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SOLITARY ALFVÉN WAVE ENVELOPES AND THE MODULATIONAL INSTABILITY

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ABSTRACT

The derivative nonlinear Schrödinger equation describes the modulational instability of circularly polarized dispersive Alfvén wave envelopes. It also may be used to determine the properties of finite amplitude localized stationary wave envelopes. Such envelope solitons exist only in conditions of modulational stability. This leaves open the question of whether, and if so, how, the modulational instability produces envelope solitons.

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I. INTRODUCTION

Gul'yel'mi and Bondarenko (1978) and Ovenden et al. (1983) have proposed that modulational Alfvén solitons may populate the solar wind and the region upstream of the bow shock. In our view, such envelope solitons may be a useful description of the nonlinear dispersive circularly polarized turbulence often observed in space. It is plausible to suggest that solitons would be the eventual outcome of the modulational instability of finite amplitude dispersive Alfvén waves (e.g., Sakai and Sonnerup, 1983, Wang and Goldstein, 1986, and references therein). However, in this paper, we revisit the theory of Alfvén modulational solitons (Ichikawa et al., 1980), to show that solitary wave envelopes exist only for conditions of modulational stability.

II. THE DERIVATIVE NONLINEAR SCHRÖDINGER EQUATION

Plane circularly polarized parallel Alfvén waves obey the derivative nonlinear Schrödinger (DNLS) equation,

$$\frac{\partial \phi}{\partial \tau} + \alpha \frac{\partial}{\partial \eta} \{\phi \, (|\phi|^2 - |\phi_o|^2)\} = iu \frac{\partial^2 \phi}{\partial \eta^2} \quad , \tag{1}$$

where $\phi = (B_y \mp iB_z)/B_x$ for right-handed waves ($u > 0$) and left-handed circularly polarized waves ($u < 0$), respectively. B_x is the constant component of magnetic field in the direction of propagation. When terms of order the ratio of the square root of the electron to ion mass ratio are neglected, $|u| = 1/2 \; c/\omega_{pi}$ is the ion inertial scale length, where c is the speed of light, and ω_{pi} is the ion plasma frequency. The quantity $|\phi_o|^2$ is a constant which we will identify with the circularly polarized amplitude at $\eta = \infty$; it may always be removed from Eq. (1) by Galilean transformation.

Equation (1) is written in a coordinate system moving with the Alfvén speed C_A to the right, where

$$\eta = x - C_A t, \qquad \tau = C_A t \tag{2a, b}$$

$$C_A^2 = B_x^2/(4\pi\rho_o) \quad , \tag{2c}$$

where ρ_o is the mass density at the point where $|\phi|^2 = |\phi_o|^2$. The quantity α is defined by

$$\alpha = \frac{1}{4}\,\frac{C_A{}^2}{C_A{}^2 - C_S{}^2} \quad , \quad C_S{}^2 = \gamma P_o/\rho_o \quad , \tag{3a, b}$$

where C_S is the sound speed.

The DNLS equation describes the coupling between two parallel propagating Alfvén waves whose frequency shifts due to dispersion are comparable to those due to nonlinearity. Equation (1) was first derived by Rogister (1971) who used kinetic theory. It was derived in nondispersive fluid theory ($\mu = 0$) by Cohen and Kulsrud (1974), and in quasi-neutral two-fluid theory by Mio et al. (1976) for $C_S{}^2 = 0$. The generalization to finite $C_S{}^2$ is trivial, so long as the "quasi-static" approximation holds for the density ρ:

$$\frac{\rho - \rho_o}{\rho_o} \doteq \frac{\alpha(|\phi|^2 - |\phi_o|^2)}{2} \quad . \tag{4}$$

Equations (3) and (4) are not valid if $C_A{}^2 - C_S{}^2$ is of order $(|\phi|^2 - |\phi_o|^2)$, but it is valid if $C_A{}^2$ differs enough from $C_S{}^2$ that nonlinear couplings to sound waves may be neglected. For a detailed discussion, see Sakai and Sonnerup (1983).

The complex differential equation (1) is equivalent to two real differential equations, which may be obtained by substituting $\phi = b(\eta,\tau)\exp i\theta(\eta,\tau)$, where b is real and positive, and θ is real,

$$\frac{\partial u}{\partial \tau} + \alpha\,(3b^2 - b_o^{\,2})\,\frac{\partial b}{\partial \eta} + \frac{\mu}{b}\,\partial/\partial\eta\,(b^2\,\partial\theta/\partial\eta) = 0 \quad , \tag{4a}$$

$$\frac{\partial \theta}{\partial \tau} + \alpha\,(b^2 - b_o^{\,2})\,\frac{\partial \theta}{\partial \eta} + \mu\,\left(\frac{\partial u}{\partial \eta}\right)^2 = \frac{\mu}{b}\,\frac{\partial^2 b}{\partial \eta^2} \quad . \tag{4b}$$

We will consider local circularly polarized modulations of a finite amplitude carrier wave of the same sense of polarization. We force satisfaction of this boundary condition by writing

$$\theta = k_o\eta - \Omega_o\tau + \psi(\eta,\tau) \quad , \tag{5a}$$

where (k_o,Ω_o) are the wave number and frequency of the carrier measured in the Alfvén frame, and $\psi(\eta,\tau)$ is the phase modulation. We assume that Ω_o obeys the carrier wave dispersion relation when $b^2 = b_o^{\,2}$,

$$\Omega_o = \mu k_o^{\,2} \quad . \tag{5b}$$

The relation (5b) is valid so long as $b = b_o$ = constant, and does not necessarily require that b is small.

For calculational convenience, we will make a further Galilean transformation to the coordinate system

$$\xi = \eta - (2\mu k_o - \alpha b_o^2)\tau, \quad \tau' = \tau \quad , \tag{6}$$

although we will return to the Alfvén frame (2 a,b) to express our final results. Note that $2\mu k_o = d\Omega_o/dk_o$ is the group velocity of the carrier wave. Henceforth we will drop the superscript prime on τ. We also define normalized quantities

$$b = b_o w \quad , \qquad A = \alpha b_o^2 \quad , \tag{7 a,b}$$

and multiply (4b) by μ to obtain

$$\frac{1}{2}\frac{\partial w^2}{\partial \tau} + \frac{\partial}{\partial \xi}\left(\frac{3Aw^4}{4}\right) + \frac{\partial}{\partial \xi}\left(w^2 \mu \frac{\partial \psi}{\partial \xi}\right) = 0 \quad , \tag{8a}$$

$$\mu \frac{\partial \psi}{\partial \tau} + Aw^2 \left(\mu \frac{\partial \psi}{\partial \xi}\right) + \left(\mu \frac{\partial \psi}{\partial \xi}\right)^2 + \mu k_o A (w^2 - 1) = \frac{\mu^2}{w}\frac{\partial^2 w}{\partial \xi^2} \quad . \tag{8b}$$

III. THE MODULATIONAL INSTABILITY

We substitute

$$W = 1 + \delta W \exp i\,(k\xi - \omega\tau), \quad \psi = \delta\psi \exp i\,(k\xi - \omega\tau) \quad , \tag{9 a,b}$$

into (8 a,b) to obtain the small amplitude dispersion relation,

$$(V - 2A)^2 - \left[A(A + 2\,\mu k_o) + k^2\mu^2\right] = 0 \quad , \tag{10a}$$

where $V = \omega/k$ is the phase velocity in the ξ-frame. We now take $k^2\mu^2 \ll 1$ to obtain the linear dispersion relation for long wavelength modulations of the carrier wave,

$$D_L = (V-2A)^2 - \left[A(A + 2\mu k_o)\right] = 0 \quad , \tag{11a}$$

which has solutions,

$$V = 2A \pm \sqrt{A(A + 2\mu k_o)} \quad , \tag{11b}$$

that transform to

$$V_A = A + d\Omega_o/dk_o \pm \sqrt{A(A + d\Omega_o/dk_o)} \quad , \tag{11c}$$

where V_A is the phase velocity in the Alfvén frame.

Equations (11 a,b,c) predict instability when $A(A + 2\mu k_o) < 0$. When $C_S^2 < C_A^2$, the RHCP mode ($\mu > 0$) is stable, and the LHCP mode ($\mu < 0$) is unstable for $0 < A < 2|\mu|k_o$. When $C_S^2 > C_A^2$, the LHCP wave is stable and the RHCP mode is unstable for $0 < |A| < 2\mu k_o$. As is well known, a finite amplitude $|A| > 2|\mu|k_o$ stabilizes the modulational instability. More complete derivations may be found in Sakai and Sonnerup (1983) and Wang and Goldstein (1986).

IV. MODULATIONAL SOLITONS

In this section, we extend the derivation of Ichikawa et al. (1980). Our principal contributions will be two-fold. We will include the effects of a nonzero sound speed, which are a trivial addition calculationally, but a substantial addition physically. Secondly, we will find a transparent notation which will enable us to relate the conditions for soliton formation to those for modulational instability.

We seek stationary nonlinear solutions in the form,

$$w = w(y), \quad \psi = \psi(y), \quad y = \xi - V\tau \quad , \tag{12}$$

where V is the propagation velocity in the ξ-frame. Substituting (12) into (8a), integrating, and applying the boundary condition for a localized solution, that $\partial\psi/\partial y = 0$ at $w = 1$, we obtain

$$\mu \frac{d\psi}{dy} = \frac{w^2 - 1}{2w^2} \left[V - \frac{3A}{2} (w^2 + 1)\right] \quad . \tag{13}$$

Inserting (13) into (8b), and applying further localization conditions that $dw/dy \to 0$ as $w \to 1$, we eventually arrive at an equation formally equivalent to that for a particle with zero total energy in an anharmonic potential well:

$$\frac{\mu^2}{2} \left(\frac{dZ}{dy}\right)^2 + U(Z) = 0 \quad , \tag{14a}$$

where $Z = w^2 - 1$ and

$$U(Z) = \frac{Z^2}{2} \left\{ \frac{A^2Z^2}{4} - AZ \left[V + 2\mu k_o - A\right] \right.$$

$$\left. + \left[(V - 2A)^2 - A (A + 2\mu k_o)\right]\right\} \quad . \tag{14b}$$

Let us now transform (14b) back to the Alfvén frame

$$U = \frac{Z^2}{2} \left\{\frac{A^2Z^2}{4} - V_A Z + D_L\right\} \quad , \tag{15a}$$

where V_A is the normalized soliton speed in the Alfvén frame, and the third term on the right hand of (14b), transformed to the Alfvén frame, is the linear dispersion function, D_L,

$$D_L = \left(V_A - \frac{d\Omega_o}{dk_o} - A\right)^2 - A\left(A + \frac{d\Omega_o}{dk_o}\right) \quad , \tag{15b}$$

A further transformation of variables,

$$\delta = \eta/\mu = \frac{x - C_A t}{\mu} \quad , \tag{16a}$$

$$S = \frac{AZ}{2} = \frac{1}{8}\left(\frac{C_A^{\,2}}{C_A^{\,2} - C_S^{\,2}}\right)\left(\frac{b^2}{b_o^{\,2}} - 1\right) \quad , \tag{16b}$$

reduces (14 a,b) to a canonical form,

$$\frac{1}{2}\left(\frac{dS}{d\delta}\right)^2 + U(S) = 0 \quad , \tag{17a}$$

$$U(S) = \frac{S^2}{2}\left(S^2 - V_A S + D_L\right) = \frac{S^2}{2} D_{NL}(S) \quad , \tag{17b}$$

which is valid for both RHCP and LHCP waves. The solutions to (17 a,b) are subject to the requirement that b be real and positive everywhere.

The condition that solitons exist is that, at S = 0,

$$\frac{d^2U}{dS^2}(0) = D_L < 0 \quad , \tag{18a}$$

which requires in turn that

$$\left(V_A - d\Omega_o/dk_o - A\right)^2 < A\left(A + \frac{d\Omega_o}{dk_o}\right) \quad . \tag{18b}$$

Since the left-hand side of (18b) is positive, we conclude that modulational solitons exist only in conditions of modulational stability.

Defining $-D_L = k^2 > 0$, we may find the soliton dispersion relation - the relation between the soliton speed and its amplitude - by solving $D_{NL}(S^*) = 0$ to obtain

$$S_\pm^* = V_A \pm \Delta \quad , \qquad \Delta = \sqrt{V_A^2 + k^2} \quad . \tag{19}$$

Since the DNLS describes two modes, there are two possible solitons. When $C_A^2 > C_S^2$, the S_+^* soliton is "bright" ($b_+^* > b_o$) and the S_-^* soliton is "dark" ($b_-^* < b_o$). When $C_S^2 > C_A^2$, the S_+^* soliton is dark and the S_-^* soliton is bright. Since, from the quasi-static relationship, $\rho/\rho_o - 1 = 4S$, the S_+^* soliton is compressional for both $C_A^2 > C_S^2$ and $C_S^2 > C_A^2$, and the S_-^* soliton is rarefactive.

Equations (17 a,b) can be integrated by quadrature, assuming that $S = S_\pm^*$ at $\delta = 0$, to yield the soliton shape:

$$S_\pm(\delta) = \frac{\pm k^2}{\Delta \cosh k\delta \mp V_A} \quad . \tag{20}$$

Since the hyperbolic cosine is an even function of its argument, the form of the soliton does not depend on whether it is RHCP or LHCP, although the sign of μ enters into the evaluation of Δ. By integrating (20) over all space, we obtain invariants of the DNLS equation

$$I_\pm \equiv \int_{-\infty}^{+\infty} S_\pm(\delta)d\delta = \pm 4\tan^{-1}\frac{k}{\Delta \mp V_A} = \pm 4\tan^{-1}\sqrt{\frac{\Delta \pm V_A}{\Delta \mp V_A}} \quad , \tag{21}$$

which measure the overall soliton strength. When $k \to 0$, I_+ approaches 4π and I_- approaches zero, so that the S_-^* soliton has zero strength at marginal stability.

As marginal stability is approached, k^2 decreases, and the scale lengths of the solitary waves described by (20) increase. Precisely at marginal stability, we may set $k^2 = 0$, and solve the resulting equation,

$$\frac{1}{2}\left(\frac{dS}{d\delta}\right)^2 + \frac{S^3}{2}\left(S-V_A\right) = 0 \quad , \tag{22}$$

by quadrature to obtain an "algebraic" soliton (Ichikawa <u>et al</u>., 1980).

$$S_0(\delta) = \frac{V_A}{1 + \delta^2 V_A^{\,2}/4} \quad , \tag{23}$$

which may be compressive or rarefactive depending on the sign of V_A. The integral invariant corresponding to (23),

$$I_0 = \int_{-\infty}^{-\infty} S_0(\delta)d\delta = 4\pi \quad , \tag{24}$$

is precisely the $k \to 0$ limit of I_+.

V. DISCUSSION

The conclusion that solitons exist only in stable conditions appears reasonable. If we write the linear dispersion relation (11c) in the limit of small carrier amplitude, $A \ll d\Omega_0/dk_0$, we find

$$V_A = d\Omega_0/dk_0 \pm \sqrt{A d\Omega_0/dk_0} \tag{25}$$

so that nonlinear steepening (A) and dispersion ($d\Omega_o/dk_o$) must reinforce one another for modulational instability to occur. For example, for the LHCP wave with $C_S^2 < C_A^2$, nonlinear steepening sharpens the envelope profile while dispersion slows down the leading edge of the steepening pulse, hastening the moment when it overturns. On the other hand, steady solitary waves are formed when steepening is counteracted by dispersion.

Since it appears that dispersion may not halt the steepening of modulationally unstable envelopes, their eventual fate is unclear to us. As they steepen to form shocklike structures, it is evident that physical processes unimportant to the instability and initial steepening may come into play - for example, nonlinear couplings via the decay instability to sound waves and backscattered waves (Wang and Goldstein, 1986) and to plasma particles (Rogister, 1971; Spangler, 1986). These processes are not described by the DNLS equation. But even at the level of description afforded by the DNLS equation, the problem is interesting. We know that another nonlinear dispersive wave equation, the Korteweg-de Vries equation, possesses an infinite set of constants of motion which enable it to be solved by inverse scattering techniques. By these means, it may be shown that any nonlinear disturbance of positive measure always evolves into a set of K-dV solitons (Karpman, 1975). The DNLS equation also possesses a similar set of invariants and can be solved by inverse scattering techniques (Kaup and Newell, 1978; Kawata and Inoue, 1978; Wadati et al. 1979). It would be interesting to examine the solutions for the explicitly unstable case to ascertain whether indeed solitons form part of the time asymptotic state.

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