IC/86/373
INTERNAL REPORT
(Limited distribution)

International Atomic Energy Agency
and
United Nations Educational Scientific and Cultural Organization

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

RESONANCE FLUORESCENCE SPECTRA OF A THREE-LEVEL ATOM DRIVEN BY TWO STRONG LASER FIELDS *

Peng Jin Sheng **
International Centre for Theoretical Physics, Trieste, Italy.

ABSTRACT

The resonance fluorescence of a three-level atom interacted with two high-power laser fields is investigated in strong field approximation. The fluorescence distribution is obtained by means of the theory of dressing transformation.

MIRAMARE – TRIESTE
December 1986

* To be submitted for publication.

** Permanent address: Department of Physics, Huazhong Normal University, Wuhan, People's Republic of China.

The fluorescence spectrum of two-level atoms irradiated with an intense resonance electromagnetic field has so far been investigated more completely [1),2),3)], and the main results have been verified by experiments [4),5)]. More recently, much attention has been devoted to the study of fluourescence distribution, emitted from a three-level atom driven by one laser field [6)] or two laser fields [7)-11)]. The experimental study [12)] has shown that the results of fluorescence distribution coincided with the theoretical prediction [13)] in the case of two driving fields, one is intense and the other weak. However the study of both theory and experiment of fluorescence spectra for two strong lasers is expanding. Since resonance fluorescence distribution has played a central role in high-resolution spectroscopy technique, therefore to study this phenomenon is very significant, both theoretically and practically.

The purpose of the present study is to investigate the fluorescence spectra arising from the interaction of three-level atom with two high-power laser fields by the theory of dressing unitary transformation [14),15)].

The model we shall adopt is illustrated in Fig.1. A three-level atom having energy levels $E_1 < E_2 < E_3$, corresponding to the three states $|1\rangle$, $|2\rangle$, $|3\rangle$, interacts with two monochromatic laser fields, described by operators α, β and with fluorescence field, described by operators α_K. The frequencies of laser fields is ω_α and ω_β respectively. Wiggly arrows indicate allowed fluorescence transition, and solid arrows indicate transitions induced by driving laser fields. The atomic operators are C_i^+ and C_i ($i = 1, 2, 3$) corresponding to the creating and annihilating processes of atomic population in energy levels i. If we choose $\frac{1}{2}(E_1 + E_2) = 0$ and $E_2 - E_1 = \omega_0$, thus the Hamiltonian of this system can be described in rotating wave approximation (R.W.A.) as ($\hbar = 1$)

$$H = \frac{1}{2}\omega_0(C_2^+C_2 - C_1^+C_1) + E_3C_3^+C_3 + \omega_\alpha\alpha^+\alpha + \omega_\beta\beta^+\beta + \varepsilon(\alpha C_2^+C_1 + \alpha^+C_1^+C_2)$$
$$+ \varepsilon(\beta C_3^+C_2 + \beta^+C_2^+C_3) + \sum_K \varepsilon_K\alpha_K^+\alpha_K + \sum_K \varepsilon_K(\alpha_K C_2^+C_1 + \alpha_K^+C_1^+C_2 + \alpha_K C_3^+C_2 + \alpha_K^+C_2^+C_3) \quad (1)$$

The algebra of atomic operators C_i^+ and C_i may be found in Hilbert space. Here we only use the following rules:

$$C_i C_j = 0 \quad (i \neq j), \quad C_i C_i C_j = C_j \quad (2)$$

In view of the high intensity of two laser fields, we shall adopt the dressing unitary transformation technique which can formally eliminate the ε -interaction term in (1) from the Hamiltonian, yielding a dressed representation for the system.

Consider that the fluorescence interaction term in Hamiltonian (1) is relatively a small contribution, we may divide Eq.(1) into two parts H_0 and W_K, here

$$H_0 = \frac{1}{2}\omega_0(c_2^+c_2 - c_1^+c_1) + E_3 c_3^+c_3 + \omega_\alpha \alpha^+\alpha + \omega_\beta \beta^+\beta + \varepsilon(\alpha c_2^+c_1 + \alpha^+c_1^+c_2) + \varepsilon(\beta c_3^+c_2 + \beta^+c_2^+c_3) + \sum_K \omega_K \alpha_K^+ \alpha_K \tag{3}$$

The remaining part of fluorescence interaction term may be treated in a second step.

First we treat H_0 by using a unitary transformation operator

$$T = \exp\left\{-\frac{\theta}{(4N)^{1/2}}\left[(\alpha c_2^+c_1 - \alpha^+c_1^+c_2) + (\beta c_3^+c_2 - \beta^+c_2^+c_3)\right]\right\} = \exp\left\{-\frac{\theta}{(4N)^{1/2}}(\bar{V}_1 + \bar{V}_2)\right\} \tag{4}$$

where

$$N = \alpha^+\alpha + \beta^+\beta + \frac{1}{2}(c_2^+c_2 - c_1^+c_1) + \frac{3}{2}c_3^+c_3 + \frac{1}{2} \tag{5}$$

is the operator associated with the total number of excitation of system (atom + driving fields), and θ is a free parameter. Operator T performs a unitary transformation on H_0, it gives

$$\tilde{H}_0 = T^{-1}H_0T = H_0 + \left(-\frac{1}{2}\theta N^{-\frac{1}{2}}\right)[H_0, \bar{V}_1+\bar{V}_2] + \frac{1}{2!}\left(-\frac{1}{2}\theta N^{-\frac{1}{2}}\right)^2[[H_0,\bar{V}_1+\bar{V}_2], \bar{V}_1+\bar{V}_2] + \cdots\cdots \tag{6}$$

Since both laser beams are high-power fields, we may assume a strong field approximation

$$\langle\alpha^+\alpha\rangle \approxeq \langle\beta^+\beta\rangle \tag{7}$$

then

$$N \approxeq \alpha^+\alpha + \beta^+\beta \approx 2\alpha^+\alpha \approx 2\beta^+\beta \tag{8}$$

Consider the commutation relations of operators α^+, α, β^+, β and C_i^+, C_i. After some algebra we obtain the following result

$$\begin{aligned}\tilde{H}_0 = {} & \frac{1}{2}\omega_0(c_2^+c_2 - c_1^+c_1) + E_3c_3^+c_3 + \omega_\alpha\alpha^+\alpha + \omega_\beta\beta^+\beta + \varepsilon(V_1+V_2)\cos\frac{\theta}{2} \\ & - \frac{\delta_\beta+\delta_\alpha}{2\sqrt{N}}(V_1+V_2)\sin\frac{\theta}{2} - \frac{\delta_\beta-\delta_\alpha}{8}(c_3^+c_3 - c_1^+c_1)(\cos\theta - 1) \\ & - \frac{\delta_\beta-\delta_\alpha}{4N}(\alpha\beta c_3^+c_1 + \alpha^+\beta^+c_1^+c_3)(\cos\theta - 1) + \frac{1}{4\sqrt{N}}(\delta_\beta-\delta_\alpha)(V_1-V_2)\sin\theta \\ & + \varepsilon\sqrt{N}(c_3^+c_3 - c_1^+c_1)\sin\frac{\theta}{2}\end{aligned} \tag{9}$$

where

$$V_1 = \alpha c_2^+c_1 + \alpha^+c_1^+c_2 \qquad V_2 = \beta c_3^+c_2 + \beta^+c_2^+c_3$$
$$\delta_\alpha = \omega_0 - \omega_\alpha \qquad \delta_\beta = E_3 - \frac{\omega_0}{2} - \omega_\beta \tag{10}$$

In order to eliminate the ε -interaction terms, we may choose the free parameter θ to satisfy the following equations:

$$\sin\frac{\theta}{2} = \frac{2\varepsilon\sqrt{N}}{\Delta}, \quad \cos\frac{\theta}{2} = \frac{\delta_\alpha + \delta_\beta}{\Delta} \tag{11}$$
$$\Delta = \sqrt{4\varepsilon^2N + (\delta_\alpha+\delta_\beta)^2} \tag{12}$$

we can obtain

$$\begin{aligned}\tilde{H}_0 = {} & \frac{1}{2}\omega_0(c_2^+c_2 - c_1^+c_1) + E_3c_3^+c_3 + \omega_\alpha\alpha^+\alpha + \omega_\beta\beta^+\beta + \frac{\delta_\alpha-\delta_\beta}{8}(c_3^+c_3 - c_1^+c_1)(\cos\theta-1) \\ & + \frac{\delta_\beta+\delta_\alpha}{8}(c_3^+c_3 - c_1^+c_1)(\cos\theta-1) - \frac{\delta_\beta-\delta_\alpha}{4N}(\alpha\beta c_3^+c_1 + \alpha^+\beta^+c_1^+c_3)(\cos\theta-1) \\ & + \frac{1}{4\sqrt{N}}(\delta_\beta-\delta_\alpha)(V_1-V_2)\sin\theta + \varepsilon\sqrt{N}(c_3^+c_3 - c_1^+c_1)\sin\frac{\theta}{2} \\ & + \sum_K \varepsilon_K\omega_K\alpha_K^+\alpha_K\end{aligned} \tag{13}$$

The part of $\varepsilon(\alpha\, C_2^+ C_1 + \alpha^+ C_1^+ C_2)$ and $\varepsilon(\beta\, C_3^+ C_2 + \beta^+ C_2^+ C_3)$ which indicates the interaction between strong laser fields and the atom has been eliminated by such dress transformation. The effects of laser fields are contained in the parameter θ. Therefore Eq.(13) may be considered as a Hamiltonian of three-level atom dressed by two intensive laser fields of frequency ω_α and ω_β. On the resonant case, we have

$$\cos\frac{\theta}{2} = 0 \quad , \quad \sin\frac{\theta}{2} = 1 \tag{14}$$

and

$$\Delta = \Delta_0 = 2\varepsilon\sqrt{N} \tag{15}$$

so that we have more simplified form of Hamiltonian $\tilde{H}_0$

$$\tilde{H}_0 = \frac{1}{2}\omega_0 C_2^+C_2 - \left(\frac{1}{2}\omega_0 + \frac{\Delta_0}{2}\right)C_1^+C_1 + \left(E_3 + \frac{\Delta_0}{2}\right)C_3^+C_3 + \omega_\alpha \alpha^+\alpha + \omega_\beta \beta^+\beta + \sum_K \omega_K \alpha_K^+\alpha_K \tag{16}$$

The next step is to consider the effect of fluorescence interaction term W_K in dressing representation

$$W_K = \sum_K \varepsilon_K(\alpha_K C_2^+C_1 + \alpha_K^+ C_1^+C_2 + \alpha_K C_3^+C_2 + \alpha_K^+ C_2^+C_3) \tag{17}$$

By means of strong field approximation, we can easily obtain

$$\begin{aligned}\widetilde{C_1^+C_2} = T^{-1}C_1^+C_2T = &\left[1 + \frac{1}{4}(\cos\theta - 1) + \frac{1}{2}\left(\cos\frac{\theta}{2} - 1\right)\right]C_1^+C_2 + \frac{1}{\sqrt{N}}\left(\frac{1}{4}\sin\theta + \frac{1}{2}\sin\frac{\theta}{2}\right)\alpha(C_2^+C_2 - C_1^+C_1)\\ &+ \frac{1}{\sqrt{N}}\left(\frac{1}{4}\sin\theta + \frac{1}{2}\sin\frac{\theta}{2}\right)\beta^+C_1^+C_3 + \frac{1}{2N}(\cos\theta - 1)\alpha^2C_2^+C_1 - \frac{1}{2N}(\cos\theta - 1)\alpha\beta^+C_2^+C_3\\ &+ \left[\frac{1}{N}\left(\cos\frac{\theta}{2} - 1\right) - \frac{1}{2N}(\cos\theta - 1)\right]\alpha\beta C_3^+C_2 + N^{-3/2}\left[\frac{1}{2}\sin\theta - \sin\frac{\theta}{2}\right]\alpha^2\beta C_3^+C_1\end{aligned} \tag{18}$$

on resonant case, it reduces to

$$\widetilde{C_1^+C_2} = \frac{1}{2\sqrt{N}}\alpha(C_2^+C_2 - C_1^+C_1) + \frac{1}{2\sqrt{N}}\beta^+C_1^+C_3 - \frac{1}{N}\alpha^2C_2^+C_1 + \frac{1}{N}\alpha\beta^+C_2^+C_3 - \frac{1}{N^{3/2}}\alpha^2\beta C_3^+C_1 \tag{19}$$

Similarly, we have

$$\widetilde{C_2^+C_3} = \frac{1}{2\sqrt{N}}\beta(C_3^+C_3 - C_2^+C_2) - \frac{1}{2\sqrt{N}}\alpha^+C_1^+C_3 + \frac{1}{N}\beta^2C_3^+C_2 - \frac{1}{N}\alpha^+\beta C_1^+C_2 + \frac{1}{N^{3/2}}\alpha\beta^2C_3^+C_1 \tag{20}$$

From Eqs.(16), (19) and (20), we obtain the dressed Hamiltonian at resonant case in the form

$$\begin{aligned}\tilde{H} = &\frac{1}{2}\omega_0 C_2^+C_2 - \frac{1}{2}(\omega_0 + \Delta_0)C_1^+C_1 + \left(E_3 + \frac{\Delta_0}{2}\right)C_3^+C_3 + \omega_\alpha\alpha^+\alpha + \omega_\beta\beta^+\beta + \sum_K \omega_K\alpha_K^+\alpha_K\\ &+ \sum_K \varepsilon_K\Big\{\alpha_K^+\Big[\frac{1}{2\sqrt{N}}\alpha(C_2^+C_2 - C_1^+C_1) + \frac{1}{2\sqrt{N}}\beta^+C_1^+C_3 - \frac{1}{N}\alpha^2C_2^+C_1 + \frac{1}{N}\alpha\beta^+C_2^+C_3 - \frac{1}{N^{3/2}}\alpha^2\beta C_3^+C_1\\ &+ \frac{1}{2\sqrt{N}}\beta(C_3^+C_3 - C_2^+C_2) - \frac{1}{2\sqrt{N}}\alpha^+C_1^+C_3 + \frac{1}{N}\beta^2C_3^+C_2 - \frac{1}{N}\alpha^+\beta C_1^+C_2 + \frac{1}{N^{3/2}}\alpha\beta^2C_3^+C_1\Big] + H.c\Big\}\end{aligned} \tag{21}$$

We see that the interaction terms between atom and the strong driving fields in Hamiltonian (1) have been eliminated by dress unitary transformation, but the interaction term between atom and the fluorescent field became more complicated. However these complicated terms just reveal obviously the fluorescence processes of a three-level atom induced by driving laser fields. In order to realize such fluorescence processes, we now discuss the fluorescence distribution of system by a perturbation approach. From the Heisenberg equation for α_K, we obtain

$$\begin{aligned}\alpha_K(t) = \alpha(0)e^{-i\omega_K t} - i\varepsilon_K\int_0^t \Big[&\frac{1}{2\sqrt{N}}\alpha(C_2^+C_2 - C_1^+C_1) + \frac{1}{2\sqrt{N}}\beta^+C_1^+C_3 - \frac{1}{N}\alpha^2C_2^+C_1 + \frac{1}{N}\alpha\beta^+C_2^+C_3\\ &- \frac{1}{N^{3/2}}\alpha^2\beta C_3^+C_1 + \frac{1}{2\sqrt{N}}\beta(C_3^+C_3 - C_2^+C_2) - \frac{1}{2\sqrt{N}}\alpha^+C_1^+C_3 + \frac{1}{N}\beta^2C_3^+C_2 - \frac{1}{N}\alpha^+\beta C_1^+C_2\\ &+ \frac{1}{N^{3/2}}\alpha\beta^2C_3^+C_1\Big]_{t-\tau} e^{-i\omega_K\tau}\,d\tau\end{aligned} \tag{22}$$

The time dependent product of the operators in an integral term can be obtained from the Heisenberg equations by neglecting in the zero-approximation the non-diagonal term in Eq.(21), it yields

$$\begin{aligned}&[\alpha(C_2^+C_2 - C_1^+C_1)]_t = [\alpha(C_2^+C_2 - C_1^+C_1)]_0\, e^{-i\omega_\alpha t}\\ &(\beta^+C_1^+C_3)_t = (\beta^+C_1^+C_3)_0\, e^{-i(\frac{1}{2}\omega_0 + \Delta_0 + E_3 - \omega_\beta)t}\\ &\qquad\vdots\end{aligned} \tag{23}$$

Substitution of Eq.(23) into Eq.(22) and subsequent integration gives

$$\begin{aligned}\alpha_K(t) = \alpha_K(0)e^{i\omega_K t} + \varepsilon_K\Big\{&\frac{1}{2\sqrt{N}}[\alpha(C_2^+C_2 - C_1^+C_1)]_t\left[P\frac{1}{\omega_\alpha - \omega_K} - i\pi\delta(\omega_\alpha - \omega_K)\right]\\ &+ \frac{1}{2\sqrt{N}}(\beta^+C_1^+C_3)_t\left[P\frac{1}{E_3 + \frac{1}{2}\omega_0 + \Delta_0 - \omega_\beta - \omega_K} + i\pi\delta(E_3 + \tfrac{1}{2}\omega_0 + \Delta_0 - \omega_\beta - \omega_K)\right] + \cdots\Big\}\end{aligned} \tag{24}$$

We may neglect the principal parts in Eq.(24) because they yield only small Lamb shifts in the energy levels. ϵ_K is the coupling constant between the K-th mode of fluorescence field and the atom, so that $\sum_K \epsilon_K a_K$ is proportional to the amplitude of the fluorescent field, thus

$$E(t) \propto \sum_K \varepsilon_K \alpha_K(t) = \sum_K \varepsilon_K \alpha_K(0) e^{-i\omega_K t} - i\gamma_1 \frac{1}{2\sqrt{N}} \alpha (C_2^+ C_2 - C_1^+ C_1) - i\gamma_2 \frac{1}{2\sqrt{N}} \beta^+ C_1^+ C_3$$
$$+ i\gamma_3 \frac{1}{N} \alpha^2 C_2^+ C_1 - i\gamma_4 \frac{1}{N} \alpha \beta^+ C_2^+ C_3 + i\gamma_5 \frac{1}{N^{3/2}} \alpha^2 \beta C_3^+ C_1 - i\gamma_6 \frac{1}{2\sqrt{N}} \beta (C_3^+ C_3 - C_2^+ C_2) \qquad (25)$$
$$+ i\gamma_7 \frac{1}{2\sqrt{N}} \alpha^+ C_1^+ C_3 - i\gamma_8 \frac{1}{N} \beta^2 C_3^+ C_2 + i\gamma_9 \frac{1}{N} \alpha^+ \beta C_1^+ C_2 - i\gamma_{10} \frac{1}{N^{3/2}} \alpha^2 \beta C_3^+ C_1$$

where

$$\gamma_1 = \pi \sum_K \varepsilon_K^2 \delta(\omega_\alpha - \omega_K) \qquad \gamma_2 = \pi \sum_K \varepsilon_K^2 \delta(E_3 + \tfrac{1}{2}\omega_0 + \Delta_0 - \omega_\beta - \omega_K)$$
$$\gamma_3 = \pi \sum_K \varepsilon_K^2 \delta(2\omega_\alpha - \omega_0 - \tfrac{\Delta_0}{2} - \omega_K) \qquad \gamma_4 = \pi \sum_K \varepsilon_K^2 \delta(E_3 + \omega_\alpha - \omega_\beta - \tfrac{1}{2}\omega_0 + \tfrac{\Delta_0}{2} - \omega_K)$$
$$\gamma_5 = \pi \sum_K \varepsilon_K^2 \delta(2\omega_\alpha + \omega_\beta - \tfrac{1}{2}\omega_0 - E_3 - \Delta_0 - \omega_K) \qquad \gamma_6 = \pi \sum_K \varepsilon_K^2 \delta(\omega_\beta - \omega_K) \qquad (26)$$
$$\gamma_7 = \pi \sum_K \varepsilon_K^2 \delta(E_3 + \tfrac{\omega_0}{2} + \Delta_0 - \omega_\alpha - \omega_K) \qquad \gamma_8 = \pi \sum_K \varepsilon_K^2 \delta(2\omega_\beta + \tfrac{\omega_0}{2} - E_3 - \tfrac{\Delta_0}{2} - \omega_K)$$
$$\gamma_9 = \pi \sum_K \varepsilon_K^2 \delta(\omega_\beta + \omega_0 - \omega_\alpha + \tfrac{\Delta_0}{2} - \omega_K) \qquad \gamma_{10} = \pi \sum_K \varepsilon_K^2 \delta(2\omega_\beta + \omega_\alpha - E_3 - \tfrac{\omega_0}{2} - \Delta_0 - \omega_K)$$

These γ_i (i = 1 - 10) obviously correspond to the spontaneous relaxation rates for transitions between the dressed levels shown in Fig.2. Here the wiggly arrows indicate allowed spontaneous emittions.

We can see from the present results, when the detunning $\delta\omega = \omega_\alpha - \omega_\beta > 2\Delta_0$ the resonance fluorescence spectrum consists of ten components symmetrical about the frequencies ω_α and ω_β respectively. Morever, when the frequency detunning $\delta\omega = \omega_\alpha - \omega_\beta$ is less than $2\Delta_0$, the fluorescence spectra will overlap, if $\delta\omega = \Delta_0$ the spectra distribution will be seven peak lines which coincide with previous results [8),10)]. When the detunning $\delta = 0$, that is $\omega_\alpha = \omega_\beta$, we find the exact five peaks fluorescence distribution

$$\gamma_1 = \gamma_6 = \pi \sum_K \varepsilon_K^2 \delta(\omega_\alpha - \omega_K)\,, \qquad \gamma_2 = \gamma_7 = \pi \sum_K \varepsilon_K^2 \delta(E_3 + \tfrac{1}{2}\omega_0 - \omega_\alpha + 2\Delta_0 - \omega_K)$$
$$\gamma_3 = \gamma_8 = \pi \sum_K \varepsilon_K^2 \delta(2\omega_\alpha - \omega_0 - \Delta_0 - \omega_K)\,, \qquad \gamma_4 = \gamma_9 = \pi \sum_K \varepsilon_K^2 \delta(\omega_0 + \Delta_0 - \omega_K)$$
$$\gamma_5 = \gamma_{10} = \pi \sum_K \varepsilon_K^2 \delta(3\omega_\alpha - 2\omega_0 - 2\Delta_0 - \omega_K) \qquad (27)$$

this is exactly equivalent to those obtained by other authors [9),11)]. In comparison with the previous results, the present conclusion of fluouressence spectrum of ten components appear more general.

In conclusion, we have obtained the resonance fluorescence distribution of a three-level atom driven by two high-power laser fields by the theory of dressing unitary transformation. It is hoped that this study will stimulate experiment work such as to confirm the present theoretical prediction.

ACKNOWLEDGEMENTS

The author would like to thank Professor Abdus Salam, the International Atomic Energy Agency and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste. He would also like to thank Professor F. Persico for helpful discussions.

REFERENCES

1) R. Mollow, Phys. Rev. 188, (1969) 1969.

2) R. Mollow, Phys. Rev. A12, (1976) 1919.

3) G. Compagno and F. Persico, Phys. Rev. A25, (1982) 3138.

4) F. Schuda, C.R. Stroud and M. Hercher, J. Phys. B7, (1974) L198.

5) S. Swain, J. Phys. B8, (1975) L437.

6) K.J. Woloschuk, S. Hontzeas and Constantine Mavroyannis, Can. J. Phys. 60, (1982) 986.

7) B.J. Feldman and M.S. Feld, Phys. Rev. A5, (1972) 899.

8) R.M. Whitley and C.R. Stroud Jr., Phys. Rev. A14, (1976) 1498.

9) C. Cohen-Tannoudji and S. Reynaud, J. Phys. B10 (1977) 2311.

10) M.P. Sharma, A. Balbin Villaverde and Constantine Mavroyannis, Can. J. Phys. 58, (1980) 1570.

11) Peng Jin Sheng, ACTA. Physica Sinica 34, (1985) 408.

12) P.T.H. Fisk, H.A. Bochor and R.J. Sandemen, Phys. Rev. A33, (1986) 2418.

13) P.T.H. Fisk, H.A. Bochor and R.J. Sandemen, Phys. Rev. A33, (1986) 2424.

14) P. Carbonaro, G. Compagno and F. Persico, Phys. Lett. A73, (1979) 97.

15) C. Compagno, J.S. Peng and F. Persico, Phys. Rev. A26, (1982) 2065.

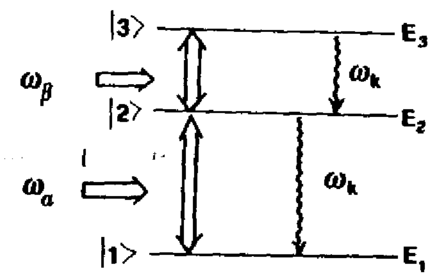

Fig.1

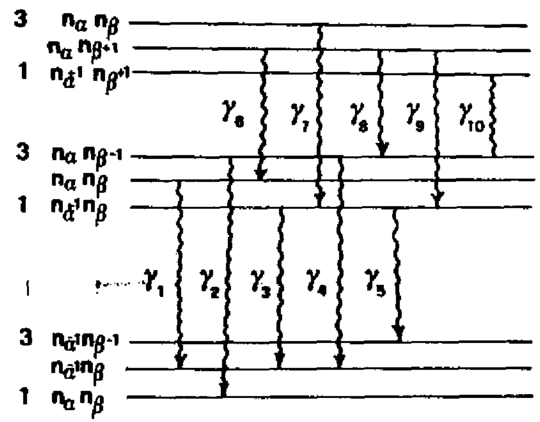
3 $n_\alpha\, n_\beta$
$n_\alpha\, n_{\beta+1}$
1
γ_6 γ_7 γ_8 γ_9 γ_{10}
3 $n_\alpha\, n_{\beta-1}$
$n_\alpha\, n_\beta$
1
γ_1 γ_2 γ_3 γ_4 γ_5
3
1 $n_\alpha\, n_\beta$

Fig.2