IC/87/119

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

MASSIVE YANG-MILLS THEORY: RENORMALIZABILITY VERSUS UNITARITY

R. Delbourgo

S. Twisk

and

G. Thompson

INTERNATIONAL ATOMIC ENERGY AGENCY

UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION

1987 MIRAMARE-TRIESTE

IC/87/119

International Atomic Energy Agency
and
United Nations Educational Scientific and Cultural Organization

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

MASSIVE YANG-MILLS THEORY: RENORMALIZABILITY VERSUS UNITARITY *

R. Delbourgo, S. Twisk
Physics Department, University of Tasmania, Hobart, Australia 7005

and

G. Thompson
International Centre for Theoretical Physics, Trieste, Italy.

ABSTRACT

Various massive Yang-Mills theories not based on the Higgs mechanism are investigated. They are subject to conflicting demands in the twin requirements of unitarity and perturbative renormalizaibility. Either one or other of these requirements is violated. Unitarity is considered in some detail.

MIRAMARE - TRIESTE
June 1987

* To be submitted for publication.

I. INTRODUCTION

The Higgs method for generating masses in non-Abelian gauge fields has now by and large universal acceptance. There are of course shortcomings in this approach for generating masses. Perhaps the most telling is the indeterminant masses and couplings of the Higgs mesons themselves. Within the context of grand unified theories, there are further problems associated with the Higgs picture, namely the hierarchy problem. The non-discovery of the Higgs mesons, and the difficulties discussed above argue for continued attempts to construct an alternative massive non-Abelian gauge model. In fact the few years before the advent of one standard electroweak model saw considerable efforts in this direction. All these efforts failed (in some sense) for either of two reasons; either the model was non-polynomial [1] and hence not perturbatively renormalizable or it was non-unitary, that is it contained physical ghosts [2]. And yet these theories (disregarding the non-unitary models) avoid the hierarchy problem and a model exists that reproduces the standard model to tree level (including θ_w). [3] The main reason for abandoning these has been our lack of technical know-how in dealing with their renormalizability. Though some advance in this direction has been made [4].

The aim of this work is to briefly review and discuss the question of unitarity in various models advanced as viable alternatives to the Higgs scheme, we find that while gauge invariance and gauge covariance (i.e. gauge fixing independence) are necessary conditions for ensuring unitarity, they are not sufficient for this purpose. The non-Abelian generalization of the Stueckelberg model for massive photon qed, is delt with in some detail, as by and large of all the models considered this remains the most promising. Even though the model remains non-renormalizable (perturbatively) because of its non-polynomial interactions. We illustrate the (one loop) unitarity of the theory by three examples.

Following this we discuss a model independently discovered by Curci and Ferrari and by Fradkin and Tyutin [2]. This model turns out to be of the regular type (i.e. polynomial) and to possess an (extended) BRST [5] invariance. Nonetheless it is not unitarity. This is displayed in various ways as it sets the scene for the discussion of a model that we recently proposed [6]. That model is dealt with in Sec.IV. It is a polynomial theorv (in a particular gauge), it is gauge invariant and as an improvement on that of Curci-Ferrari Fradkin-Tyutin it is gauge covariant. However once more unitarity is violated [7], and reasons for why this is the case are presented. The last model considered was put forward by Batalin [8]. This is also a non-polynomial theory, however here, there is no explicit Stueckelberg field, the structure of the non-polynomial terms being dictated by the requirement of gauge covariance.

We end with the same conclusion as Burnel [9], namely that on the basis of <u>these</u> models of avoiding the Higgs scalars, perturbative renormalizability may be bought at the expense of unitarity, otherwise the Stueckelberg theory is unitarity but not conventionally renormalizable.

II. *STUECKELBERG MODELS*

2.1 Introduction

We begin with a brief review of massive photon electrodynamics. This theory by naive power counting is non-renormalizable as the large momentum behaviour of the photon propagator is $k_\mu k_\nu/(k^2m^2)$. This result is misleading due to the inherent gauge invariance of the theory; because of current conservation the "bad" term in the photon propagator is not problematic. A concrete realization of the gauge invariance is afforded by the Stueckelberg form of the theory (for a review of this and much of what follows in this section, one can do no better than to read the articles by Boulware and Slavnov [1]).

The Lagrangian (without the matter fields) is

$$\mathcal{L} = -\frac{1}{4}F^2 + \frac{m^2}{2}\left(A_\mu - \frac{\partial_\mu \phi}{m}\right)^2 \tag{2.1}$$

which is obtained from the conventional massive Lagrangian by the (gauge) shift $A_\mu \to A_\mu - \frac{\partial_\mu \phi}{m}$. This does not affect the form of the coupling to the matter fields, for, by gauge invariance, such a shift can be compensated. This Lagrangian is now gauge invariant under the transformations

$$\delta A_\mu = \partial_\mu \Lambda \quad ; \quad \delta\phi = m\Lambda \tag{2.2}$$

where ϕ is now taken to be a bonafide field. As is usual for theories exhibiting a gauge invariance, propagators cannot be derived from (2.1) until a gauge has been chosen. If the condition $\phi = 0$ is adopted, then the original theory is recovered. Alternatives include the conventional covariant gauges $(\partial\cdot A)^2/2\alpha$ or 't Hooft gauges $(\partial\cdot A + \alpha m\phi)^2/2\alpha$ which are useful in cases of spontaneous symmetry breaking. Renormalizability becomes clear in the latter gauges, as there the propagator for the photon behaves as $\eta_{\mu\nu}/k^2$ in the ultraviolet.

Now if we can show that the model is gauge covariant (independent of choice of gauge) then the two gauge choices lead to the same theory, thereby establishing that massive photon electrodynamics is a renormalizable theory. The easiest way to do this is to make use of the BRST invariance,

$$\begin{aligned} &\delta A_\mu = \partial_\mu \omega \quad , \quad \delta\phi = m\omega \quad , \quad \delta\omega = 0 \\ &\delta\bar{\omega} = B \quad , \quad \delta B = 0 \quad , \quad \delta^2 = 0 \end{aligned} \tag{2.3}$$

The appropriate gauge fixing and ghost Lagrangian is

$$\mathcal{L}' = \delta\left(F(A_\mu, \phi)\bar{\omega} + \alpha B\bar{\omega}\right) \tag{2.4}$$

Now as $\mathcal{L}'$ is a total variation (of some function K) and is itself BRST invariant, we may use an argument attributed to Lee [10] (see also Ref.11) which states that adding a term which is invariant under a BRST transformation to the action is equivalent to a re-definition of the fields coupled to the sources in the generating functional. The S matrices, of theories, which only differ in their source terms, coincide [10]. Different choices of F in (2.4) will give either $(\partial\cdot A)^2/2\alpha$, $(\partial\cdot A + \alpha m\phi)^2/2\alpha$ or $\phi^2/2\alpha$ (and of course many others). But the important point is that the $\phi = 0$ gauge (the original massive model) where the theory is apparently power counting non-renormalizable has the same S matrix as in (say) the Landau gauge $\partial\cdot A = 0$ where renormalizaibility is explicit.

The massive Yang-Mills Lagrangian,

$$\mathcal{L} = -\frac{1}{4}(F^a_{\mu\nu})^2 + \frac{m^2}{2}(A^a_\mu)^2 \tag{2.5}$$

like the U(1) case has bad high energy behaviour for the gauge field propagator. Unlike the U(1) theory, it is not evident whether this is not a problem, say, as a consequence of some current conservation. To address this question the analogue of (2.1) for non-Abelian theories is essential. Kunimasa and Goto first wrote down the appropriate generalization (though it was also later rediscovered independently and developed subsequently [1])

$$\mathcal{L} = -\frac{1}{4}F^2 + \frac{m^2}{2}\,\mathrm{Tr}\left[(A_\mu - U^{-1}i\partial_\mu U/g)^2\right] \tag{2.6}$$

with U a matrix-valued field in the adjoint representation

$$U = \exp(ig\,\phi^a T^a / m) \tag{2.7}$$

(other parametrizations are possible, this one is chosen for convenience). It is easy to see that as in (2.1) the mass term here is obtained by shifting A_μ by a gauge transformation

$$A_\mu \longrightarrow U\,(A_\mu - U^{-1} i\partial_\mu U/g)\,U^{-1} \tag{2.8}$$

The Lagrangian (2.6) exhibits the gauge invariance

$$\delta A_\mu = D_\mu \Lambda \quad , \quad \delta\phi = m\Lambda \tag{2.9}$$

On choosing the gauge $\phi = 0$ (allowed by the invariance (2.9)) one recovers (2.5). Generalized 't Hooft gauges, covariant gauges and so on lead to the same S matrix (if it can be defined) as for $\phi = 0$ by arguments identical to those given previously for the U(1) group.

The caveat is important. By inspecting (2.6), we see that in such desired gauges as the Landau gauge, the boson propagator is well behaved, however the action remains non-polynomial. The theory turns out not to be conventionally renormalizable [1]. Though there is some hope that if one moves outside of conventional perturbation theory, by using the methods introduced by Efimov [12] for dealing with non-polynomial interactions, sense may still be made of the theory [4].

2.2 Unitarity

Leaving the question of renormalizability aside for the moment, let us look slightly more closely at the question of unitarity. Firstly, as we have already indicated, the S matrix is formally independent of the gauge fixing. This and BRST invariance are pre-requisites of unitarity, however as we shall see later they are not sufficient conditions. So to display unitarity, we establish this for the gauge propagator to one loop (Slavnov and Faddeev [1]). To the required order in g and in the Landau gauge the action is

$$\mathcal{L} = -\tfrac{1}{4}F^2 + \tfrac{m^2}{2}A^2 + \tfrac{1}{2}\phi\,\partial_\mu D^\mu\phi + B\,\partial\cdot A + \bar\omega\,\partial_\mu D^\mu\omega + \cdots \tag{2.10}$$

Integrating out the ghosts leads to a contribution $\ln \partial\cdot D$ in the effective action, while the ϕ fields give $-\frac{1}{2}\ln \partial\cdot D$. Hence the net effect is to give a contribution, exactly 1/2 of that of the ghost term. This means

for example, that as $m \to 0$, one does not recover massless Yang-Mills theory. However this 1/2 factor is correct in the massive version.

Break up the Landau propagator into spin 1 and spin 0 parts according to

$$\begin{aligned}\Delta_{\mu\nu}(k) &= (-\eta_{\mu\nu} + k_\mu k_\nu/k^2)/(k^2-m^2)\\ &= (-\eta_{\mu\nu} + k_\mu k_\nu/m^2)/(k^2-m^2) - k_\mu k_\nu/k^2m^2\\ &\equiv \Delta^{(1)}_{\mu\nu}(k) + \Delta^{(0)}_{\mu\nu}(k)\end{aligned} \tag{2.11}$$

Then the vector contribution to $\pi_{\mu\nu}$ (disregarding tadpole graphs which give no cuts) can be broken up into three pieces as shown in Fig.1. The (1)(1) cut has no zero mass singularities and can be safely disregarded at the external $p^2 = m^2$, if contracted over $\sum_\lambda \epsilon_k^{(\lambda)}(p)\,\epsilon_{k'}^{(\lambda)}(p) = (-\eta_{kk'} + p_k p_{k'}/m^2)$. The (1)(0) contribution can be reduced to

$$\pi^{(1)(0)}_{kk'} = g^2/m^2 \cdot \int d^4q/q^2 \cdot q^\lambda\,\Gamma_{k\lambda\mu}(p,q,r)\,\frac{(\eta^{\mu\mu'} - r^\mu r^{\mu'}/m^2)}{(r^2-m^2)}\,q^{\lambda'}\,\Gamma_{k'\lambda'\mu'}(-p,-q,-r)$$

$$\to \quad -g^2/m^2 \cdot \int\frac{d^4q}{q^2}\,[(r^2-m^2)\eta_{k\mu} - r_k r_\mu]\,\frac{(\eta^{\mu\mu'} - r^\mu r^{\mu'}/m^2)}{(r^2-m^2)}\,[(r^2-m^2)\eta_{k'\mu'} - r_{k'}r_{\mu'}]$$

but

$$\frac{r_\mu\,(\eta^{\mu\mu'} - r^\mu r^{\mu'}/m^2)}{(r^2-m^2)} = \frac{-r^{\mu'}}{m^2}$$

eliminates the massive pole; hence the cut starting at $p^2 = m^2$ gives no contribution at the end point. There remains the (0)(0) cut from

$$\pi^{(0)(0)}_{kk'} = g^2/m^4 \cdot \int\frac{d^4q}{q^2r^2}\,q^\lambda r^\mu\,\Gamma_{k\lambda\mu}(p,q,r)\,q^{\lambda'}r^{\mu'}\,\Gamma_{k'\lambda'\mu'}(-p,-q,-r).$$

$$\to -g^2\int\frac{d^4q}{r^2q^2}\,r_k r_{k'} \longrightarrow g^2\int\frac{d^4q}{r^2q^2}\,q_k r_{k'} \tag{2.12}$$

The ghost contribution is precisely of this form and comes with the opposite sign. The two-vector contribution (2.12) must be multiplied by a factor of 1/2 (due to identical particles) and <u>cancels against half the ghost contribution</u>. Hence we see that the ϕ field contribution is exactly right to cancel off the left over half.

To further emphasize the need for the 1/2 factor, consider the scattering process $f(p)\,\bar{f}(Q) \to f(p')\,\bar{f}(Q')$, where f stands for a fermion in the fundamental representation of the gauge group, say SU(2) for simplicity. At tree level, or to order g^2, there is no problem with the zero mass pole in the Landau propagator (2.11); the mass degeneracy ensures that the longitudinal pieces of the vector propagator do not contribute anyway; hence, the problem with unitarity appears to order g^4. In this order there are numerous diagrams to grapple with; however in the forward or "s channel" we can regard the graphs as a folding of the tree level process drawn in Fig.2, with their conjugates (as well as a corresponding set for the exchange or "t-channel") plus the one-loop ghost contributions. To check unitarity we need to show that the cuts in the s channel start at $4m^2$, or that the lower mass cuts cancel out for on-shell fermions.

In Fig.3 are sketched the folded tree diagrams with zero-zero, zero-one and one-one contributions separated out (including the identical particle factor of 1/2), plus the ghost graph. With the labelling of momenta as drawn and using current conservation

$$\bar{V}(Q)(\not{P} + \not{Q})\,U(P) = \bar{V}(Q)(\not{p} + \not{q})\,U(P) = 0$$

we need only concentrate on those portions which have cuts below $4m^2$, namely:

<u>(O)(O) contribution</u>

$$\frac{1}{2}\cdot\int d^4p\;\bar{V}(Q)\begin{pmatrix}\gamma_\lambda\tfrac{1}{2}\tau_b\dfrac{1}{(\not{P}-\not{p}-M)}\gamma_\kappa\tfrac{1}{2}\tau_a+\gamma_\kappa\tfrac{1}{2}\tau_a\dfrac{1}{(\not{P}-\not{q}-M)}\gamma_\lambda\tfrac{1}{2}\tau_b\\ +\,i\,\epsilon_{abc}\,\Gamma_{\kappa\lambda\mu}\,\Delta^{\mu\mu'}(p+q)\,\gamma_{\mu'}\tfrac{1}{2}\tau_c\end{pmatrix}U(P)\cdot\frac{p^\kappa q^\lambda p^{\kappa'}q^{\lambda'}}{m^2p^2m^2q^2}$$

$$\cdot\,\bar{U}(P')\,(\quad\text{conjugate expression}\quad)\,V(Q')$$

$$=\frac{1}{2}\int\frac{d^4p}{p^2q^2}\;\frac{1}{4}\,i\,\epsilon_{abc}\,\bar{V}(Q)\,\tau_c\frac{(\not{q}-\not{p})}{(p+q)^2-m^2}\,U(P)\;\frac{1}{4}\,i\,\epsilon_{abd}\,\bar{U}(P')\,\tau_d\frac{(\not{q}-\not{p})}{(p+q)^2-m^2}\,V(Q')$$

<u>(O)(1) contribution</u>

Here we have only one contraction over internal momenta corresponding to the spin-zero line. But in addition we should observe that

$$q^\lambda\,\Delta_{\lambda\lambda'}(q)\,q^{\lambda'} = q^2/m^2$$

kills the massive pole. This miracle indeed happens and it removes the m^2 cut in the variable $(P + Q)^2$. Thus the amplitude becomes

$$-\int d^4p\;\bar{V}(Q)\begin{pmatrix}\tfrac{1}{2}\gamma_\lambda\tau_b\dfrac{1}{(\not{P}-\not{p}-M)}\tfrac{1}{2}\not{p}\tau_a+\tfrac{1}{2}\not{p}\tau_a\dfrac{1}{(\not{p}-\not{Q}-M)}\tfrac{1}{2}\gamma_\lambda\tau_b\end{pmatrix}U(P)\cdot\frac{\Delta^{\lambda\lambda'(1)}(q)}{m^2p^2}$$

$$\cdot\;\bar{U}(P')\,(\quad\text{conjugate expression}\quad)\,V(Q')$$

$$=-\int d^4p\;\tfrac{1}{2}\,i\,\epsilon_{abc}\,\tau_c\,\bar{V}(Q)\,\not{q}\,U(P)\cdot q^\lambda\frac{\Delta^{(1)}_{\lambda\lambda'}(q)}{m^2p^2}q^{\lambda'}\tfrac{1}{2}\,i\,\epsilon_{abd}\,\tau_d\,\bar{U}(P')\,\not{q}\,V(Q')$$

$\longrightarrow$ no cut.

<u>Ghost contribution</u>

This is the simplest amplitude to evaluate

$$-\int d^4p\;\bar{V}(Q)\tfrac{1}{2}\tau_c\gamma_\mu\Delta^{\mu\mu'}(p+q)\,U(P)\cdot p'_\mu q'_\nu\;\epsilon_{abc}\,\epsilon_{abd}\,\bar{U}(P')\tfrac{1}{2}\tau_d\gamma_\nu\Delta^{\nu\nu'}(p+q)\,V(Q')/p^2q^2$$

$$=\int d^4p\;\tfrac{1}{2}\,i\,\epsilon_{abc}\,\bar{V}(Q)\,\tau_c\,\not{p}\,U(P)\cdot\tfrac{1}{2}\,i\,\epsilon_{abd}\,\bar{U}(P')\,\tau_d\,\not{q}\,V(Q')/\left[p^2q^2((p+q)^2-m^2)^2\right]$$

<u>Barring the factor of 1/2</u> it precisely equals the (O)(O) contribution.

The remaining unphysical cut, starting at $(p+q)^2 = 0$ can only be removed if one included the Stueckelberg scalar with its own contribution (Fig.4). Again this is blessed with the correct (+ve) sign and the factor of 1/2 (identical ϕ) to cure the unitarity disease.

Similar considerations apply to the unitarity (to order g^4) of massive vector elastic scattering. Unless the Stueckelberg contribution is included, zero mass cuts will persist and lead to unphysical results. The need for only a 1/2 the usual ghost contribution is thus pervasive.

How are we to understand the fact that the 1/2 factor which is *essential* for unitarity in the massive case (for arbitrarily small mass) but is quite incorrect for the massless theory? Faddeev and Slavnov [1] point out that massless vector particles have two degrees of freedom or polarization states while massive vectors have three. For the Yang-Mills field, unlike U(1), the longitudinal "photon" does not vanish in the $m \to 0$ limit, instead the S matrix describes simultaneously "photons" with transverse polarization and also scalar particles of zero mass. Roughly speaking, in the Landau gauge ($m \to 0$ limit giving the usual massless propagator) ϕ decouples for U(1) but not for non-Abelian groups. To recover the correct massless model, the zero mass scalar particle's contribution must be eliminated.

A general proof of the unitarity of (2.6) *a la* Kugo and Ojima [13], relying on the BRST structure has also been presented [4]. There must be some reservations though, for a proof in the same vain is also possible for the model discussed in Sec.IV, but as we will see there, unitarity is explicitly broken.

2.3 Renormalization

We turn to the problem of high energy behaviour for longitudinally polarized vector bosons. This is usually taken as a test case for the standard Higgs mechanism [14]; one looks for cancellation of large energy (E) behaviour order by order in g^2 to ensure one stays within the unitarity bounds [15]. It is of interest to see what happens in the corresponding Stueckelberg variant.

Consider then a model with O(3) symmetry for simplicity and focus upon longitudinal-longitudinal elastic scattering, $1+2 \to 1' + 2'$. First look at the spontaneously broken case with residual O(2) symmetry corresponding to rotational invariance about the third internal axis. We encounter the trilinear Higgs (H) coupling

$$e m H \left(A_1^2 + A_2^2 \right)$$

among others, where m is the mass of A_1 and A_2; A_3 of course remaining massless. There are four diagrams to be combined (Fig.5); we quote the answers for each of them, without giving the elementary calculational details, in terms of the energy E, momentum p and scattering angle θ in the c.m. frame - remember that we are contracting over ϵ_L polarization vectors at each leg. The total amplitude is $T^{tot} = T^a + T^b + T^c + T^H$ with

$$T^a = \frac{p^2 \cos\theta \,(3E^2 - p^2)^2}{m^4 E^2} \underset{E\to\infty}{\longrightarrow} \frac{c\,E^4}{m^4} \left(4 + O(r^2) \right)$$

$$T^b = \frac{1}{m^4(1+c)2p^2} \begin{bmatrix} (4E^2 + 2p^2(1-c))(p^2+E^2c)^2 + 4E^2p^2(1+c)^2(4p^2+2E^2(1-c)) \\ - 8E^2p^2(1+c)(p^2+E^2c)(3-c) \end{bmatrix}$$

$$\underset{E\to\infty}{\longrightarrow} \frac{E^4}{m^4} \left[(3-c)(1+c) - 8rc + O(r^2) \right]$$

$$T^c = \frac{2}{m^4}(p^2 - E^2c)^2 - \frac{1}{m^4}(p^2+E^2)^2 - \frac{1}{m^4}(p^2+E^2c)^2$$

$$\underset{E\to\infty}{\longrightarrow} \frac{E^4}{m^4} \left[-3 - 6c + c^2 + r(2+6c) + O(r) \right]$$

$$T^H = \frac{4(p^2 - E^2c)^2}{m^2\left[-2p^2(1-c) - m_H^2\right]} \underset{E\to\infty}{\longrightarrow} -2\frac{E^4}{m^4} \left(r(1-c) + O(r^2) \right)$$

where $c \equiv \cos\theta$, $r = m/E$ or m_H/E. It is simple to check that $T^{tot} \to O(r^2)\, E^4/m^4 = O(1)$ as was emphasized early, by the proponents of the electroweak scheme.

Now contrast the situation with the Stueckelberg variant, even with ϕ incorporated. This time we are dealing with a mass degenerate case ($A_{1,2,3}$ all have mass m) and there is no Higgs term. Furthermore, there is no Stueckelberg field exchange to this order. After working some minor changes to the energy denominators in the previous calculations we find

$$T^a \to c\,E^4/m^4 \,.\, (4 + r) + O(1)$$

$$T^b \to E^4/m^4 \,.\, \left[(1+c)(3-c) + \tfrac{1}{2}r(1+2c)(3-c) - r(3+10c-c^2) \right] + O(1)$$

$$T^c \to E^4/m^4 \,.\, \left[-3 - 6c + c^2 + r(2+6c) \right] + O(1)$$

This time the total amplitude behaves at high energy like

$$T \to E^4/2m^4 \,.\, r(1-c) = E^3/2m^3 \,.\, (1-c)$$

No more is it unitarily bounded. The reason for this bad asymptotic behaviour is possibly the failure of perturbative renormalizability to all orders in g due to the non-polynomiality occasioned by ϕ. Whereas we would find that $SS^+ = 1$ is satisfied order by order in g^2 (providing ϕ is properly included) we would discover that each order in g^2 has increasing powers of E^2/m^2. Since renormalizability would force us to add increasing numbers of subtraction constants at each order, this would ultimately lead to a theory with no predictive power and thus of no value.

Nevertheless, this spells doom at the perturbative level only, and as we have already mentioned there is some progress outside this confine [4]. Indeed Shizuya [16] has been able to establish the renormalizaibility of massive QCD in two dimensions (albeit in a non-conventional manner).

III. MODELS OF CURCI-FERRARI AND FRADKIN-TYUTIN

Curci and Ferrari and Fradkin and Tyutin [2] proposed the following action as a possible candidate for a theory of massive Yang-Mills fields;

$$\mathcal{L} = -\frac{1}{4}F^2 + \frac{m^2}{2}A^2 - \frac{(\partial A)^2}{2\alpha} + \alpha m^2\bar{\omega}\omega + \bar{\omega}\overleftrightarrow{\partial}\cdot D\omega + \frac{\alpha}{8}(\bar{\omega}\times\omega)^2 \tag{3.1}$$

(more correctly Fradkin and Tyutin have this for $\alpha = 0$). How such quartic ghost terms may arise in general is interesting in itself, but lies outside our main aim, details may be found in Ref.5.

(3.1) is invariant under the extended BRST transformations

$$\delta A_\mu = D_\mu\omega, \quad \delta\omega = \frac{\omega\times\omega}{2}, \quad \delta\bar{\omega} = -\frac{\partial.A}{\alpha} + \bar{\omega}\times\omega, \quad \delta^2 \neq 0 \tag{3.2}$$

This theory then comes complete with a gauge invariance and good high energy behaviour (in the Landau gauge) so that it is power counting renormalizable, yet it is not unitary. There are various ways of establishing this and it is instructive to consider more than one.

Firstly, recall that gauge covariance was assured providing one could write the ghost and gauge fixing terms in the action as a variation of some function. In (3.1) all ghosts and gauge fixing terms are of this form, except for the ghost mass term

$$\delta\left(\frac{\partial.A}{2}.\bar{\omega} + \alpha\bar{\omega}.(\bar{\omega}\times\omega)\right) = -\frac{(\partial.A)^2}{2\alpha} + \bar{\omega}\overleftrightarrow{\partial}D\omega + \frac{\alpha}{8}(\bar{\omega}\times\omega)^2$$

and

$$\delta^2\left(\frac{\partial.A}{2}.\bar{\omega} + \alpha\bar{\omega}.(\bar{\omega}\times\omega)\right) = 0$$

Hence all gauge choices are gauge equivalent to

$$\mathcal{L} = -\frac{1}{4}F^2 + \frac{m^2}{2}A^2 + \alpha m^2\bar{\omega}\omega$$

which is gauge dependent. The reason for this is clear, from the transformation rules (3.2) the variation of the gauge boson mass term can only cancel against the variation of the ghost mass term. The lesson is that gauge invariance of the complete theory is not enough, one needs the physical part to be invariant by itself (as for example the A_μ, ϕ system of the previous section is) or covariance, and hence unitarity, will fail.

An alternative way to understand the failure is already indicated in Eqs.(3.2). The BRST proof of unitarity for massive gauge theories [13] rests on the nilpotency of the BRST charge. Here, the BRST charge is not nilpotent, as $\delta^2\bar{\omega} \neq 0$. Actually this is always the situation if one does not introduce an auxiliary field B, so that $\delta\bar{\omega} = B$, $\delta B = 0$ for the conventional BRST transformations or $\delta\bar{\omega} = B + \bar{\omega}\times\omega$, $\delta B = B\times\omega$ for the extended set. Unfortunately, in this example one may not introducce the B field into the transformations and maintain the invariance of the action (as the reader may verify easily).

Finally, consider (3.1) in the Landau gauge. Then it is identical to the action (2.10) except for the Stueckelberg terms. But as we have already seen, the ϕ couplings are absolutely necessary to cancel half the ghost contribution and thus to ensure unitarity. These extra couplings are missing in (3.1) rendering this theory non-unitary.

IV. A VARIANT OF THE STUECKELBERG MODEL

In a recent note [6] we suggested it might be possible to set up a sensible Yang-Mills theory of massive vectors without invoking the Higgs mechanism. We did this by adopting a variant of the Stueckelberg models described in Sec.II, wherein the scalar Stueckelberg field was eliminated in favour of a (gauge-fixing) functional of the vector field in such a way that the gauge invariance of the mass term was preserved; the inherent non-polynomiality of (2.6) could then be disregarded in some particular gauge. While the renormalizaibility of the scheme was not in doubt, it was subsequently shown that unitarity was neverthless violated [7),17]. Here we will review this model and display, in line with the preceding sections, the cause of the failure of unitarity.

Our original suggestion [6] consisted in spotting that the equation of motion for the Stueckelberg field ϕ, obtained by the variation of the mass term in (2.6),

$$\mathcal{L}_m = m^2\,\mathrm{Tr}\,(A_\mu - U^{-1}i\partial_\mu U/g)^2 \tag{4.1}$$

is

$$D_\mu\,(A_\mu - U^{-1}i\partial_\mu U/g) = 0 \tag{4.2}$$

Thus we proposed to eliminate ϕ by its dependence on A_μ via (4.2). The starting Lagrangian is taken to be [18)]

$$\mathcal{L} = -\frac{1}{4}F^2 + m^2\,\mathrm{Tr}\,(A_\mu - U^{-1}i\partial_\mu U/g)^2 + \bar{\omega}'\,\partial\cdot D\omega'$$

$$+\; C[\,\partial\cdot A - D_\mu(U^{-1}i\partial_\mu U/g)\,] \tag{4.3}$$

where C imposes (4.2). The theory as presented here is not in the form given in Ref.6. The two formulations are equivalent, the present one being the more useful for our purposes. (4.3) is invariant under the gauge transformations (2.9) ($\delta C = 0$) even though there is an additional constraint. On choosing the Landau gauge, by the constraint (4.2) we see that the A_μ and ϕ fields decouple in the mass term (4.1), further (4.2) has the (perturbative) solution $\phi = 0$ (disregarding the homogeneous term).

We now turn to the question of unitarity and discuss the objections raised against the Curci-Ferrari, Fradkin-Tyutin models as they apply to this model. Firstly, the problem of requiring a ghost term to cancel part of the variation of the (A_μ,ϕ) system does not arise here as (4.3) is gauge (and consequently BRST) invariant. The second objection leveled, is also avoided here, for the BRST transformations of this model are the conventional ones, so that the BRST charge is nilpotent. Let us take stock. As presented the model is power counting renormalizable and gauge covariant. It is power counting renormalizable, as (in the Landau gauge) the high energy behaviour of the vector propagator is "good" and the non-polynomial interactions decouple. It is gauge invariant as the gauge fixing and ghost terms are the same as in the conventional Stueckelberg formulation and so may be written as the variation of some function, the variation is nilpotent, and as we have previously argued this guarantees gauge covariance. The final test of unitarity, was to see if the correct factor of 1/2 the ghost contribution was engendered. The relevant terms in our model are

$$\mathcal{L} = \frac{1}{2}\phi\,\partial_\mu D^\mu\phi + B\,\partial A + \bar{\omega}\,\partial D\omega + C(\partial A - D_\mu\partial_\mu\phi)$$

$$+\; \bar{\omega}'\,\partial\cdot D\omega' \tag{4.4}$$

The ϕ, C integrals yield a contribution $-\,\mathrm{Tr}\,\ln\partial\cdot D$ exactly cancelling one of the ghost terms and unfortunately lead to <u>a violation of unitarity</u> as we are left with precisely one ghost contribution and not with the correct 1/2 contribution. We learn that gauge invariance and gauge covariance of a theory are not strong enough conditions to ensure unitarity.

Let us try to understand this failure in terms of the original massive theory (2.5). Instead of choosing the Landau gauge in (4.3) pick on the gauge $\phi = 0$. The Lagrangian simplifies to (neglecting inessential ghost terms)

$$\mathcal{L} = -\frac{1}{4}F^2 + \frac{m^2}{2}A^2 + C(\partial A) + \bar{\omega}'\,\partial\cdot D\omega' \tag{4.5}$$

and <u>not to</u> (4.5), whereas the original Stueckelberg model has this limit. That is, while the theory presented in this section is gauge covariant, it is in fact not directly related to the original massive Yang-Mills theory, but bears closer resemblance to models recently advanced [18)]. The conclusion appears to be that the original Stueckelberg model is just right to ensure (one loop) unitarity and any tampering leads to non-unitarity. [In fact it is possible to save this model, by re-introducing the homogeneous part of the ϕ field as suggested by Kubo [7)], however the interactions become non-polynomial once more, and the net effect is to reobtain the original Stueckelberg model.]

V. BATALIN'S MODEL

In a rather interesting paper, Batalin [8)] proposed the following programme for developing a massive Yang-Mills theory. The idea is to work with the following action;

$$\mathcal{L} = -\frac{1}{4}F^2 + \frac{m^2}{2}A^2 + \frac{\alpha}{2}(\partial\cdot A)^2 + G(\alpha|A) \tag{5.1}$$

with the following conditions placed on $G(\alpha|A)$;

i) When the gauge fixing is turned off $\alpha = 0$, $G(\alpha|A)$ vanishes, thus leading to the original massive action (2.5)

$$G(0|A) = 0 \tag{5.2}$$

and

ii) That under the gauge transformation

$$\delta A^a_\mu(x) = \delta\alpha\;\Omega^a_\mu(x;\alpha|A) \tag{5.3}$$

G transforms as

$$\frac{dG(\alpha|A)}{d\alpha} = \int d^4x\;\Omega^a_\mu(x,\alpha|A)\frac{\delta G(\alpha|A)}{\delta A^a_\mu(x)} + R(\alpha|A) \tag{5.4}$$

where

$$\Omega^a_\mu(x;\alpha|A) = \frac{1}{2\alpha}\nabla^{ab}_\mu(A)\int \mathcal{D}^{bc}(x,y|A)\,\partial.A^c(y)\,d^4y \tag{5.5}$$

with

$$\nabla^{ab}_\mu(A) = \partial_\mu\delta^{ab} + \epsilon^{abc}A^c_\mu$$

$$\left(\nabla^{ab}_\mu\nabla^{\mu bc} - \frac{m^2}{\alpha}\delta^{ac}\right)\mathcal{D}^{cd}(x,y|A) = \delta^{ad}\delta(x-y)$$

and $R(\alpha|A)$ represents the charge in the A measure under the transformation (5.3). This second condition is imposed because the transformation (5.3) has the effect of simply charging α to $\alpha+\delta\alpha$ in (5.1). The corresponding transformation of G (5.4) is designed so that the complete action, transforms to

$$\mathcal{L} = -\frac{1}{4}F^2 + \frac{m^2}{2}A^2 + \frac{(\alpha+\delta\alpha)}{2}(\partial.A)^2 + G(\alpha+\delta\alpha|A) \tag{5.6}$$

thus keeping the original form.

This formulation clearly does not suffer from the non-gauge covariance of the Curci-Ferrari, Fradkin-Tyutin theories, being designed as it were to avoid exactly this drawback. A procedure for solving for G is given in some detail in the paper by Batalin.

Some features worth noting are that Faddeev-Popov ghosts make no explicit appearance, nor do Stueckelberg fields. Indeed Batalin argues against their introduction altogether, in that the theory may be non-analytic in the coupling around zero coupling, which is difficult to deal with in the Stueckelberg form. A second point is that as specified $G(\alpha|A)$ is not suitable for dealing with the massless theory, a minor modification is required. Thirdly, Batalin points out that the theory is <u>not</u> perturbatively renormalizable, but suggeststhat, that problem, is best tackled non-perturbatively here.

While, these considerations lead us to "abandon" perturbation theory, it must be said that the only check on unitarity we have is at a perturbative level (otherwise purely formal). To date as far as we are aware, the one-loop unitarity of Batalin's model has not been established, though the formal gauge covariance (and equivalence with the action (2.5)) suggest that unitarity ought to hold. The equivalence with (2.5) is of course an important criterion, to avoid the problems faced by the model of the previous section.

VI. CONCLUSIONS

The main conclusion that we come to from the studies of these various models is that renormalizability and unitarity seem to be competing qualities. The original Stueckelberg formulation, with its inherent non-polynomiality, is unitary but not renormalizable. This is in itself quite interesting, implying that the naive massive Yang-Mills action (2.5) is of the correct form to ensure unitarity, and as we have seen any tampering with this leads us astray.

Batalin's approach also looks promising, but its true value will not be known until some checks on its unitarity are made.

On the question of renormalizability, Burnel [9] has recently argued that it is possible to find gauges in the Stueckelberg model where all difficulties with renormalizaibility are confined to vertices that always involve unphysical fields. This makes it plausible that in the physical sector all worst divergences cancel.

Finally, it must be admitted that the Higgs mechanism remains the most complete method for giving mass to the vector bosons.

ACKNOWLEDGMENTS

The authors would like to thank Dr. P.D. Jarvis for discussions. G.T. would also like to thank Drs. L. Masperi and R. Zhang for many enlightening conversations and Professor Abdus Salam, the International Atomic Energy Agency and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste.

REFERENCES

1) T. Kunimasa and T. Goto, Progr. Theor. Phys. 37, 452 (1967);
M. Veltman, Nucl. Phys. B7, 637 (1968);
J. Reiff and M. Veltman, Nucl. Phys. B13, 545 (1969);
D. Boulware, Ann. Phys. 56, 140 (1970);
A.A. Slavnov and L.D. Faddeev, Teor. Mat. Fiz. 3, 18 (1970);
A. Salam and J. Strathdee, Phys. Rev. D2, 2869 (1970);
A. Vainshtein and I. Khriplovich, Sov. J. Nucl. Phys. 13, 111 (1971);
A.A. Slavnov, Teor. Mat. Fiz. 10, 201 (1972).

2) E. Fradkin and I. Tyutin, Phys. Lett. 30B, 562 (1969);
G. Curci and R. Ferrari, Nuovo Cimento 32A, 151 (1976);
ibid. 35A, 1 (1976).

3) T. Sonoda and S. Tsai, Prog. Theor. Phys. 71, 878 (1984).

4) T. Fukuda, M. Monda, M. Takeda and K. Yokoyama, Progr. Theor. Phys. 66, 1827 (1981); ibid. 67, 1206 (1982); ibid. 70, 284 (1983).

5) G. Curci and R. Ferrari, Ref.2;
L. Bonora and M. Tonin, Phys. Lett. 98B, 48 (1981);
R. Delbourgo and P. Jarvis, J. Phys. A15, 611 (1982).

6) R. Delbourgo and G. Thompson, Phys. Rev. Lett. 57, 2610 (1986).

7) J. Kubo, Phys. Rev. Lett. 58, 2000 (1987);
P. Kosinski and L. Szymanowski, Phys. Rev. Lett. 58, 2001 (1987).

8) I. Batalin, Nucl. Phys. B76, 347 (1974).

9) A. Burnel, Phys. Rev. D 33, 2981 (1986); ibid. D 33, 2985 (1986).

10) B. Lee, "Gauge theories", in Methods in Field Theory, Eds. R. Balain and J. Zinn-Justin (Les Houches, 1976).

11) L. Baulieu and J. Theirry-Meig, Nucl. Phys. B 197, 477 (1982).

12) G. Efimov, Soc. Phys.-JETP 17, 1417 (1963);
R. Delbourgo, A Salam and J. Strathdee, Phys. Rev. 187, 1999 (1969).

13) T. Kugo and I. Ojima, Suppl. Progr. Theor. Phys. 66, 1 (1979).

14) C. Llewellyn-Smith, Phys. Lett. 46B, 233 (1973);
J. Cornwall, D. Levin and G. Tikttopoulos, Phys. Rev. D10, 1145 (1974).

15) I. Aitchison and A. Hey, Gauge Theories in Particle Physics (Adam Hilger, 1982).

16) K. Shizuya, Nucl. Phys. B87, 255 (1975); ibid. B94, 260 (1975);
ibid. B121, 125 (1977).

17) A.A. Slavnov, private communication. We would like to thank Professor Slavnov for bringing some of the relevant references to our attention.

18) M. Carena, C. Wagner and L. Masperi, ICTP, Trieste, preprint IC/87/19.

FIGURE CAPTIONS

Fig.1 Vector self-energy divided into massive and massless contributions.

Fig.2 Tree level diagrams for fermion-antifermion annihilation into two massive mesons.

Fig.3 g^4 contribution to $f\bar{f} \to f\bar{f}$, associated with cuts in the forward scattering channel and in the absence of the Stueckelberg in-field There are corresponding graphs in the exchange channel connected with the crossing P'<-> -Q.

Fig.4 A g^4 contribution to $f\bar{f} \to f\bar{f}$ from the Stueckelberg scalar.

Fig.5 Boson-boson scattering in spontaneously broken O(3) Higgs model.

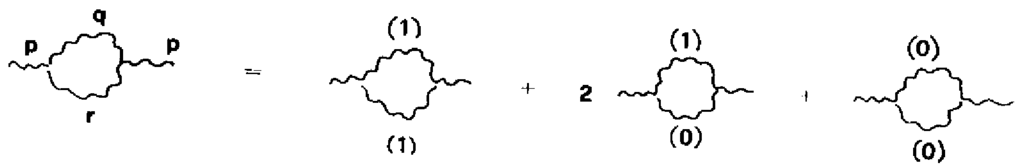

Fig.1

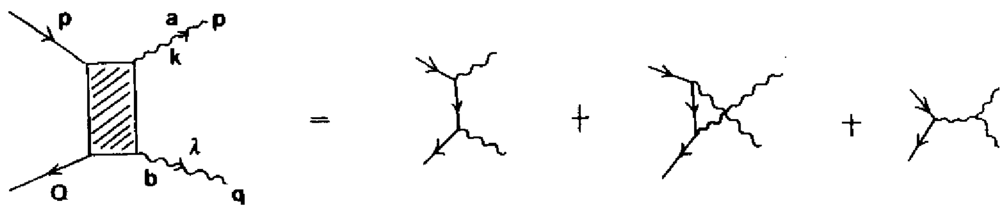

Fig.2

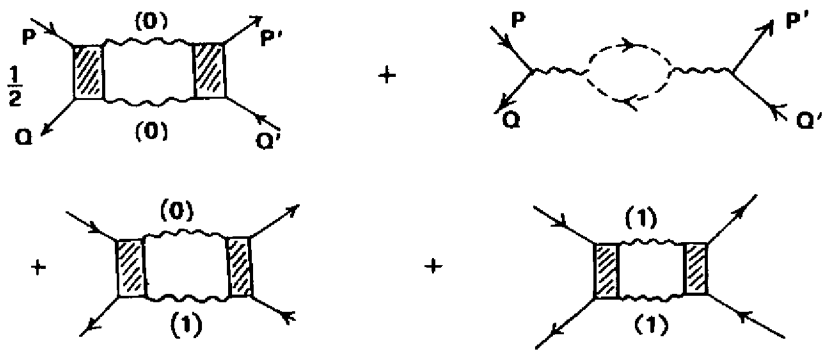

Fig.3

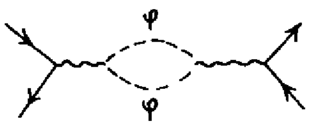

Fig.4

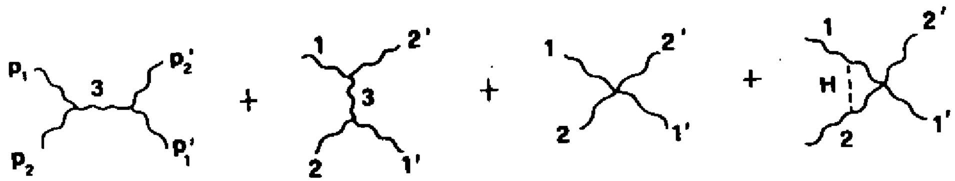

Fig.5