load vector for every load incre ment
ESTIF	- element stiff ness matrix
S	- assembled stiffness matrix
DIS	- nodal displacements
STRS	- Gauss point stresses
SIGBAR	- stress intensity for different gauss points
DEP	- $[D]^{ep}$ matri
W	- $\lfloor W \rfloor$ matrix for plasticity calculation
UNBFRC	- Unbalanced force
FACLOD	- fraction of loads for every iteration
SE	- strain energy density for all gauss points
NPJINT	- identification of element and gauss points
DUDVDX	- Computation of $\frac{\partial u}{\partial x}$, $\frac{\partial v}{\partial x}$ from nodal values

7. Solved Cases -

Many cases were solved to test the propr implementation of above theory and also test the accuracy of the results obtained by the present code.

Case - 1 :- This is a case of linear elastic fracture mechanics analysis of bar with cracks using quarter tip finite elements as shown in fig. Bar is assumed to be in plane strain condition. In all 20, eight noded elements connected by 67 nodes are used for discretization. J integral is calculated for four paths. The values obtained are 223, 224, 221 and 225. Corresponding stress intensities are 21.55, 21.60, 21.45 and 21.65. These values compare well with the theoretical value of 20.4. Little difference are attributed to the coarse discretization.

Case -2: This case is the analysis of a compact test specimen under plane strain condition [7] as shown in fig.2. Only elastic case is considered here for computing J integral. The theoretical solution for stress intensity factor for this case is given by

$\frac{K_I BW}{\sqrt{a}\, P} = 13.58$ which computes the value of K_I as 1347·267 for p=1000. Corresponding J value is 78.65. Two paths are used in finite element analysis to compute J. These values are obtained as 79.4 and 78.2 which match excellently with the above quoted theoretical value.

Case 3 - This is the case of a cracked bar under plane stress condition [7] as shown in fig.3. Isotropic material with elastic deformation are considered in the analysis. The theoretical value of $K_I = 1.1721\sigma\sqrt{\pi a} = 20.77493$, which in turn gives $J_I = 0.75719$. Three paths are considered for computing J and values obtained are 0.758, 0.756 and 0.760, which again have excellent matching with the theoretical value.

Case 4 : This is the case of an orthotropic material. The geometrical and material properties are shown in fig.4. J values are calculated for three paths surrounding crack tip. The values obtained are 0.1372 0.1342 and 0.1338.

Case 5 - The case of compact test specimen defined in case 2 above is analysed for elastic - plastic case as shown in fig.5. The applied loading is displacement control. J values for different crack opening displacements for this case are quoted in ref. 6. Fig. 6 shows the variation of J with δ as obtained in this analysis and also as given in[6]. A close trend can be seen between the two variations. The difference is attributed to the different types of elements used in these two analyses. Ref. 6 used C.S.T. element, whereas in the present case 8 noded isoparametric elements have been used.

8. Future work

To enhance the capability in this area, it is decided to formulate and develop necessary computer codes for the following cases -

(a) Elasto plastic interaction of two neighbouring cracks.

(b) Analysis of kinked cracks.

(c) Interaction of two neighbouring kinked cracks.

(d) Experimental verification of results obtained for above cases.

9. Reference

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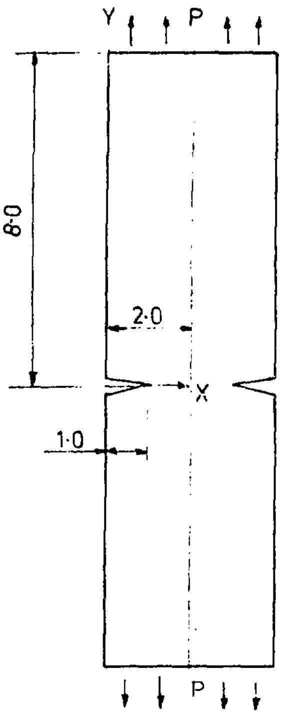


INPUTS –

E = 1·0

ν = 0·2

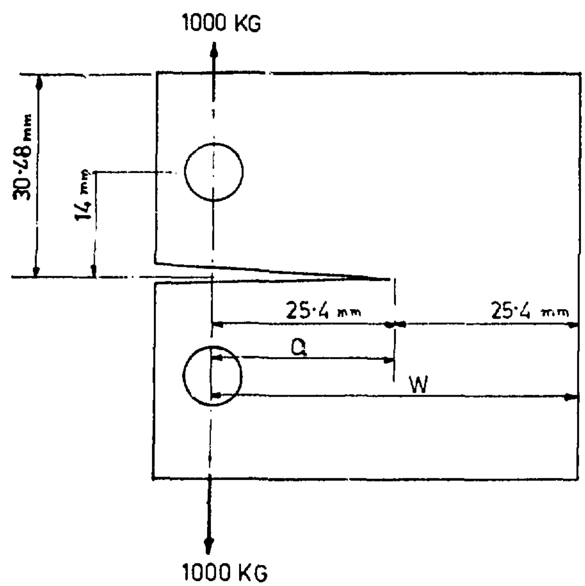
P = 10·0

K_I | THEORETICAL = 20·4

RESULTS –

PATH NO	J-VALUE	K_I
1	223	21·5547
2	224	21·602
3	221	21·456
4	225	21·650

FIG.1 LINEAR ELASTIC FRACTURE MECHANIC ANALYSIS OF BAR WITH CRACKS (A PLANE STRAIN CASE)



<u>INPUTS</u>

$E = 2 \cdot 1 \times 10^4 \; KG/mm^2$

$\nu = 0 \cdot 3$

$B = 1 \cdot 0$

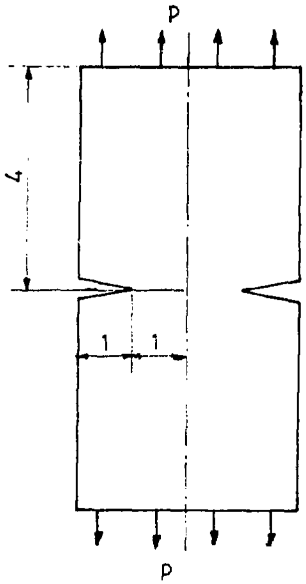
$K_I \mid THEORETICAL = \dfrac{13 \cdot 58 \sqrt{a} \; P}{BW} = 1347 \cdot 267$

$J \mid THEORETICAL = 78 \cdot 65$

<u>RESULTS</u>

PATH NO	J
1	79·4
2	78·2

FIG. 2 ANALYSIS OF COMPACT TEST SPECIMEN.
(A PLANE STRAIN CASE)



INPUTS

$E = 570\ N/mm^2$

$\nu = 0.0178$

$P = 10\ N/mm^2$

$K_I|$ THEORETICAL $= 1.1721\ P\sqrt{\pi a}$

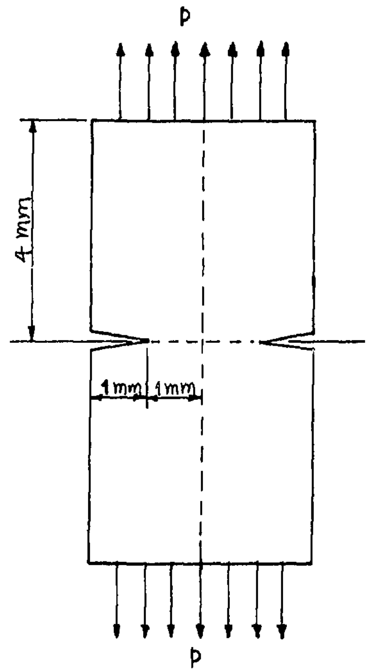
$= 20.77493$

$J|$ THEORETICAL $= 0.75719$

RESULTS

PATH NO	J
1	0·758
2	0·756
3	0·760

FIG.3 ANALYSIS OF A CRACKED BAR.
(A PLANE STRESS CASE)



INPUTS —

$E_x = 570 \ N/mm^2$

$E_y = 16300 \ N/mm^2$

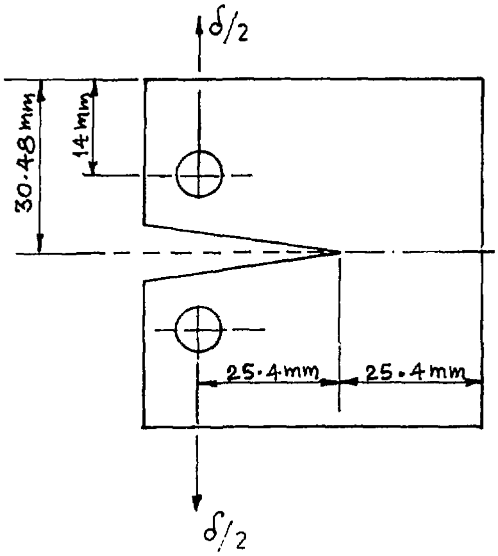
$\nu_{xy} = 0{\cdot}0178$

$P = 10 \ N/mm^2$

RESULTS —

PATH NO.	J - VALUE
1	0·1372
2	0·1342
3	0·1338

FIG. 4 ANALYSIS OF AN ORTHOTROPIC CRACKED BAR (A PLANE STRESS CASE)



INPUTS ——

$$E = 2{\cdot}1 \times 10^4 \ Kg/mm^2$$

$$\nu = 0{\cdot}3$$

$$\sigma_y = 95 \ Kg/mm^2$$

$$H'/E = 0{\cdot}01$$

RESULTS —— ATTACHED GRAPH SHOWS VARIATION OF J WITH δ

FIG.5 ELASTOPLASTIC ANALYSIS OF COMPACT TEST SPECIMEN (A PLANE STRAIN CASE)

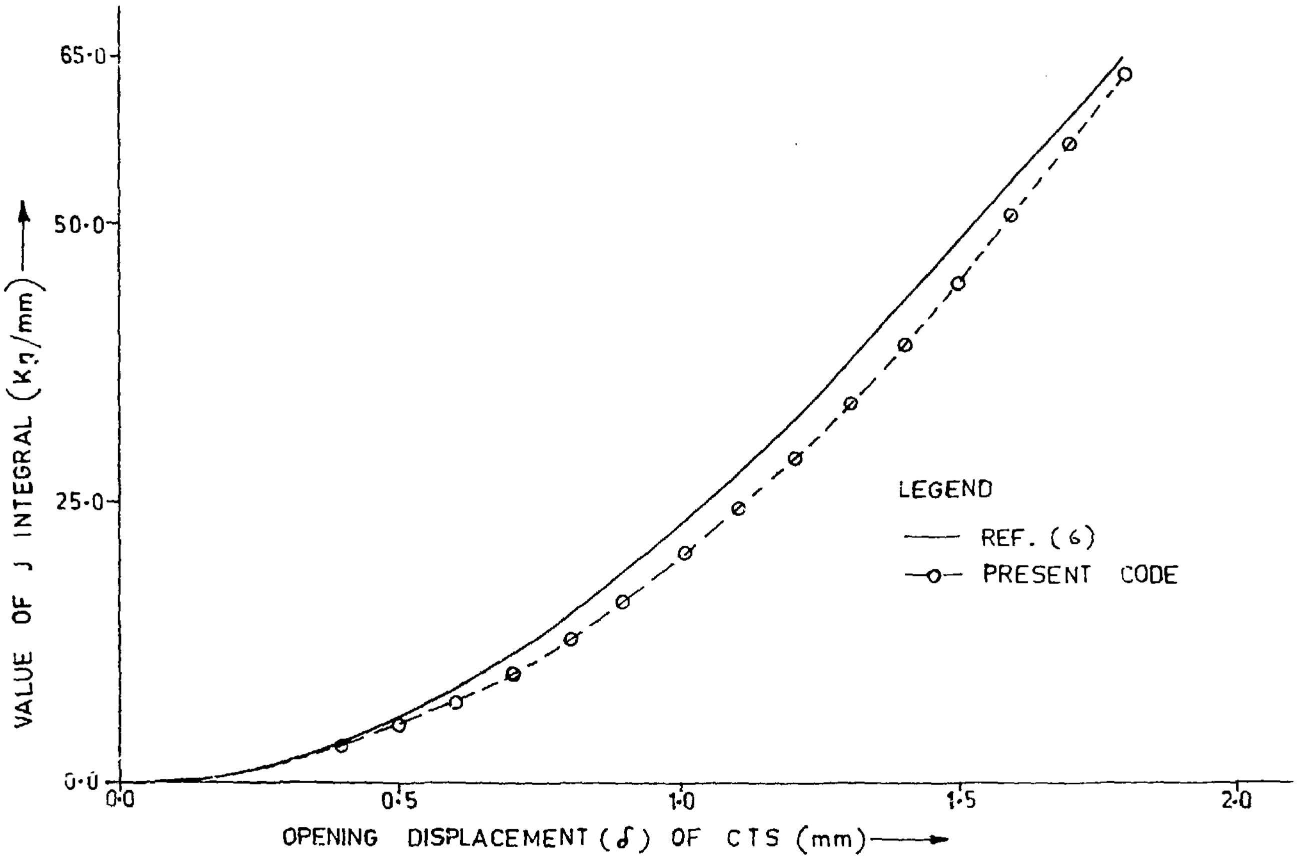


FIG. 6 — J-δ ELASTOPLASTIC VARIATION OF COMPACT TEST SPECIMEN

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