IC/86/49

INTERNATIONAL CENTRE FOR
THEORETICAL PHYSICS

THE EFFECT OF TRANSITIONAL PARTICLES DRIVEN BY SINGLE WAVE

Qiu Yun-qing

and

Xia Meng-fen

INTERNATIONAL
ATOMIC ENERGY
AGENCY

UNITED NATIONS
EDUCATIONAL,
SCIENTIFIC
AND CULTURAL
ORGANIZATION

1986 MIRAMARE-TRIESTE

IC/86/49

International Atomic Energy Agency
and
United Nations Educational Scientific and Cultural Organization

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

THE EFFECT OF TRANSITIONAL PARTICLES DRIVEN BY SINGLE WAVE *

Qiu Yun-qing **
International Centre for Theoretical Physics, Trieste, Italy,

and

Xia Meng-fen
Department of Physics, Peking University, Beijing,
People's Republic of China.

ABSTRACT

The unperturbed separatrix crossing driven by a single wave in a tokamak plasma is discussed. The separatrix crossing is followed by a mixing process, and a small-scale structure occurs in the distribution function in h-ψ plane. The separatrix crossing is a convective process in h-ψ plane, and there is a definite crossing channel. The convective flux and the net flux in h-direction are calculated. The separatrix crossing is accompanied by a radial flux, which is composed of a directional flux and a diffusion flux.

MIRAMARE – TRIESTE
May 1987

* To be submitted for publication.

** Permanent address:Department of Physics, Peking University, Beijing, People's Republic of China.

1. INTRODUCTION

In a tokamak in high temperature regimes some properties of the plasma are strongly dependent on the appearance of the transforms between the trapped and passing particles. Such unperturbed separatrix crossing are usually driven by Coulomb collisions, turbulence,etc.(Galeev and Sagdeev,1968; Dobrowolny, Orefice and Pozzoli, 1973; Casati and Lazzaro, 1981; Hastings,1984)

In this paper, we point out that the stochastic transforms across the trapped/passing boundary may be driven by a single wave, i.e., they may occur in a dynamic system. The mechanism of the unperturbed separatrix crossing is concerned with the dynamic stochasticity (Chirikov, 1979; Escande, 1985). Based on a simplified dynamic model of the toroidal plasma interacting with a single wave, we discuss the characteristics of the wave-driven separatrix crossing, especially the behaviors of the transitional particles, which are the particles situated in the crossing stages.

This work was motivated by that the effects of interaction of the toroidal plasma with the waves are getting increasingly important. In addition, the separatrix crossing effects are also noticeable in a series of important problems (Tennyson, Cary and Escande, 1986).

In Sec.2, we give a dynamic model for our system, in Sec.3, the processes of unperturbed separatrix crossing are discussed, in Sec.4, the channel and the flux of the separatrix crossing are investigated, in Sec.5, a small-scale structure in the distribution function is obtained, in Sec.6, the radial flux of the transitional particles is calculated, and the conclusions are summarized in Sec.7.

2. DYNAMIC MODEL

The magnetic field is given in toroidal coodinates (r, θ, φ) as

$$\underset{\sim}{B} = (0,\ \epsilon/q\ ,\ 1)\ \frac{R_0}{R} B_0 \qquad (1)$$

where

$$R = R_0(1+\epsilon\cos\theta),\quad \epsilon = r/R_0\ ,\quad q = rB_\varphi/R_0B_\theta \quad B_\varphi>0$$

and $B_\theta>0$. The electrostatic wave is described by

$$\phi = \phi_0 \cos(\underset{\sim}{k}\cdot\underset{\sim}{r} - \omega t + \psi_0). \qquad (2)$$

Assuming $\omega \ll \Omega_e = eB_0/m_e c$, $k_\perp v_\perp/\Omega_e \ll 1$ and $k_\parallel q R_0 \gg 1$, where $//$ and $\perp$ stand for the longitudinal and perpendicular components, respectively, the motion equations of an electron (mass m_e and charge $-e$) can be obtained in drift approximation as

$$\frac{d\theta}{d\tau} = V, \qquad (3)$$

$$\frac{dV}{d\tau} = -\sin\theta - Ak\sin(k\theta - \nu\tau + \psi_0), \qquad (4)$$

$$\frac{dX}{d\tau} = \sin\theta\ . \qquad (5)$$

where

$$\tau = \omega_{b0} t,\quad \omega_{b0} = \epsilon^{1/2} v_t/qR_0\ ,\quad v_t = (T_e/m_e)^{1/2},$$

$$V = v_\parallel/\epsilon^{\frac{1}{2}} v_t\ ,\quad k = k_\parallel q R_0\ ,\quad \nu = \omega/\omega_{b0},$$

$$A = e\phi_0/\epsilon T_e \quad X = r/\Delta r\ ,\quad \Delta r = \epsilon^{-1/2} q\rho\ ,\quad \rho = v_\perp/\Omega_e.$$

ψ_0 in R.H.S. of eq.(4) is the initial phase of the wave at $\theta=0$. Eq.(5) describes the radial motion of the particle.

The effective Hamiltonian corresponding to eqs.(3) and (4) may be written in the following form

$$\begin{aligned} H &= H_0 + H_1\ , \\ H_0 &= V^2/2 - \cos\theta\ , \\ H_1 &= -A\cos(k\theta - \nu\tau + \psi). \end{aligned} \qquad (6)$$

Such a kind of dynamic system has been extensively discussed in the field of dynamic stochasticity. In the vicinity of the unperturbed separatrix $h = H_0 - 1$, (7)

the motion of the particle may be described by the following mapping equations

$$h_{\ell+1} = h_\ell - \mathcal{S}_\ell A\nu A_{2k}(\mathcal{S}_\ell \nu)\sin\psi_\ell, \qquad (8)$$

$$\psi_{\ell+1} = \psi_\ell - \nu\Delta\tau_{\ell+1}\ , \qquad (9)$$

$$\mathcal{S}_{\ell+1} = \mathcal{S}_\ell \,\mathrm{Sgn}\, h_{\ell+1}\ , \qquad (10)$$

$$Y_{\ell+1} = Y_\ell + [1 - \mathrm{Sgn}(h_\ell h_{\ell+1})]\mathcal{S}_\ell \,\mathrm{Sgn}\, h_\ell, \qquad (11)$$

where h_ℓ are the values of h taking at $\theta=\pm\pi$ or at the turning points and ψ_ℓ are the phase of the wave at $\theta=0$ where the particle is just going through. $\mathcal{S}_\ell=\pm1$ are the signs of V , $\Delta\tau_\ell$ are the period of passing particle or the half period of trapped particle

$$\Delta\tau_\ell = \ell_n \frac{32}{|h_\ell|}. \qquad (12)$$

$A_m(\nu)$ is Melnikov-Arnold integral (Chirikov, 1979), and

$$Y_{\ell+1} = \frac{1}{2}(X_\ell + X_{\ell+1}), \qquad (13)$$

are the average radial positions of the particle.

An important character of the mapping equations (8)-(11) is the presence of a series of resonance lines in h-ψ plane:

$$h = h_n = \pm 32\, e^{-2\pi n/\nu}, \tag{14}$$

where n is an interger. The density of the resonance lines increases infinitely as the unperturbed separatrix is reached. The resonance lines play an important role in the mapping. The crossing points of the resonance lines with lines $\psi = 0$ or $\psi = \pm\pi$ are the fixed points of the mapping. The map of a curve h = constant is strongly dependent on the numbers of the resonance lines in the region $[h-\beta,\ h+\beta]$, where

$$\beta = A\nu A_{2k}(\nu). \tag{15}$$

In Fig.1, we show the map of a resonance line for various cases: (a) $\Delta h_n > \beta$, (b) $\Delta h_n < \beta < |h_n|$ and (c) $|h_n| < \beta$, where Δh_n is the interval between adjacent resonance lines, and we assume $\beta > 0$.

Because $|A_{2k}(-\nu)/A_{2k}(\nu)| \ll 1$ as $\nu \gg 1$, the change of h takes place mainly for $V > 0$ case. The mapping equations (8)-(10) reduce to

$$h' = h - \beta \sin\psi, \tag{16}$$

$$\psi' = \psi - \nu\Delta\tau', \tag{17}$$

where

$$\Delta\tau' = \lambda(h')\ln\frac{32}{|h'|},$$

$$\lambda(h') = \left[1 + \tfrac{1}{2}(1 - \mathrm{sgn}\, h')\right]. \tag{18}$$

Fig.1. The maps of resonances lines. $\nu = 38$, $\beta = 0.013$.

(a) $n = 30$, $\Delta h_n > \beta$. ψ-axis stands for the resonance line $n = 30$.

(b) $n = 37$, $\Delta h_n < \beta < |h_n|$. ψ-axis stands for the resonance line $n = 37$.

(c) $n = 50$, $|h_n| < \beta$. The curve 1 stands for the resonance line $n = 50$.

---

3. THE UNPERTURBED SEPARATRIX CROSSING

The separatrix crossing driven by a single wave is related to the presence of a stochastic region across the unpertubed separatrix. When $\beta < \nu^{-1}$, the width of the stochastic region may be estimated at $\Delta h \sim \nu\beta$ (for $\xi = 1$ in region $h > 0$). Our numerical calculations show that the correlation effect is unimportant in the central part of the stochastic region, and the motion of the particle in h-ψ plane is Markovian, approximately. We can introduce a local diffusion coefficient D_0 in h-direction to describe the motion. We have

$$D_0 \simeq \frac{\beta^2}{4\Delta\tau}, \tag{19}$$

for $h > 0$ with $\xi = 1$ and $h < 0$, and $D_0 = 0$ for $h > 0$ with $\xi = -1$. $\Delta\tau$ in R.H.S. of eq.(19) is given by eq.(18). From eqs.(19) and (18), we can see that D_0 decreases when h closes in on the separatrix $h = 0$.

Now we consider an ensemble of particles with initial distribution function

$$f(h, \psi, 0) = \frac{1}{2\pi}\,\delta(h - h_0) \tag{20}$$

1

Let N be the number of periods, after which the separatrix crossing occurs. The ensemble average of N is denoted by $\langle N\rangle$. In order to calculate $\langle N\rangle$ we use an average diffusion coefficient $\bar{D}_0$, which is the average value of D_0 given by eq.(19), and we estimate approximately

$$\bar{D}_0 \simeq \frac{\beta^2}{4\,\overline{\Delta\tau}} \quad . \tag{21}$$

where $\overline{\Delta\tau}$ is the average period over the region from $\hbar=\hbar_0$ to $\hbar=0$, and we have

$$\overline{\Delta\tau} = \left[1+\frac{1}{2}(1-\mathrm{sgn}\,\hbar_0)\right]\left(1+\ln\frac{32}{|\hbar_0|}\right).$$

From eq.(21), we obtain

$$\langle N\rangle \simeq \frac{\hbar_0^2}{2\,\bar{D}_0\,\overline{\Delta\tau}} = 2\hbar_0^2/\beta^2 \quad . \tag{22}$$

In our numerical calculations, the average value $\langle N\rangle$ is calculated for half of the particles in ensemble, which cross the separatrix earlier than the others. In Fig.2, we show the numerical results of $\langle N\rangle$ and those given by eq.(22).

Fig.2. Comparison of the numerical results of $\langle N\rangle$ (crosses) with those given by eq.(22) (dotted line), $\beta=0.013$.

From Fig.2, we can see that $\langle N\rangle \gg 1$ for the most part of the stochastic region where the initial value $\hbar_0$ is localized. It means

that in these regions the motion of particles still maintains the basic pattern of the trapped or passing particles. However, there is a layer with width $\Delta\hbar\sim\beta$ across the separatrix, where $\langle N\rangle\sim 1$. For these particles, on an average, the separatrix crossing occurs in a duration shorter than the bounce period. Then, these particles are neither trapped nor passing particles, and they may be called the transitional particles. The single-wave-induced transitional particles and transitional layer are different from those induced by Coulomb collisions in neoclassical theory.

The characteristics of the separatrix crossing are mainly related to the behaviors above-mentioned anomalous transitional particles.

4. CHANNEL AND FLUX OF THE SEPARATRIX CROSSING

For our dynamic system, the stochastic unperturbed separatrix crossing is not a "pure" random process. Here, we point out that there is a definite crossing channel in $\hbar-\psi$ plane shown in Fig.3 (for the case of $\beta>0$). The crossing channel in $\hbar-\psi$ plane is the area surrounded by the curves $\hbar=0$ and $\hbar=\beta\sin\psi$, and it is composed of two parts, i.e. D-channel and U-channel. A particle crosses the unperturbed separatrix from the trapped region ($\hbar<0$) to the passing region ($\hbar>0$) via U-channel, and the inverse crossing is via D-channel.

Fig.3. The crossing channel (D-channel and U-channel) in h-ψ plane.

In Fig.4, we show the map of D-channel, i.e. the distribution of the particles, which have just crossed the separatrix from D-channel to the region $h<0$.

Fig.4.The map of D-channel.

In order to illustrate the distribution of the particles after separatrix crossing via the channel clearly, we use an auxiliary picture, i.e., the map of a line segment $\psi=\psi_0$, $B\sin\psi_0 \ge h \ge 0$ given in Fig.5. The map of this line segment extends in ψ-direction infinitely and distributes over the whole region $-\pi \le \psi \le \pi$, $0 \ge h \ge -B\sin\psi_0$. Such a picture is due to that the original line segment crosses with infinitely great number of the resonance lines, and the map of the line segment between adjacent crossed points is a line segment extended over the region $-\pi \le \psi \le \pi$. If there is another line segment $\psi=\psi_0'$, $B\sin\psi_0' \ge h \ge 0$ in D-channel near the original line segment, the maps of both line segments will have similar structure, and they will distribute in h-ψ plane alternately. This means that the unperturbed separatrix crossing is followed by an infinite mixing process. Such a mixing is closely related to the dynamic stochasticity. In fact, because $\Delta h_n \sim \frac{2\pi}{\nu}|h_n|$,

the mixing condition $\Delta h_n < 2B$ may be rewritten as $|h_n| < \frac{\nu B}{\pi}$, which coincides with the stochasticity condition approximately. However, the mixing related to the separatrix crossing is typical and strongest.

Fig.5. The map of the line segment $\psi=\psi_0$, $B\sin\psi_0 \ge h \ge 0$.

$B=0.013$.

Now we calculate the flux of unperturbed separatrix crossing driven by a single wave. From eqs.(8) and (12) the interval between h and its map h' is

$$\tau \simeq \ln \frac{32}{\sqrt{|h(h-B\sin\psi)|}} . \qquad (23)$$

Let J_D be the flux from region $h>0$ to $h<0$ via D-channel, and J_U be the flux from region $h<0$ to $h>0$ via U-channel. We have

$$J_D = \int_0^{B} dh \int_{\sin^{-1}\frac{h}{B}}^{\pi-\sin^{-1}\frac{h}{B}} d\psi \, \frac{f(h,\psi)}{\ln \frac{32}{\sqrt{|h(h-B\sin\psi)|}}} , \qquad (24)$$

and

$$J_U = \int_0^{-B} dh \int_{-\sin^{-1}\frac{|h|}{B}}^{-(\pi-\sin^{-1}\frac{|h|}{B})} d\psi \, \frac{f(h,\psi)}{\ln \frac{32}{\sqrt{|h(h-B\sin\psi)|}}} . \qquad (25)$$

Where $f(h,\psi)$ is the distribution function in h-ψ plane. J_D and

J_U are the intensity of convective flow across the separatrix $h=0$ in h-direction in h-ψ plane and the net flux is

$$\Delta J = J_U - J_D . \tag{26}$$

If the distribution function may be expressed as

$$f(h,\psi) \cong f_0(h) \cong f_0(0) + f_0'(0)\, h , \tag{27}$$

approximately, we have

$$J_D \cong J_U \cong J_0 = 2B\, f_0(0) \left\langle \frac{1}{\tau} \right\rangle . \tag{28}$$

in the zero-order, where

$$\left\langle \frac{1}{\tau} \right\rangle = \frac{1}{2B} \int_0^B dh \int_{\sin^{-1}\frac{h}{B}}^{\pi-\sin^{-1}\frac{h}{B}} d\psi \frac{1}{\ln \dfrac{32}{\sqrt{|h(h-B\sin\psi)|}}} . \tag{29}$$

The order of magnitude of $\langle \frac{1}{\tau} \rangle$ can be estimated by

$$\left\langle \frac{1}{\tau} \right\rangle \sim \frac{1}{\langle \tau \rangle} \tag{30}$$

where $\langle\tau\rangle$ is the average period (half period for region $h<0$) of particles in the channel, we have

$$\langle \tau \rangle = 2 + \ln \frac{16}{B} = \ln \frac{16 e^2}{B} \tag{31}$$

From eqs.(28),(30) and (31), we obtain the convective flux

$$J_0 \simeq 2B\, f_0(0) \big/ \langle \tau \rangle , \tag{32}$$

and the net flux may be calculated from eqs.(24)-(27)

$$\Delta J = -G\, 2\pi f_0'(0) \tag{33}$$

where

$$G = \int_0^B dh \int_{\sin^{-1}\frac{h}{B}}^{\pi-\sin^{-1}\frac{h}{B}} d\psi \frac{h/\pi}{\ln \dfrac{32}{\sqrt{|h(h-B\sin\psi)|}}} . \tag{34}$$

which may be estimated by

$$G \cong 2B \frac{\langle h/\pi \rangle}{\langle \tau \rangle} = \frac{B^2}{4\langle\tau\rangle} , \tag{35}$$

then

$$\Delta J \cong -2\pi f_0'(0) \frac{B^2}{4\langle\tau\rangle} . \tag{36}$$

Eq.(36) means that the net flux is a diffusion-like flux in h-direction, and the diffusion coefficient is $B^2/4\langle\tau\rangle$.

In preceding discussions we have assumed that $f_0(h)$ is a smooth function. However, we will point out that there is a complex small-scale structure in the distribution function. So the above-mentioned expressions for the flux must be understood in the meaning of the coarse-grained average.

5.SMALL-SCALE STRUCTURE IN THE DISTRIBUTION FUNCTION

The mapping described by eqs.(8)-(11) is area-conserved in h-ψ plane, i.e.

$$f(h', \psi', \tau+\Delta\tau) = f(h, \psi, \tau) \tag{37}$$

where

$$h' = h - B \sin\psi \tag{38}$$

$$\psi' = \psi - \nu_0 \tau' \tag{39}$$

and

$$\Delta\tau' = \lambda(h') \ln\frac{32}{|h'|}, \qquad \lambda(h') = 1 + \frac{1}{2}(1 - \mathrm{sgn}\, h'). \tag{40}$$

We have neglected the change of h when $\zeta = -1$.

The evolution equation of the distribution function may be deduced from eqs.(37)-(40):

$$f(h, \psi, \tau) = f\left[h + B \sin(\psi + \nu_0 \Delta\tau),\ \psi + \nu_0\Delta\tau,\ \tau - \Delta\tau\right], \tag{41}$$

where $\Delta\tau$ is the function of h. We now split off from distribution funtion a term $f_0(h)$ being independent of ψ and τ ,we thus put

$$f(h, \psi, \tau) = f_0(h) + f_1(h, \psi, \tau). \tag{42}$$

From eqs.(41) and (42), we get

$$f_1(h, \psi, \tau) = f_0[h + B \sin(\psi + \nu_0 \Delta\tau)] - f_0(h) + f_1[h + B\sin(\psi + \nu_0\Delta\tau),\ \psi + \nu_0\Delta\tau,\ \tau - \Delta\tau]. \tag{43}$$

Eq.(43) shows that we can split off from $f_1(h, \psi, \tau)$ a term

$$f_{10}(h, \psi) = f_0[h + B \sin(\psi + \nu_0\Delta\tau)] - f_0(h), \tag{44}$$

which is independent of time. If the function $f_0(h)$ is smooth enough we have

$$f_{10}(h, \psi) \simeq f_0'(h) B \sin(\psi + \nu_0\Delta\tau) + \frac{1}{2} f_0''(h) B^2 \sin^2(\psi + \nu_0\Delta\tau), \tag{45}$$

approximately. Because $(\psi + \nu_0\Delta\tau)$ is the most sensitive factor in the R.H.S. of eq.(44) or (45), $f_{10}(h, \psi)$ shows a special structure in $h - \psi$ plane. The contour lines are given by equation

$$\psi + \lambda(h) \ln\frac{32}{|h|} = const. \tag{46}$$

The period of f_{10} is 2π in ψ-direction, and in h-direction f_{01} has a quasi-periodic structure, the period is nearly equal to

$$\Delta h_n = |h_{n+1} - h_n|, \tag{47}$$

where

$$h_n = \pm 32\, e^{-2\pi n/\lambda\nu}. \tag{48}$$

When $h \to 0$, the period $\Delta h_n \to 0$. If $f_0(h)$ has a finite slope in the vicinity of $h = 0$, f_{10} will have a finite oscillating amplitude $\Delta f_{10} \sim |f_0'(0) B|$. Then , in the vicinity of the separatrix, f_{10} may have a strong-oscillating short-period structure.

The above-mentioned time-independent small-scale structure in the distribution function in $h - \psi$ plane is driven by the zero-order slope of the distribution function. The mechanism may be attributed to the strong mixing process of the area-conserved mapping in the vicinity of the unperturbed separatrix. In principle, the first-order slope of distribution function may drive the next order small-scale structure, and so on. So the distribution function will have a infinitely complex structure. However, the lowest order structure is most important.

In Fig.6 we give a sketch map showing the first-order small-scale structure driven by the zero-order slope of the distribution function . Fig.6 (a) shows the inhomogeneous part of the zero-order distribution function, and Fig.6 (b) gives the maps of those points. From Fig.6 (b), we can see that the slope of the distribution function is reserved roughly in the meaning of coarse-grained average, however, a small-scale quasi-periodic structure appears in the distribution function.

Fig.6. The sketch map of the small-scale structure in distribution function in $\hbar$-ψ plane.

(a) The inhomogeneous part of the zero-order distribution function.

(b) The small-scale structure in the distribution function.

---

6. THE RADIAL FLUX

Eq.(11) shows that the unperturbed separatrix crossing is accompanied by a displacement of the average radial position of the particle:

$$\delta y = (1 - \mathrm{sgn}(\hbar \hbar'))\, \mathrm{sgn}\, \hbar . \tag{49}$$

For the separatrix crossing via D-channel ($\hbar > 0, \hbar' < 0$)

$$\delta y_D = 2 . \tag{50}$$

and for the separatrix crossing via U-channel ($\hbar < 0, \hbar' > 0$)

$$\delta y_U = -2 = -\delta y_D . \tag{51}$$

Because the separatrix crossing is a convective process, from eqs.(50) and (51), the radial motion is also convective.

Now let us assume the zero-order distribution function be also dependent on the average radial position y : $f_0 = f_0(y, \hbar)$, and B be independent of y. Then the fluxes of separatrix crossing will be dependent on y : $J_D(y)$ and $J_U(y)$. From eqs.(32),(38),(50) and (51) we obtain the expression for net radial flux as

$$\Gamma = \Gamma_1 + \Gamma_2 , \tag{52}$$

where

$$\Gamma_1 = -\Delta J \cdot \delta y_D = \frac{\pi B^2}{\langle \tau \rangle} \frac{\partial f_0}{\partial \hbar}\bigg|_{\hbar=0} , \tag{53}$$

and

$$\Gamma_2 = -\frac{dJ}{dy} (\delta y_D)^2 = -\frac{8B}{\langle \tau \rangle} \frac{\partial f_0}{\partial y}\bigg|_{\hbar=0} , \tag{54}$$

where $\langle \tau \rangle$ is given by eq.(31). Let

$$f_0(y, \hbar) = n(y)\, g(\hbar) , \tag{55}$$

where n is the density of the particles. The radial fluxes may be expressed by the dimensional quantities as

$$\tilde{\Gamma}_1 = n v_t \frac{f}{K_0} \frac{\pi B^2}{\langle \tau \rangle} \frac{dg}{d\hbar}\bigg|_{\hbar=0} , \tag{56}$$

and

$$\tilde{\Gamma}_2 = - D_\perp \frac{dn}{dr} , \qquad (57)$$

where

$$D_\perp = \epsilon^{-1/2} \frac{\rho^2 q v_t}{R_0} \cdot \frac{\delta B g(0)}{\langle \tau \rangle} . \qquad (58)$$

Then, the radial flux is composed of two parts. $\tilde{\Gamma}_1$ is a directional flux, which is proportional to the slope of distribution function in h-direction; and $\tilde{\Gamma}_2$ is a diffusion flux, which is proportional to the gradient of the density dn/dr and the radial diffusion coefficient $D_\perp$ is given by eq.(58). The direction of Γ_1 and Γ_2 are determined by the signs of dg/dh and dn/dr, respectively. Comparing eqs.(53) and (54), we get

$$\frac{\Gamma_1}{\Gamma_2} = - \frac{\pi}{\delta} \frac{\partial f_0 / \partial (h/B)}{\partial f_0 / \partial (r/\Delta r)} , \qquad (59)$$

i.e., the ratio Γ_1/Γ_2 is related to the ratio of the change rate of f_0 in h-direction (with unit B) to that in r-direction (with unit $\Delta r = \epsilon^{1/2} \rho q$).

Comparing with the neoclassical diffusion coefficient

$$D_{NC} = \epsilon^{-3/2} \rho^2 q^2 \nu_e , \qquad (60)$$

we obtain

$$\frac{D_\perp}{D_{NC}} \sim \frac{\delta B g(0)}{\epsilon^{1/2} \langle \tau \rangle} \cdot \frac{\nu_1}{\nu_e} , \qquad (61)$$

where $\nu_1 = \epsilon^{3/2} v_t / q R_0$ and $\langle \tau \rangle$ is given by eq.(31). It is possible that $D_\perp / D_{NC} \gg 1$ for high-temperature, low-density and strong-wave regimes. For example, if we take $\epsilon = 0.2$, $q = 3$, $R = 1\,m$, $n = 3 \times 10^{12} cm^{-3}$, $T_e = 5\,keV$, $B = 0.1$, we have $D_\perp / D_{NC} \sim 10^2$.

7.CONCLUSION

In a tokamak plasma interacting with a single wave, the stochastic transforms across the trapped / passing boundary may be driven by a single wave. Such unperturbed separatrix crossing is related to the dynamic stochasticity, and an anomalous transitional layer occurs in the vicinity of the separatrix. The separatrix crossing is followed by a mixing process, which leads to a small-scale structure in the distribution function in h-ψ plane in the presence of the slope of the distribution function in h-direction. The separatrix crossing is a convective process via a definite channel in h-ψ plane, we have calculated the convective flux and the net flux in h-direction in h-ψ plane. The separatrix crossing is accompanied by a radial flux, which is composed of a directional flux and a diffusion flux.

ACKNOWLEDGMENTS

The author would like to thank Professor Abdus Salam, the International Atomic Energy Agency and UNESCO for hospitality at the International Centre for Thoretical Physics,Trieste.

REFERENCES

Galeev A.A. and Sagdeev R.Z. (1968) Sov. Phys. JETP 26, 233.

Dobrovolny M., Orefice A. and Pozzoli R. (1973) Nucl. Fusion 13, 485.

Casati G. and Lazzaro E. (1981) Phys. Fluids 24, 1579

Hastings D.E. (1984) Phys. Fluids 27, 935.

Chirikov B.V. (1979) Phys. Reports 52, 263

Escande D.F. (1985) Phys. Reports 121, 165.

Tennyson J.L., Cary J.R. and Escande D.F. (1986) Phys. Rev. Lett. 56, 2117.

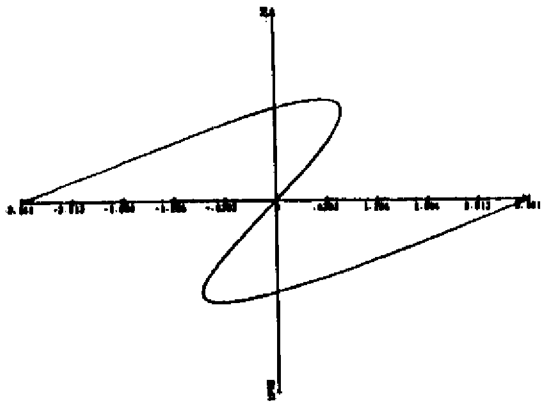

Fig.1(a)

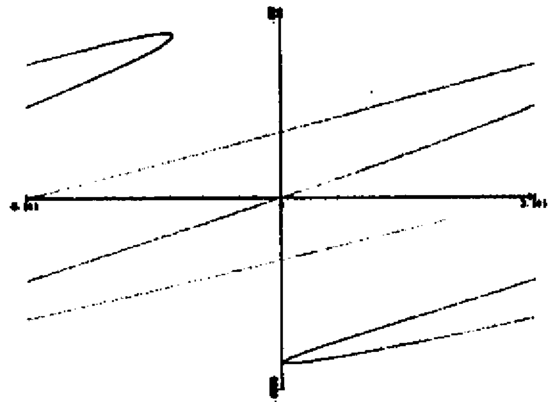

Fig.1(b)

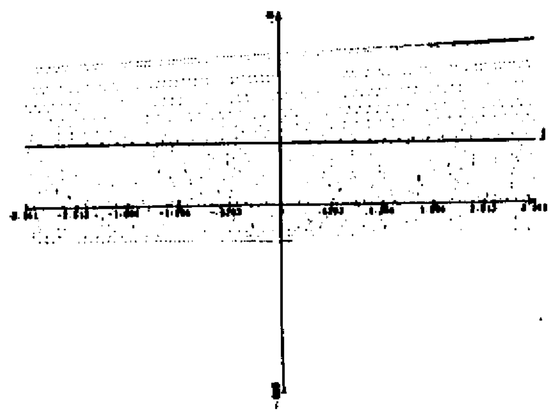

Fig.1(c)

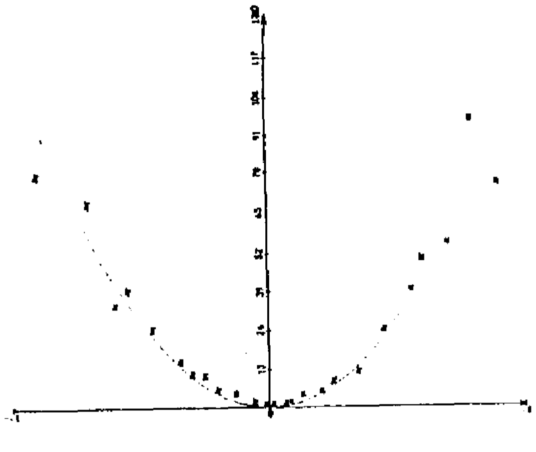

Fig.2

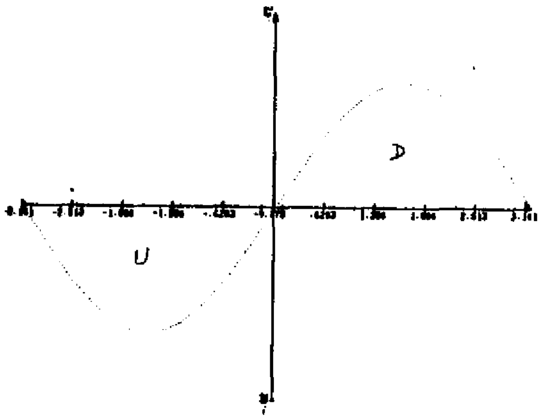

Fig.3

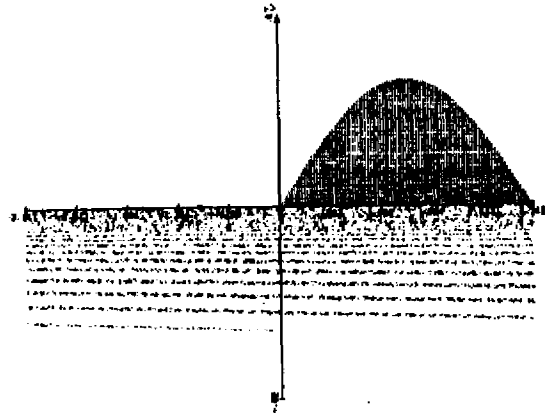

Fig.4

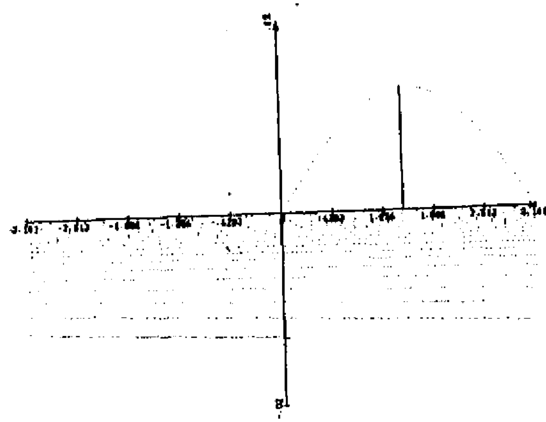

Fig.5

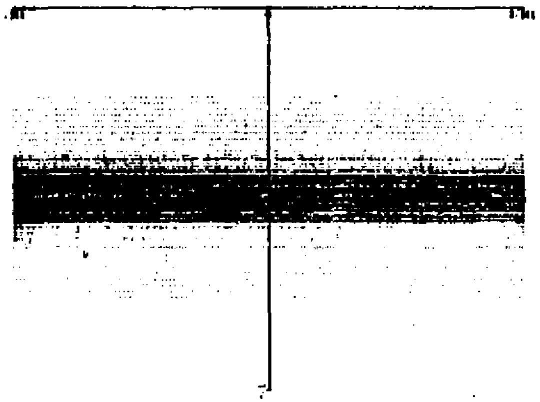

Fig.6(a)

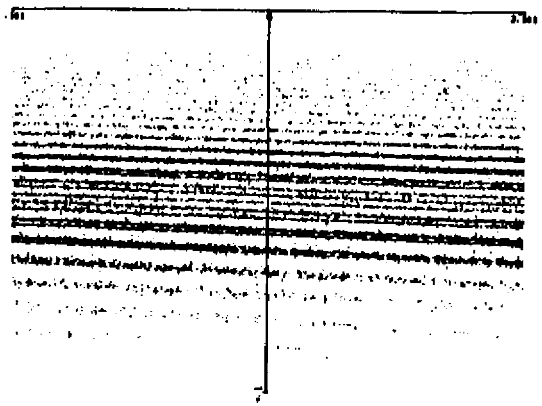

Fig.6(b)