IC/87/127

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

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INTERNATIONAL ATOMIC ENERGY AGENCY

UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION

1987 MIRAMARE-TRIESTE

International Atomic Energy Agency
and
United Nations Educational Scientific and Cultural Organization

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ON THE SELF-FOCUSING OF ELECTRIC HELICONS *

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ABSTRACT

The present work is devoted to the investigation of the stationary self-focusing of circularly polarized helicons in a magnetized plasma in the case of ultra-relativistic intensities. It is shown that the larger intensity and effective width at the boundary is the much more faster growing self-focusing.

MIRAMARE - TRIESTE
July 1987

* To be submitted for publication.

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Recently, interest in effects that can be ascribed to the nonlinear behavior of plasma waves and their influence on the containment and heating of laboratory plasma has grown. Of particular interest in this aspect is the investigation of the interaction of the high-power electromagnetic radiation with plasma. Such interest is mainly caused by the possibility of plasma heating up to high temperature in the installation for CTR.

Nowadays, due to the problem of the plasma particle acceleration to very high velocities by means of the powerful laser, the interest in this problem increases significantly. In the irradiation experiments [1] of special targets by powerful laser, transverse inhomogeneities are observed in the plasma corona. The reason for such an effect can be the beam self-focusing and its breaking for the so called "filamentation"; which causes inhomogeneity and significantly influences the effective energy absorption of the pumping wave. The idea of the possibility of self-focusing was stated by Askaryan[2] Talanov[3], the dynamic of the process was discussed first, and critical power self-focusing length and their dependence upon the medium and beam parameters have been found. It is well known now that at not very high intensities of the electromagnetic waves, the main mechanism of nonlinearity causing the self-focusing and filamentation in the collisionless plasma, is striction, and in collision plasma, is a thermal effect [5-6].

The recent achievment in laser techniques gives the possibility to obtain such fields, in which (ω_E / ω) ~ 1 (where $\omega_E = e|E| / m_o c$), and accordingly, it is necessary to study the relativistic oscillation of plasma electron mass in the field of the pumping wave. Consequently we have to take into account the fact that the medium becomes optically inhomogeneous.

The stationary self-focusing and filamentation instability of relativistic intense electromagnetic wave in an unmagnetized plasma have been studied recently in [4], where it was shown that the propagation of such waves in the relativistic case is quite different from that of the non-relativistic case. That is the reason why it follows in this paper to investigate the propagation of electromagnetic waves of relativistic intensities in a magnetized plasma; the problem in which the electron oscillation, due to the resonance character, become ultra-relativistic even for "weak" fields of the pumping wave i.e.

$$\nu^2 = \frac{p^2}{m_0^2 c^2} = \left(\frac{\omega_E\sqrt{1+\nu^2}}{\omega\sqrt{1+\nu^2}-\omega_c}\right)^2 \tag{1}$$

where $\omega_c = eB/m_0 c$ is the electron cyclotron frequency. In the case when $\nu^2 \gg 1$, ($\omega < \omega_c$), then

$$\nu = |\omega_c \pm \omega_E|$$

and

$$N^2 = 1 \mp (\omega_{p_e}^2/\omega\,\omega_E)$$

where(-) indicates the high-frequency (HF) wave, the spectrum of which is given by

$$\omega = \frac{\omega_{p_e}^2}{\omega_E}\left(1 + \frac{k^2c^2}{\omega_{p_e}^2}\cdot\frac{\omega_E^2}{\omega_{p_e}^2}\right) \tag{2}$$

and the (+) indicates the so called electric helicons, the spectrum of which is given by

$$\omega = \left(k^2c^2/\omega_{p_e}^2\right)\,\omega_E \tag{3}$$

As is well known, in the case of weak relativism ($\nu \ll 1$) [8] as well as in the case of ultra-relativism ($\nu \gg 1$), the nonlinearity connected with the electron mass oscillation in a magnetized plasma does not lead to HF wave spreading. The waves are broadening due to diffraction. But, for electric helicons, in the case of ultra-relativistic intensities, the beam self-focusing or its propagation in the regime of the self-trapping is possible.

The present paper is devoted to the investigation of the stationary self-focusing of circulary polarized electric helicons in a magnetized plasma in the case of ultra-relativistic intensities.

It is possible, from the equation written in Nashikawa, Tsintsadze and Watanab [7] to obtain an equation with square nonlinearity to describe the stationary electric helicons in the approximation that ($\partial^2/\partial z^2 \ll k^2$) in the following form:

$$2i\frac{\partial E}{\partial z} + \alpha\frac{\partial^2 E}{\partial x^2} + \beta\frac{|E| - E_0}{E_0}E = 0 \tag{4}$$

where $E = E_x - iE_y$, $\alpha = \frac{1}{k}\left(1 + \frac{k^2c^2\sqrt{1+\nu^2}}{(\omega^2\sqrt{1+\nu^2} - \omega_{p_e}^2)}\right)$,

$$\beta = 2k\left(\frac{m_0}{8m_i}\right)\left\{\frac{1}{1-(V_s/V_g)^2}\left(\frac{\omega_p^2}{\omega^2}\right) - 1\right\},$$

V_g is the group velocity, and V_s is the electron thermal velocity.

Assuming the distribution of the beam energy at the boundary to be of the Gaussian form, the intensity of the electric field could be represented as

$$E(x,z) = |E(x,z)|\exp\left\{\frac{i}{\alpha}S(x,z)\right\} \tag{5}$$

Substituting this expression into (4) to obtain the following set of equations:

$$\left.\begin{aligned} &\frac{\partial|E|^2}{\partial z} + \frac{\partial}{\partial x}\left(|E|^2\frac{\partial S}{\partial x}\right) = 0 \\ &2\frac{\partial S}{\partial z} + \left(\frac{\partial S}{\partial x}\right)^2 = \alpha^2\frac{1}{|E|}\frac{\partial^2|E|}{\partial x^2} + \alpha\beta\frac{|E| - E_0}{E_0} \end{aligned}\right\} \tag{6}$$

the solution of which, according to [7], are written in the form:

$$
\left.
\begin{aligned}
E &= \frac{E_0}{f(z)} \exp\left\{-\frac{x^2}{2a^2 f^2(z)}\right\} \\
S' &= \frac{x^2}{2} \frac{1}{f(z)} f'(z) + \Psi(z)
\end{aligned}
\right\} \qquad (7)
$$

In the paraxial approximation, i.e., in the case when $x^2 \ll a^2 f^2$, it is easy to obtain the following equation for the dimensionless beam width:

$$
f''(z) = \frac{\alpha^2}{a^4} \frac{1}{f^3(z)} - \alpha\beta \frac{1}{f^2(z)} \qquad (8)
$$

where a is the effective width of the beam at the boundary. The boundary conditions are $f(z=0)=1$, $f'(z=0)=\beta_0$ where β_0 is the curvature of the phase front at the boundary.

From equation (8) we obtain :

$$
z = \pm\left\{\frac{F(z)}{C_1} - \frac{\alpha\beta}{2a^2 C_1^{3/2}} \ln\left|\frac{2C_1 f(z) + \frac{\alpha\beta}{a^2}}{2\sqrt{C_1}} + \sqrt[2]{F(z)}\right|\right\} \pm C_2 \qquad (9)
$$

where $C_1 = \beta_0^2 + (\alpha^2/a^4) - (\alpha\beta/a^2)$;

$$
C_2 = \frac{1}{C_1}\sqrt{C_1 + (\alpha\beta/a^2) - (\alpha^2/a^4)} - (\alpha\beta/2a^2 C_1^{3/2}) \cdot
$$

$$
\cdot \ln \cdot \left|\frac{2C_1 + (\alpha\beta/a^2)}{2\sqrt{C_1}} + C_1 + (\alpha\beta/a^2) - (\alpha^2/a^4)\right| ;
$$

$$
F(z) = C_1 f^2(z) + (\alpha\beta/a^2) f(z) - (\alpha^2/a^4)
$$

To find $f(z)$ from this transcendental solution, in general, is really difficult and impossible. That is the reason why we have to use the approximation method.

As is clear from the analysis of equation (8), the propagation of electric helicons depends on the value of the parameter $\gamma = (\beta a^2/2\alpha)$. When $\gamma \gg 1$, the fast self-focusing takes place on a comparatively small length. When $\gamma \ll 1$ or $\gamma < 0$, the beam broadens. In the case when $\gamma \sim 1$, $f(z)$ starts to oscillate with growing up amplitude. This means that the wave propagates in the form of wave guide, the transverse size of which periodically changes.

With the help of the perturbation theory it is possible to state the value of the characteristic length of the transverse size of the wave guide (Δz). To do so, it is necessary to express $f(z)$ in terms of the summation of a constant averaged quantity f_0 and perturbed (δf) quantity depending on z, i.e.,

$$
f(z) = f_0 + \delta f(z) \quad ; \quad |\delta f| \ll f_0
$$

Substituting in equation (8), we get in the linear approximation

$$
\delta f''(z) = \frac{1}{f_0^3}(\alpha^2/a^4)\left\{1 - \gamma f_0 - \left(\frac{3}{f_0} - 2\gamma\right)\delta f(z)\right\} \qquad (10)
$$

According to the boundary condition $f_0 = 1$ and $\delta f(z=0) = 0$, we have $\alpha = \beta a^2/2$, and consequently the characteristic size of the wave guide oscillation is $\Delta z = 4\pi/|\beta|$.

Although γ is a complicated function of the plasma beam parameters, and the external magnetic field, it is possible in some particular cases to express it in a simple form. For example, we shall consider two cases:

(i) the case when $\left|\omega_{p_e}^2/\omega(\omega_c - \omega_E)\right| \gg 1$, then

$$
\gamma \simeq k^2 a^2 (\omega_E/\omega_c)\left\{1 - \frac{m_0}{8m_i} \frac{1}{1-(v_s/v_g)^2}(\omega_{p_e}^2/\omega^2)\right\} \qquad (11)
$$

It is clear from equation (11) that the self-focusing is possible only when

$$\left(\omega_{pe}^2 / \omega^2\right) / \left[1 - (v_s/v_g)^2\right] < 1 \qquad (12)$$

Also, it is clear that the self-focusing length rapidly decreases with the quantity (ω_E / ω_c) and $k^2 a^2$, and it has less dependence on the quantity $(\omega_{pe}^2 / \omega^2)$.

(ii) The case when

$$\left| \omega_{pe}^2 / \omega(\omega_c - \omega_E) \right| \ll 1,$$

then

$$\gamma \simeq k^2 a^2 \left(\omega_E \omega / \omega_{pe}^2\right) \left\{ \frac{m_0}{8 m_i} \frac{1}{1 - (v_s/v_g)^2} \left(\omega_{pe}^2 / \omega^2\right) - 1 \right\} \qquad (13)$$

In this case, for self-focusing, it is necessary to satisfy the condition

$$\left(\omega_{pe}^2 / \omega^2\right) > \frac{8 m_i}{m_0} \left(1 - \frac{V_s^2}{V_g^2}\right)$$

We can conclude from equation (11) and (13) that the larger intensity and effective width at the boundary, is the much more faster growing self-focusing. As they increase the non-linearity of the refractive index of the medium rapidly increases. On the other hand, in increasing the magnetic field along the direction of the wave propagation, the electrons become more "frozen" and it becomes difficult to push them out from the regime of a localised radiation.

Thus, on the basic analysis of the obtained results, it is possible to describe the regime of propagation of the electric helicons in the ultra-relativistic case.

ACKNOWLEDGEMENTS

Two of the authors (N.L.T. and M.Y.E.) would like to thank Professor Abdus Salam, the International Atomic Energy Agency and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste.

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