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MODULATIONAL INSTABILITY OF ELECTRIC HELICONS IN A MAGNETIZED COLLISIONAL PLASMA *

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ABSTRACT

The interaction of a rf electromagnetic wave with a magnetized collisional plasma in the ultra-relativistic case has been investigated to show the effect of the collisions on the modulational instability growth rate.

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Recently, there has been considerable interest in the effects on plasma of an intense electromagnetic field which causes a relativistic oscillation of the electron mass. Non-linear processes caused by relativistic effects were studied by Drake et al [1], whose results show that the relativistic electron motion can lead to the formation of shock waves. Tsintsadze and Tskhakaya [2] studied how the dependence of the electron mass on the amplitude of the pump field affects the propagation of electron -acoustic waves in a plasma with a negative dielectric constant. They showed that if the rf pressure force exceeds the gas pressure the solitary wave is a compressional wave.

Recently, Papuashvili et al [3] studied the interaction of a magnetized collisionless plasma with circularly polarized high-frequency wave, in whose field electrons can acquire relativistic velocities. Their results show that in the ultra-relativistic case there are new types of waves one of which may be called an "electron helicon". Also they show that the external magnetic field strongly affects the interaction .

In the present paper we will study a circularly polarized rf wave interacting with a magnetized collisional plasma, in which the plasma electrons acquire relativistic velocities under the effect of such a wave.

To formulate the basic equations, let us consider the case in which intense electromagnetic radiation is propagated along the constant magnetic field $\vec{B}(0,0,B)$ in a transparent plasma, and the wave length is much smaller than the characteristic distance over which the plasma density changes appreciably. The system of equations that describe the motion of particle α $(= i, e)$ and the field is :

$$\frac{\partial \vec{P}_\alpha}{\partial t} + \vec{v}_\alpha \frac{\partial \vec{P}}{\partial \vec{r}} = e_\alpha \vec{E} + \frac{e_\alpha}{m_\alpha}\left[\vec{v}_\alpha \times \vec{B}\right] - \delta \vec{P}_\alpha$$

$$\frac{\partial n_\alpha}{\partial t} + \mathrm{div}\; n_\alpha \vec{v}_\alpha = 0$$

$$(\vec{\nabla}\cdot\vec{E}) = 4\pi \sum_\alpha e_\alpha n_\alpha$$

$$(\vec{\nabla}\times\vec{E}) = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}\; ; \quad \vec{\nabla}\times\vec{B} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c}\sum_\alpha e_\alpha n_\alpha \vec{v}_\alpha$$

$$\vec{P}_\alpha = m_{o\alpha} \vec{v}_\alpha \Big/ \left(1 - \frac{v_\alpha^2}{c^2}\right)^{1/2} \tag{1}$$

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where δ is the frequency of weak collision between electrons and ions.

It is well known [4] that an intense rf electromagnetic wave, under the effect of its field the plasma electrons acquire relativistic velocities, may be purely transverse, if it is circularly polarised. Therefore we shall be interested in studying the circularly polarized rf field propagating along a constant, uniform magnetic field $\vec{B}_0$.

Analysing system (1) by assuming $n(z,t) = n_0 + \delta n(z,t)$, where n_0 is the unperturbed density and δn is the perturbed density caused by the pondromotive force (also, of course, all the other hydrodynamic quantities should be expanded in the same way), and using the same method and approximation expanded in [3], one can come to the following equation which describes the plasma behavior in the field of rf wave:

$$\nabla^2 E_+ - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} E_+ = \frac{\omega\, \omega_p \left(1 + \frac{\delta n}{n_0}\right) E_+}{\left(\omega\sqrt{1+\nu^2} - \omega_c + i\,\delta\sqrt{1+\nu^2}\right)} \tag{2}$$

where $E_+ = E_x + i E_y$, $\omega_p = (4\pi n_0 e^2/m_e)^{1/2}$ is the electron langmuir frequency, δn is a slowly-varying density perturbation, $\omega_c = (eB_0/m_e c)$ is the electron cycltron frequency, $\omega = (k^2c^2/\omega_p^2)\,\omega_E$ is the electric helicon frequency, $\omega_E = (eE_+/m_e c)$, and

$$\nu^2 = \frac{P_e^2}{m_{oe}^2 c^2} = \frac{\omega_E^2\,(1+\nu^2)}{\left(\omega\sqrt{1+\nu^2} - \omega_c\right)^2 + \delta^2(1+\nu^2)} \tag{3}$$

Choosing a condition that

$$\left(\omega\sqrt{1+\nu^2} - \omega_c\right)^2 \gg \delta\,(1+\nu^2) \tag{4}$$

it is easy from (3), in the ultra-relativistic case ($\nu \gg 1$), to obtain the following expression for the electric helicon:

$$\nu = \frac{\omega_c - \sqrt{\omega_E^2 - \delta^2}}{\omega} \simeq \frac{\omega_c}{\omega} \tag{5}$$

providing that $\omega_c \gg (\omega_E^2 - \delta^2)^{1/2}$.

It is important to point out here that the quantity $(\omega\sqrt{1+\nu^2} - \omega_c)$ as it is shown in [3] should not be less than ω_E. This condition differs from the non-relativistic case, in which ω can reach the value ω_c.

Thus, using the above mentioned notations and considering that all the hydrodynamic quantities and the field depend only on a single coordinate z, in addition to the time t, the following equation for the amplitude of the electric helicon is obtained from Eqn.(2):

$$2i\omega\left(\frac{\partial}{\partial t} + v_g\frac{\partial}{\partial z}\right)E' + c^2\frac{\partial^2}{\partial z^2}E' + \frac{\omega_p^2\,\omega}{\omega_{E_0}}\left(\frac{\delta n}{n_0} - \frac{|E| - E_0}{E_0}\right)E' + i\delta\,\frac{\omega_c\,\omega_p^2}{\omega_{E_0}^2}\left(\frac{\delta n}{n_0} - 2\,\frac{|E| - E_0}{E_0}\right)E' = 0 \tag{6}$$

where $v_g = (kc^2/\omega)$, $\omega_{E_0} = (e|E|/m_{oe}c)$ is the initial amplitude of the electromagnetic wave before the modulation, and $(|E| - E_0)/E_0 << 1$.

Expressing E' as $E' = a e^{i\varphi}$,where a and φ are real functions of z and t ,to obtain from (6) the following set of equations for a and φ :

$$-2\omega\left(\frac{\partial\varphi}{\partial t} + \frac{kc^2}{\omega}\frac{\partial\varphi}{\partial t}\right)a + c^2\frac{\partial^2 a}{\partial z^2} - c^2 a\left(\frac{\partial\varphi}{\partial z}\right)^2 + \frac{\omega_p^2\omega}{\omega_{E_0}}\left(\frac{\delta n}{n_0} - \frac{|a|-a_0}{a_0}\right)a = 0 \qquad (7)$$

and

$$2\omega\left(\frac{\partial a}{\partial t} + \frac{kc^2}{\omega}\frac{\partial a}{\partial z}\right) + 2c^2\frac{\partial\varphi}{\partial z}\frac{\partial a}{\partial z} + c^2 a\frac{\partial^2\varphi}{\partial z^2} + \delta\frac{\omega_c\omega_p^2}{\omega_{E_0}^2}\left(\frac{\delta n}{n_0} - 2\frac{|a|-a_0}{a_0}\right)a = 0 \qquad (8)$$

Linearizing (7) and (8) together with the continuity equation and the equation of ion motion , and seeking a solution of the form $\sim \exp(iqz - i\Omega t)$,to obtain the dispersion relation for the electric helicon in a magnetized collisional plasma:

$$\left\{(\Omega - v_g q)^2 + i\delta\frac{\omega_c\omega_p}{\omega\,\omega_{E_0}^2}(\Omega - v_g q) - \frac{1}{4\omega^2}c^2q^2 - \frac{c^2q^2}{4\omega^2}\frac{\omega_p^2\omega}{\omega_{E_0}}\right\}(\Omega^2 - v_s^2q^2) = \left\{c^2q^2\frac{\omega_p^2\omega}{\omega_{E_0}} + 2i\omega(\Omega - v_g q)\delta\frac{\omega_p^2\omega_c}{\omega_{E_0}^2}\right\}\frac{eca_0}{m_i\omega}\left(q^2 - i\delta\frac{kq}{\omega}\right)\frac{1}{4\omega^2} \qquad (9)$$

Substituting $\Omega = \Omega' + i\gamma$ where Ω' and γ are real values, and $\Omega' \gg \gamma$,into equation(9) to obtain the following equations governing Ω' and γ :

$$\left\{(\Omega' - v_g q)^2 - \frac{c^2q^2}{4\omega^2}\frac{\omega_p^2\omega}{\omega_{E_0}}\right\}(\Omega'^2 - v_s^2q^2) = \frac{m_{oe}}{m_i}(c^2q^2)^2\frac{k^2c^2}{4\omega^2} \qquad (10)$$

and

$$\gamma^2 + \delta\frac{\omega_c\omega_p^2}{2\omega\,\omega_{E_0}^2}\gamma - \frac{1}{8}\delta\frac{k^2c^2}{\omega^2}\left(\frac{m_e}{m_i}\right)\left(\frac{qc}{\omega}\right)\left(\frac{c}{v_s}\right)\omega_E = 0 \qquad (11)$$

From (11) we can get γ :

$$\gamma = -\delta\frac{\omega_c\omega_p^2}{2\omega\,\omega_{E_0}^2} \pm \frac{1}{2}\sqrt{\frac{\delta^2\omega_c^2\omega_p^4}{\omega^2\omega_{E_0}^4} + \frac{1}{2}\delta\frac{k^2c^2}{\omega^2}\left(\frac{m_{oe}}{m_i}\right)\frac{qc}{\omega}\frac{c}{v_s}\omega_E} \qquad (12)$$

Since ,it is necessary that γ should be positive to get a modulational instability , then we shall consider only the positive sign(+) in equation (12) to describe and discuss the following two cases :

(i) The case in which

$$\delta\frac{\omega_c\omega_p^4}{\omega_E^4} > \frac{1}{2}k^2c^2\left(\frac{m_{oe}}{m_i}\right)\frac{qc}{\omega}\left(\frac{c}{v_s}\right)\omega_E \qquad (13)$$

equation (12) simplifies to

$$\gamma_+ = \frac{1}{8}\frac{k^2c^2}{\omega^2}qc\left(\frac{m_{oe}}{m_i}\right)\frac{c}{v_s}\frac{\omega_E^2}{\omega_c\omega_p^2} \qquad (14)$$

As is clear from (14), in this case the growth rate (γ_+) does not depend on the collision frequency, in spite of the fact that collision plays a significant role in the interaction. Also ,it seems that the growth rate given by (14) is larger than that obtained in [5].

(ii) The case when

$$\frac{\delta}{qc} < \frac{1}{2}\left(\frac{\omega_E}{\omega_p}\right)^2\left(\frac{\omega_E}{\omega_c}\right)^2\left(\frac{c}{v_{T_e}}\right)\left(\frac{m_{oe}}{m_i}\right)^{1/2} \qquad (15)$$

equation (12) becomes :

$$\gamma_+ = \frac{1}{2}\left(\frac{kc}{\omega}\right)\sqrt{\delta\left(\frac{m_{oe}}{m_i}\right)\left(\frac{qc}{\omega}\right)\left(\frac{c}{v_s}\right)\omega_E} \qquad (16)$$

which shows the dependence of the growth rate on the collision frequency. It is also important to note here that the condition(15) is an easy condition to fulfill experimently.

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