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FACTORS WHICH AFFECT OPERATION OF A PLASMA FOCUS IN VARIOUS GASES *

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ABSTRACT

It is shown that the axial transit time t_a and γ, the ratio of specific heats, are the main factors which affect the operation of a plasma focus in various gases. An energy balance theory is used to explore this dependence. The results are consistent with previous calculations and with experiment.

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Introduction

There has been some renewed interest in the plasma focus lately as a source of themonuclear neutrons, hard and soft x-rays and a relativistic electron beam. The yield of the device in such applications depends on the intensity of its focus action. Experimentally the focus is quite well understood, yet there is no comprehensive theory which fully describes all phases of the device. Porter (1) proposed a two dimensional code which describes both the axial and radial phases of the focus. The axial phase computation by this code gives the same results as a simplilfied one dimensional snow-plough model; however there is no similar simplified model for radial phase analysis.

The snow-plough model gives a zero final radius for the radial phase compression, which is non-physical. Lee (2,3,4) has used an energy/pressure balance theory coupled with the snow-plough model to compute the radius ratio for various pinches including the plasma focus. His calculations for a constant current constant length pinch and the varying current varying length pinch are in agreement with experiment.

In the process of tuning the UNU/ICTP Plasma Fusion Facility (5) it was necessary to establish the various pressure regimes of operarion of the device (6,7). It is also important to investigate the operation of a plasma focus devices in various gases as this is expected to lead to a better understanding of the phenomenon of thermodynamic neutron yield enhancement effect. In this paper we apply the Lee energy balance theory to the focus in order to understand the factors which affect the focus intensity when a plasma focus is operated in various gases.

The axial transit time t_a is one of the factors expected to affect the intensity of a plasma focus. The snow-plough model gives

$$t_a = \left[\frac{4\pi(b^2-a^2)}{\mu \ln(b/a)}\right]^{1/2} \frac{z_o \rho_o^{1/2}}{I_o} \qquad (1)$$

where ρ_o is the ambient density, z_o is the length of the inner electrode, b/a is the outer to inner electrode radius ratio, I is the peak surge current. In the design of a plasma focus it is necessary to match t_a to the rise time of the current (5); this ensures that the current attains a high enough value at the time when the axial phase ends and the radial implosion phase begins. It is shown later that although the final radius ratio is independent of the maximum current, it depends on the current time shape. As t_a depends on the ambient density, the nature of the filling is expected to affect focus intensity.

For the radial implosive phase of the focus, the length of the pinching column increases as its radius decreases. The magnetic force which pushes the plasma in both the radial and axial directions is given by

$$F_B = (B_\theta^2/2\mu)\, 2\pi r \ell \qquad (2)$$

where

$$B_\theta = \mu I/2\pi r$$

is the azimuthal magnetic field, I the current, r and l the radius and length of the pinch column and μ the plasma permeability. The work per unit mass done by the magnetic piston is given by

$$W = \frac{1}{\rho \pi r_m^2 l_m} \int_{r_m}^{r_0} \frac{\mu I^2(r)}{4\pi r^2 (2\mu)} \cdot 2\pi r l(r)\, dr \qquad (3)$$

At the end of the radial phase the magnetic pressure is balanced by the thermal pressure and the plasma is in a quasi-equilibrium state. The enthalpy of the plasma in this quasi-static state is given by

$$h = \left(R_0/M\right) T_m \chi \frac{\gamma}{(\gamma-1)} \qquad (4)$$

where T is the temperature, χ the departure coefficient, γ is the ratio of specific heats while R_0 is the universal gas constant and M the molecular weight of the filling gas. If the work done per unit mass by the piston is converted without loss to enthalpy then we may derive from eqations 3 and 4

$$T_m = \frac{(\gamma-1)}{\gamma} \left(M/R_0\chi\right) \frac{\mu}{4\pi^2 \rho r_m^2 l_m} \int_{r_m}^{r_0} \frac{I^2(r) l(r)}{r} dr \qquad (5)$$

Setting the magnetic pressure

$$P_B = \frac{\mu I_m^2}{8\pi^2 r_m^2} \qquad (6)$$

equal to the thermal pressure

$$P = \left(R_0/M\right) \rho T_m \chi \qquad (7)$$

we get an independent expression for T_m thus

$$T_m = \frac{\mu I_m^2}{8\pi r_m^2} \left(M/R_0 \chi \rho\right) \qquad (8)$$

Equations 5 and 8 give the so called energy integral as

$$I_m^2 = \frac{2(\gamma-1)}{\gamma l_m} \int_{r_m}^{r_0} \frac{I^2(r) l(r)}{r} dr \qquad (9)$$

If in the energy integral the current is represented as

$$I^2(r) = I_m^2 f^2(r) \qquad (10)$$

where f(r) is some current profile which can be determined from the snow-plough model, it is immediately seen that the radius ratio r_m/r_0 is indeppendent of the peak current I_m at the time the quasi-equilibrium is first established; but that it depends on the current-time shape function f(r). Similarly it is seen that the radius ratio depends on the length-time shape function.

Equation 9 also shows that the radius ratio depends very strongly on the ratio of specific heats of the filling gas. The ratio γ is related to the number of degrees of freedom f of the gas as: $\gamma = (2 + f)/f$. For an ideal gas such as argon at room temperature or for a fully ionized gas $\gamma = 5/3$. At higher temperatures collisions between argon atoms can become energetic enough to effect transitions between various electronic states of the gas. Under these conditions any energy absorbed by the gas can go either into changing the three modes of translational motion or into changing the electronic configuration of the gas. The number of degrees of freedom is then considerabely higher; for example it is found that at 10,000 K, argon is about 10% singly ionized and it has an f = 7.5 per original argon atom.

In general the ratio γ can only be determined if the various stages of ionization of the gas at the given temperature are known. To do this it is first necessary to apply a set of Saha equations in order to determine the degrees of ionization of the gas at these temperatures. The Saha equation is essentially the law of mass action which relates the densities of reactants to the reaction energies and temperatures as well as the partition functions of the reactants. Applied to the s-th state of ionization of a gas, the Saha equation takes the form

$$\frac{\alpha_s}{\alpha_{s-1}} \sum_{r=1}^{r=j} r\alpha_r = 7.7 \times 10^{-6} M \frac{Z_s}{Z_{s-1}} \frac{T^{3/2}}{\rho} \exp\left(-u_s/kT\right) \qquad (11)$$

where Z_s and α_s are the partition function and ionization fraction of the s-th ionization state, u_s is the ionization energy, M, ρ and j are the molecular weight, density and the highest level of ionization of the gas at temperature T. To solve for j stages of ionization we set s = 1,2,.....j and solve the resulting system of non-linear equations. It is usually only possible to do this computationally by some residue minimization technique.

The partition functions and statistical weights are obtained from tabulations of atomic energy levels (8) or by the use of an approximate hydrogenic summation formula with cut-off procedure based on reduced ioniaztion potential (9). The ionization potentials may be obtained from tabulations or from the use of the iso-electrinic sequence.

Solution of the Saha equations gives the ionization fractions at various temperatures from which the departure coefficient as a function of temperature

can be calculated. The ratio of specific heats are then found as functions of temperature.

These computations have been carried out for argon (9); the results indicate that for a freely ionizing argon gas the ratio = 1.14 is true over a large temperature range (10^4 - 10^5 K). In an argon plasma focus in which the shock velocity is 30 cm/ sec and the temperature is 3 x 10^6 K, γ was found to be 1.3. This value has been used by Tou and Lee (10) to predict the final radius ratio of an argon plasma focus of 0.06. For deuterium which is fully ionized at the relevant temperature and for which the ratio γ=5/3, the predicted final radius ratio was 0.14; both these predictions are in agreement with measurements.

It is thus seen that operation of the plasma focus in various gases which can have higher degrees of ionization than deuterium can lead to higher electromagnetic compression (lower final radius ratios) during the radial phase. It can thus be expected that when these gases are added in trace ammounts to a deuterium focus, the ratio = 5/3 for a fully ionized deuterium can be modified, and this could lead to higher compression of the deuterium and a higher yield of neutrons. This phenomenon is well known experimentally (11,12). The analysis also explains why xenon and krypton are preferred for use in pinches operated as x-ray sources; these two gases are expected to have even lower values of γ and therefore more intense electromechanical compression at the end of the pinch and a higher yield of x-rays.

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