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UNIFORM SEMICLASSICAL APPROXIMATION FOR ABSORPTIVE SCATTERING SYSTEMS

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ABSTRACT:

The uniform semiclassical approximation of the elastic scattering amplitude is generalized to absorptive systems. An integral equation is derived which connects the absorption modified amplitude to the absorption free one. Division of the amplitude into a diffractive and refractive components is then made possible.

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The Uniform Semiclassical Approximation (USCA) to quantal scattering problems has been employed extensively in several branches of Physics[1]. This approximation, based on the important work of Chester, Friedman and Ursell[2], supplies a powerful tool to deal with caustics problems such as encountered in rainbow scattering.

In applying these methods to problems involving absorption, such as the case in particle and nuclear physics, one has to cope with the problem of strong absorption and accordingly with diffraction. Though treatable with the powerful complex angular momenta method (CAMM), a more direct way, of relating the absorptive scattering amplitude to the purely refraction scattering (no absorption) is called for. In so doing one is then able to clearly separate what one may call the diffraction component of total elastic amplitude. The remaining piece contains the (absorption) modified refraction effects. Such a decomposition seems to be of great use in analysing recent intermediate energy heavy-ion elastic scattering data such as the one reported in Ref. 3).

The purpose of the present letter is to supply a general theory of the elastic scattering amplitude, applicable in cases where semiclassical conditions are satisfied, which enable its calculation, given the absorption-free amplitude. The USCM is then easily generalized to general absorptive medium. Application to simple cases is presented.

We use below the notation employed by Berry[illegible].
The elastic scattering amplitude $f(\theta)$,

$$f(\theta)=\frac{1}{ik}\sum_{\ell=0}^{\infty}(\ell+1/2)\left[|S_\ell|\,e^{2i\delta_\ell}-1\right]P_\ell(\cos\theta) \tag{1}$$

Eq. (1) differs from the one used by Berry in an important aspect, namely the partial wave amplitude, S , is allowed to have a modulus smaller than one, as unitarity requires in absorptive scattering.

$$|S_\ell| \leq 1 \tag{2}$$

We now proceed and decompose $f(\theta)$ into its near and far components througly the use of the following asymptotic form of the Legendre function

$$P_\ell(\cos\theta) \simeq \sqrt{\frac{2}{\pi(\ell+1/2)\sin\theta}}\,\cos\left[(\ell+1/2)\theta-\pi/4\right] \tag{3}$$

valid for $\ell^{-1}<\theta<\pi-\ell^{-1}$. The near $f^{(+)}$ and far $f^{(-)}$ side components of $f(\theta)$ are just obtained from the $e^{-i(\ell+1/2)\theta}$ and $e^{i(\ell+1/2)\theta}$ branches of the cosine function in eq. (3), respectively.

$$f^{(\pm)}(\theta)=\frac{1}{ik\sqrt{2\pi\sin\theta}}\sum_{\lambda=1/2}^{\infty}\lambda^{1/2}\,e^{2i\delta(\lambda)}\,|S(\lambda)|\,e^{\mp i\lambda\theta}\;e^{\pm i\pi/4} \tag{4}$$

where the factor (-1) is dropped as it contributes only at $\theta=0$. The above equation can be Poisson-decomposed as

$$f^{(\pm)}(\theta)=\frac{1}{ik\sqrt{2\pi\sin\theta}}\sum_{m=-\infty}^{\infty}e^{-im\pi}\int_{0}^{\infty}d\lambda\,\lambda^{1/2}|S(\lambda)|\,e^{2i\delta(\lambda)+2m\pi i\lambda}\cdot e^{\mp i\lambda\theta}\,e^{\pm i\pi/4} \qquad (5)$$

For simplicity we consider the case in which the deflection function $2(d\delta(\lambda)/d\lambda)$ never exceeds $\pm\pi$. Then the m=0 term in the above sum approximates very well $f^{\pm}$. Introducing the notation $\sqrt{\mathrm{sen}\theta}\; f^{\pm}(\theta)\equiv I^{\pm}(\theta)$, we have

$$I^{(\pm)}(\theta)=\frac{e^{\pm i\pi/4}}{ik\sqrt{2\pi}}\int_{-\infty}^{\infty}d\lambda\,\lambda^{1/2}|S(\lambda)|\,e^{2i\delta(\lambda)\mp i\lambda\theta} \qquad (6)$$

where we have extended the lower limits of the integrals to $-\infty$ (with $|S(\lambda)|=0$ for $\lambda<0$).

It is clear from Eq. (6) that $I^{+}(\theta)$ is simply (aside from a constant) the Fourrier transform of $\lambda^{1/2}|S(\lambda)|e^{2i\delta(\lambda)}$. $I^{-}(\theta)$ is the Fourrier transform taken as a function of $(-\theta)$ of the same λ-function. We now introduce the absorption free amplitude $I_o(\theta)$ as being the Fourrier transform of $\lambda^{1/2}e^{2i\delta(\lambda)}$

$$I_o^{(\pm)}(\theta) = \frac{e^{\pm i\pi/4}}{ik\sqrt{2\pi}}\int_{-\infty}^{\infty}d\lambda\,\lambda^{1/2}\,e^{2i\delta(\lambda)\mp i\lambda\theta} \qquad (7)$$

In order to obtain an equation which relates $I^{\pm}(\theta)$ to $I^{\pm}_o(\theta)$ we use a three-step procedure; we first inverse Fourrier transform (6), divide over $|S(\lambda)|$ and finally Fourrier transform back. Denoting the Fourrier transformation of a function G(x) by $F_{x\to p}G$, we have, for the near-side amplitude

$$F_{\lambda\to\theta}\left(|S(\lambda)|^{-1}\right)F_{\theta\to\lambda}\,I^{(+)}(\theta) = I_0^{(+)}(\theta) \tag{8}$$

The operator $(F_{\lambda\to\theta}(|S(\lambda)|^{-1}F_{\theta\to\lambda})$ is an example of a class of operators called pseudo-differential[5]. As long as $|S(\lambda)|$ is representable as a polynomial in λ, the following relation holds

$$F_{\lambda\to\theta}\,|S(\lambda)|^{-1}\,F_{\theta\to\lambda} = \left|S\left(i\frac{d}{d\theta}\right)\right|^{-1} \tag{9}$$

A similar analysis follows for the far-side amplitude, in which the pseudo-differential operator is

$$F_{\lambda\to-\theta}\,|S(\lambda)|^{-1}F_{-\theta\to\lambda} = \left|S\left(i\frac{d}{d\theta}\right)\right|^{-1} \tag{10}$$

We thus find the following important relation

$$\left|S\left(i\frac{d}{d\theta}\right)\right|^{-1} I(\theta) = I_0(\theta) \tag{11}$$

which formally solves to

$$I(\theta) = \left|S\left(i\frac{d}{d\theta}\right)\right| I_0(\theta) \tag{12}$$

Eq. (12) shows how diffraction comes into the picture as a result of the application of $|S(i(d/d\theta))|$ on the otherwise purely refractive amplitude $I_0(\theta)$. It seems therefore that an appropriate name to be given to our pseudo-differential operator, $|S(i(d/d\theta))|$, is the "diffraction operator" $\hat{D}$. For convenience, we introduce the notation

$$I^{(\pm)}(\theta) \equiv \langle\theta|I^{\pm}\rangle$$
$$I_o^{(\pm)}(\theta) \equiv \langle\theta|I_o^{\pm}\rangle \tag{13}$$

Thus

$$|I^{\pm}\rangle = \hat{D}\,|I_o\rangle = \int d\theta'\,\hat{D}\,|\theta'\rangle\langle\theta'|I_o^{\pm}\rangle \tag{14}$$

Thus

$$\langle\theta|I^{\pm}\rangle = \int d\theta'\,\langle\theta|\hat{D}|\theta'\rangle\langle\theta'|I_o^{\pm}\rangle \tag{14}$$

The matrix element $\langle\theta|\hat{D}|\theta'\rangle$ is just the Green function corresponding to Eq. (11). It is the degree of non-locality in $\theta-\theta'$ that determines how diffractive the scattering is. In fact, as we show below, most of the diffractive effects in $I^{\pm}(\theta)$ are contained in the principal part of the θ'integral.

We present now an analysis of the angle Green function $\langle\theta|\hat{D}|\theta\rangle \equiv G(\theta-\theta')$. Owing to the unitarity limit of $|S\ |$, namely $|S_{\ell}|=1$ for $\ell > \ell_R$, where ℓ_R characterizes the extention of the scatterer, it is safer to express $G(\theta-\theta')$ as a Fourrier transform of $(d/d\lambda)|S(\lambda)|$. This involves explicitly extracting a pole term, $(\theta-\theta'+i\varepsilon)^{-1}$ with the small imaginary part used to guarantee convergence. We obtain (this relation was originally obtained by Frahn and Gross[6]) in a slightly different manner)

$$G^{(+)}(\theta-\theta')=\frac{i}{2\pi}\,\frac{1}{\theta'-\theta+i\varepsilon}\,F_{\lambda\to(\theta'-\theta)}\left(\frac{d}{d\lambda}|S(\lambda)|\right) \tag{16}$$

The Fourrier transform $F_{\lambda\to\theta'-\theta}((d/d\lambda)|S(\lambda)|)$ measures the contribution of the surface. Sharp surfaces are characterized by δ-like behaviour of $(d/d\lambda)|S(\lambda)|$, resulting in a constant δ-like behaviour of $F_{\lambda\to\theta'-\theta}(d/d\lambda)|S(\lambda)|$. Diffused surfaces give rise to a wider distribution in $\theta'-\theta$. For the purpose of illustration we take $|S(\lambda)|$ to be a Fermi function $|S(\lambda)|=[\exp[\lambda_R-\lambda]/\Delta+1]^{-1}$ *). Δ here measures the extent of the surface region in angular momentum space. Semiclassicaly, it is approximately given by k a with a being the diffuseness of the density profile of the scatterer.

$$F_{\lambda\to\theta'-\theta}\,\frac{d}{d\lambda}|S(\lambda)|=\frac{\pi\Delta(\theta'-\theta)}{\sinh\pi\Delta(\theta'-\theta)}\exp[i\lambda_R(\theta'-\theta)] \tag{17}$$

$$=F(\Delta(\theta'-\theta))\exp[i\lambda_R(\theta'-\theta)]$$

Thus, very diffused systems (>>0) are characterized by a small non-locality in $G(\theta-\theta')$, since one has

$$G(\theta-\theta')=\frac{i}{2\pi}\lim_{\varepsilon\to 0}\frac{1}{\theta'-\theta+i\varepsilon}\,\pi\Delta(\theta'-\theta)\,e^{-\pi\Delta|\theta'-\theta|}\cdot e^{i\lambda_R(\theta'-\theta)} \tag{18}$$

even for θ' very close to θ. The degree of non-locality in θ and accordingly the degree of diffraction is measured by $(1/\pi\Delta)$. The above situation represents a case of weak diffraction (the largeness of Δ forces G to be dominated by its on-shell part).

The other extreme, $\Delta \to 0$ gives

$$G(\theta-\theta') \longrightarrow \frac{i}{2\pi} \lim_{\varepsilon\to 0} \frac{1}{\theta'-\theta+i\varepsilon} e^{i\lambda_R(\theta'-\theta)} \tag{19}$$

namely "infinite" non-locality.

From the above discussion it would seem natural to identify the on-shell and off-shell (principal part) contributions to $G(\theta'-\theta)$ with what we may call, refractive and diffractive propagation, respectively. Namely

$$G^{(+)}(\theta-\theta') = G_R(\theta-\theta') + G_D(\theta-\theta') \tag{20}$$

$$G_R(\theta-\theta') = \frac{1}{2}\,\delta(\theta'-\theta)\, F_{\lambda\to(\theta'-\theta)}\left[\frac{d}{d\lambda}|S(\lambda)|\right] \tag{21}$$

$$G_D(\theta-\theta') = \frac{i}{2\pi}\, P\frac{1}{\theta'-\theta}\, F_{\lambda\to(\theta'-\theta)}\left[\frac{d}{d\lambda}|S(\lambda)|\right] \tag{22}$$

We now use the following identity

$$P\,\frac{1}{x} = \frac{d}{dx}\ln|x| \tag{23}$$

to rewrite Eq. (22) as

$$\begin{aligned} G_D(\theta-\theta') &= \frac{i}{2\pi}\,\frac{d}{d\theta'}\ln|\theta'-\theta|\; F_{\lambda\to(\theta'-\theta)}\left(\frac{d}{d\lambda}|S(\lambda)|\right) \\ &= \frac{i}{2\pi}\left(\frac{d}{d\theta'}\ln|\theta'-\theta|\right) F(\Delta(\theta'-\theta))\; e^{i\lambda_R(\theta'-\theta)} \end{aligned} \tag{24}$$

where we have used Eq. (17). When inserting G_{eff} in I, and integrating by parts, we obtain

$$I_{eff}(\theta) = \frac{-i}{2\pi} \cdot \int d\theta' \ln|\theta'-\theta| \left[\frac{d}{d\theta'} F(\Delta(\theta'-\theta))\right] \cdot e^{i\lambda_R(\theta'-\theta)} I_0(\theta')$$

$$- \frac{i}{2\pi} \int d\theta' \ln|\theta'-\theta| \, F(\Delta(\theta'-\theta)) \frac{d}{d\theta'}\left(e^{i\lambda_R(\theta'-\theta)} I_0(\theta')\right) \qquad (25)$$

The second term, containning the singular $\ln|\theta'-\theta|$, contributes to the refractive I while the first term is identified with the diffractive piece, since $\ln|\theta'-\theta|(d/d\theta')F(\Delta(\theta'-\theta))$ is <u>not singular</u> ($\ln x(d/dx)F(x) \xrightarrow[x\to 0]{} 0$), and therefore the non-locality of the integrand (in $\theta-\theta'$) is predominant. Thus we have finally for $I_R(\theta)$ and $I_D(\theta)$

$$I_R(\theta) = \frac{I_0(\theta)}{2} - \frac{i}{2\pi} \int d\theta' \ln|\theta'-\theta| \, F(\Delta(\theta'-\theta)) \cdot \frac{d}{d\theta'}\left(e^{2i\lambda_R(\theta'-\theta)} I_0(\theta')\right)$$

(26)

$$I_D(\theta) = -\frac{i}{2\pi} \int d\theta' \ln|\theta'-\theta| \frac{d}{d\theta'} F(\Delta(\theta'-\theta)) \cdot e^{i\lambda_R(\theta'-\theta)} I_0(\theta') \qquad (27)$$

The expression we have obtained for $I_R(\theta)$, Eq. (26) contains the on-shell piece of $I(\theta)$, namely one half the absorption-free $I_o(\theta)$ plus a correction arising from absorption. This correction depends on the absorption profile function $F((\theta'-\theta)$, and the first derivation of $I_o(\theta')$ (in the integrand). The diffractive component $I_D(\theta)$ contains $I_o(\theta')$ inside the integrand multiplied by the non-singular function $\ln|\theta'-\theta|(d/d\theta')F(\Delta(\theta'-\theta))$. A reasonable approximation would be to take $\ln|\theta'-\theta|(d/d\theta')F(\Delta(\theta'-\theta))$ outside the integrand and set $\theta'=\theta_o$ with θ_o being the stationary point angle obtained from the condition $(d/d\theta')[e^{i\lambda_R\theta'}\cdot I_o(\theta')]_{\theta'=\theta_o}=0$, $\theta'=D_o$.

Equations (26) and (27), are the principal results of this paper. Given $I_o(\theta')$, the absorption modified amplitude immediately follows. In particular the modification of the well-known uniform semiclassical approximation to $I_o(\theta)$, which is normally used to deal with cases involving rainbow scattering, is now clear. $I(\theta)$ would then involve absorption modified Airy function and its first derivative[4], through Eqs. (15) and (16) or, if the situation requires, Eqs. (26) and (27). Further developments and applications of our theory is in progress[8].

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7. With a Fermi-shape $|S(i(d/d\theta))|$, Eq. (11) takes the appealing simple form $e^{\alpha/\Delta} I(\theta - i/\Delta) + I(\theta) = I_0(\theta)$, which can be solved for $I(\theta)$ as $I(\theta) = \sum_{n=1}^{\infty} I_0(\theta + (ni/\Delta)) e^{-n\alpha/\Delta} (-)^n$. We have opted for the integral equation approach to avoid dealing with complex angles.

8. M.S. Hussein and M.P. Pato
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