UNIVERSIDADE DE SÃO PAULO

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IFUSP/P-626

THE GENTILIONIC THEORY FOR QUARKS: MANIFESTLY CONFINING FOR QUARKS AND MANIFESTLY NON-COALESCENT FOR HADRONS

M. Cattani
Instituto de Física, Universidade de São Paulo

Janeiro/1987

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M. Cattani
Instituto de Física da Universidade de São Paulo
C.P. 20516, 01498, São Paulo, SP, Brasil

SUMMARY

In this paper we show that the gentilionic theory for quarks is manifestly confining for quarks and manifestly non-coalescent for hadrons. That is, it is shown that these properties are rigorously deduced only from first principles. To prove them no arguments involving the intrinsic nature of gentileons or dynamical hypothesis are necessary to be adopted. We also show that, in the context of the quantum field theory, gentileons can be taken approximately as fermions and that the usual quantum chromodynamics can be used to calculate the properties of gentilionic hadrons.

1. Introduction

During the last two decades, the theory of elementary particles has been developed in terms of the quark hypothesis of Gell-Mann-Zweig. The SU(3) (or SU(6)) symmetry combined with several other group transformations has served as a guide to classify all known strongly interacting particles. Nevertheless, in spite of its great sucess in classifying the several multiplets found in nature, Gell-Mann's theory leaves some major questions open for investigation. Among these great problems we are interested in, we can quote the statistical one which arises with the fermionic character of quarks and the two intriguing peculiar problems posed by the quark hypothesis: the quark confinement and the non-coalescence of hadrons. These are the most important problems in particle physics. Although being powerfully attacked by the Quantum Chromodynamics (QCD), some aspects of these problems remain unsolved. In the conventional approach, it is believed that these properties are achieved if one can show that the potential between quarks has confining and saturating properties. We must note, however, that in this approach such a behaviour cannot be established without doing dynamical hypothesis, which inevitably necessitates using some approximation. Then it is very difficult to confirm that these properties do not depend on the specific approximation adopted.

Since the quark confinement and the non-coalescence of hadrons are the very fundamental properties of the quarks it is more natural to expect that their realization are manifest(1), that is, we must be able to show them without analysing any dynamic details. In other words, in a quark theory these properties must be manifest, because it intrinsically guarantees confinement and non-coalescent just as manifest covariance does the Lorentz invariance of a relativistic theory. With this in mind we have proposed, within the last few years(2-4), an alternative approach to the statistical problem of the quarks, assuming that they obey Gentile statistics instead of Fermi statistics.

In section 2 we present our main results about Gentile statistics and gentilionic systems(2,3).

In section 3 we make a summary of our preceding paper(4) where it was shown, taking quarks as spin 1/2 gentileons, that fundamental properties of hadrons can be deduced from first principles. It will become apparent from these results that the gentilionic theory is _manifestly_ confining for quarks and _manifestly_ non-coalescent for hadrons.

In section 4 we show that, in the context of the quantum field theory, gentileons can be taken approximately as fermions and that the usual QCD can used to calculate the properties of the gentilionic hadrons.

2. The Statistical Principle and the Gentileons

In a preceding work(2) we have shown rigorously, according to the postulates of quantum mechanics and to the principle of indistinguishability, that three kinds of particles could exist in nature: bosons, fermions and gentileons These results can be synthesized in terms of the following principle (Statistical Principle): "Bosons, fermions ande gentileons are represented by horizontal, vertical and intermediate Young shapes, respectively". Bosonic and fermionic systems are represented by one-dimensional totally symmetric and totally anti-symmetric wavefunctions, respectively. Subsystems of bosonic or fermionic states are also bosonic or fermionic in the sense that they have the same symmetry properties of the original systems Since the commutation relations for the creation and annihilation operators are bi-linear, bosons and fermions obey the spin-statistics theorem(5): bosons have integer spin and fermions half-integer spin.

Because gentilionic systems are represented by intermediate Young shapes, their wavefunctions are multi-dimensional with mixed symmetries. Only three or more gentileons can form a system of indistinguishable particles. This means that two identical gentileons are prohibited to constitute a system of indistinguishable particles This implies that a gentileon cannot appear as a free particle. Indeed, if this were possible, two gentileons could interact constituting, consequently, such a system. With regard to symmetry properties, a subsystem of a gentilionic system cannot be defined(2). As the creation and annihilation operators for gentileons obey multi-linear commutation relations(2), the spin-statistics theorem is meaningless for gentileons. Finally, due to intrinsic geometric features of the intermediate state-vectors, there appear <u>selection rules</u> confining the gentileons and prohibiting the coalescence of gentilionic systems(2). Two systems like (ggg) and (gggg), for instance, cannot coalesce into a composite system of indistinguishable particles (ggggggg). Only bound states (ggg) - (gggg) could be possible. The gentileon confinement appears as a consequence of the selection rule wich prohibits the decomposition of a system (qqq...q) into (qqq...) and (q).

In our papers(2,3) only systems of identical gentileons have been considered. Let us consider now systems formed by two different gentileons, g and G. Taking into account the Statistical Principle we must expect that systems like (gG), (gggG), (gggGGG), and so on ,are allowed On the other hand, systems like (ggG), (ggGG), (gggGG)... are prohibited because (gg) and (GG) are not allowed. Of course, the coalescence of mixed system is also forbidden, as can easily be verified. It is important to note that the commutation relations for the creation and annihilation operators for mixed cases, like (gG), are not defined in the gentilionic theory. It seems natural to assume them as bi-linear, commutative or anti-commutative, if the gentileon has an integer spin or half-integer spin, respectively(5-7). Since confinement and non-coalescence are properties of pure and mixed systems we see that these properties cannot be explained by the multi-linear character of the commutation relations.

Another requirement of the Statistical Principle is that only bosons and fermions can appear as free particles in nature. Confinement and non-coalescence are intrinsic properties of gentileons. These features are intrinsic to gentileons as the total symmetrisation (anti-symmetrisation) is inherent to the bosons (fermions), not depending on their physical interpretation. Thus, they could be assimilated to individual real particles or to dynamical entities as quantum collective states. However, sticking to the strange properties of the gentileons, that are quite different from the usual particles and quantum collective states, we think that they could be a new and more complex entity.

The Gentilionic Theory for Hadrons

Since gentileons are "confined entities" and their systems e "non-coalescent" it seemed natural to think quarks as spin 1/2 ntileons$^{(2,3)}$. With this hypothesis we have shown$^{(4)}$ that the (3)$_{flavour}$ ⊗ SU(3)$_{colour}$ representation, both 3 and $\bar{3}$, can be naturally corporated into the gentilionic $S^{(3)}$ symmetry. In view of these sults the mesons are composed by a quark-antiquark pair ($q\bar{q}$). According the Statistical Principle, systems like (q), (qq), ($qq\bar{q}$) and ($qq\bar{q}\bar{q}$) e prohibited. Thus the ($qq\bar{q}\bar{q}$) could exist only as bound states ($q\bar{q}$)-$\bar{q}$) of the mesons ($q\bar{q}$). Assuming the baryons as formed by ree identical gentileons (qqq) we have shown that their wavefunctions e given by $\psi = \varphi .\ Y(\text{colour})$. The one-dimensional wavefunction $(SU(6) \times O_3)_{symmetric}$ corresponds, according to the symmetric quark del of baryons, to a totally symmetric state. The four-dimensional ate Y(colour), that corresponds to the intermediate representation the symmetric group $S^{(3)}$ and depends on the SU(3)$_{colour}$ eigenstates ue, red and yellow, is given by$^{(4)}$:

$$\text{olour)} = Y(bry) = Y(123) = \frac{1}{\sqrt{4}} \begin{pmatrix} Y_1(123) \\ Y_2(123) \\ Y_3(123) \\ Y_4(123) \end{pmatrix} = \frac{1}{\sqrt{4}} \begin{pmatrix} Y_+ \\ Y_- \end{pmatrix} \qquad (3.1)$$

ere, $Y_+ = \begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix}$, $Y_- \begin{pmatrix} Y_3 \\ Y_4 \end{pmatrix}$,

$Y_1(123)= (|bry\rangle + |rby\rangle - |ybr\rangle - |yrb\rangle)/\sqrt{4}$,
$Y_2(123)= (|bry\rangle + 2|byr\rangle + |rby\rangle + |ybr\rangle - 2|ryb\rangle - |yrb\rangle)\sqrt{12}$
$Y_3(123)= (-|bry\rangle + 2|byr\rangle - |rby\rangle - |ybr\rangle + 2|ryb\rangle - |yrb\rangle)/\sqrt{12}$,
and $Y_4(123)= (|bry\rangle - |rby\rangle - |ybr\rangle + |yrb\rangle)/\sqrt{4}$.

The state function $Y(bry)$ has been named colourspinor because we have shown that it is a bi-spinor in a colour space where the axes X and Z correspond to the colour isospin $\hat{I}_3$ and to the colour hypercharge $\hat{Y}$, respectively. The bi-spinotial character of $Y(bry)$ is responsible for selection rules predicting baryon number conservation, quark confinement and non-coalescence of hadrons. It is worthwhile to note that, in this context, our theory differs drastically from parastatistics$^{(8-12)}$ and fermionic theories of quarks. In the fermionic case, $Y(123)$ would be given by $Y(123)=(|bry\rangle - |byr\rangle - |rby\rangle + |ybr\rangle + |ryb\rangle - |yrb\rangle)\sqrt{6}$ and in parastatistics case $Y(123)$ would be written as, $Y(123) = a\,Y_1 + b\,Y_2 + c\,Y_3 + d\,Y_4$, where $\underline{a}$, $\underline{b}$, $\underline{c}$ and $\underline{d}$ are arbitrary constants. For these last theories the wavefunction $Y(123)$ is one-dimensional, from which the selection rules, above mentioned, cannot be deduced.

In that paper$^{(4)}$ we have also shown that the colour Casimir $K^{[2,1]}_{(2,1)} = 0$, which is the invariant of the algebra associated with the $S^{(3)}$ gentilionic state, could be identified with the total baryon colour charge operator $\hat{Q}$. Then by using an extended form of Noether's theorem it was proved that the baryon colour charge is an equal to zero constant of motion, that is, $\langle \hat{Q} \rangle =$ constant $= 0$. This has led us to very important consequences:

(a) The colour spinor $Y(123)$ must be necessarily composed by three different colours, that is, $Y(\text{colour})= Y(bry)$. Then, the baryon wavefunctions are given by $\psi = \varphi . Y(bry)$.

(b) The conservation law $\langle \bar{Q} \rangle$ = constant= 0, that can be interpreted as a selection rule for "colour confinement", implies in quark confinement. In contrast to previous papers$^{(2,3)}$ where the confinement selection rule has been obtained based on arguments involving dimensionality and symmetry properties of $Y(123)$, here it is given simply by the condition $\langle \bar{Q} \rangle$= constant= 0.

(c) The generalized Gell-Mann-Nishijima relation is automatically satisfied.

Let us make now a summary of the fundamental properties of the composed hadrons that have been deduced assuming quarks as spin 1/2 gentileons and incorporating the $SU(3)_{flavour} \otimes SU(3)_{colour}$ representation, both 3 and $\bar{3}$, into the $S^{(3)}$ gentilionic symmetry:

(1) Systems like (q), (qq), $(qq\bar{q})$ and $(qq\bar{q}\bar{q})$ are prohibited; mesons $(q\bar{q})$ and baryons (qqq) are allowed.

(2) Quarks are confined.

(3) Baryons and mesons cannot coalesce.

(4) The baryon colour charge $\langle \bar{Q} \rangle$ is an equal to zero constant of motion.

The above mentioned properties have been obtained independently of the intrinsic nature of the gentileons; they could be particles, quantum collective excitations or something else. Consequently, no dynamical hypothesis, phenomenological or approximate arguments have been used to prove them. They have been deduced from first principles: from the Statistical Principle or by using the symmetries of the $S^{(3)}$ intermediate representation. Then, in this sense, the gentilionic theory for quarks is "manifestly confining for quarks and manifestly non-coalescent for hadrons".

In spite of these general and very important results, there remains the crucial problem of determining the intrinsic nature of the quarks and their dynamical properties. According to the current theoretical ideas, quarks are elementary particles. Since they must have spin 1/2 they are assumed as obeying the Fermi-Dirac statistics. The mathematical formulation of the fermionic quark model, the QCD, is a successful modern field theory since it is able to explain very well many properties of the hadrons. In next section we show that, within the framework of the quantum field theory, the gentileons can be approximated by fermions. Consequently, the properties of the gentilionic hadrons can be calculated, in a first and good approximation, by using the QCD

4. The Fermionic Approximation

We analyse, in this section, the gentilionic hadrons in the spirit of the quantum field theory. Adopting the current Drell-Yan model(13) for quarks we show that gentileons can be taken as fermions from the algebraic point of view. Then the gentilionic hadrons properties can be calculate, in a first approximation, by using the QCD.

Let us consider first the baryons. In these systems, represented by the wavefunctions $\psi = \varphi.Y(rby)$, quarks can assume an infinite of quantum states, according to φ and $Y(rby)$. However, as in colour space the quarks must necessarily have three different colours(4), two quarks in (qqq) cannot occupy the same quantum state. Although it is a fermionic characteristic, they cannot be rigorously taken as fermions because the creation and annihilation operators obey in this case, the following relations;

$$
\begin{aligned}
a_\alpha\, a^*_\beta + a^*_\beta\, a_\alpha &= \delta_{\alpha\beta} \\
a^*_i\, a^*_j\, a^*_k &= G\binom{\alpha\beta\gamma}{ijk}\, a^*_\alpha\, a^*_\beta\, a^*_\gamma \\
a_i\, a_j\, a_k &= G\binom{kji}{\gamma\beta\alpha}\, a_\alpha\, a_\beta\, a_\gamma \qquad (4.1) \\
a_\alpha\, a^*_\beta\, a_\gamma &= G\binom{\gamma\alpha\beta}{\gamma\beta\alpha}\, a^*_\beta\, a_\alpha\, a_\gamma \text{ , and so on...}
\end{aligned}
$$

as one can easily verify following our general results on Gentile statistics(2). We see from Eqs. (4.1) that only the first relation $[a_\alpha\,,\, a^*_\beta\,]_+ = \delta_{\alpha\beta}$, is bi-linear fermionic and that the remaining ones are tri-linear. It is important to remark that the tri-linear relations need to employed only when we intend to take into account properties which are common to three or to pairs of particles in the (qqq) system. Thus, if during the time of interaction only one quark participates of the process, and the remaining two as spectators, as assumed in the Drell-Yan model, the tri-linear feature of the commutation relations are irrelevant for the cross section calculations. In these circunstances gentileons in baryons can be approximated by fermions.

Let us consider now the mesons ($q\bar{q}$). According to section 2 the commutations relations obeyed by q and $\bar{q}$ are determined by the spin of these particles. Since q and $\bar{q}$ have spin 1/2, they can be taken as fermions from the algebraic point of view.

Thus, from the above analysis we see that, in the context of the quantum field theory and assuming the Drell-Yan model, gentileons can be approximated by fermions. Consequently, the properties of gentilionic hadrons could be calculated, in a first approximation, by using the QCD. The tri-linear commutation relations would be necessary in hadronic processes only when corr lations between quarks in baryons are considered (14). Although the calculations become somewhat involved with tri-linear relations we believe that the cross sections in these conditions will be only slightly different from those obtained with the fermionic ones.

Under these conditions we conclude that, from the point of view of the quantum field theory, the properties of the gentilionic hadrons could be calculated, in a good approximation, by using the usual QCD. However, some additional conditions which appear naturally for gentileons must be obeyed in the QCD approximation: (a) confinement, (b) non-coalescence, (c) only the existence of colour singlet hadrons is admitted and (d) the total colour charge of baryons is a constant of motion equal to zero. These conditions show the distinguishing features between the two approaches: in the fermionic QCD they are imposed "ad hoc", whereas in the gentilionic theory they are deduced in a simple way from the Statistical Principle and using the symmetry properties of the intermediate $S^{(3)}$ representation only with the assumption that quarks are gentileons.

If quarks, beeing gentileons, are really elementary particles it must exist some kind of mechanisms responsible for confinement and non-coalescence. We do not know, at the moment, the exact mechanism. They could be produced by a very peculiar interaction between quarks, by an impermeable bag as proposed in the bag model, or something else. But any acceptable mechanism must be conceived in order to be preserved the fundamental properties of the intermediate states.

Acknowledgements

The author thanks Prof. J. Frenkel and Dr. D. Spehler for helpful discussions on QCD and the Conselho Nacional de Pesquisas (CNPq) for the finantial support.

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